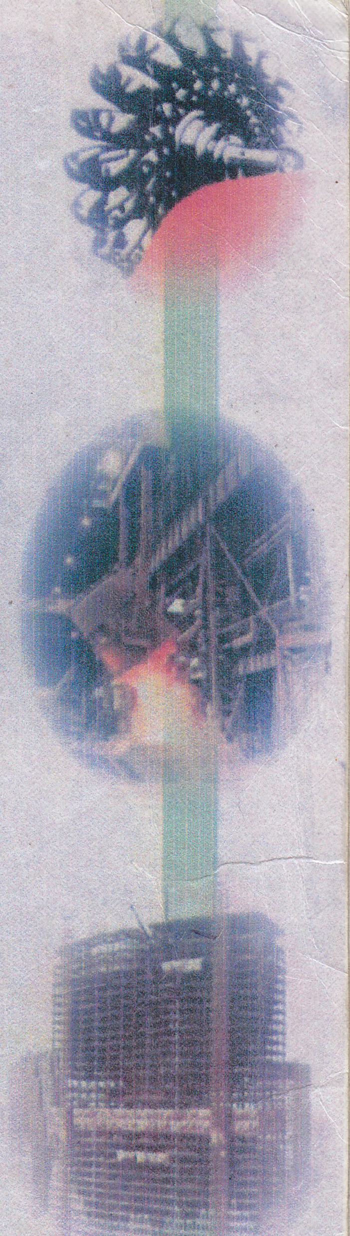
A TEXTBOOK OF

FLUID MECHANICS AND HYDRAULIC MACHINES

S.I. Units



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A TextBook of Fluid Mechanics and Hydraulic Machines

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CHAPTER

Properties of Fluids

▶ 1.1 INTRODUCTION

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

▶ 1.2 PROPERTIES OF FLUIDS

1.2.1 Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m³. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density of water is 1 gm/cm³ or 1000 kg/m³.

1.2.2 Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically, $$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$\therefore \quad w = \rho g \qquad \text{...(1.1)}$$

The value of specific weight or weight density (w) for water is 9.81×1000 Newton/m^3 in SI units.

1.2.3 Specific Volume. Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

1.2.4 Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

$$\text{Mathematically, } S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81$ N/m^3

The density of a liquid $= S \times$ Density of water

$= S \times 1000$ kg/m^3. ...(1.1A)

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water. For example the specific gravity of mercury is 13.6, hence density of mercury $= 13.6 \times 1000 = 13600$ kg/m^3.

Problem 1.1 *Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.*

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

Weight = 7 N

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \text{ N/m}^3. \text{ Ans.}}$$

$$(ii) \text{ Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3. \text{ Ans.}}$$

$$(iii) \text{ Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \text{ Density of water} = 1000 \text{ kg/m}^3\}$$

$= \mathbf{0.7135. \text{ Ans.}}$

Problem 1.21 *Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(i) Density (ρ)

Using equation (1.1.A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(ii) Specific weight (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(iii) Weight (W)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

∴ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

▶ 1.3 VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and $u + du$ as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ called Tau.

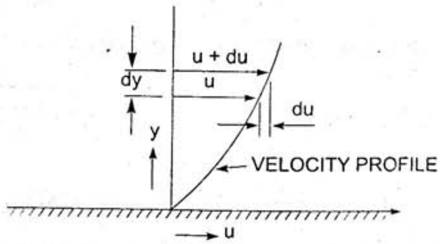

Fig. 1.1 *Velocity variation near a solid boundary.*

Mathematically, $\tau \propto \frac{du}{dy}$

or $$\tau = \mu \frac{du}{dy} \quad \text{...(1.2)}$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \quad \text{...(1.3)}$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

1.3.1 Units of Viscosity. The units of viscosity is obtained by putting the dimensions of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In MKS system, force is represented by kgf and length by metre (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by metre (m).

$\therefore$ MKS unit of viscosity $= \dfrac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \dfrac{\text{dyne-sec}}{\text{cm}^2}$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence N/m^2 = Pa = Pascal

$\therefore$ SI unit of viscosity = Ns/m^2 = Pa s.

SI unit of viscosity $= \dfrac{\text{Newton-sec}}{\text{m}^2} = \dfrac{\text{Ns}}{\text{m}^2}$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

The numerical conversion of the unit of viscosity from MKS unit to CGS unit is given below :

$$\frac{\text{one kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ N-sec}}{\text{m}^2} \qquad \{\because 1 \text{ kgf} = 9.81 \text{ Newton}\}$$

But one Newton = one kg (mass) $\times$ one $\left(\dfrac{\text{m}}{\text{sec}^2}\right)$ (acceleration)

$$= \frac{(1000 \text{ gm}) \times (100 \text{ cm})}{\text{sec}^2} = 1000 \times 100 \frac{\text{gm-cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \qquad \left\{\because \text{dyne} = \text{gm} \times \frac{\text{cm}}{\text{sec}^2}\right\}$$

$$\therefore \quad \frac{\text{one kgf-sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{\text{cm}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \frac{\text{dyne-sec}}{\text{cm}^2} = 98.1 \text{ poise} \qquad \left\{\because \frac{\text{dyne-sec}}{\text{cm}^2} = \text{Poise}\right\}$$

Thus for solving numerical problems, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in MKS.

But $\quad \dfrac{\text{one kgf-sec}}{\text{m}^2} = \dfrac{9.81 \text{ Ns}}{\text{m}^2} = 98.1$ poise

$\therefore \quad \dfrac{\text{one Ns}}{\text{m}^2} = \dfrac{98.1}{9.81}$ poise = 10 poise $\quad$ or $\quad$ One poise $= \dfrac{1}{10} \dfrac{\text{Ns}}{\text{m}^2}$.

Alternate Method. One poise $= \dfrac{\text{dyne} \times \text{s}}{\text{cm}^2} = \left(\dfrac{1 \text{ gm} \times 1 \text{ cm}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{cm}^2}$

But dyne $= 1 \text{ gm} \times \dfrac{1 \text{ cm}}{s^2}$

$$\therefore \text{ One poise} = \frac{1 \text{ gm}}{\text{s cm}} = \frac{\frac{1}{1000}\text{ kg}}{\text{s}\,\frac{1}{100}\text{ m}}$$

$$= \frac{1}{1000} \times 100 \frac{\text{kg}}{\text{sm}} = \frac{1}{10}\frac{\text{kg}}{\text{sm}} \quad \text{or} \quad 1\frac{\text{kg}}{\text{sm}} = 10 \text{ poise.}$$

Note. (*i*) In SI units second is represented by 's' and not by 'sec'.

(*ii*) If viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units.

Sometimes a unit of viscosity as centipoise is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \quad \text{or} \quad 1 \text{ cP} = \frac{1}{100} \text{ P} \qquad [\text{cP = Centipoise, P = Poise}]$$

The viscosity of water at 20°C is 0.01 poise or 1.0 centipoise.

1.3.2 Kinematic Viscosity. It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called 'nu'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{ Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\dfrac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$

Centistoke means $= \dfrac{1}{100}$ stoke.

1.3.3 Newton's Law of Viscosity. It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation (1.2) or as

$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as **Newtonian** fluids and the fluids which do not obey the above relation are called **Non-newtonian** fluids.

1.3.4 Variation of Viscosity with Temperature. Temperatur affects the viscosity. The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with the increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids the cohesive forces predominates the molecular momentum transfer due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

(*i*) **For liquids,** $\mu = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$ ...(1.4A)

where μ = Viscosity of liquid at t°C, in poise

μ_0 = Viscosity of liquid at 0°C, in poise

α, β = are constants for the liquid

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$.

The equation (1.4 *A*) shows that with the increase of temperature, the viscosity decreases.

(*ii*) **For a gas,** $\mu = \mu_0 + \alpha t - \beta t^2$ ...(1.4*B*)

where for air $\mu_0 = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$.

The equation (1.4 *B*) shows that with the increase of temperature, the viscosity increases.

1.3.5 Types of Fluids. The fluids may be classified into the following five types :

1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

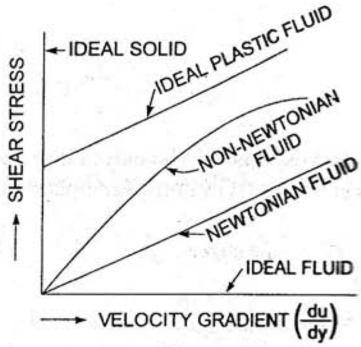

Fig. 1.2 *Types of fluids.*

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

Problem 1.3 *If the velocity distribution over a plate is given by $u = \frac{2}{3} y - y^2$ in which **u** is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.*

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2}.\ \textbf{Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2}.\ \textbf{Ans.}$$

Problem 1.4 *A plate, 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.*

Solution. Given :

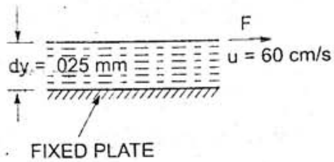

Fig. 1.3

Distance between plates, $dy = .025$ mm $= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \frac{\text{N}}{\text{m}^2}$.

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \frac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \frac{\text{N}}{\text{m}^2}$.

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = \mathbf{8.33 \times 10^{-4}\ poise}.\ \textbf{Ans.}$$

Problem 1.5 *A flat plate of area 1.5×10^6 mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.*

Solution. Given :

Area of the plate, $A = 1.5 \times 10^6 \text{ mm}^2 = 1.5 \text{ m}^2$

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance between the plates, $dy = 0.15 \text{ mm} = 0.15 \times 10^{-3}$ m

Viscosity $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$.

Using equation (1.2) we have $\tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{.15 \times 10^{-3}} = 266.66 \frac{\text{N}}{\text{m}^2}$

(*i*) ∴ Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 =$ **400 N. Ans.**

(*ii*) Power* required to move the plate at the speed 0.4 m/sec

$= F \times u = 400 \times 0.4 =$ **160 W. Ans.**

Problem 1.6 *Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.*

Solution. Given : $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{N s}}{\text{m}^2}$

Dia. of shaft, $D = 10 \text{ cm} = 0.1$ m

Distance between shaft and journal bearing,

$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Speed of shaft, $N = 150$ r.p.m.

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785 \text{ m/s}$$

Using equation (1.2), $\tau = \mu \frac{du}{dy}$,

where du = change of velocity between shaft and bearing $= u - 0 = u$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = \mathbf{52.33\ N/m^2.\ Ans.}$$

Problem 1.7 *Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.*

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

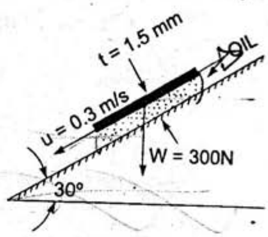

Fig. 1.4

* Power = $F \times u$ N m/s = $F \times u$ W (∵ Nm/s = Watt)

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane = $W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress,
$$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity = $u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise. Ans.}}$$

Problem 1.8 *Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.* (A.M.I.E.. 1972)

Solution. Given :

Distance between plates, $dy = 1.25 \text{ cm} = 0.0125$ m

Viscosity, $\mu = 14 \text{ poise} = \frac{14}{10} \text{ N s/m}^2$

Velocity of upper plate, $u = 2.5$ m/sec.

Shear stress is given by equation (1.2) as, $\tau = \mu \frac{du}{dy}$

where du = Change of velocity between plates = $u - 0 = u = 2.5$ m/sec.

$$dy = 0.0125 \text{ m.}$$

$$\therefore \quad \tau = \frac{14}{10} \times \frac{2.5}{.0125} = \mathbf{280 \text{ N/m}^2\text{. Ans.}}$$

Problem 1.9 *The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :*

(i) the dynamic viscosity of the oil in poise, and

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

(A.M.I.E., Winter 1977)

Solution. Given :

Each side of a square plate $= 60 \text{ cm} = 0.60$ m

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36 \text{ m}^2$

Thickness of oil film, $dy = 12.5 \text{ mm} = 12.5 \times 10^{-3}$ m

Velocity of upper plate, $u = 2.5$ m/sec

∴ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

∴ Shear stress, $\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1 \text{ N}}{0.36 \text{ m}^2}$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $\tau = \mu \frac{du}{dy}$ or $\frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$

∴ $\mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2}$ $\left(\because \frac{1 \text{ Ns}}{\text{m}^2} = 10 \text{ poise}\right)$

$= 1.3635 \times 10 =$ **13.635 poise. Ans.**

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2\text{/sec} = .001435 \times 10^4 \text{ cm}^2\text{/s}$

$=$ **14.35 stokes. Ans.** ($\because$ cm²/s = stoke)

Problem 1.10 *Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second.*

Solution. Given :

Mass density, $\rho = 981 \text{ kg/m}^3$

Shear stress, $\tau = 0.2452 \text{ N/m}^2$

Velocity gradient, $\frac{du}{dy} = 0.2$ s

Using the equation (1.2), $\tau = \mu \frac{du}{dy}$ or $0.2452 = \mu \times 0.2$

∴ $\mu = \frac{0.245}{0.200} = 1.226 \text{ Ns/m}^2$

Kinematic viscosity ν is given by

∴ $\nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2} \text{ m}^2\text{/sec}$

$= 0.125 \times 10^{-2} \times 10^4 \text{ cm}^2\text{/s} = 0.125 \times 10^2 \text{ cm}^2\text{/s}$

$= 12.5 \text{ cm}^2\text{/s} =$ **12.5 stoke. Ans.** ($\because$ cm²/s = stoke)

Problem 1.11. *Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.*

Solution. Given :

Viscosity, $\mu = 0.05 \text{ poise} = \frac{0.05}{10} \text{ N s/m}^2$

Kinematic viscosity. $\nu = 0.035$ stokes

$= 0.035\ cm^2/s$ $\{\because$ Stoke $= cm^2/s\}$

$= 0.035 \times 10^{-4}\ m^2/s$

Using the relation $\nu = \frac{\mu}{\rho}$, we get $0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{\rho}$

$$\therefore \quad \rho = \frac{0.05}{10} \times \frac{1}{.035 \times 10^{-4}} = 1428.5\ kg/m^3$$

$\therefore$ Sp. gr. of liquid $= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000} = 1.4285 \simeq$ **1.43. Ans.**

Problem 1.12 *Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.*

Solution. Given :

Kinematic viscosity $\nu = 6$ stokes $= 6\ cm^2/s = 6 \times 10^{-4}\ m^2/s$

Sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

Now sp. gr. of a liquid $= \frac{\text{Density of the liquid}}{\text{Density of water}}$

or $$1.9 = \frac{\text{Density of liquid}}{1000}$$

$\therefore$ Density of liquid $= 1000 \times 1.9 = 1900\ \frac{kg}{m^3}$

$\therefore$ Using the relation $\nu = \frac{\mu}{\rho}$, we get

$$6 \times 10^{-4} = \frac{\mu}{1900}$$

$\because$ $\mu = 6 \times 10^{-4} \times 1900 = 1.14\ Ns/m^2$

$= 1.14 \times 10 =$ **11.40 poise. Ans.**

Problem 1.13 *The velocity distribution for flow over a flat plate is given by $u = \frac{3}{4}y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at y = 0.15 m. Take dynamic viscosity of fluid as 8.6 poise.*

Solution. Given : $u = \frac{3}{4}y - y^2$

$$\therefore \quad \frac{du}{dy} = \frac{3}{4} - 2y$$

At $y = 0.15$, $\frac{du}{dy} = \frac{3}{4} - 2 \times 0.15 = 0.75 - 0.30 = 0.45$

Viscosity. $\mu = 8.5$ poise $= \frac{8.5}{10}\frac{N\,s}{m^2}$ $\left(\because 10 \text{ poise} = 1\ \frac{Ns}{m^2}\right)$

Using equation (1.2), $\tau = \mu \dfrac{du}{dy} = \dfrac{8.5}{10} \times 0.45 \dfrac{\text{N}}{\text{m}^2} = \mathbf{0.3825} \dfrac{\mathbf{N}}{\mathbf{m^2}}$. **Ans.**

Problem 1.14 *The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.*

Solution. Given :

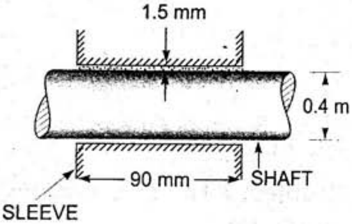

Fig. 1.5

Viscosity $\mu = 6$ poise

$$= \frac{6}{10} \frac{\text{Ns}}{\text{m}^2} = 0.6 \frac{\text{Ns}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} = \mathbf{716.48}$ **W. Ans.**

Problem 1.15 *If the velocity profile of a fluid over a plate is a parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.*

Solution. Given :

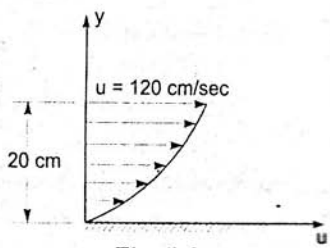

Fig. 1.6

Distance of vertex from plate = 20 cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10} \dfrac{\text{Ns}}{\text{m}^2} = 0.85.$

---

* Power in S.I. unit $= T * \omega = T \times \dfrac{2\pi N}{60}$ Watt $= \dfrac{2\pi NT}{60}$ Watt

The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \quad \text{...}(i)$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/sec

(c) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \quad \text{...}(ii)$$

Boundary condition (c) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \quad \text{...}(iii)$$

or

$$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (ii) and (iii) for a and b

From equation (iii), $b = -40a$

Substituting this value in equation (ii), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (i),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\therefore \quad \frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 =$ **12/s. Ans.**

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 =$ **6/s. Ans.**

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 =$ **0. Ans.**

Shear Stresses

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(i) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2.$

(ii) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2.$

(iii) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = \mathbf{0}.$ **Ans.**

Problem 1.16 *A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200 N is applied.* (A.M.I.E., Summer 1980)

Solution. Given : Speed of sleeve, $u_1 = 50$ cm/s

when force, $F_1 = 40$ N.

Let speed of sleeve is u_2 when force, $F_2 = 200$ **N.**

Using relation $$\tau = \mu \frac{du}{dy}$$

where τ = Shear stress = $\frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

du = Change of velocity = $u - 0 = u$

dy = Clearance = y

$$\therefore \quad \frac{F}{A} = \mu \frac{u}{y}$$

$$\therefore \quad F = \frac{A\mu u}{y} \alpha u \qquad \{\because \text{ A, } \mu \text{ and } y \text{ are constant}\}$$

$$\therefore \quad \frac{F_1}{u_1} = \frac{F_2}{u_2}$$

Substituting values, we get $\frac{40}{50} = \frac{200}{u_2}$

$$\therefore \quad u_2 = \frac{50 \times 200}{40} = 50 \times 5 = \mathbf{250 \text{ cm/s.}} \text{ **Ans.**}$$

Problem 1.17 *A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m., determine the viscosity of the fluid.* (A.M.I.E., Winter 1979)

Solution. Given :

Diameter of cylinder = 15 cm = 0.15 m

Dia. of outer cylinder = 15.10 cm = 0.151 m

Length of cylinders, L = 25 cm = 0.25 m

Torque, T = 12.0 Nm

Speed, $N = 100$ r.p.m.

Let the viscosity $= \mu$

Tangential velocity of cylinder, $u = \dfrac{\pi DN}{60} = \dfrac{\pi \times 0.15 \times 100}{60} = 0.7854$ m/s

Surface area of cylinder, $A = \pi D \times L = \pi \times 0.15 \times 0.25 = .1178 \text{ m}^2$

Now using relation $\tau = \mu \dfrac{du}{dy}$

where $du = u - 0 = u = .7854$ m/s

$$dy = \frac{0.151 - 0.150}{2} \text{ m} = .0005 \text{ m}$$

$$\tau = \frac{\mu \times .7854}{.0005}$$

$\therefore$ Shear force, $F = \text{Shear stress} \times \text{Area} = \dfrac{\mu \times .7854}{.0005} \times .1178$

$\therefore$ Torque, $T = F \times \dfrac{D}{2}$

$$12.0 = \frac{\mu \times .7854}{.0005} \times .1178 \times \frac{.15}{2}$$

$$\therefore \quad \mu = \frac{12.0 \times .0005 \times 2}{.7854 \times .1178 \times .15} = 0.864 \text{ N s/m}^2$$

$= 0.864 \times 10 =$ **8.64 poise. Ans.**

Problem 1.18 *Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if :*

(i) the thin plate is in the middle of the two plane surfaces, and

(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ? Take the dynamic viscosity of glycerine = 8.10×10^{-1} *N s/m*^{2}.

Solution. Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1} \text{ N s/m}^2$

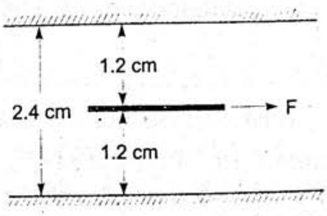

Fig. 1.7 (*a*)

Case I. When the thin plate is in the middle of the two plane surfaces [Refer to Fig. 1.7 (*a*)]

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then $F = F_1 + F_2$

The shear stress (τ_2) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu \left(\frac{du}{dv}\right)_1$$

where du = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\therefore \quad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area
$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu \left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

∴ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

∴ Total force, $F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

Case II. When the thin plate is at a distance of 0.8 cm from one of the plane surfaces [Refer to Fig. 1.7 (*b*)].

Let the thin plate is a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface
= 2.4 − 0.8 = 1.6 cm = .016 m
(Neglecting thickness of the plate)

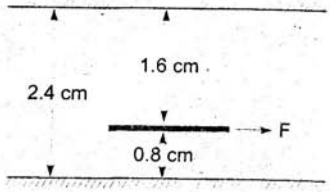

Fig. 1.7 (*b*)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu \left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

∴ Total force required $= F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

Problem 1.19 *A vertical gap 2.2 cm wide of infinite extent contains a fluid of viscosity 2.0 N s/m² and specific gravity 0.9. A metallic plate 1.2 m × 1.2 m × 0.2 cm is to be lifted up with a constant velocity of 0.15 m/sec, through the gap. If the plate is in the middle of the gap, find the force required. The weight of the plate is 40 N.*

Solution. Given :

Width of gap = 2.2 cm, viscosity, $\mu = 2.0$ N s/m²

Sq. gr. of fluid = 0.9

∴ Weight density of fluid

$= 0.9 \times 1000 = 900 \text{ kgf/m}^3 = 900 \times 9.81 \text{ N/m}^3$

($\because$ 1 kgf = 9.81 N)

Volume of plate $= 1.2 \text{ m} \times 1.2 \text{ m} \times 0.2 \text{ cm}$

$= 1.2 \times 1.2 \times .002 \text{ m}^3 = .00288 \text{ m}^3$

Thickness of plate = 0.2 cm

Velocity of plate = 0.15 m/sec

Weight of plate = 40 N.

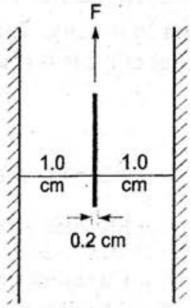

Fig. 1.8

When plate is in the middle of the gap, the distance of the plate of plate from vertical surface, of the gap

$$= \left(\frac{\text{Width of gap} - \text{Thickness of plate}}{2}\right)$$

$$= \frac{(2.2 - 0.2)}{2} = 1 \text{ cm} = .01 \text{ m}.$$

Now the shear force on the left side of the metallic plate,

$$F_1 = \text{Shear stress} \times \text{Area}$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times \text{Area} = 2.0 \times \left(\frac{0.15}{.01}\right) \times 1.2 \times 1.2 \text{ N}$$

($\because$ Area $= 1.2 \times 1.2 \text{ m}^2$)

$= 43.2$ N.

Similarly, the shear force on the right side of the metallic plate,

$$F_2 = \text{Shear stress} \times \text{Area} = 2.0 \times \left(\frac{0.15}{.01}\right) \times 1.2 \times 1.2 = 43.2 \text{ N}$$

∴ Total shear force $= F_1 + F_2 = 43.2 + 43.2 = 86.4$ N.

In this case the weight of plate (which is acting vertically downward) and upward thrust is also to be taken into account.

The upward thrust = Weight of fluid displaced

= (Weight density of fluid) × Volume of fluid displaced

$= 9.81 \times 900 \times .00288$ N

($\because$ Volume of fluid displaced = Volume of plate = .00288)

= 25.43 N.

The net force acting in the downward direction due to weight of the plate and upward thrust

= Weight of plate – Upward thrust = 40 – 25.43 = 14.57 N

∴ Total force required to lift the plate up

= Total shear force + 14.57 = 86.4 + 14.57 = **100.97 N. Ans.**

1.4 THERMODYNAMIC PROPERTIES

Fluids consist of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role. With the change of pressure and temperature, the gases undergo large variation in density. The relationship between pressure (absolute), specific volume and temperature (absolute) of a gas is given by the equation of state as

$$p\forall = RT \text{ or } \frac{p}{\rho} = RT \qquad ...(1.5)$$

where p = Absolute pressure of a gas in N/m^2

$\forall$ = Specific volume = $\frac{1}{\rho}$

R = Gas constant

T = Absolute temperature in °K

ρ = Density of a gas.

1.4.1 Dimension of R. The gas constant, R, depends upon the particular gas. The dimension of R is obtained from equation (1.5) as

$$R = \frac{p}{\rho T}$$

(*i*) In MKS units $\quad R = \dfrac{\text{kgf/m}^2}{\left(\dfrac{\text{kg}}{\text{m}^3}\right)\text{°K}} = \dfrac{\text{kgf-m}}{\text{kg °K}}$

(*ii*) In SI units, p is expressed in Newton/m^2 or N/m^2.

$$\therefore \quad R = \frac{\text{N/m}^2}{\dfrac{\text{kg}}{\text{m}^3} \times \text{K}} = \frac{\text{N-m}}{\text{kg-K}} = \frac{\text{Joule}}{\text{kg-K}} \quad [\text{Joule} = \text{N-m}]$$

$$= \frac{\text{J}}{\text{kg-K}}$$

For air, $\quad R$ in MKS $= 29.3 \dfrac{\text{kgf-m}}{\text{kg °K}}$

$$R \text{ in SI} = 29.3 \times 9.81 \frac{\text{N-m}}{\text{kg°K}} = 287 \frac{\text{J}}{\text{kg-K}}$$

1.4.2 Isothermal Process. If the changes in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (p) and density (ρ) is given by

$$\frac{p}{\rho} = \text{Constant} \qquad ...(1.6)$$

1.4.3 Adiabatic Process. If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^k} = \text{Constant} \qquad ...(1.7)$$

where k = Ratio of specific heat of a gas at constant pressure and constant volume.

= 1.4 for air.

1.4.4 Universal Gas Constant.

Let m = Mass of a gas in kg

$\forall$ = Volume of gas of mass m

p = Absolute pressure

T = Absolute temperature

Then, we have
$$p\forall = mRT \quad \text{...(1.8)}$$
where R = Gas constant.

Equation (1.8) can be made universal, *i.e.*, applicable to all gases if it is expressed **in mole-basis.**

Let n = Number of moles in volume of a gas

$\forall$ = Volume of the gas

$$M = \frac{\text{Mass of the gas molecules}}{\text{Mass of a hydrogen atom}}$$

m = Mass of a gas in kg

Then, we have $n \times M = m$.

Substituting the value of m in equation (1.8), we get
$$p\forall = n \times M \times RT \quad \text{...(1.9)}$$

The product $M \times R$ is called universal gas constant and is equal to $848 \dfrac{\text{kgf-m}}{\text{kg-mole °K}}$ in MKS units and 8314 J/kg-mole K in SI units.

One kilogram mole is defined as the product of one kilogram mass of the gas and its molecular weight.

Problem 1.20 *A gas weighs 16 N/m³ at 25°C and at an absolute pressure of 0.25 N/mm². Determine the gas constant and density of the gas.*

Solution. Given :

Weight density, $w = 16 \text{ N/m}^2$

Temperature, $t = 25°\text{C}$

$\therefore$ $T = 273 + t = 273 + 25 = 288°\text{K}$

$p = 0.25 \text{ N/mm}^2 \text{ (abs.)} = 0.25 \times 10^6 \text{ N/m}^2 = 25 \times 10^4 \text{ N/m}^2$

(*i*) Using relation $w = \rho g$, density is obtained as
$$\rho = \frac{w}{g} = \frac{16}{9.81} = \mathbf{1.63\ kg/m^3.\ Ans.}$$

(*ii*) Using equation (1.5), $\dfrac{p}{\rho} = RT$

$$\therefore \quad R = \frac{p}{\rho T} = \frac{25 \times 10^4}{1.63 \times 288} = \mathbf{532.55 \frac{Nm}{kg\ K}.\ Ans.}$$

Problem 1.21 *A cylinder of 0.6 m³ in volume contains air at 50°C and 0.3 N/mm² absolute pressure. The air is compressed to 0.3 m³. Find (i) pressure inside the cylinder assuming isothermal process and (ii) pressure and temperature assuming adiabatic process. Take k = 1.4.*

Solution. Given :

Initial volume, $\forall_1 = 0.6 \text{ m}^3$

Temperature $t_1 = 50°C$

$\therefore$ $T_1 = 273 + 50 = 323°K$

Pressure $p_1 = 0.3 \text{ N/mm}^2 = 0.3 \times 10^6 \text{ N/m}^2 = 30 \times 10^4 \text{ N/m}^2$

Final volume $\forall_2 = 0.3 \text{ m}^3$

$k = 1.4$

(*i*) Isothermal process :

Using equation (1.6), $\frac{p}{\rho}$ = Constant or $p\forall$ = Constant.

$$\therefore \quad p_1\forall_1 = p_2\forall_2$$

$$\therefore \quad p_2 = \frac{p_1\forall_1}{\forall_2} = \frac{30 \times 10^4 \times 0.6}{0.3} = 0.6 \times 10^6 \text{ N/m}^2 = \mathbf{0.6 \text{ N/mm}^2. \text{ Ans.}}$$

(*ii*) Adiabatic process :

Using equation (1.7), $\frac{p}{\rho^k}$ = Constant or $p\,\forall^k$ = Constant

$$\therefore \quad p_1\forall_1^k = p_2\forall_2^k.$$

$$\therefore \quad p_2 = p_1\frac{\forall_1^k}{\forall_2^k} = 30 \times 10^4 \times \left(\frac{0.6}{0.3}\right)^{1.4} = 30 \times 10^4 \times 2^{1.4}$$

$$= 0.791 \times 10^6 \text{ N/m}^2 = \mathbf{0.791 \text{ N/mm}^2. \text{ Ans.}}$$

For temperature, using equation (1.5), we get

$$p\forall = RT \text{ and also } p\,\forall^k = \text{Constant}$$

$$\therefore \quad p = \frac{RT}{\forall} \text{ and } \frac{RT}{\forall} \times \forall^k = \text{Constant}$$

or $$RT\forall^{k-1} = \text{Constant}$$

or $$T\,\forall^{k-1} = \text{Constant} \qquad \{\because R \text{ is also constant}\}$$

$$\therefore \quad T_1\,\forall_1^{k-1} = T_2\,\forall_2^{k-1}$$

$$\therefore \quad T_2 = T_1\left(\frac{\forall_1}{\forall_2}\right)^{k-1} = 323\left(\frac{0.6}{0.3}\right)^{1.4-1.0} = 323 \times 2^{0.4} = 426.2°K$$

$$\therefore \quad t_2 = 426.2 - 273 = \mathbf{153.2°C. \text{ Ans.}}$$

Problem 1.22 *Calculate the pressure exerted by 5 kg of nitrogen gas at a temperature of 10°C if the volume is 0.4 m³. Molecular weight of nitrogen is 28. Assume, ideal gas laws are applicable.*

Solution. Given :

Mass of nitrogen $= 5$ kg

Temperature, $t = 10°C$

$\therefore$ $T = 273 + 10 = 283°K$

Volume of nitrogen, $\forall = 0.4 \text{ m}^3$

Molecular weight $= 28$

Using equation (1.9), we have $p\forall = n \times M \times RT$

where $M \times R$ = Universal gas constant = $8314 \dfrac{\text{N-m}}{\text{kg-mole}\,^\circ\text{K}}$

and one kg-mole = (kg-mass) × Molecular weight = (kg-mass) × 28

$$\therefore \qquad R \text{ for nitrogen} = \frac{8314}{28} = 296.9 \frac{\text{N-m}}{\text{kg}\,^\circ\text{K}}$$

The gas laws for nitrogen is $p\forall = mRT$, where R = Characteristic gas constant

or
$$p \times 0.4 = 5 \times 296.9 \times 283$$

$$\therefore \qquad p = \frac{5 \times 296.9 \times 283}{0.4} = 1050283.7 \text{ N/m}^2 = \mathbf{1.05\ N/mm^2}.\ \textbf{Ans.}$$

▶ 1.5 COMPRESSIBILITY AND BULK MODULUS

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in Fig. 1.9.

Let $\forall$ = Volume of a gas enclosed in the cylinder

p = Pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

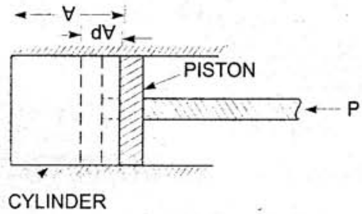

Fig. 1.9

Then increase in pressure $= dp$ kgf/m^2

Decrease in volume $= d\forall$

$$\therefore \quad \text{Volumetric strain} = -\frac{d\forall}{\forall}$$

– ve sign means the volume decreases with increase of pressure.

$$\therefore \quad \text{Bulk modulus} \qquad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\dfrac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall \qquad \text{...(1.10)}$$

$$\text{Compressibility is given by} = \frac{1}{K} \qquad \text{...(1.11)}$$

Relationship between Bulk Modulus (K) and Pressure (p) for a Gas

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of compression are as :

(*i*) **For Isothermal Process.** Equation (1.6) gives the relationship between pressure (p) and density (ρ) of a gas as

$$\frac{p}{\rho} = \text{Constant}$$

or $$p\forall = \text{Constant} \qquad \left\{\because \forall = \frac{1}{\rho}\right\}$$

Differentiating this equation, we get (p and $\forall$ both are variables)

$$pd\forall + \forall dp = 0 \quad \text{or} \quad pd\forall = -\forall dp \quad \text{or} \quad p = \frac{-\forall dp}{d\forall}$$

Substituting this value in equation (1.10), we get

$$K = p \qquad \text{...(1.12)}$$

(*ii*) **For Adiabatic Process.** Using equation (1.7) for adiabatic process

$$\frac{p}{\rho^k} = \text{Constant or } p\,\forall^k = \text{Constant}$$

Differentiating, we get $pd(\forall^k) + \forall^k(dp) = 0$

or $$p \times k \times \forall^{k-1}\, d\forall + \forall^k dp = 0$$

or $$pkd\forall + \forall dp = 0 \qquad \text{[Cancelling } \forall^{k-1} \text{ to both sides]}$$

or $$pkd\forall = -\forall dp \quad \text{or} \quad pk = -\frac{\forall dp}{d\forall}$$

Hence from equation (1.10), we have

$$K = pk \qquad \text{...(1.13)}$$

where K = Bulk modulus and k = Ratio of specific heats.

Problem 1.23 *Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15 per cent.*

Solution. Given :

Initial pressure $= 70\ \text{N/cm}^2$

Final pressure $= 130\ \text{N/cm}^2$

$\therefore$ dp = Increase in pressure $= 130 - 70 = 60\ \text{N/cm}^2$

Decrease in volume $= 0.15\%$

$$\therefore \qquad -\frac{d\forall}{\forall} = +\frac{0.15}{100}$$

Bulk modulus, K is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{60\ \text{N/cm}^2}{\frac{.15}{100}} = \frac{60 \times 100}{.15} = \mathbf{4 \times 10^4\ N/cm^2.\ Ans.}$$

Problem 1.24 *What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m³ at 80 N/cm² pressure to a volume of 0.0124 m³ at 150 N/cm² pressure ?*

Solution. Given :

Initial volume, $\forall = 0.0125\ \text{m}^3$

Final volume $= 0.0124\ \text{m}^3$

$\therefore$ Decrease in volume, $d\forall = .0125 - .0124 = .0001\ \text{m}^3$

$$\therefore \qquad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

Initial pressure $= 80 \text{ N/cm}^2$

Final pressure $= 150 \text{ N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70 \text{ N/cm}^2$

Bulk modulus is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{70}{\frac{.0001}{.0125}} = 70 \times 125 \text{ N/cm}^2$$

$$= \mathbf{8.75 \times 10^3 \text{ N/cm}^2. \text{ Ans.}}$$

1.6 SURFACE TENSION AND CAPILLARITY

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrance under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules *A*, *B*, *C* of a liquid in a mass of liquid. The molecule *A* is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule *A* is zero. But the molecule *B*, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule *B* is acting in the downward direction. The molecule *C*, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrance under tension.

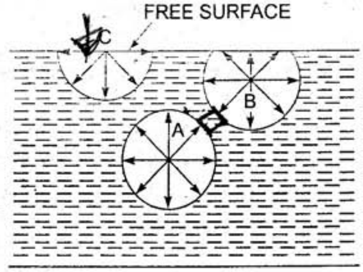

Fig. 1.10 *Surface tension.*

1.6.1 Surface Tension on Liquid Droplet. Consider a small spherical droplet of a liquid of radius '*r*'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(*i*) tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 1.11 (*b*) and this is equal to

$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$

(*ii*) pressure force on the area $\frac{\pi}{4}d^2$ and $= p \times \frac{\pi}{4} d^2$ as shown in Fig. 1.11 (*c*). These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

or

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d} \quad \text{...(1.14)}$$

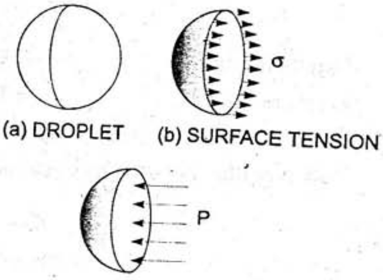

Fig. 1.11 *Forces on droplet.*

Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

1.6.2 Surface Tension on a Hollow Bubble. A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$\therefore$

$$p = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad \text{...(1.15)}$$

1.6.3 Surface Tension on a Liquid Jet. Consider a liquid jet of diameter '*d*' and length 'L' as shown in Fig. 1.12.

Let p = Pressure intensity inside the liquid jet above the outside pressure

σ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure $= p \times$ area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$.

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$\therefore$

$$p = \frac{\sigma \times 2L}{L \times d} \quad \text{...(1.16)}$$

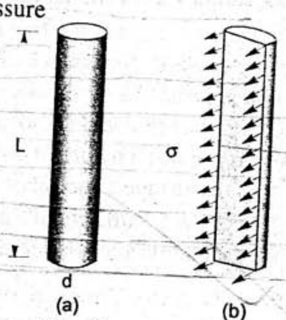

Fig. 1.12 *Forces on liquid jet.*

Problem 1.25 *The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.*

Solution. Given :

Surface tension, $\sigma = 0.0725$ N/m

Pressure intensity, p in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

Let d = dia. of the droplet

Using equation (1.14), we get $p = \frac{4\sigma}{d}$ or $0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$

$$\therefore \quad d = \frac{4 \times 0.0725}{0.02 \times (10)^4} = .00145 \text{ m} = .00145 \times 1000 = \textbf{1.45 mm. Ans.}$$

Problem 1.26 *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.*

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \textbf{0.0125 N/m. Ans.}$$

Problem 1.27 *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3}$ m

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725$ N/m

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$$= 0.725 + 10.32 = \textbf{11.045 N/cm}^2\textbf{. Ans.}$$

1.6.4 Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise. Consider a glass tube of small diameter '*d*' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

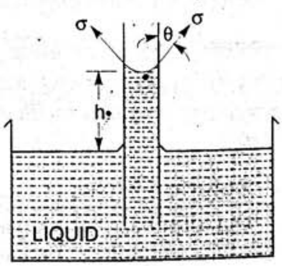

Fig. 1.13 *Capillary rise.*

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad ...(1.17)$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$
$$= \sigma \times \pi d \times \cos\theta \quad ...(1.18)$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d} \quad ...(1.19)$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \quad ...(1.20)$$

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \quad \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \quad h = \frac{4\sigma\cos\theta}{\rho g d} \quad ...(1.21)$$

Value of θ for mercury and glass tube is 128°.

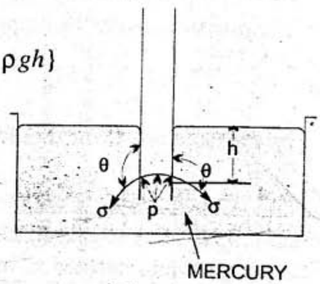

Fig. 1.14

Problem 1.28 *Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.*

Solution. Given :

Dia. of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

Surface tenstion, σ for water = 0.0725 N/m

σ for mercury = 0.52 N/m

Sp. gr. of mercury = 13.6

∴ Density $= 13.6 \times 1000$ kg/m³.

(*a*) **Capillary rise for water (θ = 0)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118$ m = **1.18 cm. Ans.**

(*b*) **For mercury**

Angle of constant between mercury and glass tube, θ = 130°

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004$ m = **– 0.4 cm. Ans.**

The negative sign indicates the capillary depression.

Problem 1.29 *Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 1.30°. Take density of water at 20°C as equal to 998 kg/m³.* (U.P.S.C. Engg. Exam., 1974)

Solution. Given :

Dia of tube, $d = 4 \text{ mm} = 4 \times 10^{-3}$ m

The capillary effect (*i.e.*, capillary rise or depression) is given by equation (1.20) as

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tension in kgf/m

θ = angle of contact, and ρ = density

(*i*) **Capillary effect for water**

σ = 0.073575 N/m, θ = 0°

ρ = 998 kg/m³ at 20°C

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \textbf{7.51 mm. Ans.}$$

(*ii*) **Capillary effect for mercury**

σ = 0.51 N/m, θ = 130° and

ρ = sp. gr. × 1000 = 13.6 × 1000 = 13600 kg/m²

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \textbf{– 2.46 mm. Ans.}$$

The negative sign indicates the capillary depression.

Problem 1.30. *The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.*

Solution. Given :

Capillary rise, $h = 0.2 \text{ mm} = 0.2 \times 10^{3}$ m

Surface tension, σ = 0.0725 N/m

Let dia. of tube $= d$

The angle θ for water $= 0$

Density (ρ) for water $= 1000$ kg/m^3

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.0725}{1000 \times 9.81 \times .2 \times 10^{-3}} = 0.148 \text{ m} = \textbf{14.8 cm. Ans.}$$

Thus minimum diameter of the tube should be 14.8 cm.

Problem 1.31 *Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.* (Converted to SI Units, A.M.I.E., Summer 1985)

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m

Surface tension, $\sigma = 0.073575$ N/m

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000$ kg/m^3

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \textbf{1.5 cm. Ans.}$$

Thus minimum diameter of the tube should be 1.5.cm.

Problem 1.32 *An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve..The diameter of the shaft is 0.5 m and it rotates at 200 r.p.m. Calculate the power lost in oil for a sleeve length of 100 mm. The thickness of oil film is 1.0 mm.* [Delhi University, December, 1992 (NS)]

Solution. Given :

Viscosity, $\mu = 5$ poise

$$= \frac{5}{10} = 0.5 \text{ N s/m}^2$$

Dia. of shaft, $D = 0.5$ m

Speed of shaft, $N = 200$ r.p.m.

Sleeve length, $L = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1$ m

Thickness of oil film, $t = 1.0 \text{ mm} = 1 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60} = 5.235$ m/s

Using the relation, $\tau = \mu \frac{du}{dy}$

where, du = Change of velocity $= u - 0 = u = 5.235$ m/s

dy = Change of distance $= t = 1 \times 10^{-3}$ m

$$\therefore \quad \tau = \frac{0.5 \times 5.235}{1 \times 10^{-3}} = 2617.5 \text{ N/m}^2$$

This is the shear stress on the shaft

∴ Shear force on the shaft, F = Shear stress × Area $= 2617.5 \times \pi D \times L$ $\quad (\because \text{ Area} = \pi D \times L)$

$$= 2617.5 \times \pi \times 0.5 \times 0.1 = 410.95 \text{ N}$$

Torque on the shaft, $$T = \text{Force} \times \frac{D}{2} = 410.95 \times \frac{0.5}{2} = 102.74 \text{ Nm}$$

$$\therefore \quad \text{Power* lost} = T \times \omega \text{ Watts} = T \times \frac{2\pi N}{60} \text{ W}$$

$$= 102.74 \times \frac{2\pi \times 200}{60} = 2150 \text{ W} = \textbf{2.15 kW. Ans.}$$

▶ 1.7 VAPOUR PRESSURE AND CAVITATION

A change from the liquid state to the gaseous state is known as vaporization. The vaporization (which depends upon the prevailing pressure and temperature condition) occurs because of continuous escaping of the molecules through the free liquid surface.

Consider a liquid (say water) which is confined in a closed vessel. Let the temperature of liquid is 20°C and pressure is atmospheric. This liquid will vaporise at 100°C. When vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as **vapour pressure** of the liquid. Or this is the pressure at which the liquid is converted into vapours.

Again consider the same liquid at 20°C at atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means, the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20°C. Thus a liquid may boil even at ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as **cavitation.**

Hence the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which the liquid is flowing, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and hence the name is cavitation.

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* Power in case of S.I. Unit $= T \times \omega$ or $\frac{2\pi NT}{60}$ Watts or $\frac{2\pi NT}{60{,}000}$ kW. The angular velocity $\omega = \frac{2\pi N}{60}$.

HIGHLIGHTS

1. The weight density or specific weight of a fluid is equal to weight per unit volume. It is also equal to,
$$w = \rho \times g.$$
2. Specific volume is the reciprocal of mass density.
3. The shear stress is proportional to the velocity gradient $\frac{du}{dy}$. Mathematically, $\tau = \mu \frac{du}{dy}$.
4. Kinematic viscosity ν is given by $\nu = \frac{\mu}{\rho}$.
5. Poise and stokes are the units of viscosity and kinematic viscosity respectively.
6. To convert the unit of viscosity from poise to MKS units, poise should be divided by 98.1 and to convert poise into SI units, the poise should be divided by 10. SI unit of viscosity is Ns/m^2 or Pa s, where N/m^2 = Pa = Pascal.
7. For a perfect gas, the equation of state is $\frac{p}{\rho} = RT$

   where R = gas constant and for air $= 29.3 \frac{\text{kgf-m}}{\text{kg}^\circ\text{K}} = 287$ J/kg °K.
8. For isothermal process, $\frac{p}{\rho}$ = Constant whereas for adiabatic process, $\frac{p}{\rho^k}$ = constant.
9. Bulk modulus of elasticity is given as $K = \frac{-dp}{\left(\frac{d\forall}{\forall}\right)}$.
10. Compressibility is the reciprocal of bulk modulus of elasticity or $= \frac{1}{K}$.
11. Surface tension is expressed in N/m or dyne/cm. The relation between surface tension (σ) and difference of pressure (p) between the inside and outside of a liquid drop is given as $p = \frac{4\sigma}{d}$

    For a soap bubble, $p = \frac{8\sigma}{d}$.

    For a liquid jet, $p = \frac{2\sigma}{d}$.
12. Capillary rise or fall of a liquid is given by $h = \frac{4\sigma \cos\theta}{wd}$.

    The value of θ for water is taken equal to zero and for mercury equal to 128°.

EXERCISE 1

(A) THEORETICAL PROBLEMS

1. Define the following fluid properties :
   Density, weight density, specific volume and specific gravity of a fluid.
2. Differentiate between : (*i*) Liquids and gases, (*ii*) Real fluids and ideal fluids, (*iii*) Specific weight and specific volume of a fluid.
3. What is the difference between dynamic viscosity and kinematic viscosity ? State their units of measurements.

4. Explain the terms : (*i*) Dynamic viscosity, and (*ii*) Kinematic viscosity. Give their dimensions. (*A.M.I.E., Summer 1988*)

5. State the Newton's law of viscosity and give examples of its application. (*Delhi University, June 1996*)

6. Enunciate Newton's law of viscosity. Explain the importance of viscosity in fluid motion. What is the effect of temperature on viscosity of water and that of air? (*A.M.I.E., Winter 1987*)

7. Define Newtonian and Non-Newtonian fluids.

8. What do you understand by terms : (*i*) Isothermal process, (*ii*) Adiabatic process, and (*iii*) Universal-gas constant.

9. Define compressibility. Prove that compressibility for a perfect gas undergoing isothermal compression is $\frac{1}{p}$ while for a perfect gas undergoing isentropic compression is $\frac{1}{wp}$.

10. Define surface tension. Prove that the relationship between surface tension and pressure inside a droplet of liquid in excess of outside pressure is given by $p = \frac{4\sigma}{d}$.

11. Explain the phenomenon of capillarity. Obtain an expression for capillary rise of a liquid.

12. (*a*) Distinguish between ideal fluids and real fluids. Explain the importance of compressibility in fluid flow. (*A.M.I.E., Summer 1988*)

(*b*) Define the terms : density, specific volume, specific gravity, vacuum pressure, compressible and in-compressible fluids. (*R.G.P. Vishwavidyalaya, Bhopal S 2002*)

13. Define and explain Newton's law of viscosity. (*Delhi University, April 1992*)

14. Convert 1 kg/s-m dynamic viscosity in poise. (*A.M.I.E., Winter 1991*)

15. Why does the viscosity of a gas increases with the increase in temperature while that of a liquid decreases with increase in temperature ? (*A.M.I.E.. Winter 1990*)

16. (*a*) How does viscosity of a fluid vary with temperature ?

(*b*) Cite examples where surface tension effects play a prominent role. (*J.N.T.U., Hyderabad S 2002*)

17. (*i*) Develop the expression for the relation between gauge pressure P inside a droplet of liquid and the surface tension.

(*ii*) Explain the following :

Newtonian and Non-Newtonian fluids, vapour pressure, and compressibility. (*R.G.P.V., Bhopal S 2001*)

(B) NUMERICAL PROBLEMS

1. One litre of crude oil weighs 9.6 N. Calculate its specific weight, density and specific gravity. (*A.M.I.E., Summer 1986*) [**Ans.** 9600 N/m^3, 978.6 kg/m^3, 0.978]

2. The velocity distribution for flow over a flat plate is given by $u = \frac{3}{2}\ y - y^{3/2}$, where u is the point velocity in metre per second at a distance y metre above the plate. Determine the shear stress at $y = 9$ cm. Assume dynamic viscosity as 8 poise. (*Nagpur University*) [**Ans.** 0.839 N/m^2]

3. A plate, 0.025 mm distant form a fixed plate, moves at 50 cm/s and requires a force of 1.471 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates in the poise. [**Ans.** 7.357×10^{-4}]

4. Determine the intensity of shear of an oil having viscosity = 1.2 poise and is used for lubrication in the clearance between a 10 cm diameter shaft and its journal bearing. The clearance is 1.0 mm and shaft rotates at 200 r.p.m. [**Ans.** 125.56 N/m^2]

5. Two plates are placed at a distance of 0.15 mm apart. The lower plate is fixed while the upper plate having surface area 1.0 m^2 is pulled at 0.3 m/s. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity 1.5 poise. [**Ans.** 300 N, 89.8 W]

6. An oil film of thickness 1.5 mm is used for lubrication between a square plate of size 0.9 m × 0.9 m and an inclined plane having an angle of inclination 20°. The weight of the square is 392.4 N and it slides down the plane with a uniform velocity of 0.2 m/s. Find the dynamic viscosity of the oil. [**Ans.** 12.42 poise]

7. In a stream of glycerine in motion, at a certain point the velocity gradient is 0.25 metre per sec per metre. The mass density of fluid is 1268.4 kg per cubic metre and kinematic viscosity is 6.30×10^{-4} square metre per second. Calculate the shear stress at the point. (*U.P.S.C., 1975*) [**Ans.** 0.2 N/m^2]
8. Find the kinematic viscosity of an oil having density 980 kg/m^2 when at a certain point in the oil, the shear stress is 0.25 N/m^2 and velocity gradient 0.3/s. $\left[\textbf{Ans. } 0.000849 \frac{m^2}{sec} \text{ or } 8.49 \text{ stokes}\right]$
9. Determine the specific gravity of a fluid having viscosity 0.07 poise and kinematic viscosity 0.042 stokes. [**Ans.** 1.667]
10. Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 2.0. [**Ans.** 11.99 poise]
11. If the velocity distribution of a fluid over a plate is given by $u = (3/4)y - y^2$, where u is the velocity in metre per second at a distance of y metres above the plate, determine the shear stress at $y = 0.15$ metre. Take dynamic viscosity of the fluid as 8.5×10^{-5} kg-sec/m^2. (*A.M.I.E., Winter 1974*) [**Ans.** 3.825×10^{-5} kgf/m^2]
12. An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of shaft is 0.5 m and it rotates at 200 r.p.m. Calculate the power lost in the oil for a sleeve length of 100 mm. The thickness of the oil film is 1.0 mm. [**Ans.** 2.15 kW]
13. The velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in m/sec at a distance of y m above the plate. Determine the shear stress at y = 0, 0.1 and 0.2 m. Take μ = 6 poise. [**Ans.** 0.4, 0.028 and 0.159 N/m^2]
14. In question 13, find the distance in metres above the plate, at which the shear stress is zero. [**Ans.** 0.333 m]
15. The velocity profile of a viscous fluid over a plate is parabolic with vertex 20 cm from the plate, where the velocity is 120 cm/s. Calculate the velocity gradient and shear stress at distances of 0, 5 and 15 cm from the plate, given the viscosity of the fluid = 6 poise.[**Ans.** 12/s, 7.18 N/m^2; 9/s, 5.385 N/m^2 ; 3/s, 1.795 N/m^2]
16. The weight of a gas is given as 17.658 N/m^3 at 30°C and at an absolute pressure of 29.43 N/cm^2. Determine the gas constant and also the density of the gas. $\left[\textbf{Ans. } \frac{1.8 \text{ kg}}{m^3}, \frac{539.55 \text{ N-m}}{\text{kg°K}}\right]$
17. A cylinder of 0.9 m^3 in volume contains air at 0°C and 39.24 N/cm^2 absolute pressure. The air is compressed to 0.45 m^3. Find (*i*) the pressure inside the cylinder assuming isothermal process, (*ii*) pressure and temperature assuming adiabatic process. Take k = 1.4 for air. [**Ans.** (*i*) 78.48 N/cm^2, (*ii*) 103.5 N/m^2, 140°C]
18. Calculate the pressure exerted by 4 kg mass of nitrogen gas at a temperature of 15°C if the volume is 0.35 m^3. Molecular weight of nitrogen is 28. [**Ans.** 97.8 N/cm^2]
19. The pressure of a liquid is increased from 60 N/cm^2 to 100 N/cm^2 and volume decreases by 0.2 per cent. Determine the bulk modulus of elasticity. [**Ans.** 2×10^4 N/cm^2]
20. Determine the bulk modulus of elasticity of a fluid which is compressed in a cylinder from a volume of 0.009 m^3 at 70 N/cm^2 pressure to a volume of 0.0085 m^3 at 270 N/cm^2 pressure. [**Ans.** 3.6×10^3 N/cm^2]
21. The surface tension of water in contact with air at 20°C is given as 0.0716 N/m. The pressure inside a droplet of water is to be 0.0147 N/cm^2 greater than the outside pressure, calculate the diameter of the droplet of water. [**Ans.** 1.94 mm]
22. Find the surface tension in a soap bubble of 30 mm diameter when the inside pressure is 1.962 N/m^2 above atmosphere. [**Ans.** 0.00735 N/m]
23. The surface tension of water in contact with air is given as 0.0725 N/m. The pressure outside the droplet of water of diameter 0.02 mm is atmospheric $\left(10.32 \frac{N}{cm^2}\right)$. Calculate the pressure within the droplet of water. [**Ans.** 11.77 N/cm^2]

24. Calculate the capillary rise in a glass tube of 3.0 mm diameter when immersed vertically in (*a*) water, and (*b*) mercury. Take surface tensions for mercury and water as 0.0725 N/m and 0.52 N/m respectively in contact with air. Specific gravity for mercury is given as 13.6. **[Ans.** 0.966 cm, 0.3275 cm]

25. The capillary rise in the glass tube used for measuring water level is not to exceed 0.5 mm. Determine its minimum size, given that surface tension for water in contact with air = 0.07112 N/m. **[Ans.** 5.8 cm]

26. **(SI Units)**. One litre of crude oil weighs 9.6 N. Calculate its specific weight, density and specific gravity. (*Converted to SI units, A.M.I.E., Summer 1986*) **[Ans.** 9600 N/m^3; 979.6 kg/m^3 ; 0.9786]

27. **(SI Units)**. A piston 796 mm diameter and 200 mm long works in a cylinder of 800 mm diameter. If the annular space is filled with a lubricating oil of viscosity 5 cp (centi-poise), calculate the speed of descent of the piston in vertical position. The weight of the piston and axial load are 9.81 N. **[Ans.** 7.84 m/s]

28. **(SI Units)**. Find the capillary rise of water in a tube 0.03 cm diameter. The surface tension of water is 0.0735 N/m. **[Ans.** 9.99 cm]

29. Calculate the specific weight, density and specific gravity of two litres of a liquid which weight 15 N. (*Delhi University, April 1992*) **[Ans.** 7500 N/m^3, 764.5 kg/m^3, 0.764]

30. A 150 mm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 151 mm. Both the cylinders are of 250 mm height. The space between the cylinders is filled with a liquid of viscosity 10 poise. Determine the torque required to rotate the inner cylinder at 100 r.p.m. (*Delhi University, April 1992*) **[Ans.** 13.87 Nm]

31. A shaft of diameter 120 mm is rotating inside a journal bearing of diameter 122 mm at a speed of 360 r.p.m. The space between the shaft and the bearing is filled with a lubricating oil of viscosity 6 poise. Find the power absorbed in oil if the length of bearing is 100 mm. (*Delhi University, May 1998*) **[Ans.** 115.73 W]

32. A shaft of diameter 100 mm is rotating inside a journal bearing of diameter 102 mm at a space of 360 r.p.m. The space between the shaft and bearing is filled with a lubricating oil of viscosity 5 poise. The length of the bearing is 200 mm. Find the power absorbed in the lubricating oil. (*Delhi University, June 1996*) **[Ans.** 111.58 W]

33. Assuming that the bulk modulus of elasticity of water is 2.07×10^6 kN/m^2 at standard atmospheric conditions, determine the increase of pressure necessary to produce 1% reduction in volume at the same temperature. (*Delhi University, June 1997*)

[Hint. $K = 2.07 \times 10^6 \text{ kN/m}^2 ; \dfrac{-d\forall}{\forall} = \dfrac{1}{100} = 0.01.$

Increase in pressure $(dp) = K \times \left(\dfrac{-d\forall}{\forall}\right) = 2.07 \times 10^6 \times 0.01 = 2.07 \times 10^4 \text{ kN/m}^2$. **Ans.**]

34. A square plate of size 1 m× 1 m and weighing 350 N slides down an inclined plane with a uniform velocity of 1.5 m/s. The inclined plane is laid on a slope of 5 vertical to 12 horizontal and has an oil film of 1 mm thickness. Calculate the dynamic viscosity of oil. [*J.N.T.U., Hyderabad, S 2002*]

[**Hint.** $A = 1 \times 1 = 1 \text{ m}^2$, W = 350 N, u = 1.5 m/s, $\tan\theta = \dfrac{5}{12} = \dfrac{BC}{AB}$

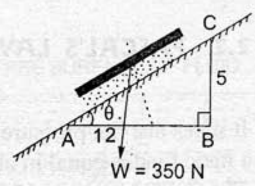


Fig. 1.15

Component of weight along the plane = W × sin θ

where $\sin\theta = \dfrac{BC}{AC} = \dfrac{5}{13}$ $\left(\because AC = \sqrt{AB^2 + BC^2} = \sqrt{12^2 + 5^2} = 13\right)$

$$\therefore \quad F = W \sin\theta = 350 \times \frac{5}{13} = 134.615$$

Now $\tau = \mu \dfrac{du}{dy}$, where $du = u - 0 = u = 1.5$ m/s and

$dy = 1 \text{ mm} = 1 \times 10^{-3}$ m

or $\dfrac{F}{A} = \mu \dfrac{du}{dy}$, $\therefore \mu = \dfrac{F}{A} \times \dfrac{dy}{du} = \dfrac{134.615}{1} \times \dfrac{1 \times 10^{-3}}{1.5} = 0.0897 \dfrac{Ns}{m^2} = 0.897$ poise **Ans.**]

2

CHAPTER

Pressure and its Measurement

▶ 2.1 FLUID PRESSURE AT A POINT

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

The units of pressure are : (*i*) kgf/m^2 and kgf/cm^2 in MKS units, (*ii*) Newton/m^2 or N/m^2 and N/mm^2 in SI units. N/m^2 is known as Pascal and is represented by Pa. Other commonly used units of pressure are :

$$\text{kPa} = \text{kilo pascal} = 1000 \text{ N/m}^2$$
$$\text{bar} = 100 \text{ kPa} = 10^5 \text{ N/m}^2.$$

▶ 2.2 PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, dx, dy and ds.

Consider an **arbitrary** fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x,

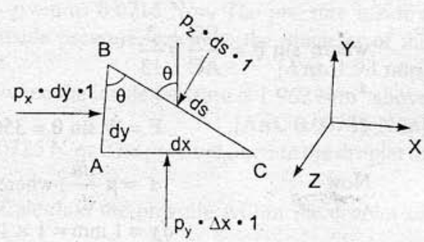


Fig. 2.1 *Forces on a fluid element.*

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces.
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element = (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1) \sin (90° - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos \theta = 0.$$

But from Fig. 2.1, $ds \cos \theta = AB = dy$

$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z \quad \text{...(2.1)}$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos (90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin \theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin \theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

or $$p_y = p_z \quad \text{...(2.2)}$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \quad \text{...(2.3)}$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

▶ 2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are :

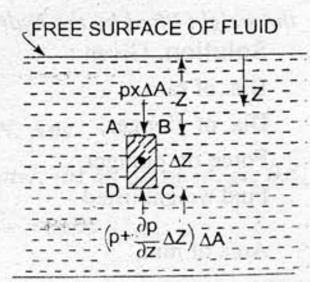


Fig. 2.2 *Forces on a fluid element.*

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.

2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.

3. Weight of fluid element = Density × g × Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or
$$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or
$$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or
$$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

∴
$$\frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w) \qquad \text{...(2.4)}$$

where w = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g Z$$

or
$$p = \rho g Z \qquad \text{...(2.5)}$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have $Z = \frac{p}{\rho \times g}$ ...(2.6)

Here Z is called **pressure head.**

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

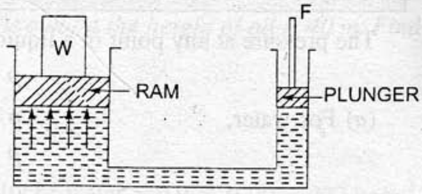


Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068}$ N/m^2

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222$ N $=$ **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20$ cm $= 0.2$ m

$\therefore$ Area of ram, $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3$ cm $= 0.03$ m

$\therefore$ Area of plunger, $a = \dfrac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000$ N.

See Fig. 2.3.

Pressure intensity developed due to plunger $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \dfrac{F}{a}$

$\therefore$ Force acting on ram = Pressure intensity × Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \textbf{675.2 N. Ans.}$$

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000$ kg/m^3.*

Solution. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2. \text{ Ans.}}$$

(*b*) For oil of sp. gr. 0.8,

From equation (1.1*A*), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}$$

$$= \mathbf{0.2354 \frac{N}{cm^2}. \text{ Ans.}}$$

(*c*) For mercury, sp. gr. = 13.6

From equation (1.1A) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, $\rho_s = \text{Specific gravity of mercury} \times \text{Density of water}$

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$\therefore$ $p = \rho_s \times g \times Z$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}. \text{ Ans}}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}$.

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \mathbf{4 \text{ m of water. Ans.}}$

(*b*) For oil, sp. gr. = 0.9

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$\therefore$ $Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \mathbf{4.44 \text{ m of oil. Ans.}}$

Problem 2.5 *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Solution. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40$ m

Density of oil, ρ_0 = Sp. gr. of oil × Density of water = 0.9 × 1000 = 900 kg/m³

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \ \frac{\text{N}}{\text{m}^2}$

$$\therefore \text{ Corresponding height of water} = \frac{p}{\text{Density of water} \times g}$$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \textbf{36 m of water. Ans.}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m³

Density of oil, ρ_2 = Sp. gr. of oil × Density of water = 0.9 × 1000 = 900 kg/m³

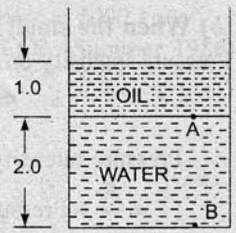


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \ \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \textbf{0.8829 N/cm}^2\textbf{. Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$
$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \textbf{2.8449 N/cm}^2\textbf{. Ans.}$$

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack ate 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m³.

Solution. Given :

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(*a*) **When the pistons are at the same level**

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

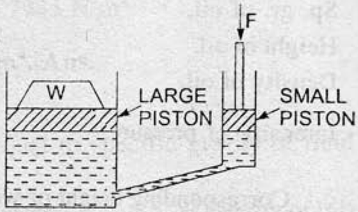


Fig. 2.5

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

(*b*) **When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section $A - A$

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

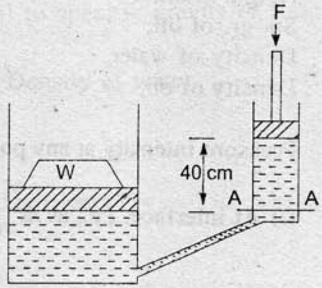


Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

$\therefore$ Pressure intensity at section

$$A - A = \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston $= 11.71 \text{ N/cm}^2$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▶ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. **Vacuum pressure** is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure − Absolute pressure.

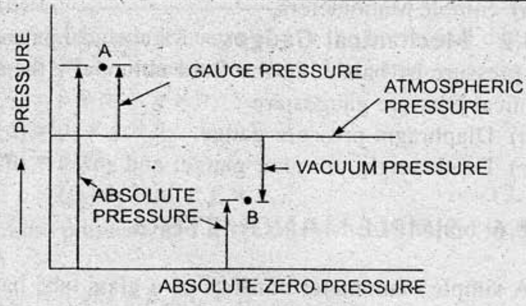


Fig. 2.7 *Relationship between pressures.*

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53 × 10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

(A.M.I.E., Summer 1986)

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

∴ $$p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, p = **45028 N/m^2. Ans**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

▶ 2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

▶ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point *A*, the height of liquid say water is *h* in piezometer tube, then pressure at *A*

$$= \rho \times g \times h \ \frac{N}{m^2}.$$

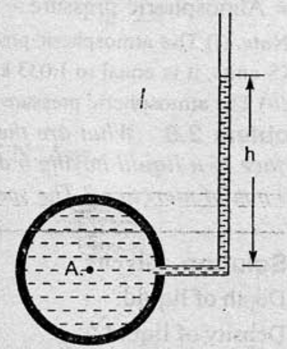
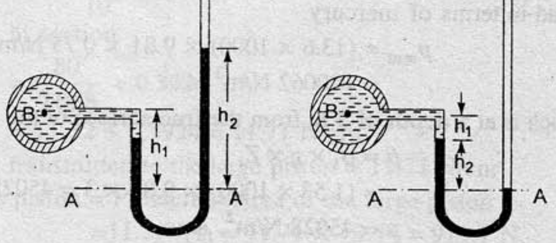


Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



(a) For gauge pressure (b) For vacuum pressure

Fig. 2.9 *U-tube Manometer.*

(*a*) **For Gauge Pressure.** Let *B* is the point at which pressure is to be measured, whose value is *p*. The datum line is *A-A*.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1). \qquad ...(2.7)$$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A = 0

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1). \qquad ...(2.8)$$

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from *A-A*, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above *A-A*, we get

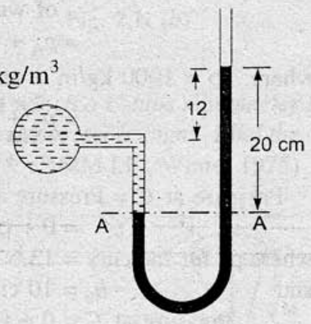


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Difference of mercury level, $h_2 = 40 \text{ cm} = 0.4 \text{ m}$. Height of liquid in left limb, $h_1 = 15 \text{ cm} = 0.15 \text{ m}$. Let the pressure in pipe = p. Equating pressure above datum line *A-A*, we get

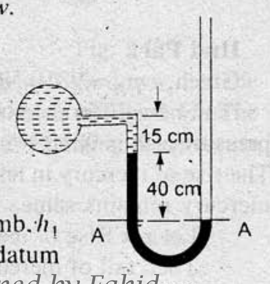


Fig. 2.11

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -[\rho_2 g h_2 + \rho_1 g h_1]$$
$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$
$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2}. \textbf{ Ans.}$$

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.* (A.M.I.E., Winter 1989)

Solution. Given :

Difference of mercury $= 10 \text{ cm} = 0.1 \text{ m}$

The arrangement is shown in Fig. 2.11 (*a*)

Let p_A = (pressure of water in pipe line (*i.e.*, at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*

= Pressure at *A* + Pressure due to 10 cm (or 0.1 m) of water

$$= p_A + \rho \times g \times h$$

where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1$ m

$$= p_A + 1000 \times 9.81 \times 0.1$$
$$= p_A + 981 \text{ N/m}^2 \quad \text{...}(i)$$

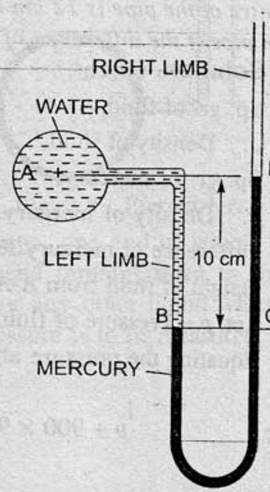


Fig. 2.11 (*a*)

Pressure at *C* = Pressure at *D* + Pressure due to 10 cm of mercury

$$= 0 + \rho_0 \times g \times h_0$$

where ρ_0 for mercury $= 13.6 \times 1000 \text{ kg/m}^3$

and $h_0 = 10 \text{ cm} = 0.1$ m

$$\therefore \quad \text{Pressure at } C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$$
$$= 13341.6 \text{ N} \quad \text{...}(ii)$$

But pressure at *B* is equal to pressure at *C*. Hence equating the equations (*i*) and (*ii*), we get

$$p_A + 981 = 13341.6$$
$$\therefore \quad p_A = 13341.6 - 981$$
$$= \mathbf{12360.6 \frac{N}{m^2}}. \textbf{ Ans.}$$

IInd Part

Given, $p_A = 9810 \text{ N/m}^2$

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (*b*). In this case the pressure at *A* is 9810 N/m² which is less than the 12360.6 N/m². Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points *B*, *C* and *D* show the initial conditions whereas points *B**, *C** and *D** show the final conditions.

The pressure at B^* = Pressure at C^*

or Pressure at A + Pressure due to $(10 - x)$ cm of water

= Pressure at D^* + Pressure due to $(10 - 2x)$ cm of mercury

or $$p_A + \rho_1 \times g \times h_1 = p_D^* + \rho_2 \times g \times h_2$$

or $$1910 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$$

$$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$$

Dividing by 9.81, we get

or $$1000 + 100 - 10x = 1360 - 272x$$

or $$272x - 10x = 1360 - 1100$$

or $$262x = 260$$

$$\therefore \quad x = \frac{260}{262} = 0.992 \text{ cm}$$

$\therefore$ New difference of mercury $= 10 - 2x$ cm $= 10 - 2 \times 0.992$

$= $ **8.016 cm. Ans.**

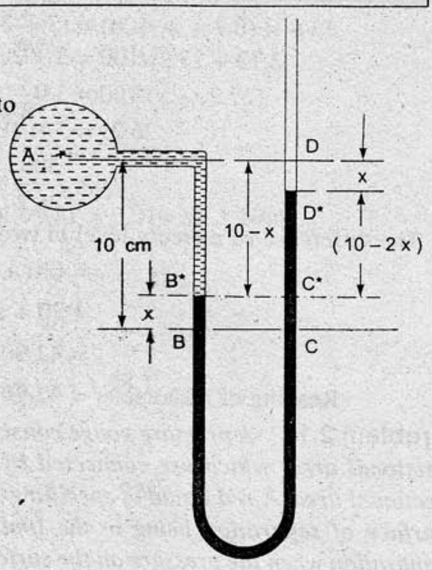


Fig. 2.11 (*b*)

Problem 2.12 *Fig. 2.12 shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.* (A.M.I.E., Winter 1975)

Solution. Vessel is empty. Given :

Difference of mercury level $h_2 = 20$ cm

Let h_1 = Height of water above X-X

Sp. gr. of mercury, $S_2 = 13.6$

Sp. gr. of water, $S_1 = 1.0$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Density of water, $\rho_1 = 1000$

Equating the pressure above datum line X-X, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

or $$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water.}$$

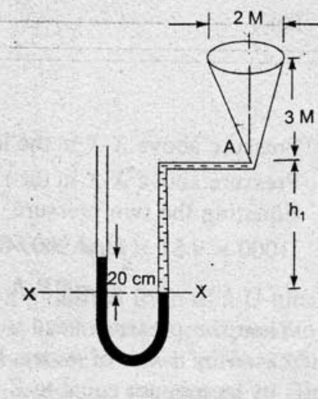


Fig. 2.12

Vessel is full of water. When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in Fig. 2.13. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.

Pressure in left limb = Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100)$$

$$= 1000 \times 9.81 \times (3 + h_1 + y/100)$$

or $\quad 13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \quad (\because \quad h_1 = 2.72 \text{ cm})$

or $\quad 2.72 + 27.2y/100 = 3 + 2.72 + y/100$

or $\quad (27.2y - y)/100 = 3.0$

or $\quad 26.2y = 3 \times 100 = 300$

$$\therefore \quad y = \frac{300}{26.2} = 11.45 \text{ cm}$$

The difference of mercury level in two limbs

$= (20 + 2y)$ cm of mercury

$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90$ cm of mercury

$\therefore$ Reading of manometer = **42.90 cm. Ans.**

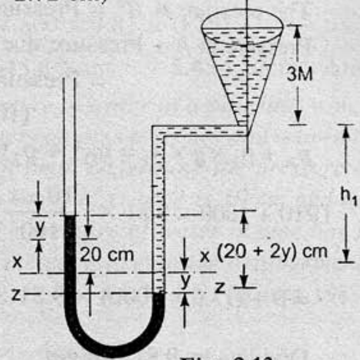


Fig. 2.13

Problem 2.13 *A pressure gauge consists of two cylindrical bulbs B and C each of 10 sq. cm cross-sectional area, which are connected by a U-tube with vertical limbs each of 0.25 sq. cm cross-sectional area. A red liquid of specific gravity 0.9 is filled into C and clear water is filled into B, the surface of separation being in the limb attached to C. Find the displacement of the surface of separation when the pressure on the surface in C is greater than that in B by an amount equal to 1 cm head of water.* (A.M.I.E., Summer, 1978)

Solution. Given :

Area of each bulb B and C, $\quad A = 10 \text{ cm}^2$

Area of each vertical limb, $\quad a = 0.25 \text{ cm}^2$

Sp. gr. of red liquid $\quad = 0.9 \quad \therefore$ Its density $= 900 \text{ kg/m}^3$

Let $\quad X\text{-}X$ = Initial separation level

h_C = Height of red liquid above $X\text{-}X$

h_B = Height of water above $X\text{-}X$

Pressure above $X\text{-}X$ in the left limb $= 1000 \times 9.81 \times h_B$

Pressure above $X\text{-}X$ in the right limb $= 900 \times 9.81 \times h_C$

Equating the two pressure, we get

$$1000 \times 9.81 \times h_B = 900 \times 9.81 \times h_C$$

$$\therefore \quad h_B = 0.9\ h_C \qquad \ldots(i)$$

When the pressure head over the surface in C is increased by 1 cm of water, let the separation level falls by an amount equal to Z. Then $Y\text{-}Y$ becomes the final separation level.

Now fall in surface level of C multiplied by cross-sectional area of bulb C must be equal to the fall in separation level multiplied by cross-sectional area of limb.

$\therefore$ Fall in surface level of C

$$= \frac{\text{Fall in separation level} \times a}{A}$$

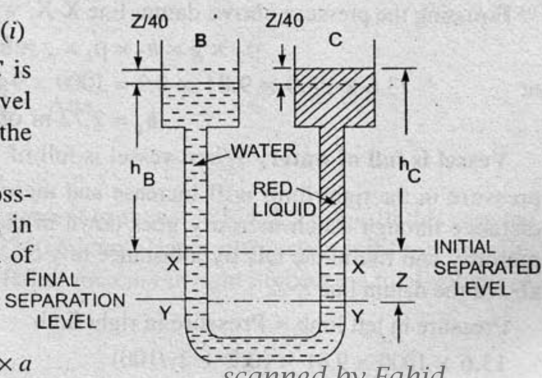


Fig. 2.14

$$= \frac{Z \times a}{A} = \frac{Z \times 0.25}{10} = \frac{Z}{40}$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water = $\rho g h = 1000 \times 9.81 \times 0.01 = 98.1 \text{ N/m}^2$

Consider final separation level Y-Y

Pressure above Y-Y in the left limb = $1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right)$

Pressure above Y-Y in the right limb = $900 \times 9.81 \left(Z + h_C - \frac{Z}{40}\right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right) = \left(Z + h_C - \frac{Z}{40}\right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40}\right) = 900 \left(Z + h_C - \frac{Z}{40}\right) + 10$$

Dividing by 1000, we get $Z + h_B + \frac{Z}{40} = 0.9 \left(Z + h_C - \frac{Z}{40}\right) + 0.01$

But from equation (i), $\quad h_B = 0.9\ h_C$

$$\therefore \quad Z + 0.9\ h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\ h_C + 0.01$$

or
$$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or
$$Z\left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z\left(\frac{41 - 35.1}{40}\right) = .01$$

$$\therefore \quad Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

2.6.3 Single Column Manometer. Single column manomter is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

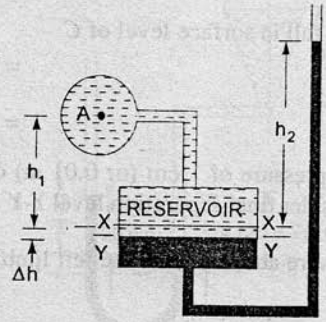


Fig. 2.15 *Vertical single column manometer.*

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \quad \text{...(i)}$$

Now consider the datum line Y-Y as shown in Fig. 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above $Y\text{-}Y = \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (*i*), $\quad \Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g \quad \text{...(2.9)}$$

As the area A is very large as compared to a, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g$ ...(2.10)

From equation (2.10), it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

2. Inclined Single Column Manometer

Fig. 2.16 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

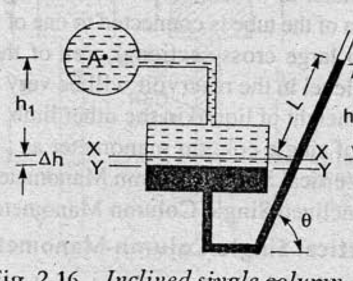


Fig. 2.16 *Inclined single column manometer.*

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X = $L \times \sin\theta$

From equation (2.10), the pressure at A is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of h_2, we get

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g. \qquad \text{...(2.11)}$$

Problem 2.14 *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

∴ Density $\rho_1 = 900$ kg/m^3

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20$ cm $= 0.2$ m

Rise of mercury in right limb,

$h_2 = 40$ cm $= 0.4$ m

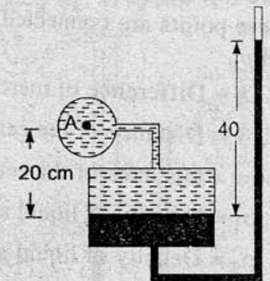


Fig. 2.17

Let p_A = Pressure in pipe

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}[133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8$ N/m^2 = 52134 N/m^2 = **5.21 N/cm^2. Ans.**

▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1 U-tube Differential Manometer. Fig. 2.18 shows the differential manometers of U-tube type.

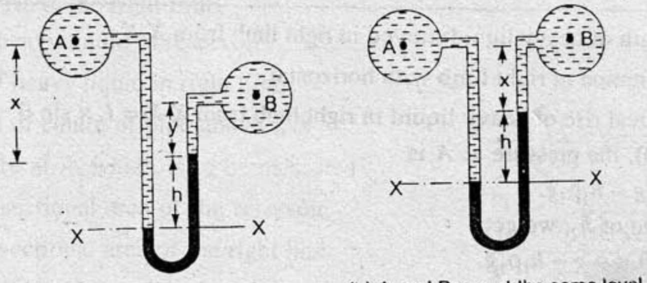


Fig. 2.18 *U-tube differential manometers.*

Fig. 2.18 (*a*). Let the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of *B*, from the mercury level in the right limb.

x = Distance of the centre of *A*, from the mercury level in the right limb.

ρ_1 = Density of liquid at *A*.

ρ_2 = Density of liquid at *B*.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at *X-X*.

Pressure above *X-X* in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at *A*.

Pressure above *X-X* in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at *B*.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \quad \text{...(2.12)}$$

$\therefore$ Difference of pressure at *A* and *B* $= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

Fig. 2.18 (*b*). *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \quad \text{...(2.13)}$$

Problem 2.15 *A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.*

Solution. Given :

Sp. gr. of oil, $S_1 = 0.9$ $\therefore$ Density, $\rho_1 = 0.9 \times 1000 = 900$ kg/m³

Difference in mercury level, $h = 15$ cm $= 0.15$ m

Sp. gr. of mercury, $S_g = 13.6$ $\therefore$ Density, $\rho_g = 13.6 \times 1000$ kg/m³

The difference of pressure is given by equation (2.13)

or
$$p_A - p_B = g \times h(\rho_g - \rho_1)$$
$$= 9.81 \times 0.15\ (13600 - 900) = \mathbf{18688\ N/m^2.\ Ans.}$$

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.*

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm² $= 1 \times 10^4$ kgf/m²

$= 10^4 \times 9.81$ N/m² ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm²

$= 1.8 \times 10^4$ kgf/m²

$= 1.8 \times 10^4 \times 9.81$ N/m² ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m³

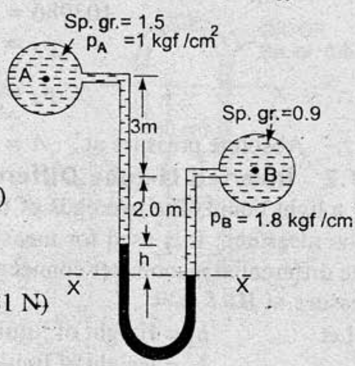


Fig. 2.19

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or
$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or
$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$
$$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1\ cm.\ Ans.}$$

Problem 2.17 *A differential manometer is connected at the two points A and B as shown in Fig. 2.20. At B air pressure is 9.81 N/cm² (abs), find the absolute pressure at A.*

Solution. Given :

Air pressure at $B = 9.81$ N/cm²

or $p_B = 9.81 \times 10^4$ N/m²

Density of oil $= 0.9 \times 1000 = 900 \text{ kg/m}^3$
Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$
Let the pressure at A is p_A
Taking datum line at X-X
Pressure above X-X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B$$
$$= 5886 + 98100 = 103986$$

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$
$$= 13341.6 + 1765.8 + p_A$$

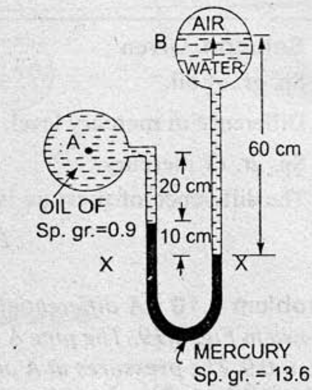


Fig. 2.20

Equating the two pressure head

$$103986 = 13341.6 + 1765.8 + p_A$$
$$\therefore \quad p_A = 103986 - 15107.4 = 88876.8$$
$$\therefore \quad p_A = 88876.8 \text{ N/m}^2 = \frac{88876.8\text{N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}.$$

$\therefore$ Absolute pressure at A = **8.887 N/cm². Ans.**

2.7.2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X-X
h_2 = Height of liquid in right limb
h = Difference of light liquid
ρ_1 = Density of liquid at A
ρ_2 = Density of liquid at B
ρ_s = Density of light liquid
p_A = Pressure at A
p_B = Pressure at B.

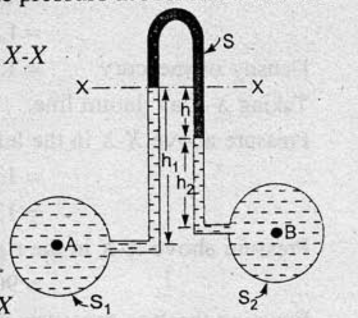


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or
$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18 *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Solution. Given :

Pressure head at $A = \dfrac{p_A}{\rho g} = 2$ m of water

$$\therefore \quad p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$$

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb $= p_A - \rho_1 \times g \times h_1$

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $$p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$$

or $$p_B = \mathbf{1.8599 \text{ N/cm}^2. \text{ Ans.}}$$

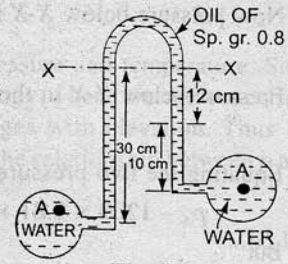


Fig. 2.22

Problem 2.19 *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

Solution. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$$= (30 + 20) - 30 = 20 \text{ cm}$$

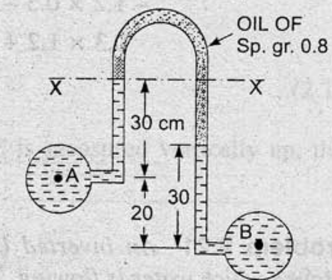


Fig. 2.23

Taking datum line at X-X

Pressure in the left limb below X-X

$$= p_A - 1000 \times 9.81 \times 0$$

$$= p_A - 2943$$

Pressure in the right limb below X-X

$$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$$

$$= p_B - 2943 - 1569.6 = p_B - 4512.6$$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore$ $$p_B - p_A = 4512.6 - 2943 = \mathbf{1569.6 \text{ N/m}^2. \text{ Ans.}}$$

Problem 2.20 *Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in Fig. 2.24 below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.*

(A.M.I.E., Winter 1985)

Solution. Given :

Fig. 2.24 shows the arrangement. Taking X-X as datum line.

Let p_A = Pressure at A

p_B = Pressure at B

Density of liquid in pipe A = Sp. gr. × 1000

$= 1.2 \times 1000$

$= 1200 \text{ kg/m}^2$

Density of liquid in pipe B $= 1 \times 1000 = 1000 \text{ kg/m}^3$

Density of oil $= 0.7 \times 1000 = 700 \text{ kg/m}^3$

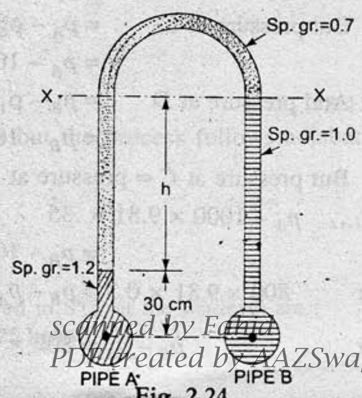


Fig. 2.24

Now pressure below X-X in the left limb

$$= p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times (h + 0.3)$$

Equating the two pressure, we get

$$p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = p_B - 1000 \times 9.81\ (h + 0.3)$$

But $\quad p_A = p_B$ (given)

$$\therefore \quad - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = - 1000 \times 9.81\ (h + 0.3)$$

Dividing by 1000×9.81

$$- 1.2 \times 0.3 - 0.7h = - (h + 0.3)$$

or $\quad 0.3 \times 1.2 + 0.7h = h + 0.3$ or $0.36 - 0.3 = h - 0.7h = 0.3h$

$$\therefore \quad h = \frac{0.36 - 0.30}{0.30} = \frac{0.06}{0.30} \text{ m}$$

$$= \frac{1}{5} \text{ m} = \frac{1}{5} \times 100 = \textbf{20 cm. Ans.}$$

Problem 2.21 *An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30 cm. When an oil of specific gravity 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from tlhe respective centre lines of the pipes) are found to be same and equal to 35 cm. Determine the difference of pressure between the pipes.*

(A.M.I.E., Summer 1990)

Solution. Given :

Specific gravity of measuring liquid = 0.8

The arrangement is shown in Fig. 2.24 (a).

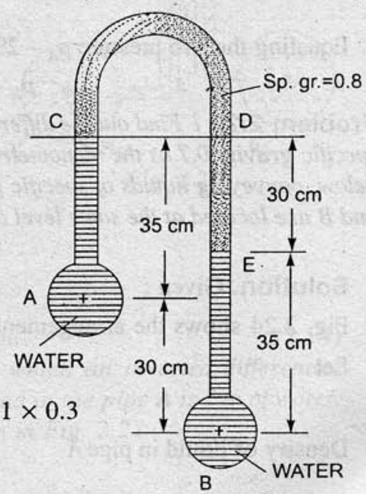


Fig. 2.24 (a)

Let $\quad p_A$ = pressure at A

p_B = pressure at B.

The points C and D lie on the same horizontal line.

Hence pressure at C should be equal to pressure at D.

But pressure at C $\quad = p_A - \rho g\, h$

$= p_A - 1000 \times 9.81 \times (0.35)$

And pressure at D $\quad = p_B - \rho_1 g h_1 - \rho_2 g h_2$

$= p_B - 1000 \times 9.81 \times (0.35) - 800 \times 9.81 \times 0.3$

But pressure at C = pressure at D

$$\therefore \quad p_A - 1000 \times 9.81 \times .35$$

$$= p_B - 1000 \times 9.81 \times 0.35 - 800 \times 9.81 \times 0.3$$

or $\quad 800 \times 9.81 \times 0.3 = p_B - p_A$

or $\quad p_B - p_A = 800 \times 9.81 \times 0.3 = \textbf{2354.4} \ \dfrac{\textbf{N}}{\textbf{m}^2}$. **Ans.**

▶ 2.8 PRESSURE AT A POINT IN COMPRESSIBLE FLUID

For compressible fluids, density (ρ) changes with the change of pressure and temperature. Such problems are encountered in aeronautics, oceanography and meteorology where we are concerned with atmospheric* air where density, pressure and temperature changes with elevation. Thus for fluids with variable density, equation (2.4) cannot be integrated, unless the relationship between p and ρ is known. For gases the equation of state is

$$\frac{p}{\rho} = RT \quad \text{...(2.15)}$$

or

$$\rho = \frac{p}{RT}$$

Now equation (2.4) is

$$\frac{dp}{dZ} = w = \rho g = \frac{p}{RT} \times g$$

$$\therefore \quad \frac{dp}{p} = \frac{g}{RT} dZ \quad \text{...(2.16)}$$

In equation (2.4), Z is measured vertically downward. But if Z is measured vertically up, then $\frac{dp}{dZ} = -\rho g$ and hence equation (2.16) becomes

$$\frac{dp}{p} = \frac{-g}{RT} dZ \quad \text{...(2.17)}$$

2.8.1 Isothermal Process. Case I. If temperature T is constant which is true for **isothermal process**, equation (2.17) can be integrated as

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT} dz = -\frac{g}{RT}\int_{Z_0}^{Z} dz$$

or

$$\log \frac{p}{p_0} = \frac{-g}{RT}[Z - Z_0]$$

where p_0 is the pressure where height is Z_0. If the datum line is taken at Z_0, then $Z_0 = 0$ and p_0 becomes the pressure at datum line.

$$\therefore \quad \log \frac{p}{p_0} = \frac{-g}{RT} Z$$

$$\frac{p}{p_0} = e^{-gZ/RT}$$

or pressure at a height Z is given by $p = p_0 e^{-gZ/RT}$ ...(2.18)

2.8.2 Adiabatic Process. If temperature T is not constant but the process follows adiabatic law then the relation between pressure and density is given by

$$\frac{p}{\rho^k} = \text{Constant} = C \quad \text{...}(i)$$

---

* The standard atmospheric pressure, temperature and density referred to STP at the sea-level are : Pressure = 101.325 kN/m^2 ; Temperature = 15°C and Density = 1.225 kg/m^3.

where k is ratio of specific constant.

$$\therefore \quad \rho^k = \frac{p}{C}$$

or $$\rho = \left(\frac{p}{C}\right) \quad \ldots(ii)$$

Then equation (2.4) for Z measured vertically up becomes,

$$\frac{dp}{dZ} = -\rho g = -\left(\frac{p}{C}\right)^{1/k} g$$

or $$\frac{dp}{\left(\frac{p}{C}\right)^{1/k}} = -gdZ \text{ or } C^{1/k}\frac{dp}{\frac{1}{p^k}} = -gdZ$$

Integrating, we get $\int_{p_0}^{p} C^{1/k}\, p^{-1/k}\, dp = \int_{Z_0}^{Z} -gdZ$

or $$C^{1/k}\left[\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

or $$\left[\frac{C^{1/k}\, p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z} \qquad [C \text{ is a constant, can be taken inside}]$$

But from equation (i), $$C^{1/k} = \left(\frac{p}{\rho^k}\right)^{1/k} = \frac{p^{1/k}}{\rho}$$

Substituting this value of $C^{1/k}$ above, we get

$$\left[\frac{p^{1/k}}{\rho}\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g[Z - Z_0]$$

or $$\left[\frac{p^{\frac{1-1+k}{k}}}{\rho^{\frac{k-1}{k}}}\right]_{p_0}^{p} = -g[Z - Z_0] \text{ or } \left[\frac{k}{k-1}\frac{p}{\rho}\right]_{p_0}^{p} = -g[Z - Z_0]$$

or $$\frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -g[Z - Z_0]$$

If datum line is taken at Z_0, where pressure, temperature and density are p_0, T_0 and ρ_0, then $Z_0 = 0$.

$$\therefore \quad \frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -gZ \text{ or } \frac{p}{\rho} - \frac{p_0}{\rho_0} = -gZ\frac{(k-1)}{k}$$

or $$\frac{p}{\rho} = \frac{p_0}{\rho_0} - gZ\frac{(k-1)}{k} = \frac{p_0}{\rho_0}\left[1 - \frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{\rho}\times\frac{\rho_0}{p_0}=\left[1+\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right] \quad ...(iii)$$

But from equation (*i*), $\frac{p}{\rho^k}=\frac{p_0}{\rho_0^k}$ or $\left(\frac{\rho_0}{\rho}\right)^k=\frac{p_0}{p}$ or $\frac{\rho_0}{\rho}=\left(\frac{p_0}{p}\right)^{1/k}$

Substituting the value of $\frac{\rho_0}{\rho}$ in equation (*iii*), we get

$$\frac{p}{p_0}\times\left(\frac{p_0}{p}\right)^{1/k}=\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{p_0}\times\left(\frac{p}{p_0}\right)^{-1/k}=\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]$$

or $$\left(\frac{p}{p_0}\right)^{1-\frac{1}{k}}=\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}=\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]$$

$\therefore$ $$\frac{p}{p_0}=\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}}$$

$\therefore$ Pressure at a height Z from ground level is given by

$$p=p_0\left[1-\frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} \quad ...(2.19)$$

in equation (2.19), p_0 = pressure at ground level, where $Z_0 = 0$
ρ_0 = density of air at ground level

Equation of state is $\frac{p_0}{\rho_0}=RT_0$ or $\frac{\rho_0}{p_0}=\frac{1}{RT_0}$

Substituting the values of $\frac{\rho_0}{p_0}$ in equation (2.19), we get

$$p=p_0\left[1-\frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \quad ...(2.20)$$

2.8.3 Temperature at any Point in Compressible Fluid. For the adiabatic process, the temperature at any height in air is calculated as :

Equation of state at ground level and at a height Z from ground level is written as

$$\frac{p_0}{\rho_0}=RT_0 \text{ and } \frac{p}{\rho}=RT$$

Dividing these equations, we get

$$\left(\frac{p_0}{\rho_0}\right)\div\frac{p}{\rho}=\frac{RT_0}{RT}=\frac{T_0}{T} \text{ or } \frac{p_0}{\rho_0}\times\frac{\rho}{p}=\frac{T_0}{T}$$

or $$\frac{T}{T_0}=\frac{\rho_0}{p_0}\times\frac{p}{\rho}=\frac{p}{p_0}\times\frac{\rho_0}{\rho} \quad ...(i)$$

But $\frac{p}{p_0}$ from equation (2.20) is given by

$$\frac{p}{p_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

Also for adiabatic process $\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k}$ or $\left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p}$

or
$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{-\frac{1}{k}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\left(\frac{k}{k-1}\right)\times\left(-\frac{1}{k}\right)} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

Substituting the values of $\frac{p}{p_0}$ and $\frac{\rho_0}{\rho}$ in equation (*i*), we get

$$\frac{T}{T_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \times \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}-\frac{1}{k-1}} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

$$\therefore \quad T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right] \qquad \text{...(2.21)}$$

2.8.4 Temperature Lapse-Rate (L). It is defined as the rate at which the temperature changes with elevation. To obtain an expression for the temperature lapse-rate, the temperature given by equation (2.21) is differentiated with respect to Z as

$$\frac{dT}{dZ} = \frac{d}{dZ}\left[T_0\left(1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right)\right]$$

where T_0, K, g and R are constant

$$\therefore \quad \frac{dT}{dZ} = -\frac{k-1}{k} \times \frac{g}{RT_0} \times T_0 = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

The temperature lapse-rate is denoted by L and hence

$$L = \frac{dT}{dZ} = \frac{-g}{R}\left(\frac{k-1}{k}\right) \qquad \text{...(2.22)}$$

In equation (2.22), if (*i*) $k = 1$ which means isothermal process, $\frac{dT}{dZ} = 0$, which means temperature is constant with height.

(*ii*) If $k > 1$, the lapse-rate is negative which means temperature decreases with the increase of height.

In atmosphere, the value of k varies with height and hence the value of temperature lapse-rate also varies. From the sea-level upto an elevation of about 11000 m (or 11 km), the temperature of air decreases uniformly at the rate of 0.0065°C/m. from 11000 m to 32000 m, the temperature remains constant at – 56.5°C and hence in this range lapse-rate is zero. Temperature rises again after 32000 m in air.

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*i*) **Pressure by hydrostatic law.** For hydrostatic law, ρ is assumed constant and hence p is given by equation $\frac{dp}{dZ} = -\rho g$

Integrating, we get $\int_{p_0}^{p} dp = \int -\rho g\, dZ = -\rho g \int_{Z_0}^{Z} dZ$

or $p - p_0 = -\rho g\,[Z - Z_0]$

For datum line at sea-level, $Z_0 = 0$

$$\therefore \quad p - p_0 = -\rho g Z \quad \text{or} \quad p = p_0 - \rho g Z$$

$$= 10.143 \times 10^4 - 1.208 \times 9.81 \times 2500 \quad [\because \rho = \rho_0 = 1.208]$$

$$= 101430 - 29626 = 71804 \frac{\text{N}}{\text{m}^2} \text{ or } \frac{71804}{10^4} \text{ N/cm}^2$$

$$= \mathbf{7.18 \text{ N/cm}^2. \text{ Ans.}}$$

(*ii*) **Pressure by Isothermal Law.** Pressure at any height Z by isothermal law is given by equation (2.18) as

$$p = p_0\, e^{-gZ/RT}$$

$$= 10.143 \times 10^4\, e^{-\frac{gZ \times \rho_0}{p_0}} \qquad \left[\because \frac{p_0}{\rho_0} = RT \text{ and } \rho_0 g = w_0\right]$$

$$= 10.143 \times 10^4\, e^{-\frac{Z\rho_0 \times g}{p_0}}$$

$$= 10.143 \times 10^4\, e^{(-2500 \times 1.208 \times 9.81)/10.143 \times 10^4}$$

$$= 101430 \times e^{-.292} = 101430 \times \frac{1}{1.3391} = 75743 \text{ N/m}^2$$

$$= \frac{75743}{10^4} \text{ N/cm}^2 = \mathbf{7.574 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.23 *The barometric pressure at sea level is 760 mm of mercury while that on a mountain top is 735 mm, if the density of air is assumed constant at 1.2 kg/m³, what is the elevation of the mountain top.* (A.M.I.E., Summer, 1988)

Solution. Given :

Pressure* at sea, $p_0 = 760$ mm of Hg

$$= \frac{760}{1000} \times 13.6 \times 1000 \times 9.81 \text{ N/m}^2 = 101396 \text{ N/m}^2$$

---

* Here pressure head (Z) is given as 760 mm of Hg. Hence ($p/\rho g$) = 760 mm of Hg. The density (ρ) for mercury = 13.6 × 1000 kg/m³. Hence pressure (p) will be equal to $\rho \times g \times Z$ *i.e.*, $13.6 \times 1000 \times 9.81 \times \frac{760}{1000}$ N/m².

Pressure at mountain. $p = 735$ mm of Hg

$$= \frac{735}{1000} \times 13.6 \times 1000 \times 9.81 = 98060 \text{ N/m}^2$$

Density of air, $\rho = 1.2 \text{ kg/m}^3$

Let h = Height of the mountain from sea-level.

We know that as the elevation above the sea-level increases, the atmospheric pressure decreases. Here the density of air is given constant, hence the pressure at any height 'h' above the sea-level is given by the equation,

$$p = p_0 - \rho \times g \times h$$

or

$$h = \frac{p_0 - p}{\rho \times g} = \frac{101396 - 98060}{1.2 \times 9.81} = \mathbf{283.33 \text{ m. Ans.}}$$

Problem 2.24 *Calculate the pressure at a height of 7500 m above sea level if the atmospheric pressure is 10.143 N/cm² and temperature is 15°C at the sea-level, assuming (i) air is incompressible, (ii) pressure variation follows isothermal law, and (iii) pressure variation follows adiabatic law. Take the density of air at the sea-level as equal to 1.285 kg/m³. Neglect variation of g with altitude.*

Solution. Given :

Height above sea-level, $Z = 7500$ m

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Temperature at sea-level, $t_0 = 15°C$

$\therefore$ $T_0 = 273 + 15 = 288°K$

Density of air, $\rho = \rho_0 = 1.285 \text{ kg/m}^3$

(*i*) Pressure when air is incompressible :

$$\frac{dp}{dZ} = -\rho g$$

$$\therefore \int_{p_0}^{p} dp = -\int_{Z_0}^{Z} \rho g dz \quad \text{or} \quad p - p_0 = -\rho g[Z - Z_0]$$

or

$$p = p_0 - \rho g Z \qquad \{\because Z_0 = \text{datum line} = 0\}$$

$$= 10.143 \times 10^4 - 1.285 \times 9.81 \times 7500$$

$$= 101430 - 94543 = 6887 \text{ N/m}^2 = \mathbf{0.688 \frac{N}{cm^2}. \text{ Ans.}}$$

(*ii*) Pressure variation follows isothermal law :

Using equation (2.18), we have $p = p_0 e^{-gZ/RT}$

$$= p_0 \, e^{-gZ\rho_0/p_0} \qquad \left\{\because \frac{p_0}{\rho_0} = RT \therefore \frac{\rho_0}{p_0} = \frac{1}{RT}\right\}$$

$$= 101430 \, e^{-gZ\rho_0/p_0} = 101430 \, e^{-7500 \times 1.285 \times 9.81/101430}$$

$$= 101430 \, e^{-.9320} = 101430 \times .39376$$

$$= \mathbf{39939 \text{ N/m}^2 \text{ or } 3.993 \text{ N/cm}^2. \text{ Ans.}}$$

(*iii*) Pressure variation follows adiabatic law : [$k = 1.4$]

Using equation (2.19), we have $p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{k/(k-1)}$, where $\rho_0 = 1.285 \text{ kg/m}^3$

$$\therefore \quad p = 101430\left[1 - \frac{(1.4-1.0)}{1.4} \times 9.81 \times \frac{(7500 \times 1.285)}{101430}\right]^{\frac{1.4}{1.4-1.0}}$$

$$= 101430\,[1 - .2662]^{1.4/.4} = 101430 \times (.7337)^{3.5}$$

$$= \mathbf{34310\ N/m^2\ or\ 3.431\ \frac{N}{cm^2}.\ Ans.}$$

Problem 2.25 *Calculate the pressure and density of air at a height of 4000 m from sea-level where pressure and temperature of the air are 10.143 N/cm² and 15°C respectively. The temperature lapse rate is given as 0.0065°C/m. Take density of air at sea-level equal to 1.285 kg/m³.*

Solution. Given :

Height, $Z = 4000$ m

Pressure at sea-level, $p_0 = 10.143\ \text{N/cm}^2 = 10.143 \times 10^4 = 101430\ \frac{\text{N}}{\text{m}^2}$

Temperature at sea-level, $t_0 = 15°\text{C}$

$\therefore \quad T_0 = 273 + 15 = 288°\text{K}$

Temperature lapse-rate, $L = \frac{dT}{dZ} = -0.0065°\text{K/m}$

$\rho_0 = 1.285\ \text{kg/m}^3$

Using equation (2.22), we have $L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{k-1}{k}\right)$

or $$-0.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right), \text{ where } R = \frac{p_0}{\rho_0 T_0} = \frac{101430}{1.285 \times 288} = 274.09$$

$$\therefore \quad -0.0065 = \frac{-9.81}{274.09} \times \left(\frac{k-1}{k}\right)$$

$$\therefore \quad \frac{k-1}{k} = \frac{0.0065 \times 274.09}{9.81} = 0.1815$$

$$\therefore \quad k[1 - .1815] = 1$$

$$\therefore \quad k = \frac{1}{1 - .1815} = \frac{1}{.8184} = 1.222$$

This means that the value of power index $k = 1.222$.

(*i*) **Pressure** at 4000 m height is given by equation (2.19) as

$$p = p_0\left[1 - \frac{k-1}{k}\, gZ\, \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}}, \text{ where } k = 1.222 \text{ and } \rho_0 = 1.285$$

$$\therefore \quad p = 101430\left[1 - \left(\frac{1.222-1.0}{1.222}\right) \times 9.81 \times \frac{4000 \times 1.285}{101430}\right]^{\frac{1.222}{1.222-1.0}}$$

$$= 101430\,[1 - 0.09]^{5.50} = 101430 \times .595$$

$$= 60350\ \text{N/m}^2 = \mathbf{6.035\ \frac{N}{cm^2}.\ Ans.}$$

(*ii*) **Density.** Using equation of state, we get

$$\frac{p}{\rho} = RT$$

where p = Pressure at 4000 m height
ρ = Density at 4000 m height
T = Temperature at 4000 m height

Now T is calculated from temperature lapse-rate as

$$t \text{ at } 4000 \text{ m} = t_0 + \frac{dT}{dZ} \times 4000 = 15 - .0065 \times 4000 = 15 - 26 = -11°\text{C}$$

$$\therefore \quad T = 273 + t = 273 - 11 = 262°\text{K}$$

$\therefore$ Density is given by $\rho = \frac{p}{RT} = \frac{60350}{274.09 \times 262}$ kg/m^3 = **0.84 kg/m^3. Ans.**

Problem 2.26 *An aeroplane is flying at an altitude of 5000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065°K/m. Neglect variation of g with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C and density of air as 1.285 kg/cm^3.*

Solution. Given :

Height, $Z = 5000$ m

Lapse-rate, $L = \frac{dT}{dZ} = -.0065°\text{K/m}$

Pressure at ground level, $p_0 = 10.143 \times 10^4$ N/m^2

$t_0 = 15°\text{C}$

$\therefore$ $T_0 = 273 + 15 = 288°\text{K}$

Density, $\rho_0 = 1.285$ kg/m^3

$\therefore$ Temperature at 5000 m height $= T_0 + \frac{dT}{dZ} \times \text{Height} = 288 - .0065 \times 5000$

$= 288 - 32.5 = 255.5°\text{K}$

First find the value of power index k as

From equation (2.22), we have $L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{k-1}{k}\right)$

or $$-.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right)$$

where $R = \frac{p_0}{\rho_0 T_0} = \frac{101430}{1.285 \times 288} = 274.09$

$$\therefore \quad -.0065 = -\frac{9.81}{274.09}\left(\frac{k-1}{k}\right)$$

$$\therefore \quad k = 1.222$$

The pressure is given by equation (2.19) as

$$p = p_0\left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\left(\frac{k}{k-1}\right)}$$

$$= 101430\left[1-\left(\frac{1.222-1.0}{1.222}\right)\times 9.81\times\frac{5000\times 1.285}{101430}\right]^{\frac{1.222}{1.222-1.0}}$$

$$= 101430\left[1-\frac{.222}{1.222}\times 9.81\times\frac{5000\times 1.285}{101430}\right]^{\frac{1.222}{.222}}$$

$$= 101430\,[1-0.11288]^{5.50} = 101430\times 0.5175 = 52490 \text{ N/m}^3$$

$= 5.249$ **N/cm². Ans.**

HIGHLIGHTS

1. The pressure at any point in a fluid is defined as the force per unit area.
2. The Pascal's law states that intensity of pressure for a fluid at rest is equal in all directions.
3. Pressure variation at a point in a fluid at rest is given by the hydrostatic law which states that the rate of increase of pressure in the vertically downward direction is equal the specific weight of the fluid, $\frac{dp}{dZ} = w = \rho \times g$.
4. The pressure at any point in a incompressible fluid (liquid) is equal to the product of density of fluid at that point, acceleration due to gravity and vertical height from free surface of fluid, $p = \rho \times g \times Z$.
5. Absolute pressure is the pressure in which absolute vacuum pressure is taken as datum while gauge pressure is the pressure in which the atmospheric pressure is taken as datum,
$$p_{abs.} = p_{atm} + p_{gauge}.$$
6. Manometer is a device used for measuring pressure at a point in a fluid.
7. Manometers are classified as (*a*) Simple manometers and (*b*) Differential manometers.
8. Simple manometers are used for measuring pressure at a point while differential manometers are used for measuring the difference of pressures between the two points in a pipe, or two different pipes.
9. A single column manometer (or micrometer) is used for measuring small pressures, where accuracy is required.
10. The pressure at a point in static compressible fluid is obtained by combining two equations, *i.e.*, equation of state for a gas and equation given by hydrostatic law.
11. The pressure at a height Z in a static compressible fluid (gas) under going isothermal compression $\left(\frac{p}{\rho} = \text{const.}\right)$
$$p = p_0\, e^{-gZ/RT}$$
where p_0 = Absolute pressure at sea-level or at ground level
Z = Height from sea or ground level
R = Gas constant
T = Absolute temperature.
12. The pressure and temperature at a height Z in a static compressible fluid (gas) undergoing adiabatic compression (p/ρ^k = const.)
$$p = p_0\left[1-\frac{k-1}{k}\,gZ\,\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} = p_0\left[1-\frac{k-1}{k}\,\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

and temperature, $$T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0, T_0 are pressure and temperature at sea-level $k = 1.4$ for air.

13. The rate at which the temperature changes with elevation is known as Temperature Lapse-Rate. It is given by

$$L = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

if (*i*) $k = 1$, temperature is zero.

(*ii*) $k > 1$, temperature decreases with the increase of height.

EXERCISE 2

(A) THEORETICAL PROBLEMS

1. Define pressure. Obtain an expression for the pressure intensity at a point in a fluid.
2. State and prove the Pascal's law.
3. What do you understand by Hydrostatic Law ?
4. Differentiate between : (*i*) Absolute and gauge pressure, (*ii*) Simple manometer and differential manometer, and (*iii*) Piezometer and pressure gauges.
5. What do you mean by vacuum pressure ?
6. What is a manometer ? How are they classified ?
7. What do you mean by single column manometers ? How are they used for the measurement of pressure ?
8. What is the difference between U-tube differential manometers and inverted U-tube differential manometers ? Where are they used ?
9. Distinguish between manometers and mechanical gauges. What are the different types of mechanical pressure gauges ?
10. Derive an expression for the pressure at a height Z from sea-level for a static air when the compression of the air is assumed isothermal. The pressure and temperature at sea-levels are p_0 and T_0 respectively.
11. Prove that the pressure and temperature for an adiabatic process at a height Z from sea-level for a static air are :

$$p_0 = p_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \text{ and } T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0 and T_0 are the pressure and temperature at sea-level.

12. What do you understand by the term, 'Temperature Lapse-Rate' ? Obtain an expression for the temperature Lapse-Rate.
13. What is hydrostatic pressure distribution ? Give one example where pressure distribution is non-hydrostatic. *(A.M.I.E., Winter 1990)*
14. Explain briefly the working principle of Bourdon Pressure Gauge with a neat sketch. *(J.N.T.U., Hyderabad, S 2002)*

(B) NUMERICAL PROBLEMS

1. A hydraulic press has a ram of 30 cm diameter and a plunger of 5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N. **[Ans. 14.4 kN]**
2. A hydraulic press has a ram of 20 cm diameter and a plunger of 4 cm diameter. It is used for lifting a weight of 20 kN. Find the force required at the plunger. **[Ans. 800 N]**

3. Calculate the pressure due to a column of 0.4 m of (*a*) water, (*b*) an oil of sp. gr. 0.9, and (*c*) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000 \frac{kg}{m^3}$. **[Ans.** (*a*) 0.3924 N/cm^2, (*b*) 0.353 N/cm^2, (*c*) 5.33 N/cm^2]

4. The pressure intensity at a point in a fluid is given 4.9 N/cm^2. Find the corresponding height of fluid when it is : (*a*) water, and (*b*) an oil of sp. gr. 0.8. **[Ans.** (*a*) 5 m of water, (*b*) 6.25 m of oil]

5. An oil of sp. gr. 0.8 is contained in a vessel. At a point the height of oil is 20 m. Find the corresponding height of water at that point. **[Ans.** 16 m]

6. An open tank contains water upto a depth of 1.5 m and above it an oil of sp. gr. 0.8 for a depth of 2 m. Find the pressure intensity : (*i*) at the interface of the two liquids, and (*ii*) at the bottom of the tank. **[Ans.** (*i*) 1.57 N/cm^2, (*ii*) 3.04 N/cm^2]

7. The diameters of a small piston and a large piston of a hydraulic jack are 2 cm and 10 cm respectively. A force of 60 N is applied on the small piston. Find the load lifted by the large piston, when : (*a*) the pistons are at the same level, and (*b*) small piston is 20 cm above the large piston. The density of the liquid in the jack is given as $1000 \frac{kg}{m^2}$. **[Ans.** (*a*) 1500 N, (*b*) 1520.5 N]

8. Determine the gauge and absolute pressure at a point which is 2.0 m below the free surface of water. Take atmospheric pressure as 10.1043 N/cm^2. **[Ans.** 1.962 N/cm^2 (gauge), 12.066 N/cm^2 (abs.)]

9. A simple manometer is used to measure the pressure of oil (sp. gr. = 0.8) flowing in a pipe line. Its right limb is open to the atmosphere and left limb is connected to the pipe. The centre of the pipe is 9 cm below the level of mercury (sp. gr. 13.6) in the right limb. If the difference of mercury level in the two limbs is 15 cm, determine the absolute pressure of the oil in the pipe in N/cm^2. *(A.M.I.E., Winter, 1977)* **[Ans.** 12.058 N/cm^2]

10. A simple manometer (U-tube) containing mercury is connected to a pipe in which an oil of sp. gr. 0.8 is flowing. The pressure in the pipe is vacuum. The other end of the manometer is open to the atmosphere. Find the vacuum, pressure in pipe, if the difference of mercury level in the two limbs is 20 cm and height of oil in the left limb from the centre of the pipe is 15 cm below. **[Ans.** – 27.86 N/cm^2]

11. A single column vertical manometer (*i.e.,* micrometer) is connected to a pipe containing oil of sp. gr. 0.9. The area of the reservoir is 80 times the area of the manometer tube. The reservoir contains mercury of sp. gr. 13.6. The level of mercury in the reservoir is at a height of 30 cm below the centre of the pipe and difference of mercury levels in the reservoir and right limb is 50 cm. Find the pressure in the pipe. **[Ans.** 6.474 N/cm^2]

12. A pipe contains an oil of sp. gr. 0.8. A differential manometer connected at the two points A and B of the pipe shows a difference in mercury level as 20 cm. Find the difference of pressure at the two points. **[Ans.** 25113.6 N/m^2]

13. A U-tube differential manometer connects two pressure pipes *A* and *B*. Pipe *A* contains carbon tetrachloride having a specific gravity 1.594 under a pressure of 11.772 N/cm^2 and pipe *B* contains oil of sp. gr. 0.8 under a pressure of 11.772 N/cm^2. The pipe *A* lies 2.5 m above pipe *B*. Find the difference of pressure measured by mercury as fluid filling U-tube. *(A.M.I.E. December, 1974)* **[Ans.** 31.36 cm of mercury]

14. A differential manometer is connected at the two points *A* and *B* as shown in Fig. 2.25. At *B* air pressure is 7.848 N/cm^2 (abs.), find the absolute pressure at *A*. **[Ans.** 6.91 N/cm^2]

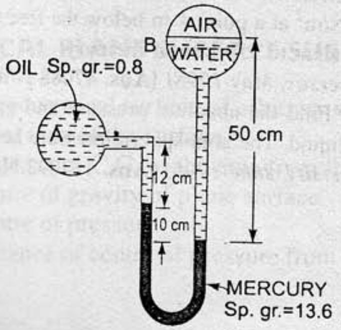


Fig. 2.25

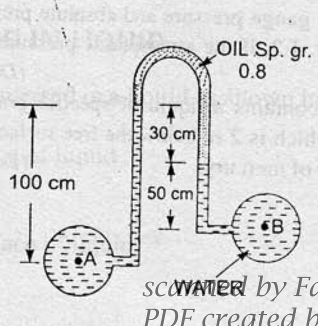


Fig. 2.26

15. An inverted differential manometer containing an oil of sp. gr. 0.9 is connected to find the difference of pressures at two points of a pipe containing water. If the manometer reading is 40 cm, find the difference of pressures. [Ans. 392.4 N/m^2]

16. In above Fig. 2.26 shows an inverted differential manometer connected to two pipes A and B containing water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the difference of pressure head between A and B. [Ans. 0.26 m of water]

17. If the atmospheric pressure at sea-level is 10.143 N/cm^2, determine the pressure at a height of 2000 m assuming that the pressure variation follows : (*i*) Hydrostatic law, and (*ii*) Isothermal law. The density of air is given as 1.208 kg/m^3. [Ans. (*i*) 7.77 N/cm^2, (*ii*) 8.03 N/cm^3]

18. Calculate the pressure at a height of 8000 m above sea-level if the atmospheric pressure is 101.3 kN/m^2 and temperature is 15°C at the sea-level assuming (*i*) air is incompressible, (*ii*) pressure variation follows adiabatic law, and (*iii*) pressure variation follows isothermal law. Take the density of air at the sea-level as equal to 1.285 kg/m^3. Neglect variation of g with altitude.
[Ans. (*i*) 607.5 N/m^2, (*ii*) 31.5 kN/m^2 (*iii*) 37.45 kN/m^2]

19. Calculate the pressure and density of air at a height of 3000 m above sea-level where pressure and temperature of the air are 10.143 N/cm^2 and 15°C respectively. The temperature lapse-rate is given as 0.0065° K/m. Take density of air at sea-level equal to 1.285 kg/m^3. [Ans. 6.896 N/cm^2, 0.937 kg/m^3]

20. An aeroplane is flying at an altitude of 4000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065° K/m. Neglect variation of g with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C respectively. The density of air at ground level is given as 1.285 kg/m^3. [Ans. 6.038 N/cm^2]

21. The atmosphere pressure at the sea-level is 101.3 kN/m^2 and the temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming (*i*) air is in compressible, (*ii*) isothermal variation of pressure and density, and (*iii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of 'g' with altitude.
[Ans. (*i*) 501.3 N/m^2, (*ii*) 37.45 kN/m^2, (*iii*) 31.5 kN/m^2]

22. An oil of sp. gr. is 0.8 under a pressure of 137.2 kN/m^2
(*i*) What is the pressure head expressed in metre of water ?
(*ii*) What is the pressure head expressed in metre of oil ? [Ans. (*i*) 14 m, (*ii*) 17.5 m]

23. The atmospheric pressure at the sea-level is 101.3 kN/m^2 and temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming : (*i*) isothermal variation of pressure and density, and (*ii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of 'g' with altitude.
Derive the formula that you may use. *(Delhi University, 1992)* [Ans. (*i*) 37.45 kN/m^2 (*ii*) 31.5 kN/m^2]

24. What are the gauge pressure and absolute pressure at a point 4 m below the free surface of a liquid of specific gravity 1.53, if atmospheric pressure is equivalent to 750 mm of mercury.
(Delhi University, 1992) [Ans. 60037 N/m^2 and 160099 N/m^2]

25. Find the gauge pressure and absolute pressure in N/m^2 at a point 4 m below the free surface of a liquid of sp. gr. 1.2, if the atmospheric pressure is equivalent to 750 mm of mercury.
(Delhi University, May 1998) [Ans. 47088 N/m^2 ; 147150 N/m^2]

26. A tank contains a liquid of specific gravity 0.8. Find the absolute pressure and gauge pressure at a point, which is 2 m below the free surface of the liquid. The atmospheric pressure head is equivalent to 760 mm of mercury. *(Delhi University, June 1996)* [Ans. 117092 N/m^2 ; 15696 N/m^2]

3
CHAPTER

Hydrostatic Forces on Surfaces

▶ 3.1 INTRODUCTION

This chapter deals with the fluids (*i.e.*, liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

▶ 3.2 TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▶ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

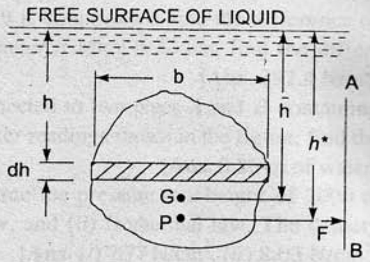


Fig. 3.1

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho gh$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho gh \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho gh \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h} \quad \text{...(3.1)}$$

For water the value of $\rho = 1000 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h*).** Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\{\because \; dF = \rho gh \times b \times dh\}$

$= \rho gh \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho gh \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int bh^2 \, dh = \rho g \int h^2 \, dA \quad (\because \; bdh = dA)$$

But $\int h^2 dA = \int bh^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \quad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \quad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_G in equation (3.4), we get

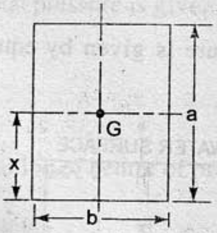
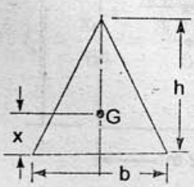
$$h^* = \frac{I_G + A\overline{h^2}}{Ah} = \frac{I_G}{A\bar{h}} + \bar{h} \quad \text{...(3.5)}$$

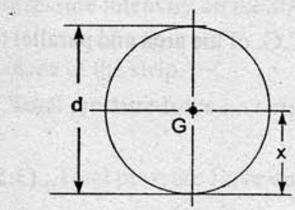
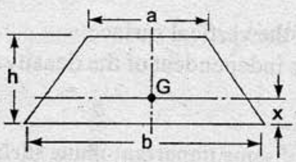
In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2

$$A = 3 \times 2 = 6 \text{ m}^2, \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

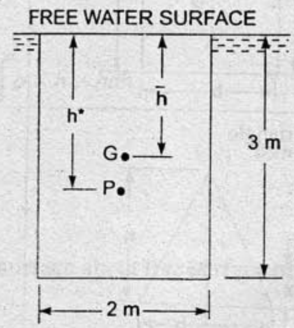


Fig. 3.2

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.\ Ans.}$$

(*b*) Upper edge is 2.5 m below water surface (Fig. 3.3). Total pressure (*F*) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

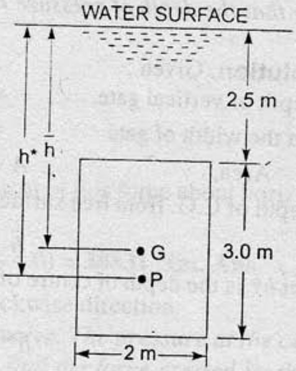


Fig. 3.3

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$
$$= \mathbf{235440\ N.\ Ans.}$$

Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5, A = 6.0, \bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.\ Ans.}$$

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Solution. Given : Dia. of plate, $d = 1.5$ m

$$\therefore \text{ Area,} \quad A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

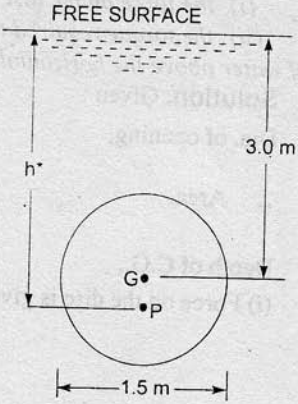


Fig. 3.4

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81\ N.\ Ans.}$$

Position of centre of pressure (h^*) is given by equation (3.5)

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$$\therefore \quad h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$
$$= \mathbf{3.0468\ m.\ Ans.}$$

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Problem 3.3 *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p+\dfrac{d^2}{12p}\right)$.

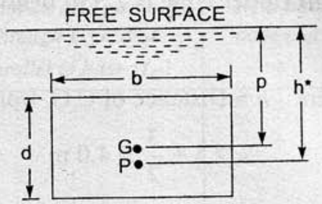


Fig. 3.5

Solution. Given :

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d \text{ m}^2$

Depth of C.G. from free surface

$\bar{h} = p$ m.

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12}$$

$$\therefore \quad h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \frac{d^2}{12p} + p \quad \text{or} \quad \mathbf{p + \frac{d^2}{12}}. \textbf{ Ans.}$$

Problem 3.4 *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

(i) the force on the disc, and

(ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m. (A.M.I.E., Winter, 1977)

Solution. Given :

Dia. of opening, $d = 3$ m

$\therefore$ Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2$.

Depth of C.G., $\bar{h} = 4$ m

(*i*) Force on the disc is given by equation (3.1) as

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$$

$$= \mathbf{277368 \text{ N} = 277.368 \text{ kN. Ans.}}$$

(*ii*) To find the torque required to maintain the disc in equilibrium, first calculate the point of application of force acting on the disc, *i.e.*, centre of pressure of the force F. The depth of centre of pressure (h^*) is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0 \qquad \left\{\because I_G = \frac{\pi}{64} d^4\right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 \text{ m}$$

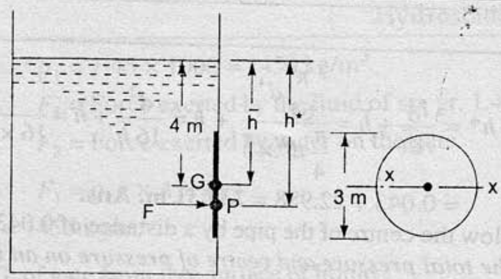


Fig. 3.6

The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

$$= F \times \left(h^* - \bar{h}\right) = 277368\ (4.14 - 4.0) = \mathbf{38831\ Nm.\ Ans.}$$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.* (Converted to SI Units, A.M.I.E., Winter, 1975)

Solution. Given :

Dia. of pipe, $d = 4$ m

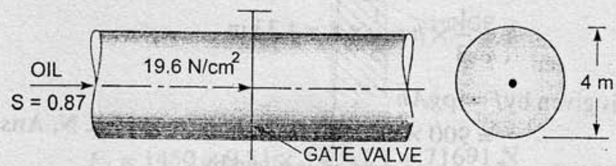


Fig. 3.7

$\therefore$ Area, $A = \dfrac{\pi}{4} \times 4^2 = 4\pi\ \text{m}^2$

Sp. gr. of oil, $S = 0.87$

$\therefore$ Density of oil, $\rho_0 = 0.87 \times 1000 = 870\ \text{kg/m}^3$

$\therefore$ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81\ \text{N/m}^3$

Pressure at the centre of pipe, $p = 19.6\ \text{N/cm}^2 = 19.6 \times 10^4\ \text{N/m}^2$

$\therefore$ Pressure head at the centre $= \dfrac{p}{w_0} = \dfrac{19.6 \times 10^4}{870 \times 9.81} = 22.988$ m

$\therefore$ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500\ N = 2.465\ MN.\ Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\,\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m.\ Ans.}$$

Or centre of pressure is below the centre of the pipe by a distance of 0.043 m. **Ans.**

Problem 3.6 *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

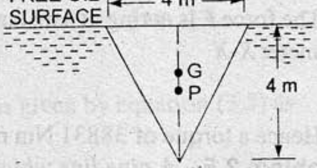


Fig. 3.8

Solution. Given :

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

$\therefore$ Area, $A = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8.0\ m^2$

Sp. gr. of oil. $S = 0.9$

$\therefore$ Density of oil, $\rho = 900\ kg/m^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33\ m.$$

Total pressure (F) is given by $F = \rho g A \bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33\ N = \mathbf{9597.6.\ N.\ Ans.}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11\ m^4$$

$$\therefore \quad h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99\ m.\ Ans.}$$

Problem 3.7 *A vertical sluice gate is used tc cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.* (A.M.I.E., May, 1975)

Solution. Given :

Width of gate, $b = 2$ m

Depth of gate, $d = 1.2$ m

$\therefore$ Area, $A = b \times d = 2 \times 1.2 = 2.4\ m^2$

Sp. gr. of liquid $= 1.45$

∴ Density of liquid, $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^3

Let F_1 = Force exerted by the fluid of sp. gr. 1.45 on gate

F_2 = Force exerted by water on the gate.

The force F_1 is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$

where $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^2

$\bar{h}_1$ = Depth of C.G. of gate from free surface of liquid

$$= 1.5 + \frac{1.2}{2} = 2.1 \text{ m.}$$

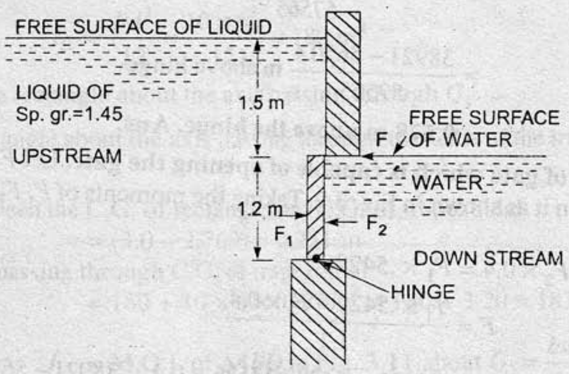


Fig. 3.9

∴ $F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691$ N

Similarly, $F_2 = \rho_2 g.A\bar{h}_2$

where $\rho_2 = 1{,}000$ kg/m^3

$\bar{h}_2$ = Depth of C.G. of gate from free surface of water

$$= \frac{1}{2} \times 1.2 = 0.6 \text{ m}$$

∴ $F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126$ N

(*i*) **Resultant force on the gate** = $F_1 - F_2 = 71691 - 14126 =$ **57565 N. Ans.**

(*ii*) **Position of centre of pressure of resultant force.** The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h_1^* = \frac{I_G}{A\bar{h}_1} + \bar{h}_1$$

where $I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288$ m^4

∴ $$h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

∴ Distance of F_1 from hinge

$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429$ m

The force F_2 will be acting at a depth of h_2^* from free surface of water and is given by

$$h_2^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

where $I_G = 0.288 \text{ m}^4$, $\bar{h}_2 = 0.6$ m, $A = 2.4 \text{ m}^2$.

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of F_2 from hinge $= 1.2 - 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

$$= \textbf{0.578 m above the hinge. Ans.}$$

(*iii*) **Force at the top of gate which is capable of opening the gate.** Let F is the force required on the top of the gate to open it as shown in Fig. 3.9. Taking the moments of F, F_1 and F_2 about the hinge, we get

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$$

or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

$$= \textbf{27725.5 N. Ans.}$$

Problem 3.8 *A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 m wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.*

Solution. Given :

Width at top $= 16$ m

Width at bottom $= 10$ m

Depth, $d = 6$ m

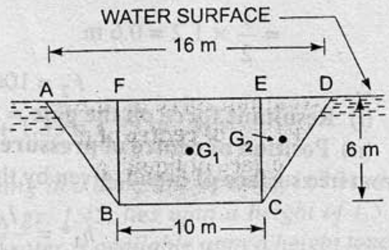


Fig. 3.10

Area of trapezoidal $ABCD$,

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10 + 16)}{2} \times 6 = 78 \text{ m}^2$$

Depth of C.G. of trapezoidal area $ABCD$ from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16-10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \text{ m from water surface.}$$

(*i*) **Total Pressure (F).** Total pressure, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 78 \times 2.769 \text{ N}$$
$$= 2118783 \text{ N} = \mathbf{2.118783 \text{ MN. Ans.}}$$

(*ii*) **Centre of Pressure (h*).** Centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of trapezoidal *ABCD* about its C.G.

Let I_{G_1} = M.O.I. of rectangle *FBCE* about its C.G.

I_{G_2} = M.O.I. of two Δs *ABF* and *ECD* about its C.G.

Then $$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ m}^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1.

∴ M.O.I. of the rectangle about the axis passing through the C.G. of the trapezoidal I_{G_1} + Area of rectangle × x_1^2

where x_1 is distance between the C.G. of rectangle and C.G. of trapezoidal

$= (3.0 - 2.769) = 0.231$ m

∴ M.O.I. of *FBCE* passing through C.G. of trapezoidal

$= 180 + 10 \times 6 \times (0.231)^2 = 180 + 3.20 = 183.20 \text{ m}^4$

Now $$I_{G_2} = \text{M.O.I. of } \Delta ABD \text{ in Fig. 3.11 about } G_2 = \frac{bd^3}{36}$$

$$= \frac{(16-10) \times 6^3}{36} = 36 \text{ m}^4$$

The distance between the C.G. of triangle and C.G. of trapezoidal

$= (2.769 - 2.0) = 0.769$

∴ M.O.I. of the two Δs about an axis passing through C.G. of trapezoidal

$= I_{G_2}$ + Area of triangles × $(.769)^2$

$= 36.0 + \frac{6 \times 6}{2} \times (.769)^2$

$= 36.0 + 10.64 = 46.64$

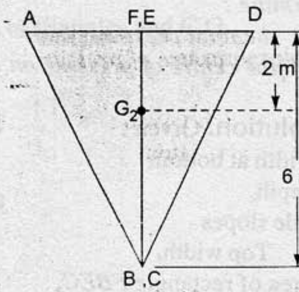


Fig. 3.11

∴ I_G = M.O.I. of trapezoidal about its C.G.

= M.O.I. of rectangle about the C.G. of trapezoidal

+ M.O.I. of triangles about the C.G. of the trapezoidal

$= 183.20 + 46.64 = 229.84 \text{ m}^4$

∴ $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $A = 78$, $\bar{h} = 2.769$

$$h^* = \frac{229.84}{78 \times 2.769} + 2.769 = 1.064 + 2.769 = \mathbf{3.833 \text{ m. Ans.}}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface *AD* is given by (refer to Table 3.1 on page 69)

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$$

$$= \frac{(2 \times 10 + 16)}{(10+16)} \times \frac{6}{3} \qquad (\because \quad a = 10, b = 16 \text{ and } h = 6)$$

$$= \frac{36}{26} \times 2 = 2.769 \text{ m}$$

This is also equal to the distance of the C.G. of the trapezoidal from free surface of water.

$$\bar{h} = 2.769 \text{ m}$$

∴ Total pressure, $F = \rho g A \bar{h}$ $\qquad (\because \quad A = 78)$

$$= 1000 \times 9.81 \times 78 \times 2.769 \text{ N} = \mathbf{2118783 \text{ N. Ans.}}$$

Centre of Pressure

$$(h^*) = \frac{I_G}{A\bar{h}} + \bar{h}$$

Now I_G from Table 3.1 is given by,

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(10^2 + 4 \times 10 \times 16 + 16^2)}{36(10+16)} \times 6^3$$

$$= \frac{(100 + 640 + 256)}{36 \times 26} \times 216 = 229.846 \text{ m}^4$$

∴ $h^* = \dfrac{229.846}{78 \times 2.769} + 2.769$ $\qquad (\because \quad A = 78 \text{ m}^2)$

$$= \mathbf{3.833 \text{ m. Ans.}}$$

Problem 3.9 *A trapezoidal channel 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine :*

(i) the total pressure, and

(ii) the centre of pressure on the vertical gate closing the channel when it is full of water.

(A.M.I.E., Summer, 1978)

Solution. Given :

Width at bottom $= 2$ m

Depth, $d = 1$ m

Side slopes $= 1 : 1$

∴ Top width, $AD = 2 + 1 + 1 = 4$ m

Area of rectangle $FBEC$, $A_1 = 2 \times 1 = 2 \text{ m}^2$

Area of two triangles ABF and ECD, $A_2 = \dfrac{(4-2)}{2} \times 1 = 1 \text{ m}^2$

∴ Area of trapezoidal $ABCD$, $A = A_1 + A_2 = 2 + 1 = 3 \text{ m}^2$

Depth of C.G. of rectangle $FBEC$ from water surface,

$$\bar{h}_1 = \frac{1}{2} = 0.5 \text{ m}$$

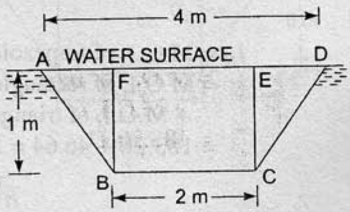


Fig. 3.12

Depth of C.G. of two triangles *ABF* and *ECD* from water surface,

$$\bar{h}_2 = \frac{1}{3} \times 1 = \frac{1}{3} \text{ m}$$

$\therefore$ Depth of C.G. of trapezoidal *ABCD* from free surface of water

$$\bar{h} = \frac{A_1 \times \bar{h}_1 + A_2 \times \bar{h}_2}{(A_1 + A_2)} = \frac{2 \times 0.5 + 1 \times 0.33333}{(2+1)} = .44444$$

(*i*) **Total Pressure (F).** Total pressure F is given by

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 3.0 \times 0.44444 = \mathbf{13079.9\ N.\ Ans.}$$

(*ii*) **Centre of Pressure (h*).** M.O.I. of rectangle *FBCE* about its C.G.,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = \frac{1}{6} \text{ m}^4$$

M.O.I. of *FBCE* about an axis passing through the C.G. of trapezoidal

or $$I_{G_1}^* = I_{G_1} + A_1 \times [\text{Distance between C.G. of rectangle and C.G. of trapezoidal}]^2$$

$$= \frac{1}{6} + 2 \times \left[\bar{h}_1 - \bar{h}\right]^2$$

$$= \frac{1}{6} + 2 \times [0.5 - .4444]^2 = .1666 + .006182 = 0.1727$$

M.O.I. of the two triangles *ABF* and *ECD* about their C.G.,

$$I_{G_2} = \frac{bd^3}{36} = \frac{(1+1) \times 1^3}{36} = \frac{2}{36} = \frac{1}{18} \text{ m}^4.$$

M.O.I. of the two triangles about the C.G. of trapezoidal,

$$I_{G_2}^* = I_{G_1} + A_2 \times [\text{Distance between C.G. of triangles and C.G. of trapezoidal}]^2$$

$$= \frac{1}{18} + 1 \times \left[\bar{h} - \bar{h}_2\right]^2 = \frac{1}{18} + 1 \times \left[.4444 - \frac{1}{3}\right]^2$$

$$= \frac{1}{18} + (.1111)^2 = 0.0555 + (.1111)^2$$

$$= .0555 + 0.01234 = 0.06789 \text{ m}^4$$

$\therefore$ M.O.I. of the trapezoidal about its C.G.

$$I_G = I_{G_1}^* + I_{G_2}^* = .1727 + .06789 = 0.24059 \text{ m}^4$$

Then centre of pressure (h^*) on the vertical trapezoidal,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.24059}{3 \times .4444} + .4444 = 0.18046 + .4444 = 0.6248$$

$$\simeq \mathbf{0.625\ m.\ Ans.}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface AD is given by (refer to Table 3.1 on page 69).

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3} = \frac{(2\times 2+4)}{(2+4)} \times \frac{1}{3} \qquad (\because \quad a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$\therefore$ $\bar{h} = x = 0.444$ m

$\therefore$ Total pressure, $F = \rho g A \bar{h} = 1000 \times 9.81 \times 3.0 \times .444$ $(\because \quad A = 3.0)$

$= \mathbf{13079\ N.\ Ans.}$

Centre of pressure, $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where I_G from Table 3.1 is given by

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(2^2 + 4\times 2\times 4 + 4^2)}{36(2+4)} \times 1^3 = \frac{52}{36\times 6} = 0.2407 \text{ m}^4$$

$\therefore$ $h^* = \dfrac{0.2407}{3.0 \times .444} + .444 = \mathbf{0.625\ m.\ Ans.}$

Problem 3.10 *A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure.* (A.M.I.E., Summer, 1986)

Solution. Given : Diagonals of aperture, $AC = BD = 2$ m

$\therefore$ Area of square aperture, A = Area of ΔACB + Area of ΔACD

$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2} = \frac{2\times 1}{2} + \frac{2\times 1}{2} = 1 + 1 = 2.0 \text{ m}^2$$

Sp. gr. of liquid $= 1.15$

$\therefore$ Density of liquid, $\rho = 1.15 \times 1000 = 1150$ kg/m^3

Depth of centre of aperture from free surface,

$\bar{h} = 1.5$ m.

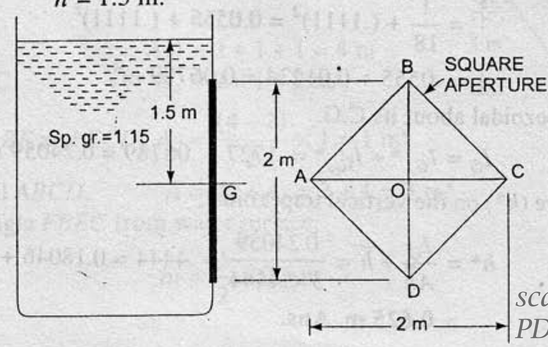


Fig. 3.13

(*i*) The thrust on the plate is given by

$$F = \rho g A \bar{h} = 1150 \times 9.81 \times 2 \times 1.5 = \mathbf{33844.5. \text{ Ans.}}$$

(*ii*) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of *ABCD* about diagonal *AC*

= M.O.I. of triangle *ABO* about *AC* + M.O.I. of triangle *ACD* about *AC*

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12} \qquad \left(\because \text{ M.O.I. of a triangle about its base} = \frac{bh^3}{12}\right)$$

$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \text{ m}^4$$

$$\therefore \qquad h^* = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = \frac{1}{3 \times 2 \times 1.5} + 1.5 = \mathbf{1.611 \text{ m. Ans.}}$$

Problem 3.11 *A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1 m height. Calculate :*

(i) total pressure on one side of the tank,

(ii) the position of centre of pressure for one side of the tank, which is 2 m wide.

Solution. Given :

Depth of water = 0.5 m

Depth of liquid = 1 m

Sp. gr. of liquid = 0.8

Density of liquid, $\rho_1 = 0.8 \times 1000 = 800 \text{ kg/m}^3$

Density of water, $\rho_2 = 1000 \text{ kg/m}^3$

Width of tank = 2 m

(*i*) **Total pressure on one side** is calculated by drawing pressure diagram, which is shown in Fig. 3.14.

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on *D* (or *DE*), $p_D = \rho_1 g . h_1$

$= 800 \times 9.81 \times 1.0 = 7848 \text{ N/m}^2$

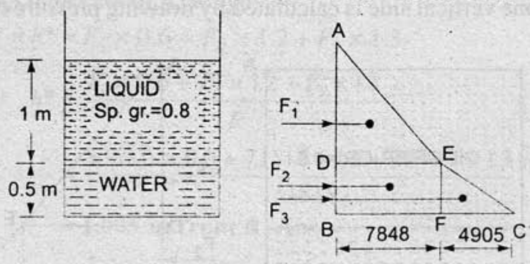


Fig. 3.14

Intensity of pressure on base (or *BC*). $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753 \text{ N/m}^2$

Now force F_1 = Area of ΔADE × Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848 \text{ N}$$

Force F_2 = Area of rectangle $DBFE$ × Width of tank

$= 0.5 \times 7848 \times 2 = 7848$ N

F_3 = Area of ΔEFC × Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

∴ Total pressure, $F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 =$ **18148.5 N. Ans.**

(*ii*) **Centre of Pressure (h*).** Taking the moments of all force about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 (AD + \frac{1}{2} BD) + F_3[AD + \frac{2}{3} BD]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5\right)$$

$$= 5232 + 9810 + 3270 = 18312$$

$$\therefore \quad h^* = \frac{18312}{18148.5} = \textbf{1.009 m from top. Ans.}$$

Problem 3.12 *A cubical tank has sides of 1.5 m. It contains water for the lower 0.6 m depth. The upper remaining part is filled with oil of specific gravity 0.9. Calculate for one vertical side of the tank :*

(*a*) *total pressure, and*

(*b*) *position of centre of pressure.* (A.M.I.E., Winter, 1987)

Solution. Given :

Cubical tank of sides 1.5 m means the dimensions of the tank are 1.5 m × 1.5 m × 1.5 m.

Depth of water = 0.6 m

Depth of liquid = 1.5 – 0.6 = 0.9 m

Sp. gr. of liquid = 0.9

Density of liquid, $\rho_1 = 0.9 \times 1000 = 900$ kg/m^3

Density of water, $\rho_2 = 1000$ kg/m^3

(*a*) **Total pressure** on one vertical side is calculated by drawing pressure diagram, which is shown in Fig. 3.15.

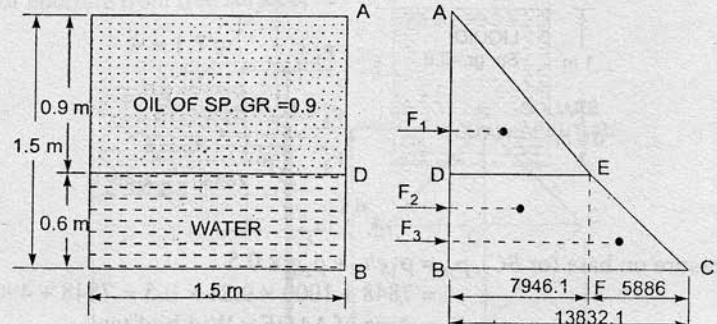


Fig. 3.15

Intensity of pressure at A, $p_A = 0$

Intensity of pressure at D, $p_D = \rho_1 g \times h = 900 \times 9.81 \times 0.9 = 7946.1 \text{ N/m}^2$

Intensity of pressure at B, $p_B = \rho_1 g h_1 + \rho_2 g h_2 = 900 \times 9.81 \times 0.9 + 1000 \times 9.81 \times 0.6$

$= 7946.1 + 5886 = 13832.1 \text{ N/m}^2$

Hence in pressure diagram :

$$DE = 7946.1 \text{ N/m}^2, BC = 13832.1 \text{ N/m}^2, FC = 5886 \text{ N/m}^2$$

The pressure diagram is split into triangle ADE, rectangle $BDEF$ and triangle EFC. The total pressure force consists of the following components :

(*i*) Force F_1 = Area of triangle $ADE \times$ Width of tank

$= \left(\frac{1}{2} \times AD \times DE\right) \times 1.5$ ($\because$ Width = 1.5 m)

$= \left(\frac{1}{2} \times 0.9 \times 7946.1\right) \times 1.5 \text{ N} = 5363.6 \text{ N}$

This force will be acting at the C.G. of the triangle ADE, *i.e.*, at a distance of $\frac{2}{3} \times 0.9 = 0.6$ m below A

(*ii*) Force F_2 = Area of rectangle $BDEF \times$ Width of tank

$= (BD \times DE) \times 1.5 = (0.6 \times 7946.1) \times 1.5 = 7151.5$

This force will be acting at the C.G. of the rectangle $BDEF$ *i.e.*, at a distance of $0.9 + \frac{0.6}{2} = 1.2$ m below A.

(*iii*) Force F_3 = Area of triangle $EFC \times$ Width of tank

$= \left(\frac{1}{2} \times EF \times FC\right) \times 1.5 = \left(\frac{1}{2} \times 0.6 \times 5886\right) \times 1.5 = 2648.7 \text{ N}$

This force will be acting at the C.G. of the triangle EFC, *i.e.*, at a distance of $0.9 + \frac{2}{3} \times 0.6 = 1.30$ m below A.

$\therefore$ Total pressure force on one vertical face of the tank,

$$F = F_1 + F_2 + F_3$$

$= 5363.6 + 7151.5 + 2648.7 =$ **15163.8 N. Ans.**

(*b*) Position of centre of pressure

Let the total force F is acting at a depth of h^* from the free surface of liquid, *i.e.*, from A. Taking the moments of all forces about A, we get

$$F \times h^* = F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3$$

or
$$h^* = \frac{F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3}{F}$$

$$= \frac{5363.6 \times 0.6 + 7151.5 \times 1.2 + 2648.7 \times 1.3}{15163.8}$$

= **1.005 m from A. Ans.**

▶ 3.4 HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, where h is depth of surface.

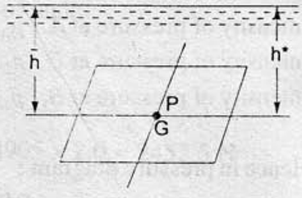


Fig. 3.16

Let A = Total area of surface

Then total force, F, on the surface

$$= p \times \text{Area} = \rho g \times h \times A = \rho g A \bar{h}$$

where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.

Problem 3.13 *Fig. 3.17 shows a tank full of water. Find :*

(i) Total pressure on the bottom of tank.

(ii) Weight of water in the tank.

(iii) Hydrostatic paradox between the results of (i) and (ii). Width of tank is 2 m.

Solution. Given :

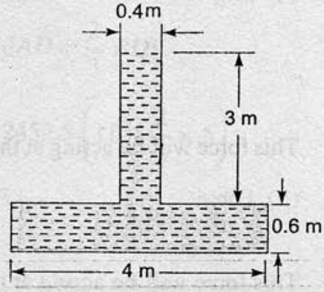


Fig. 3.17

Depth of water on bottom of tank

$$h_1 = 3 + 0.6 = 3.6 \text{ m}$$

Width of tank $= 2$ m

Length of tank at bottom $= 4$ m

$\therefore$ Area at the bottom, $A = 4 \times 2 = 8 \text{ m}^2$

(*i*) Total pressure F, on the bottom is

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 8 \times 3.6$$

$$= \mathbf{282528 \text{ N. Ans.}}$$

(*ii*) Weight of water in tank = $\rho g \times$ Volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times .6 \times 2]$$

$$= 1000 \times 9.81 \, [2.4 + 4.8] = \mathbf{70632 \text{ N. Ans.}}$$

(*iii*) From the results of (*i*) and (*ii*), it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

▶ 3.5 INCLINED PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig. 3.18.

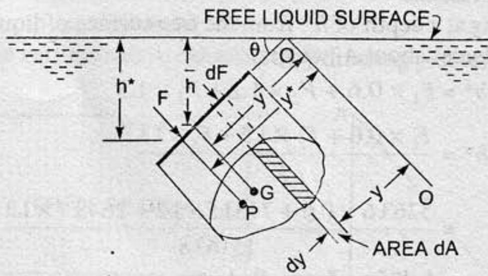


Fig. 3.18 *Inclined immersed surface.*

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h^* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho gh$

∴ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho gh \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho g h dA$

But from Fig. 3.18, $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$

∴ $h = y \sin\theta$

∴ $F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$

But $\int y dA = A\bar{y}$

where $\bar{y}$ = Distance of C.G. from axis O-O

∴ $F = \rho g \sin\theta\, \bar{y} \times A$

$= \rho g A \bar{h}$ $\quad (\because \bar{h} = \bar{y}\sin\theta)$ ...(3.6)

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho g h dA$

$= \rho g y \sin\theta\, dA$ $\quad [h = y \sin\theta]$

Moment of the force, dF, about axis O-O

$= dF \times y = \rho g y \sin\theta\, dA \times y = \rho g \sin\theta\, y^2 dA$

Sum of moments of all such forces about O-O

$= \int \rho g \sin\theta\, y^2\, dA = \rho g \sin\theta \int y^2\, dA$

But $\int y^2\, dA$ = M.O.I. of the surface about O-$O = I_0$

∴ Sum of moments of all forces about O-$O = \rho g \sin\theta\, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$= F \times y^*$ ...(3.8)

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$F \times y^* = \rho g \sin\theta\, I_0$

or $y^* = \frac{\rho g \sin\theta\, I_0}{F}$ ...(3.9)

Now $y^* = \frac{h^*}{\sin\theta}$, $F = \rho g A \bar{h}$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Substituting these values in equation (3.9), we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\therefore \quad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

But $\quad \frac{\bar{h}}{\bar{y}} = \sin\theta \quad$ or $\quad \bar{y} = \frac{\bar{h}}{\sin\theta}$

$$\therefore \quad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\times\frac{\bar{h}^2}{\sin^2\theta}\right]$$

or $$h^* = \frac{I_G\sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{...(3.10)}$$

If $\theta = 90°$, equation (3.10) becomes same as equation (3.5) which is applicable to vertically plane submerged surfaces.

In equation (3.10), I_G = M.O.I. of inclined surfaces about an axis passing through G and parallel to O-O.

Problem 3.14 (a) *A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $\quad b = 2$ m

Depth, $\quad d = 3$ m

Angle, $\quad \theta = 30°$

Distance of upper edge from free water surface = 1.5 m

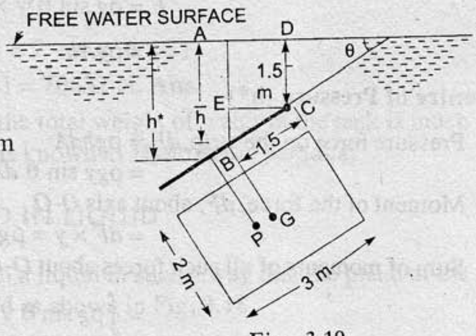


Fig. 3.19

(*i*) **Total pressure** force is given by equation (3.6) as

$$F = \rho g A\bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

$\therefore \quad \bar{h}$ = Depth of C.G. from free water surface

$= 1.5 + 1.5 \sin 30°$

$\{\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

$\therefore \quad F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G\sin^2\theta}{A\bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5\times\sin^2 30°}{6\times 2.25} + 2.25 = \frac{4.5\times\frac{1}{4}}{6\times 2.25} + 2.25$$

$= 0.0833 + 2.25 =$ **2.3333 m. Ans.**

Problem 3.14 (b) *A rectangular plane surface 3 m wide and 4 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure force and position of centre of pressure, when the upper edge is 2 m below the free surface.*

(Delhi University, Dec. 2002)

Solution. Given :

$$b = 3 \text{ m}, d = 4 \text{ m}, \theta = 30°$$

Distance of upper edge from free surface of water = 2 m

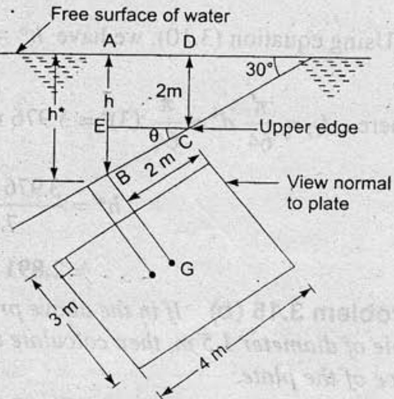


Fig. 3.19 (a)

(i) **Total pressure force** is given by equation (3.6) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$,

$A = b \times d = 3 \times 4 = 12 \text{ m}^2$

and $\bar{h}$ = Depth of C.G. of plate from free water surface

$= 2 + BE = 2 + BC \sin\theta$

$= 2 + 2 \sin 30° = 2 + 2 \times \frac{1}{2} = 3 \text{ m}$

∴ $F = 1000 \times 9.81 \times 12 \times 3 = 353167 \text{ N} = $ **353.167 kN. Ans.**

(ii) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \dfrac{bd^3}{12} = \dfrac{3 \times 4^3}{12} = 16 \text{ m}^4$

∴ $$h^* = \frac{16 \times \sin^2 30°}{12 \times 3} + 3 = \frac{16 \times \frac{1}{4}}{36} + 3 = \mathbf{3.111 \text{ m. Ans.}}$$

Problem 3.15 (a) *A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.*

Solution. Given :

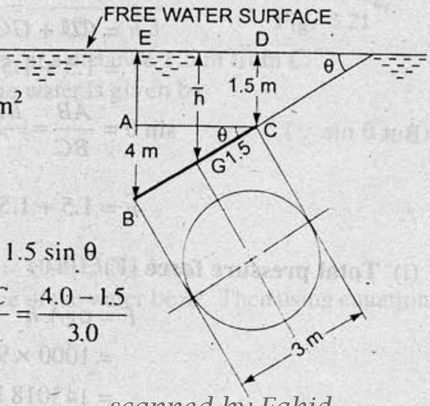
Dia. of plate, $d = 3.0$ m

∴ Area, $A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance $DC = 1.5$ m. $BE = 4$ m

Distance of C.G. from free surface

$= \bar{h} = CD + GC \sin\theta = 1.5 + 1.5 \sin\theta$



But $\sin\theta = \dfrac{AB}{BC} = \dfrac{BE - AE}{BC} = \dfrac{4.0 - DC}{3.0} = \dfrac{4.0 - 1.5}{3.0}$

$= \dfrac{2.5}{3.0} = 0.8333$

∴ $\bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749$ m

Fig. 3.20

(*i*) **Total pressure (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 7.0685 \times 2.749 = \mathbf{190621\ N.\ Ans.}$$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$

where $I_G = \dfrac{\pi}{64} d^4 = \dfrac{\pi}{64} (3)^4 = 3.976 \text{ m}^4$

$$\therefore \quad h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$
$$= \mathbf{2.891\ m.\ Ans.}$$

Problem 3.15 (*b*) *If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5 m, then calculate the total pressure and position of the centre of pressure on one face of the plate.*

Solution. Given : [Refer to Fig. 3.20 (*a*)]

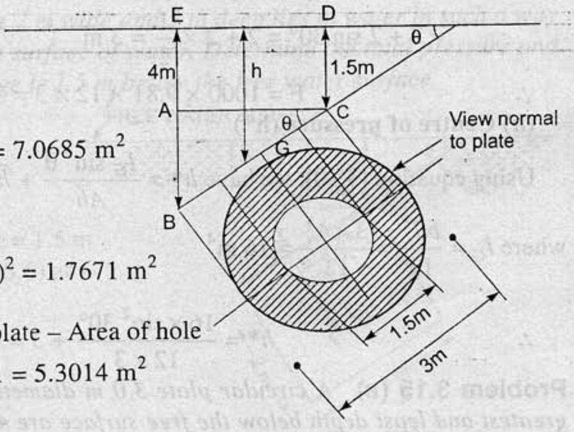


Fig. 3.20 (*a*)

Dia. of plate, $d = 3.0$ m

$\therefore$ Area of solid plate $= \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (3)^2 = 7.0685 \text{ m}^2$

Dia. of hole in the plate, $d_0 = 1.5$ m

$\therefore$ Area of hole $= \dfrac{\pi}{4} d_0^2 = \dfrac{\pi}{4} (1.5)^2 = 1.7671 \text{ m}^2$

$\therefore$ Area of the given plate, A = Area of solid plate – Area of hole

$$= 7.0685 - 1.7671 = 5.3014 \text{ m}^2$$

Distance $CD = 1.5$, $BE = 4$ m

Distance of C.G. from the free surface,

$$\bar{h} = CD + GC \sin \theta$$
$$= 1.5 + 1.5 \sin \theta$$

But $\sin \theta = \dfrac{AB}{BC} = \dfrac{BE - AE}{BC} = \dfrac{4 - 1.5}{3} = \dfrac{2.5}{3}$

$$\therefore \quad \bar{h} = 1.5 + 1.5 \times \frac{2.5}{3} = 1.5 + 1.25 = 2.75 \text{ m}$$

(*i*) **Total pressure force (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 5.3014 \times 2.75$$
$$= 143018 \text{ N} = \mathbf{143.018\ kN.\ Ans.}$$

(*ii*) **Position of centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64}[d^4 - d_0^4] = \frac{\pi}{64}[3^4 - 1.5^4] \text{ m}^4$

$$A = \frac{\pi}{4}[d^2 - d_0^2] = \frac{\pi}{4}[3^2 - 1.5^2] \text{ m}^2$$

$$\sin\theta = \frac{2.5}{3} \text{ and } \bar{h} = 2.75$$

$$\therefore \quad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right] \times \left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right] \times 2.75} + 2.75$$

$$= \frac{\frac{1}{16}\left[3^2 + 1.5^2\right] \times \left(\frac{2.5}{3}\right)^2}{2.75} + 2.75 = \frac{1 \times 11.25 \times 6.25}{16 \times 2.75 \times 9} + 2.75$$

$= 0.177 + 2.75 = $ **2.927 m. Ans.**

Problem 3.16 *A circular plate 3 metre diameter is submerged in water as shown in Fig. 3.21. Its greatest and least depths are below the surfaces being 2 metre and 1 metre respectively. Find : (i) the total pressure on front face of the plate, and (ii) the position of centre of pressure.*

(A.M.I.E., Winter, 1983)

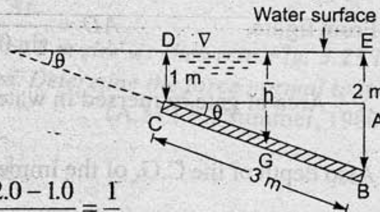


Fig. 3.21

Solution. Given :

Dia. of plate, $d = 3.0$ m

$\therefore$ Area, $A = \frac{\pi}{4}(3.0)^2 = 7.0685 \text{ m}^2$

Distance, $DC = 1$ m, $BE = 2$ m

In ΔABC, $\sin\theta = \frac{AB}{AC} = \frac{BE - AE}{BC} = \frac{BE - DC}{BC} = \frac{2.0 - 1.0}{3.0} = \frac{1}{3}$

The centre of gravity of the plate is at the middle of *BC*, *i.e.*, at a distance 1.5 m from *C*.

The distance of centre of gravity from the free surface of the water is given by

$$\bar{h} = CD + CG \sin\theta = 1.0 + 1.5 \times \tfrac{1}{3} \qquad (\because \sin\theta = \tfrac{1}{3})$$

$= 1.5$ m.

(*i*) Total pressure on the front face of the plate is given by

$$F = \rho g A \bar{h}$$

$= 1000 \times 9.81 \times 7.0685 \times 1.5 = $ **104013 N. Ans.**

Let the distance of the centre of pressure from the free surface of the water be h^*. Then using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} (3)^4, A = \frac{\pi}{4} d^2, \bar{h} = 1.5$ m and $\sin\theta = \frac{1}{3}$

Substituting the values, we get

$$h^* = \frac{\frac{\pi}{64} d^4 \times \left(\frac{1}{3}\right)^2}{\frac{\pi}{4} d^2 \times 1.5} + 1.5 = \frac{d^2}{16} \times \frac{1}{9 \times 1.5} + 1.5$$

$$= \frac{3^2}{16 \times 9 \times 1.5} + 1.5 = .0416 + 1.5 = \mathbf{1.5416\ m.\ Ans.}$$

Problem 3.17 *A rectangular gate 5 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.22. To keep the gate in a stable position, a counter weight of 5000 kgf is attached at the upper end of the gate as shown in figure. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and friction at the hinge and pulley.*

Solution. Given :

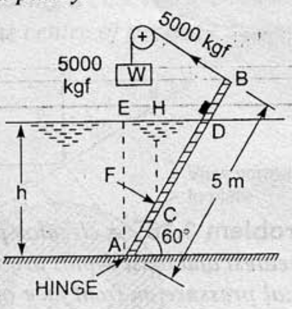


Fig. 3.22

Length of gate = 5 m

Width of gate = 2 m

$\theta = 60°$

Weight, $W = 5000$ kgf

$= 5000 \times 9.81$ N

$= 49050$ N ($\because$ 1 kgf = 9.81 N)

As the pulley is frictionless, the force acting at $B = 49050$ N. First find the total force F acting on the gate AB for a given depth of water.

From figure, $AD = \frac{AE}{\sin\theta} = \frac{h}{\sin 60} = \frac{5}{\sqrt{3}/2} = \frac{2h}{\sqrt{3}}$

$\therefore$ Area of gate immersed in water, $A = AD \times \text{Width} \times \frac{2h}{\sqrt{3}} \times 2 = \frac{4h}{\sqrt{3}}$ m^2

Also depth of the C.G. of the immersed area $= \bar{h} = \frac{h}{2} = 0.5\,h$

$\therefore$ Total force F is given by $F = \rho g A \bar{h} = 1000 \times 9.81 \times \frac{4h}{\sqrt{3}} \times \frac{h}{\sqrt{2}} = \frac{19620}{\sqrt{3}} h^2$N

The centre of pressure of the immersed surface, h^* is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of the immersed area

$$= \frac{b \times (AD)^3}{12} = \frac{2}{12} \times \left(\frac{2h}{\sqrt{3}}\right) \qquad \left\{\because AD = \frac{2h}{\sqrt{3}}\right\}$$

$$= \frac{16h^3}{12 \times 3 \times \sqrt{3}} = \frac{4h^3}{9 \times \sqrt{3}} \text{ m}^4$$

$$\therefore \quad h^* = \frac{4h^3}{9\times\sqrt{3}} \times \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{4h}{\sqrt{3}}\times\frac{h}{2}} + \frac{h}{2} = \frac{3h^3}{18h^2}\times\frac{h}{2} = \frac{h}{6}+\frac{h}{2} = \frac{h+3h}{6} = \frac{2h}{3}$$

Now in the ΔCHD, $\quad CH = h^* = \frac{2h}{3}$, $\angle CDH = 60°$

$$\therefore \quad \frac{CH}{CD} = \sin 60°$$

$$\therefore \quad CD = \frac{CH}{\sin 60°} = \frac{h^*}{\sin 60°} = \frac{2h}{3\times\frac{\sqrt{3}}{2}} = \frac{4h}{3\times\sqrt{3}}$$

$$\therefore \quad AC = AD - CD = \frac{2h}{\sqrt{3}} - \frac{4h}{3\sqrt{3}} = \frac{6h-4h}{3\sqrt{3}} = \frac{2h}{3\sqrt{3}} \text{ m}$$

Taking the moments about hinge, we get

$$49050 \times 5.0 = F \times AC = \frac{19620}{\sqrt{3}} h^2 \times \frac{2h}{3\sqrt{3}}$$

or
$$245250 = \frac{39240\, h^3}{3\times 3}$$

$$\therefore \quad h^3 = \frac{9\times 245250}{39240} = 56.25$$

$$\therefore \quad h = (56.25)^{1/3} = \mathbf{3.83 \text{ m. Ans.}}$$

Problem 3.18 *An inclined rectangular sluice gate AB 1.2 m by 5 m size as shown in Fig. 3.23 is installed to controlled the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.* (A.M.I.E., Summer, 1980)

Solution. Given :

$$A = \text{Area of gate} = 1.2 \times 5.0 = 6.0 \text{ m}^2$$

Depth of C.G. of the gate from free surface of the water $= \bar{h}$

$$= DG = BC - BE$$
$$= 5.0 - BG \sin 45°$$
$$= 5.0 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576 \text{ m}$$

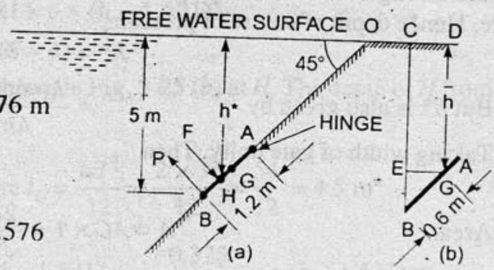


Fig. 3.23

The total pressure force (F) acting on the gate,

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 6.0 \times 4.576$$
$$= 269343 \text{ N}$$

This force is acting at H, where the depth of H from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

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where I_G = M.O.I. of gate $= \frac{bd^3}{12} = \frac{5.0 \times 1.2^3}{12} = 0.72$ m

$\therefore$ Depth of centre of pressure $h^* = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = 0.13 + 4.576 = 4.589$ m

But from Fig. 3.23 (*a*), $\frac{h^*}{OH} = \sin 45°$

$\therefore$ Distance, $OH = \frac{h^*}{\sin 45°} = \frac{4.589}{\frac{1}{\sqrt{2}}} = 4.589 \times \sqrt{2} = 6.489$ m

Distance, $BO = \frac{5}{\sin 45°} = 5 \times \sqrt{2} = 7.071$ m

Distance, $BH = BO - OH = 7.071 - 6.489 = 0.582$ m

$\therefore$ Distance $AH = AB - BH = 1.2 - 0.582 = 0.618$ m

Taking the moments about the hinge *A*

$$P \times AB = F \times (AH)$$

where *P* is the force normal to the gate applied at *B*

$\therefore$ $P \times 1.2 = 269343 \times 0.618$

$\therefore$ $P = \frac{269343 \times 0.618}{1.2}$ **= 138708 N. Ans.**

Problem 3.19 *A gate supporting water is shown in Fig. 3.24. Find the height h of the water so that the gate tips about the hinge. Take the width of the gate as unity.*

Solution. Given : $\theta = 60°$

Distance, $AC = \frac{h}{\sin 60°} = \frac{2h}{\sqrt{3}}$

where h = Depth of water.

The gate will start tipping about hinge *B* if the resultant pressure force acts at *B*. If the resultant pressure force passes through a point which is lying from *B* to *C* anywhere on the gate, the gate will tip over the hinge. Hence limiting case is when the resultant force passes through *B*. But the resultant force passes through the centre of pressure. Hence for the given position point *B* becomes the centre of pressure. Hence depth of centre of pressure,

$h^* = (h - 3)$ m

But h^* is also given by $= \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$

Taking width of gate unity. Then

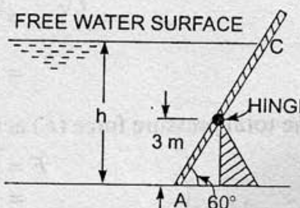

Fig. 3.24

Area, $A = AC \times 1 = \frac{2h}{\sqrt{3}} \times 1$; $\bar{h} = \frac{h}{2}$

$$I_G = \frac{bd^3}{12} = \frac{1 \times AC^3}{12} = \frac{1 \times \left(\frac{2h}{\sqrt{3}}\right)^3}{12} = \frac{8h^3}{12 \times 3 \times \sqrt{3}} = \frac{2h^3}{9 \times \sqrt{3}}$$

$$\therefore \quad h^* = \frac{2h^3}{9 \times \sqrt{3}} \times \frac{\sin^2 60°}{\frac{2h}{\sqrt{3}} \times \frac{h}{2}} + \frac{h}{2} = \frac{2h^3 \times \frac{3}{4}}{9h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{2h}{3}$$

Equating the two values of h^*,

$$h - 3 = \frac{2h}{3} \quad \text{or} \quad h - \frac{2h}{3} = 3 \quad \text{or} \quad \frac{h}{3} = 3$$

$$\therefore \quad h = 3 \times 3 = 9 \text{ m}$$

$\therefore$ Height of water for tipping the gate = **9 m. Ans.**

Problem 3.20 *A rectangular sluice gate AB, 2 m wide and 3 m long is hinged at A as shown in Fig. 3.25. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to the gate is 343350 N. Find the height of the water 'h' which will just cause the gate to open. The centre of gravity of the weight and gate is at G.*

Solution. Given :

Width of gate, $b = 2$ m ; Length of gate $L = 3$ m

$\therefore$ Area, $A = 2 \times 3 = 6 \text{ m}^2$

Weight of gate and $W = 343350$ N

Angle of inclination, $\theta = 45°$

Let h is the required height of water.

Depth of C.G. of the gate and weight = $\bar{h}$

From Fig. 3.25 (*a*),

$$\therefore \quad \bar{h} = h - ED = h - (AD - AE)$$

$$= h - (AB \sin\theta - EG \tan\theta) \quad \left\{\because \tan\theta = \frac{AE}{EG} \therefore AE = EG \tan\theta\right\}$$

$$= h - (3 \sin 45° - 0.6 \tan 45°)$$

$$= h - (2.121 - 0.6) = (h - 1.521) \text{ m}$$

The total pressure force, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 6 \times (h - 1.521)$$

$$= 58860\ (h - 1.521) \text{ N}.$$

The total force F is acting at the centre of pressure as shown in Fig. 3.25 (*b*) at H. The depth of H from free surface is given by h^* which is equal to

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = \frac{54}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5 \times \sin^2 45}{6 \times (h - 1.521)} + (h - 1.521) = \frac{0.375}{(h - 1.521)} + (h - 1.521) \text{ m}$$

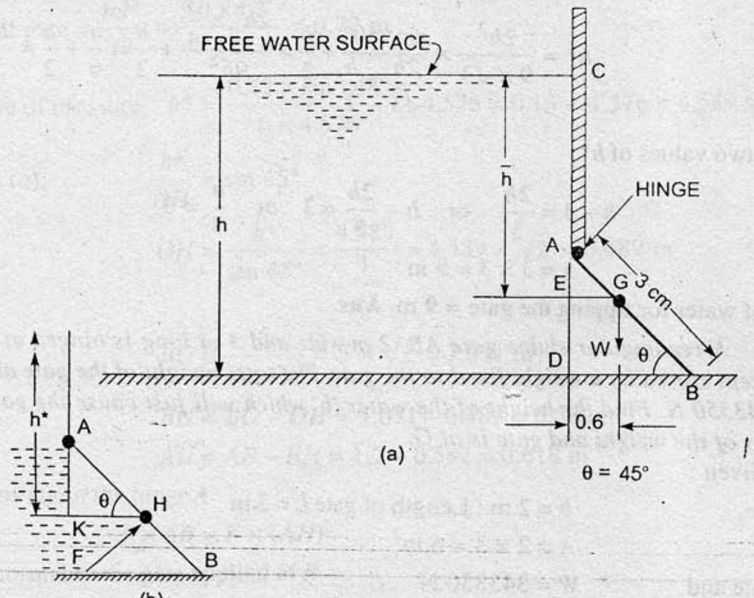

Fig. 3.25

Now taking moments about hinge A, we get

$$343350 \times EG = F \times AH$$

or
$$343350 \times 0.6 = F \times \frac{AK}{\sin 45°}$$

$$\left[\text{From } \Delta AKH, \text{ Fig. 3.24 } (b)\ AK = AH \sin\theta = AH \sin 45° \therefore AH = \frac{AK}{\sin 45°}\right]$$

$$= \frac{58860\,(h - 1.521) \times AK}{\sin 45°}$$

$\therefore$
$$AK = \frac{343350 \times 0.6 \times \sin 45}{58860\,(h - 1.521)} = \frac{0.3535 \times 7}{(h - 1.521)} \qquad ...(i)$$

But
$$AK = h^* - AC = \frac{.375}{(h - 1.521)} + (h - 1.521) - AC \qquad ...(ii)$$

But
$$AC = CD - AD = h - AB \sin 45 = h - 3 \times \sin 45° = h - 2.121$$

$\therefore$ Substituting this value in (*ii*), we get

$$AK = \frac{.375}{h - 1.521} + (h - 1.521) - (h - 2.121)$$

$$= \frac{.375}{h - 1.521} + 2.121 - 1.521 = \frac{.375}{h - 1.521} + 0.6 \qquad ...(iii)$$

Equating the two values of AK from (*i*) and (*iii*)

$$\frac{0.3535 \times 7}{h - 1.521} = \frac{0.375}{h - 1.521} + 0.6$$

or $\quad 0.3535 \times 7 = 0.375 + 0.6\,(h - 1.521) = 0.375 + 0.6\,h - 0.6 \times 1.521$

or $\quad 0.6h = 2.4745 - .375 + 0.6 \times 1.521 = 2.0995 + 0.9126 = 3.0121$

$\therefore \quad h = \dfrac{3.0121}{0.6} = $ **5.02 m. Ans.**

Problem 3.21 *Find the total pressure and position of centre of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plan of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.*

Solution. Given :

Base of plate, $\quad b = 2$ m

Height of plate, $\quad h = 3$ m

$\therefore$ Area, $\quad A = \dfrac{b \times h}{2} = \dfrac{2 \times 3}{2} = 3\ \text{m}^2$

Inclination, $\quad \theta = 60°$

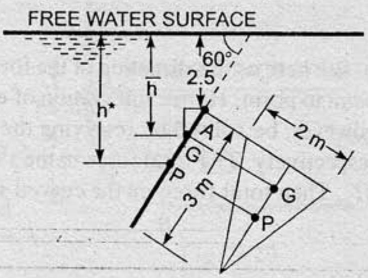

Fig. 3.26

Depth of centre of gravity from free surface of water,

$$\bar{h} = 2.5 + AG \sin 60°$$

$$= 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2} \qquad \left\{\because AG = \frac{1}{3} \text{ of height of triangle}\right\}$$

$$= 2.5 + .866 \text{ m} = 3.366 \text{ m}$$

(*i*) **Total pressure force (F)**

$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3 \times 3.366 = $ **99061.38 N. Ans.**

(*ii*) **Centre of pressure (h*).** Depth of centre of pressure from free surface of water is given by

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $\quad I_G = \dfrac{bh^3}{36} = \dfrac{2 \times 3^3}{36} = \dfrac{3}{2} = 1.5\ \text{m}^4$

$\therefore \quad h^* = \dfrac{1.5 \times \sin^2 60}{3 \times 3.366} + 3.366 = 0.1114 + 3.366 = $ **3.477 m. Ans.**

▶ 3.6 CURVED SURFACE SUB-MERGED IN LIQUID

Consider a curved surface AB, sub-merged in a static fluid as shown in Fig. 3.27. Let dA is the area of a small strip at a depth of h from water surface.

Then pressure intensity on the area dA is $= \rho g h$

and pressure force, $\quad dF = p \times \text{Area} = \rho g h \times dA \quad$ ...(3.11)

This force dF acts normal to the surface.

Hence total pressure force on the curved surface should be

$$F = \int \rho g h dA \qquad \text{...(3.12)}$$

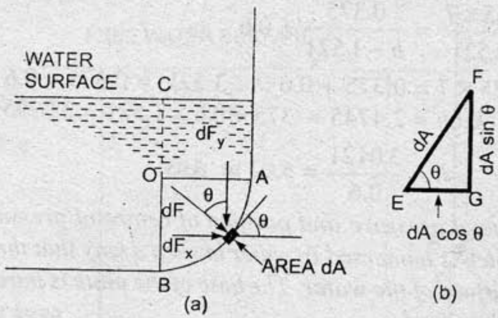

Fig. 3.27

But here as the direction of the forces on the small areas are not in the same direction, but varies from point to point. Hence integration of equation (3.11) for curved surface is impossible. The problem can, however, be solved by resolving the force dF in two components dF_x and dF_y in the x and y directions respectively. The total force in the x and y directions, *i.e.*, F_x and F_y are obtained by integrating dF_x and dF_y. Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{...(3.13)}$$

and inclination of resultant with horizontal is $\tan \phi = \dfrac{F_y}{F_x}$ ...(3.14)

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin \theta = \rho g h dA \sin \theta \qquad \{\because dF = \rho g h dA\}$$

and

$$dF_y = dF \cos \theta = \rho g h dA \cos \theta$$

Total forces in the x and y direction are :

$$F_x = \int dF_x = \int \rho g h dA \sin \theta = \rho g \int h dA \sin \theta \quad \text{...(3.15)}$$

and

$$F_y = \int dF_y = \int \rho g h dA \cos \theta = \rho g \int h dA \cos \theta \quad \text{...(3.16)}$$

Fig. 3.27 (*b*) shows the enlarged area dA. From this figure, *i.e.*, ΔEFG,

$$EF = dA$$

$$FG = dA \sin \theta$$

$$EG = dA \cos \theta$$

Thus in equation (3.15) $dA \sin \theta = FG$ = Vertical projection of the area dA and hence the expression $\rho g \int h dA \sin \theta$ represents the total pressure force on the projected area of the curved surface on the vertical plane. Thus

F_x = Total pressure force on the projected area of the curved surface on vertical plane. ...(3.17)

Also $dA \cos \theta = EG$ = horizontal projection of dA and hence $h dA \cos \theta$ is the volume of the liquid contained in the elementary area dA up to free surface of the liquid. Thus $\int h dA \cos \theta$ is the total volume contained between the curved surface extended upto free surface.

Hence $\rho g \int h dA \cos \theta$ is the total weight supported by the curved surface. Thus

$$F_y = \rho g \cdot \int h dA \cos \theta$$

= weight of liquid supported by the curved surface upto free surface of liquid. ...(3.18)

In Fig. 3.28, the curved surface AB is not supporting any fluid. In such cases, F_y is equal to the weight of the imaginary liquid supported by AB upto free surface of liquid. The direction of F_y will be taken in upward direction.

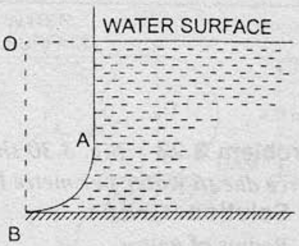

Fig. 3.28

Problem 3.22 *Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.29. Take the width of the gate as unity.*

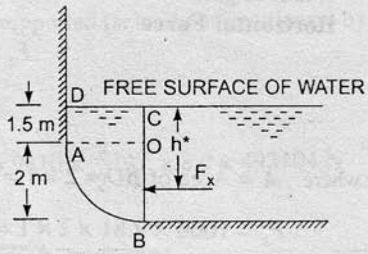

Fig. 3.29

Solution. Given :

Width of gate $= 1.0$ m

Radius of the gate $= 2.0$ m

$\therefore$ Distance $AO = OB = 2$ m

Horizontal force, F_x exerted by water on gate is given by equation (3.17) as

F_x = Total pressure force on the projected area of curved surface AB on vertical plane

= Total pressure force on OB

{projected area of curved surface on vertical plane = $OB \times 1$}

$$= \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 2 \times 1 \times \left(1.5 + \frac{2}{2}\right) \quad \{\because \text{ Area of } OB = A = BO \times 1 = 2 \times 1 = 2,$$

$\bar{h}$ = Depth of C.G. of OB from free surface $= 1.5 + \frac{2}{2}$ }

$F_x = 9.81 \times 2000 \times 2.5 =$ **49050 N. Ans.**

The point of application of F_x is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where I_G = M.O.I. of OB about its C.G. $= \dfrac{bd^3}{12} = \dfrac{1 \times 2^3}{12} = \dfrac{2}{3}\ \text{m}^4$

$$\therefore \quad h^* = \frac{\frac{2}{3}}{2 \times 2.5} + 2.5 = \frac{1}{7.5} + 2.5 \text{ m}$$

$= 0.1333 + 2.5 = 2.633$ m from free surface.

Vertical force, F_y, exerted by water is given by equation (3.18)

F_y = Weight of water supported by AB upto free surface

= Weight of portion $DABOC$

= Weight of $DAOC$ + Weight of water AOB

= ρg [Volume of $DAOC$ + Volume of AOB]

$$= 1000 \times 9.81 \left[AD \times AO \times 1 + \frac{\pi}{4}(AO)^2 \times 1\right]$$

$$= 1000 \times 9.81 \left[1.5 \times 2.0 \times 1 + \frac{\pi}{4} \times 2^2 \times 1\right]$$

$$= 1000 \times 9.81\ [3.0 + \pi]\text{N} = \mathbf{60249.1\ N.\ Ans.}$$

Problem 3.23 *Fig. 3.30 shows a gate having a quadrant shape of radius 2 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.*

Solution. Given :

Radius of gate = 2 m

Width of gate = 1 m

Horizontal Force

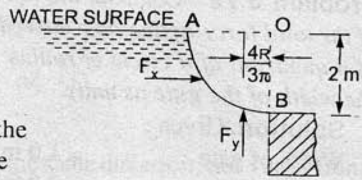

Fig. 3.30

F_x = Force on the projected area of the curved surface on vertical plane

= Force on $BO = \rho g A \bar{h}$

where A = Area of $BO = 2 \times 1 = 2\ \text{m}^2$, $\bar{h} = \frac{1}{2} \times 2 = 1$ m ;

$$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620\ \text{N}$$

This will act at a depth of $\frac{2}{3} \times 2 = \frac{4}{3}$ m from free surface of liquid.

Vertical Force, F_y

F_y = Weight of water (imagined) supported by AB

$= \rho g \times$ Area of $AOB \times 1.0$

$$= 1000 \times 9.81 \times \frac{\pi}{4} (2)^4 \times 1.0 = 30819\ \text{N}$$

This will act at a distance of $\frac{4R}{3\pi} = \frac{4 \times 2.0}{3\pi} = 0.848$ m from OB.

∴ Resultant force, F is given by

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{19620 + 30819} = \sqrt{384944400 + 949810761}$$

$$= \mathbf{36534.4\ N.\ Ans.}$$

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620} = 1.5708$$

∴ $$\theta = \tan^{-1} 1.5708 = \mathbf{57° 31'.\ Ans.}$$

Problem 3.24 *Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4.0 m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.*

Solution. Given :

Dia. of gate = 4 m

∴ Radius, $R = 2$ m

Length of gate, $l = 8$ m

Horizontal force, F_x acting on the gate is

$F_x = \rho g A \bar{h}$ = Force on projected area of curved surface ACB on vertical plane

= Force on vertical area AOB

where A = Area of $AOB = 4.0 \times 8.0 = 32.0 \text{ m}^2$

$\bar{h}$ = Depth of C.G. of $AOB = 4/2 = 2.0$ m

$\therefore \quad F_x = 1000 \times 9.81 \times 32.0 \times 2.0$

$= 627840$ N.

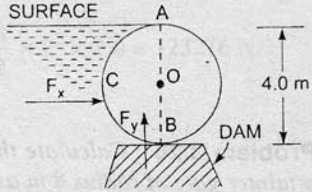

Fig. 3.31

Vertical force, F_y is given by

F_y = Weight of water enclosed or supported (actually or imaginary) by the curved surface ACB

$= \rho g \times$ Volume of portion ACB

$= \rho g \times$ Area of $ACB \times l$

$$= 1000 \times 9.81 \times \frac{\pi}{2}(R)^2 \times 8.0 = 9810 \times \frac{\pi}{2}(2)^2 \times 8.0 = 493104 \text{ N}$$

It will be acting in the upward direction.

$\therefore$ Resultant force, $F = \sqrt{F_x^2 + F_y^2} = \sqrt{627840 + 493104}$ = **798328 N. Ans.**

Direction of resultant force is given by $\tan\theta = \frac{F_y}{F_x} = \frac{493104}{627840} = 0.7853$

$\therefore \quad \theta = 31° \; 8'$. **Ans.**

Problem 3.25 *Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90° sector of radius 4 m as shown in Fig. 3.32. Take width of gate unity.*

Solution. Given :

Radius of gate, $R = 4$ m

Horizontal component, F_x of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$= \rho g A \bar{h}$

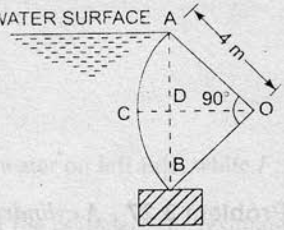

Fig. 3.32

where $A = AB \times$ Width of gate

$= 2 \times AD \times 1 \qquad (\because AB = 2AD)$

$= 2 \times 4 \times \sin 45° = 8 \times .707 = 5.656 \text{ m}^2 \qquad \{\because AD = 4 \sin 45°\}$

$$\bar{h} = \frac{AB}{2} = \frac{5.656}{2} = 2.828 \text{ m}$$

$\therefore \quad F_x = 1000 \times 9.81 \times 5.656 \times 2.828$ N = **156911 N. Ans.**

Vertical component

F_y = Weight of water supported or enclosed by the curved surface

= Weight of water in portion $ACBDA$

$= \rho g \times$ Area of $ACBDA \times$ Width of gate

$= 1000 \times 9.81 \times$ [Area of sector $ACBOA$ – Area of ΔABO] $\times 1$

$$= 9810 \times \left[\frac{\pi}{4}R^2 - \frac{AO \times BO}{2}\right] \quad [\because \Delta AOB \text{ is a right angled}]$$

$$= 9810 \times \left[\frac{\pi}{4}4^2 - \frac{4 \times 4}{2}\right] = \mathbf{44796\ N.\ Ans.}$$

Problem 3.26 *Calculate the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 8 m as shown in Fig. 3.33. Take width of gate unity.*

Solution. The horizontal component of water pressure is given by

$F_x = \rho g A \bar{h}$ = Force on the area projected on vertical plane

= Force on the vertical area of BD

where $A = BD \times$ Width of gate $= 4.0 \times 1 = 4.0$ m

$$\bar{h} = \frac{1}{2} \times 4 = 2 \text{ m}$$

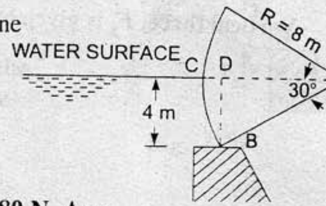

Fig. 3.33

$$F_x = 1000 \times 9.81 \times 4.0 \times 2.0 = \mathbf{78480\ N.\ Ans.}$$

Vertical component of the water pressure is given by

F_y = Weight of water supported or enclosed (imaginary) by curved surface CB

= Weight of water in the portion $CBDC$

= $\rho g \times$ [Area of portion $CBDC$] $\times$ Width of gate

= $\rho g \times$ [Area of sector CBO – Area of the triangle BOD] $\times$ 1

$$= 1000 \times 9.81 \times \left[\frac{30}{360} \times \pi R^2 - \frac{BD \times DO}{2}\right]$$

$$= 9810 \times \left[\frac{1}{12}\pi \times 8^2 - \frac{4.0 \times 8.8 \cos 30°}{2}\right]$$

$$\{\because DO = BO \cos 30° = 8 \times \cos 30°\}$$

$$= 9810 \times [16.755 - 13.856] = \mathbf{28439\ N.\ Ans.}$$

Problem 3.27 *A cylindrical gate of 4 m diameter 2 m long has water on its both sides as shown in Fig. 3.34. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinder so that it may not be lifted away from the floor.*

Solution. Given :

Dia. of gate = 4 m

Radius = 2 m

(*i*) The forces acting on the left side of the cylinder are :

The horizontal component, F_{x_1}

where F_{x_1} = Force of water on area projected on vertical plane

= Force on area AOC

$= \rho g A \bar{h}$

$= 1000 \times 9.81 \times 8 \times 2$

$= 156960$ N

where $A = AC \times$ Width $= 4 \times 2$

$= 8 \text{ m}^2$

$= \bar{h} = \frac{1}{2} \times 4 = 2$ m

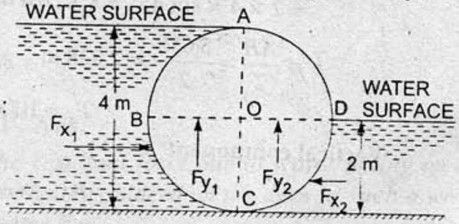

Fig. 3.34

$$F_{y_1} = \text{weight of water enclosed by } ABCOA$$

$$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = \mathbf{123276\ N.}$$

Right Side of the Cylinder

$$F_{x_2} = \rho g A_2 \bar{h}_2 = \text{Force on vertical area } CO$$

$$= 1000 \times 9.81 \times 2 \times 2 \times \frac{2}{2} \left\{\because\ A_2 = CO \times 1 = 2 \times 1 = 2\ \text{m}^2, \bar{h}_2 = \frac{2}{2} = 1.0\right\}$$

$$= 39240\ \text{N}$$

$$F_{y_2} = \text{Weight of water enclosed by } DOCD$$

$$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638\ \text{N}$$

∴ Resultant force in the direction of x,

$$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720\ \text{N}$$

Resultant force in the direction of y,

$$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914\ \text{N}$$

(*i*) **Resultant force,** F is given as

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = \mathbf{219206\ N.\ Ans.}$$

(*ii*) **Direction of resultant force** is given by

$$\tan \theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$$

$$\therefore \qquad \theta = \mathbf{57° \ 31'.\ Ans.}$$

(*iii*) **Location of the resultant force**

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67$ m from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33$ m from free surface on the right side of the cylinder. The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$$F_x \times y = F_{x_1} [4 - 2.67] - F_{x_2} [2 - 1.33]$$

or

$$117720 \times y = 156960 \times 1.33 - 39240 \times .67 = 208756.8 - 26290.8 = 182466$$

$$\therefore \qquad y = \frac{182466}{117720} = 1.55 \text{ m from the bottom.}$$

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488$ m from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488$ m from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

$$F_y \times x = F_{y_1} \times .8488 - F_{y_2} \times .8488$$

or $$184914 \times x = 123276 \times .8488 - 61638 \times .8488 = .8488\ [123276 - 61638] = 52318.4$$

$$\therefore \quad x = \frac{52318.4}{184914} = 0.2829 \text{ m from AOC.}$$

(*iv*) **Least Weight of Cylinder.** The resultant force in the upward direction is

$$F_y = 184914 \text{ N}$$

Thus the weight of cylinder should not be less than the upward force F_y. Hence least weight of cylinder should be at least.

$$= \mathbf{184914\ N.\ Ans.}$$

Problem 3.28 *Fig. 3.35 shows the cross-section of a tank full of water under pressure. The length of the tank is 2 m. An empty cylinder lies along the length of the tank on one of its corner as shown. Find the horizontal and vertical components of the force acting on the curved surface ABC of the cylinder.* (A.M.I.E., May 1974)

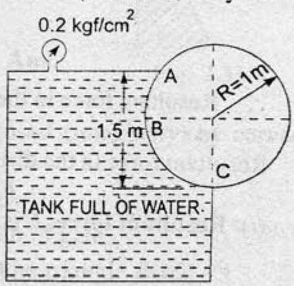

Fig. 3.35

Solution. Radius, $R = 1$ m

Length of tank, $l = 2$ m

Pressure, $p = 0.2 \text{ kgf/cm}^2 = 0.2 \times 9.81 \text{ N/cm}^2$
$= 1.962 \text{ N/cm}^2 = 1.962 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $$h = \frac{p}{\rho g} = \frac{1.962 \times 10^4}{1000 \times 9.81} = 2 \text{ m}$$

$\therefore$ Free surface of water will be at a height of 2 m from the top of the tank.

$\therefore$ Fig. 3.36 shows the equivalent free surface of water.

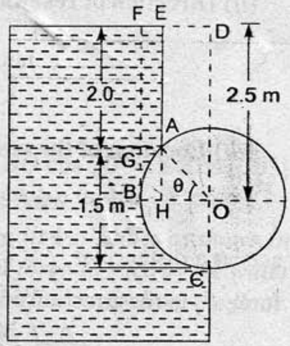

Fig. 3.36

(*i*) **Horizontal Component of Force**

$$F_x = \rho g A \bar{h}$$

where A = Area projected on vertical plane
$= 1.5 \times 2.0 = 3.0 \text{ m}^2$

$$\bar{h} = 2 + \frac{1.5}{2} = 2.75$$

$$\therefore \quad F_x = 1000 \times 9.81 \times 3.0 \times 2.75$$
$$= \mathbf{80932.5\ N.\ Ans.}$$

(*ii*) **Vertical Component of Force**

F_y = Weight of water enclosed or supported actually or imaginary by curved surface ABC
= Weight of water in the portion *CODE ABC*
= Weight of water in *CODFBC* – Weight of water in *AEFB*

But weight of water in *CODFBC*
= Weight of water in [*COB* + *ODFBO*]

$$= \rho g \left[\frac{\pi R^2}{4} + BO \times OD\right] \times 2 = 1000 \times 9.81 \left[\frac{\pi}{4} \times 1^2 + 1.0 \times 2.5\right] \times 2$$

$$= 64458.5 \text{ N}$$

Weight of water in $AEFB = \rho g$ [Area of $AEFB$] $\times 2.0$

$= 1000 \times 9.81$ [Area of $(AEFG + AGBH - AHB)$] $\times 2.0$

In ΔAHO, $\quad \sin\theta = \dfrac{AH}{AO} = \dfrac{0.5}{1.0} = 0.5 \quad \therefore \quad \theta = 30°$

$$BH = BO - HO = 1.0 - AO\cos\theta = 1.0 - 1 \times \cos 30° = 0.134$$

Area, $\quad ABH =$ Area ABO – Area AHO

$$= \pi R^2 \times \frac{30}{360} - \frac{AH \times HO}{2.0} = \frac{\pi R^2}{12} - \frac{0.5 \times .866}{2} = 0.0453$$

$\therefore$ Weight of water in $AEFB$

$$= 9810 \times [AE \times AG + AG \times AH - 0.0453] \times 2.0$$
$$= 9810 \times [2.0 \times .134 + .134 \times .5 - .0453] \times 2.0$$
$$= 9810 \times [.268 + .067 - .0453] \times 2.0 = 5684 \text{ N}$$

$\therefore \quad F_y = 64458.5 - 5684 =$ **58774.5 N. Ans.**

Problem 3.29 *Find the magnitude and direction of the resultant water pressure acting on a curved face of a dam which is shaped according to the relation* $y = \dfrac{x^2}{9}$ *as shown in Fig. 3.37. The height of the water retained by the dam is 10 m. Consider the width of the dam as unity.*

Solution. Equation of curve AB is

$$y = \frac{x^2}{9} \quad \text{or} \quad x^2 = 9y$$

$\therefore \quad x = \sqrt{9y} = 3\sqrt{y}$

Height of water, $\quad h = 10$ m

Width, $\quad b = 1$ m

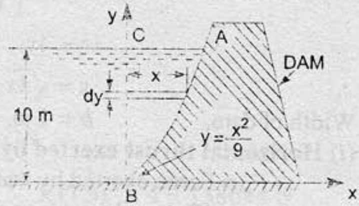

Fig. 3.37

The horizontal component, F_x is given by

$F_x =$ Pressure due to water on the curved area projected on vertical plane

$=$ Pressure on area BC

$= \rho g A \bar{h}$

where $\quad A = BC \times 1 = 10 \times 1 \text{ m}^2, \ \bar{h} = \frac{1}{2} \times 10 = 5$ m

$$F_x = 1000 \times 9.81 \times 10 \times 5 = 490500 \text{ N}$$

Vertical component, F_y is given by

$F_y =$ Weight of water supported by the curve AB

$=$ Weight of water in the portion ABC

$= \rho g$[Area of ABC] $\times$ Width of dam

$$= \rho g \left[\int_0^{10} x \times dy\right] \times 1.0 \qquad \left\{\text{Area of strip} = x\,dy \quad \therefore \text{Area } ABC = \int_0^{10} x\,dy\right\}$$

$$= 1000 \times 9.81 \times \int_0^{10} 3\sqrt{y}\,dy \qquad \{\because x = 3\sqrt{y}\}$$

$$= 29430 \left[\frac{y^{3/2}}{3/2}\right]_0^{10} = 29430 \times \frac{2}{3}\left[y^{3/2}\right]_0^{10} = 19620\,[10^{3/2}]$$

$$= 19620 \times 31.622 = 620439 \text{ N}$$

∴ Resultant water pressure on dam

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(490500)^2 + (620439)^2}$$

$$= 790907 \text{ N} = \textbf{790.907 kN. Ans.}$$

Direction of the resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{620439}{490500} = 1.265$$

∴ $\theta = \mathbf{51° 40'}$. **Ans.**

Problem 3.30 *A dam has a parabolic shape $y = y_0\left(\frac{x}{x_0}\right)^2$ as shown in Fig. 3.38 below having $x_0 = 6$ m and $y_0 = 9$ m. The fluid is water with density = 1000 kg/m³. Compute the horizontal, vertical and the resultant thrust exerted by water per metre length of the dam.* (A.M.I.E., Summer, 1985)

Solution. Given :

Equation of the curve *OA* is

$$y = y_0\left(\frac{x}{x_0}\right)^2 = 9\left(\frac{x}{6}\right)^2 = 9 \times \frac{x^2}{36} = \frac{x^2}{4}$$

or $x^2 = 4y$

∴ $x = \sqrt{4x} = 2y^{1/2}$

Width of dam, $b = 1$ m.

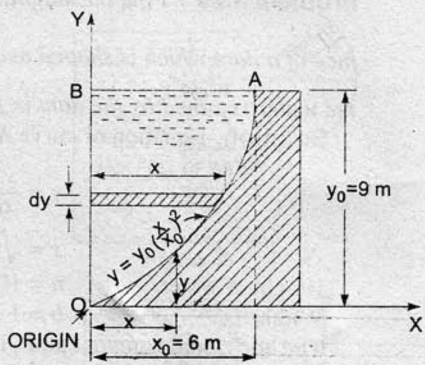

Fig. 3.38

(*i*) **Horizontal thrust exerted by water**

F_x = Force exerted by water on vertical surface *OB, i.e.*, the surface obtained by projecting the curved surface on vertical plane

$$= \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times (9 \times 1) \times \frac{9}{2} = \textbf{397305 N. Ans.}$$

(*ii*) **Vertical thrust exerted by water**

F_y = Weight of water supported by curved surface *OA* upto free surface of water

= Weight of water in the portion *ABO*

= ρg × Area of *OAB* × Width of dam

$$= 1000 \times 9.81 \times \left[\int_0^9 x \times dy\right] \times 1.0$$

$$= 1000 \times 9.81 \times \left[\int_0^9 2y^{1/2} \times dy\right] \times 1.0 \qquad (\because x = 2y^{1/2})$$

$$= 19620 \times \left[\frac{y^{3/2}}{(3/2)}\right]_0^9 = 19620 \times \frac{2}{3}[9^{3/2}]$$

$$= 19620 \times \frac{2}{3} \times 27 = \textbf{353160 N. Ans.}$$

(iii) Resultant thrust exerted by water

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{397305 + 353160} = \mathbf{531574\ N.\ Ans.}$$

Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = \mathbf{41.63°.\ Ans.}$$

Problem 3.31 *A cylinder 3 m in diameter and 4 m long retains water on one side. The cylinder is supported as shown in Fig. 3.39. Determine the horizontal reaction at A and the vertical reaction at B. The cylinder weighs 196.2 kN. Ignore friction.*

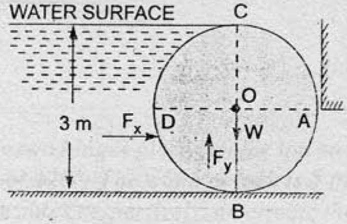

Fig. 3.39

Solution. Given :

Dia. of cylinder $= 3$ m

Length of cylinder $= 4$ m

Weight of cylinder, $W = 196.2$ kN $= 196200$ N

Horizontal force exerted by water

$F_x =$ Force on vertical area BOC

$= \rho g A\bar{h}$

where $A = BOC \times l = 3 \times 4 = 12\ \text{m}^2$, $\bar{h} = \frac{1}{2} \times 3 = 1.5$ m

$F_x = 1000 \times 9.81 \times 12 \times 1.5 = 176580$ N

The vertical force exerted by water

$F_y =$ Weight of water enclosed in $BDCOB$

$$= \rho g \times \left(\frac{\pi}{2} R^2\right) \times l = 1000 \times 9.81 \times \frac{\pi}{2} \times (1.5)^2 \times 4 = 138684 \text{ N}$$

Force F_y is acting in the upward direction.

For the equilibrium of cylinder

Horizontal reaction at $A = F_x = 176580$ N

Vertical reaction at $B =$ Weight of cylinder $- F_y$

$= 196200 - 138684 = \mathbf{57516\ N.\ Ans.}$

▶ 3.7 TOTAL PRESSURE AND CENTRE OF PRESSURE ON LOCK GATES

Lock gates are the devices used for changing the water level in a canal or a river for navigation. Fig. 3.40 shows plan and elevation of a pair of lock gates. Let AB and BC be the two lock gates. Each gate is supported on two hinges fixed on their top and bottom at the ends A and C. In the closed position, the gates meet at B.

Let F = Resultant force due to water on the gate AB or BC acting are right angles to the gate

R = Reaction at the lower and upper hinge

P = Reaction at the common contact surface of the two gates and acting perpendicular to the contact surface.

Let the force P and F meet at O. Then the reaction R must pass through O as the gate AB is in the equilibrium under the action of three forces. Let θ is the inclination of the lock gate with the normal to the side of the lock.

In $\angle ABO$, $\angle OAB = \angle ABO = \theta$.

Resolving all force along the gate AB and putting equal to zero, we get

$$R \cos \theta - P \cos \theta = 0 \text{ or } R = P \quad \text{...(3.19)}$$

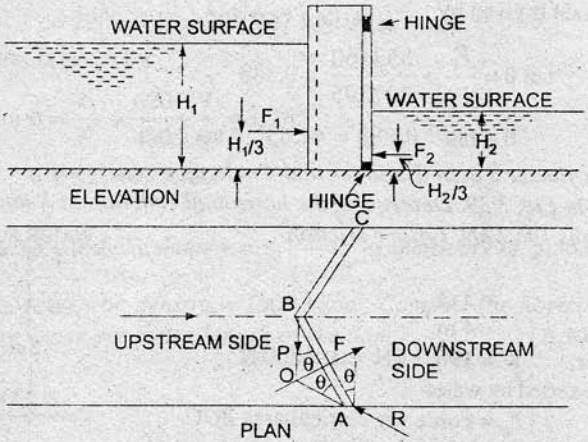

Fig. 3.40

Resolving forces normal to the gate AB

$$R \sin \theta + P \sin \theta - F = 0$$

or $$F = R \sin \theta + P \sin \theta = 2P \sin \theta \quad \{\because R = P\}$$

$\therefore$ $$P = \frac{F}{2 \sin \theta} \quad \text{...(3.20)}$$

To calculate P and R

In equation (3.20), P can be calculated if F and θ are known. The value of θ is calculated from the angle between the lock gates. The angle between the two lock gate is equal to $180 - 2\theta$. Hence θ can be calculated. The value of F is calculated as :

Let H_1 = Height of water on the upstream side

H_2 = Height of water on the downstream side

F_1 = Water pressure on the gate on upstream side

F_2 = Water pressure on the gate on downstream side of the gate

l = Width of gate

Now
$$F_1 = \rho g A_1 \bar{h}_1$$
$$= \rho g \times H_1 \times l \times \frac{H_1}{2}$$
$$= \rho g l \frac{H_1^2}{2} \qquad \left[\because A = H_1 \times l, \bar{h}_1 = \frac{H_1}{2}\right]$$

Similarly, $$F_2 = \rho g A_2 \bar{h}_2 = \rho g \times (H_2 \times l) \times \frac{H_2}{2} = \frac{\rho g l H_2^2}{2}$$

$\therefore$ Resultant force $$F = F_1 - F_2 = \frac{\rho g l H_1^2}{2} - \frac{\rho g l H_2^2}{2}$$

Substituting the value of θ and F in equation (3.20), the value of P and R can be calculated.

Reactions at the top and bottom hinges

Let R_t = Reaction of the top hinge

R_b = Reaction of the bottom hinge

Then $R = R_t + R_b$

The resultant water pressure F acts normal to the gate. Half of the value of F is resisted by the hinges of one lock gates and other half will be resisted by the hinges of other lock gate. Also F_1 acts at a distance of $\frac{H_1}{3}$ from bottom while F_2 acts at a distance of $\frac{H_1}{3}$ from bottom.

Taking moments about the lower hinge

$$R_t \times \sin\theta \times H = \frac{F_1}{2} \times \frac{H_1}{3} - \frac{F_2}{2} \times \frac{H_2}{3} \quad ...(i)$$

where H = Distance between two hinges

Resolving forces horizontally

$$R_t \sin\theta + R_b \sin\theta = \frac{F_1}{2} - \frac{F_2}{2} \quad ...(ii)$$

From equations (*i*) and (*ii*), we can find R_t and R_B.

Problem 3.32 *Each gate of a lock is 6 m high and is supported by two hinges placed on the top and bottom of the gate. When the gates are closed, they make an angle of 120°. The width of lock is 5 m. If the water levels are 4 m and 2 m on the upstream and downstream sides respectively, determine the magnitudes of the forces on the hinges due to water pressure.*

Solution. Given :

Height of lock = 6 m

Width of lock = 5 m

Width of each lock gate = AB

or $l = \frac{AD}{\cos 30^\circ} = \frac{2.5}{\cos 30^\circ}$

$= 2.887$ m

Angle between gates = 120°

$\therefore \quad \theta = \frac{180 - 120}{2} = \frac{60}{2} = 30°$

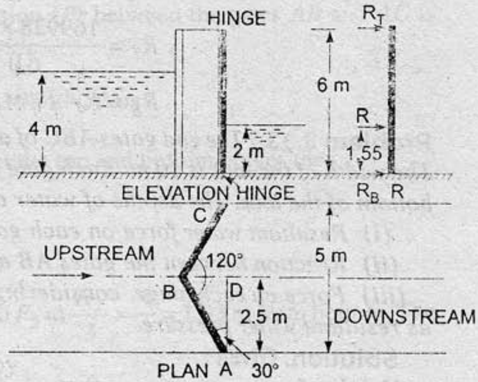

Fig. 3.41

Height of water on upstream side

$H_1 = 4$ m

and $H_2 = 2$ m

$\therefore$ Total water pressure on upstream side

$$F_1 = \rho g A_1 \bar{h}_1, \text{ where } A_1 = H_1 \times l = 4.0 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 4 \times 2.887 \times 2.0 \quad \left\{\bar{h}_1 = \frac{H_1}{2} = \frac{4}{2} = 2.0 \text{ m}\right\}$$

$$= 226571 \text{ N}$$

Force F_1 will be acting at a distance of $\frac{H_1}{3} = \frac{4}{3} = 1.33$ m from bottom.

Similarly, total water pressure on the downstream side

$$F_2 = \rho g A_2 \bar{h}_2, \text{ where } A_2 = H_2 \times l = 2 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 2 \times 2.887 \times 1.0 \quad \bar{h}_2 = \frac{H_2}{2} = \frac{2}{2} = 1.0 \text{ m}$$

$= 56643$ N

F_2 will act at a distance of $\frac{H_2}{3} = \frac{2}{3} = 0.67$ m from bottom,

Resultant water pressure on each gate

$$F = F_1 - F_2 = 226571 - 56643 = 169928 \text{ N}.$$

Let x is height of F from the bottom, then taking moments of F_1, F_2 and F about the bottom, we have

$$F \times x = F_1 \times 1.33 - F_2 \times 0.67$$

or

$$169928 \times x = 226571 \times 1.33 - 56643 \times 0.67$$

$$\therefore \quad x = \frac{226571 \times 1.33 - 56643 \times 0.67}{169928} = \frac{301339 - 37950}{169928} = 1.55 \text{ m}$$

From equation (3.20), $P = \frac{F}{2 \sin\theta} = \frac{169928}{2 \sin 30} = 169928$ N.

From equation (3.19), $R = P = 169928$ N.

If R_T and R_B are the reaction at the top and bottom hinges, then $R_T + R_B = R = 169928$ N.

Taking movements of hinge reactions R_T, R_B and R about the bottom hinges, we have

$$R_T \times 6.0 + R_B \times 0 = R \times 1.55$$

$$\therefore \quad R_T = \frac{169928 \times 1.55}{6.0} = 43898 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 169928 - 43898 = \mathbf{126030\ N.\ Ans.}$$

Problem 3.33 *The end gates ABC of a lock are 9 m high and when closed include on angle of 120°. The width of the lock is 10 m. Each gate is supported by two hinges located at 1 m and 6 m above the bottom of the lock. The depths of water on the two sides are 8 m and 4 m respectively. Find :*

(i) Resultant water force on each gate,

(ii) Reaction between the gates AB and BC, and

(iii) Force on each hinge, considering the reaction of the gate acting in the same horizontal plane as resultant water pressure.

Solution. Given :

Height of gate $= 9$ m

Inclination of gate $= 120°$

$$\therefore \quad \theta = \frac{180 - 120}{2} = 30°$$

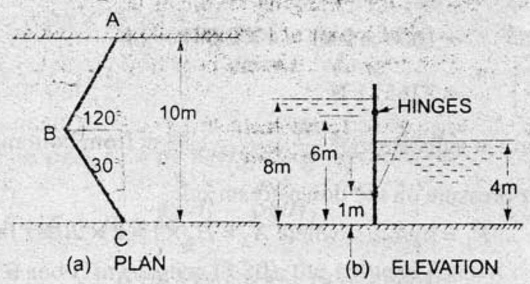

Fig. 3.42

Width of lock $= 10$ m

$\therefore$ Width of each lock $= \dfrac{5}{\cos 30}$ or $l = 5.773$ m

Depth of water on upstream side, $H_1 = 8$ m

Depth of water on downstream side, $H_2 = 4$ m

(*i*) **Water pressure on upstream side**

$$F_1 = \rho g A_1 \bar{h}_1$$

where $A_1 = l \times H_1 = 5.773 \times 8 = 46.184$ m, $\bar{h}_1 = \dfrac{H_1}{2} = \dfrac{8}{2} = 4.0$ m

$$F_1 = 1000 \times 9.81 \times 46.184 \times 4.0 = 1812260 \text{ N} = 1812.26 \text{ kN}$$

Water pressure on downstream side,

$$F_2 = \rho g A_2 \bar{h}_2$$

where $A_2 = l \times H_2 = 5.773 \times 4 = 23.092$ m, $\bar{h}_2 = \dfrac{4}{2} = 2.0$

$$F_2 = 1000 \times 9.81 \times 23.092 \times 2.0 = 453065 \text{ N} = 453.065 \text{ kN}$$

$\therefore$ Resultant water pressure

$$= F_1 - F_2 = 1812.26 - 453.065 = 1359.195 \text{ kN}$$

(*ii*) **Reaction between the gates AB and AC.** The reaction (P) between the gates AB and AC is given by equation (3.20) as

$$F = \frac{F}{2 \sin \theta} = \frac{1359.195}{2 \times \sin 30} = \mathbf{1359.195 \text{ kN. Ans.}}$$

(*iii*) **Force of each hinge.** If R_T and R_B are the reactions at the top and bottom hinges then

$$R_T + R_B = R$$

But from equation (3.19), $R = P = 1359.195$

$\therefore$ $R_T + R_B = 1359.195$

The force F_1 is acting at $\dfrac{H_1}{3} = \dfrac{8}{3} = 2.67$ m from bottom and F_2 at $\dfrac{H_2}{3} = \dfrac{4}{3} = 1.33$ m from bottom. The resultant force F will act at a distance x from bottom given by

$$F \times x = F_1 \times 2.67 - F_2 \times 1.33$$

or
$$x = \frac{F_1 \times 2.67 - F_2 \times 1.33}{F} = \frac{1812.26 \times 2.67 - 453.065 \times 1.33}{1359.195}$$

$$= \frac{4838.734 - 602.576}{1359.195} = 3.116 = 3.11 \text{ m}$$

Hence R is also acting at a distance 3.11m from bottom.

Taking moments of R_T and R about the bottom hinge

$$R_T \times [6.0 - 1.0] = R \times (x - 1.0)$$

$\therefore$
$$R_T = \frac{Rx(x - 1.0)}{5.0} = \frac{1359.195 \times 2.11}{5.0} = 573.58 \text{ N}$$

$\therefore$
$$R_B = R - R_T = 1359.195 - 573.58$$
$$= \mathbf{785.615 \text{ kN. Ans.}}$$

▶ 3.8 PRESSURE DISTRIBUTION IN A LIQUID SUBJECTED TO CONSTANT HORIZONTAL/VERTICAL ACCELERATION

In chapters 2 and 3, the containers which contains liquids, are assumed to be at rest. Hence the liquids are also at rest. They are in static equilibrium with respect to containers. But if the container containing a liquid is made to move with a constant acceleration, the liquid particles initially will move relative to each other and after some time, there will not be any relative motion between the liquid particles and boundaries of the container. The liquid will take up a new position under the effect of acceleration imparted to its container. The liquid will come to rest in this new position relative to the container. The entire fluid mass moves as a single unit. Since the liquid after attaining a new position is in static condition relative to the container, the laws of hydrostatic can be applied to determine the liquid pressure. As there is no relative motion between the liquid particles, hence the shear stresses and shear forces between liquid particles will be zero. The pressure will be normal to the surface in contact with the liquid.

The following are the important cases under consideration :

(*i*) Liquid containers subject to constant horizontal acceleration.

(*ii*) Liquid containers subject to constant vertical acceleration.

3.8.1 Liquid Containers Subject to Constant Horizontal Acceleration.

Fig. 3.43 (*a*) shows a tank containing a liquid upto a certain depth. The tank is stationary and free surface of liquid is horizontal. Let this tank is moving with a constant acceleration '*a*' in the horizontal direction towards right as shown in Fig. 3.43 (*b*). The initial free surface of liquid which was horizontal, now takes the shape as shown in Fig. 3.43 (*b*). Now *AB* represents the new free surface of the liquid. Thus the free surface of liquid due to horizontal acceleration will become a downward sloping inclined plane, with the liquid rising at the back end the liquid falling at the front end. The equation for the free liquid surface can be derived by considering the equilibrium of a fluid element C lying on the free surface. The forces acting on the element *C* are :

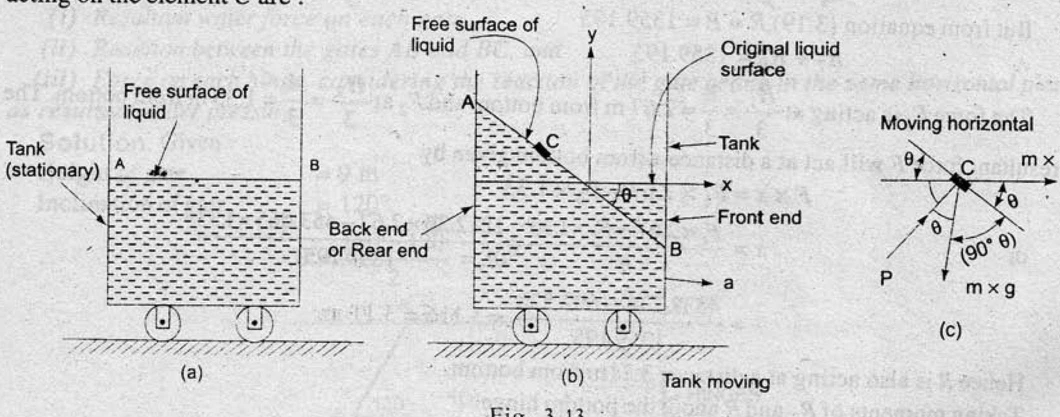

Fig. 3.43

(*i*) the pressure force *P* exerted by the surrounding fluid on the element *C*. This force is normal to the free surface.

(*ii*) the weight of the fluid element *i.e.*, $m \times g$ acting vertically downward.

(*iii*) accelerating force *i.e.*, $m \times a$ acting in horizontal direction.

Resolving the forces horizontally, we get

$$P \sin\theta + m \times a = 0$$

or $$P \sin\theta = -ma \quad ...(i)$$

Resolving the forces vertically, we get

$$P \cos\theta - mg = 0$$

or $$P \cos\theta = m \times g \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\tan\theta = -\frac{a}{g} \quad \left(\text{or } \frac{a}{g} \text{ Numerically}\right) \quad ...(3.20A)$$

The above equation, gives the slope of the free surface of the liquid which is contained in a tank which is subjected to horizontal constant acceleration. The term (a/g) is a constant and hence $\tan\theta$ will be constant. The –ve sign shows that the free surface of liquid is sloping downwards. Hence the free surface is a straight plane inclined down at an angle θ along the direction of acceleration.

Now let us find the expression for the pressure at any point *D* in the liquid mass subjected to horizontal acceleration. Let the point *D* is at a depth of '*h*' from the free surface. Consider an elementary prism *DE* of height '*h*' and cross-sectional area *dA* as shown in Fig. 3.44.

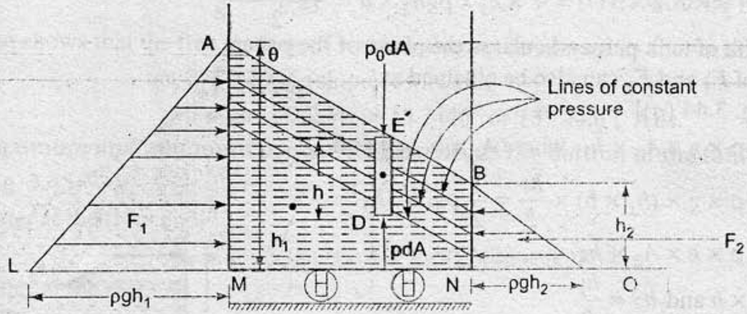

Fig. 3.44

Consider the equilibrium of the elementary prism *DE*.

The forces acting on this prism *DE* in the vertical direction are :

(*i*) the atmospheric pressure force ($p_0 \times dA$) at the top end of the prism acting downwards,

(*ii*) the weight of the element ($\rho \times g \times h \times dA$) at the C.G. of the element acting in the downward direction, and

(*iii*) the pressure force ($p \times dA$) at the bottom end of the prism acting upwards.

Since there is no vertical acceleration given to the tank, hence net force acting vertically should be zero.

$$\therefore \quad p \times dA - p_0 \times dA - \rho gh\, dA = 0$$

or $$p - p_0 - \rho gh = 0 \quad \text{or} \quad p = p_0 + \rho gh$$

or $$p - p_0 = \rho gh$$

or Gauge pressure at point *D* is given by

$$p = \rho gh$$

or pressure head at point *D*, $$\frac{p}{\rho g} = h.$$

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From the above equation, it is clear that pressure head at any point in a liquid subjected to a constant horizontal acceleration is equal to the height of the liquid column above that point. Therefore the pressure distribution in a liquid subjected to a constant horizontal acceleration is same as hydrostatic pressure distribution. The planes of constant pressure are therefore, parallel to the inclined surface as shown in Fig. 3.44. This Fig. 3.44 also shows the variation of pressure on the rear and front of the tank.

If h_1 = Depth of liquid at the rear end of the tank

h_2 = Depth of liquid at the front end of the tank

F_1 = Total pressure force exerted by liquid on the rear side of the tank

F_2 = Total pressure force exerted by liquid on the front side of the tank,

then F_1 = (Area of triangle AML) × Width

$$= (\tfrac{1}{2} \times LM \times AM \times b) = \tfrac{1}{2} \times \rho g h_1 \times h_1 \times b = \frac{\rho g . b . h_1^2}{2}$$

and F_2 = (Area of traingle BNO) × Width

$$= (\tfrac{1}{2} \times BN \times NO) = \tfrac{1}{2} \times h_2 \times \rho g h_2 \times b = \frac{\rho g . b . h_2^2}{2}$$

where b = Width of tank perpendicular to the plane of the paper.

The values of F_1 and F_2 can also be obtained as [Refer to Fig. 3.44 (a)]

$$F_1 = \rho \times g \times A_1 \times \bar{h}_1, \text{ where } A_1 = h_1 \times b \text{ and}$$

$$= \rho \times g \times (h_1 \times b) \times \frac{\bar{h}_1}{2} = \frac{1}{2} \rho g . b . h_1^2$$

and $F_2 = \rho \times g \times A_2 \times \bar{h}_2$

where $A_2 = h_2 \times b$ and $\bar{h}_2 = \dfrac{h_2}{2}$

$$= \rho \times g \times (h_2 \times b) \times \frac{h_2}{2}$$

$$= \frac{1}{2} \rho g . b \times h_2^2.$$

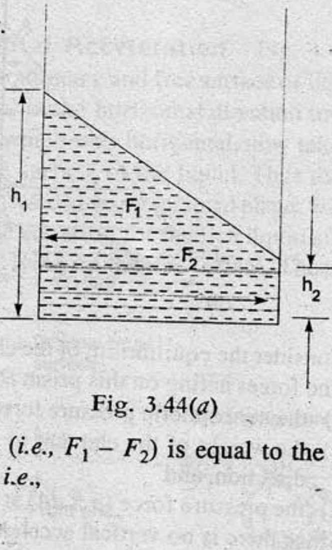

Fig. 3.44(a)

It can also be proved that the difference of these two forces ($i.e.$, $F_1 - F_2$) is equal to the force required to accelerate the mass of the liquid contained in the tank $i.e.$,

$$F_1 - F_2 = M \times a$$

where M = Total mass of the liquid contained in the tank

a = Horizontal constant acceleration.

Note : (i) If a tank completely filled with liquid and open at the top is subjected to a constant horizontal acceleration, then some of the liquid will spill out from the tank and new free surface with its slope given by equation $\tan\theta = -\dfrac{a}{g}$ will be developed.

(ii) If a tank partly filled with liquid and open at the top is subjected to a constant horizontal acceleration, spilling of the liquid may take place depending upon the magnitude of the acceleration.

(iii) If a tank completely filled with liquid and closed at the top is subjected to a constant horizontal acceleration, then the liquid would not spill out from the tank and also there will be no adjustment in the surface elevation of the liquid. But the equation $\tan\theta = -\dfrac{a}{g}$ is applicable for this case also.

(iv) The example for a tank with liquid subjected to a constant horizontal acceleration, is a fuel tank on an airplane during take off.

Problem 3.34 *A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of 2.4 m/s². The length, width and depth of the tank are 6 m, 2.5 m and 2 m respectively. If the depth of water in the tank is 1 m and tank is open at the top then calculate :*

(*i*) *the angle of the water surface to the horizontal,*

(*ii*) *the maximum and minimum pressure intensities at the bottom,*

(*iii*) *the total force due to water acting on each end of the tank.*

Solution. Given :

Constant acceleration, $a = 2.4\ m/s^2$.

Length = 6 m ; Width = 2.5 m and depth = 2 m.

Depth of water in tank, $h = 1$ m

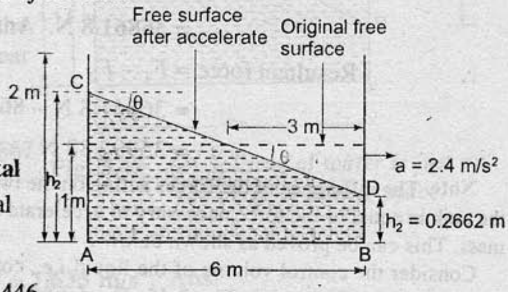

Fig. 3.45

(*i*) **The angle of the water surface to the horizontal**

Let θ = the angle of water surface to the horizontal

Using equation (3.20), we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig. 3.45)

$\therefore$ $\tan\theta = 0.2446$ (slope downward)

$\therefore$ $\theta = \tan^{-1} 0.2446 = 13.7446°$ or **13° 44.6′. Ans.**

(*ii*) **The maximum and minimum pressure intensities at the bottom of the tank**

From the Fig. 3.45,

Depth of water at the front end,

$$h_1 = 1 - 3\tan\theta = 1 - 3 \times 0.2446 = 0.2662 \text{ m}$$

Depth of water at the rear end,

$$h_2 = 1 + 3\tan\theta = 1 + 3 \times 0.2446 = 1.7338 \text{ m}$$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now the maximum pressure intensity at the bottom will be at point *A* and it is given by,

$$p_{max} = \rho \times g \times h_2$$
$$= 1000 \times 9.81 \times 1.7338 \text{ N/m}^2 = \mathbf{17008.5\ N/m^2}.\ \textbf{Ans.}$$

The minimum pressure intensity at the bottom will be at point B and it is given by

$$p_{min} = \rho \times g \times h_1$$
$$= 1000 \times 9.81 \times 0.2662 = \mathbf{2611.4\ N/m^2}.\ \textbf{Ans.}$$

(*iii*) **The total force due to water acting on each end of the tank**

Let F_1 = total force acting on the front side (*i.e.*, on face *BD*)

F_2 = total force acting on the rear side (*i.e.*, on face *AC*)

Then $F_1 = \rho g A_1 \bar{h}_1$, where $A_1 = BD \times$ width of tank $= h_1 \times 2.5 = 0.2662 \times 2.5$

and $\bar{h}_1 = \frac{BD}{2} = \frac{h_1}{2} = \frac{0.2662}{2} = 0.1331$ m

$$= 1000 \times 9.81 \times (0.2662 \times 2.5) \times 0.1331$$
$$= \textbf{868.95 N. Ans.}$$

and $F_2 = \rho.g.A_2.\bar{h}_2$, where $A_2 = AB \times$ width of tank $= h_2 \times 2.5 = 1.7338 \times 2.5$

$$\bar{h}_2 = \frac{AB}{2} = \frac{h_2}{2} = \frac{1.7338}{2} = 0.8669 \text{ m}$$

$$= 1000 \times 9.81 \times (1.7338 \times 2.5) \times 0.8669$$

$$= \mathbf{36861.8 \text{ N. Ans.}}$$

$\therefore$ Resultant force $= F_1 - F_2$

$$= 36861.8 \text{ N} - 868.95$$

$$= 35992.88 \text{ N}$$

Note. The difference of the forces acting on the two ends of the tank is equal to the force necessary to accelerate the liquid mass. This can be proved as shown below :

Consider the control volume of the liquid *i.e.*, control volume is *ACDBA* as shown in Fig. 3.46. The net force acting on the control volume in the horizontal direction must be equal to the product of mass of the liquid in control volume and acceleration of the liquid.

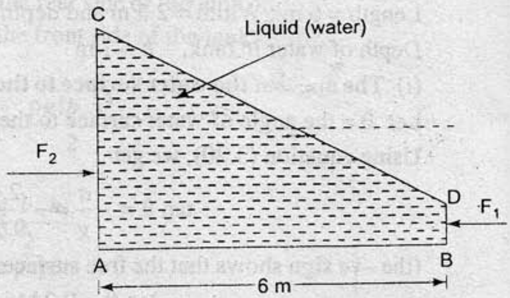

Fig. 3.46

$$\therefore \quad (F_1 - F_2) = M \times a$$

$$= (\rho \times \text{volume of control volume}) \times a$$

$$= (1000 \times \text{Area of } ABDCE \times \text{width}) \times 2.4$$

$$= \left[1000 \times \left(\frac{AC + BD}{2}\right) \times AB \times \text{width}\right] \times 2.4$$

$$\left[\because \text{Area of trapezium} = \left(\frac{AC + BD}{2}\right) \times AB\right]$$

$$= 1000 \times \left(\frac{1.7338 + 0.2662}{2}\right) \times 6 \times 2.5 \times 2.4$$

$$= 36000 \text{ N}$$

($\because AC = h_2 = 1.7338$ m, $BD = h_1 = 0.2662$ m, and $AB = 6$ m, width = 2.5 m)

The above force is nearly the same as the difference of the forces acting on the two ends of the tank. (*i.e.*, 35992.88 ≃ 36000).

Problem 3.35 *The rectangular tank of the above problem contains water to a depth of 1.5 m. Find the horizontal acceleration which may be imparted to the tank in the direction of its length so that*

(*i*) *the spilling of water from the tank is just on the verge of taking place,*

(*ii*) *the front bottom corner of the tank is just exposed,*

(*iii*) *the bottom of the tank is exposed upto its mid-point.*

Also calculate the total forces exerted by the water on each end of the tank in each case. Also prove that the difference between these forces is equal to the force necessary to accelerate the mass of water tank.

Solution. Given :

Dimensions of the tank from previous problem,

$$L = 6 \text{ m, width } (b) = 2.5 \text{ m and depth} = 2 \text{ m}$$

Depth of water in tank, $h = 1.5$ m

(*i*) **Horizontal acceleration imparted to the tank**

(*a*) *When the spilling of water from the tank is just on the verge of taking place*

Let a = required horizontal acceleration

When the spilling of water from the tank is just on the verge of taking place, the water would rise upto the rear top corner of the tank as shown in Fig. 3.47 (*a*)

$$\therefore \quad \tan\theta = \frac{AC}{AO} = \frac{(2-1.5)}{3} = \frac{0.5}{3} = 0.1667$$

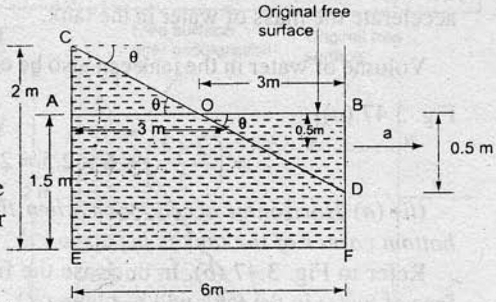

Fig. 3.47 (*a*) *Spilling of water is just on the verge of taking place.*

But from equation (3.20) $\tan\theta = \frac{a}{g}$ (Numerically)

$$\therefore \quad a = g \times \tan\theta = 9.81 \times 0.1667 = \mathbf{1.635\ m/s^2.\ Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end *CE* of the tank is

$$F_1 = \rho g A_1 \bar{h}_1, \text{ where } A_1 = CE \times \text{width of the tank} = 2 \times 2.5$$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1 \text{ m}$$

$$= 1000 \times 9.81 \times (2 \times 2.5) \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end *FD* of the tank is

$$F_2 = \rho g A_2 \times \bar{h}_2, \text{ where } A_2 = FD \times \text{width} = 1 \times 2.5$$

$$(\because AC = BD = 0.5 \text{ m}, \quad \therefore \quad FD = BF - BD = 1.5 - 0.5 = 1)$$

$$= 1000 \times 9.81 \times (1 \times 2.5) \times 0.5 \qquad \bar{h}_2 = \frac{FD}{2} = \frac{1}{2} = 0.5 \text{ m}$$

$$= \mathbf{12262.5\ N.\ Ans.}$$

(*c*) *Difference of the forces is equal to the force necessary to accelerate the mass of water in the tank*

Difference of the forces $= F_1 - F_2$

$$= 49050 - 12262.5 = 36787.5 \text{ N}$$

Volume of water in the tank before acceleration is imparted to it $= L \times b \times$ depth of water

$$= 6 \times 2.5 \times 1.5 = 22.5 \text{ m}^3.$$

The force necessary to accelerate the mass of water in the tank

= Mass of water in tank × Acceleration

$= (\rho \times \text{volume of water}) \times 1.635$ $\quad (\because a = 1.635 \text{ m/s}^2)$

$= 1000 \times 22.5 \times 1.635$ [There is no spilling of water and volume of water = 22.5 m^3]

$= 36787.5$ N

Hence the difference between the forces on the two ends of the tank is equal to the force necessary to accelerate the mass of water in the tank.

Volume of water in the tank can also be calculated as volume = $\left(\frac{CE + FD}{2}\right) \times EF \times$ Width [Refer to Fig. 3.47 (*a*)]

$$= \left(\frac{2+1}{2}\right) \times 6 \times 2.5 = 22.5 \text{ m}^3.$$

(*ii*) (*a*) *Horizontal acceleration when the front bottom corner of the tank is just exposed.*

Refer to Fig. 3.47 (*b*). In this case the free surface of water in the tank will be along *CD*.

Let a = required horizontal acceleration.

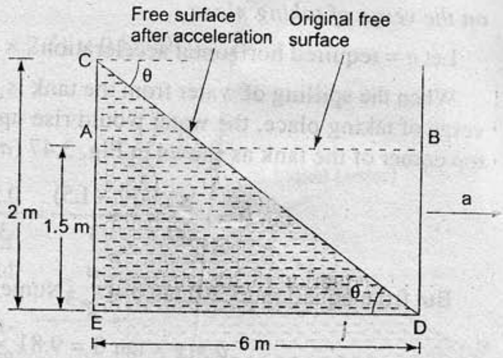

Fig. 3.47 (*b*)

In this case, $\tan\theta = \frac{CE}{ED} = \frac{2}{6} = \frac{1}{3}$

But from equation (3.17),

$$\tan\theta = \frac{a}{g} \text{ (Numerically)}$$

$\therefore$ $a = g \times \tan\theta = 9.81 \times \frac{1}{3} =$ **3.27 m/s². Ans.**

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end *CE* of the tank is

$$F_1 = \rho g \times A_1 \times \bar{h}_1$$

where $A_1 = CE \times \text{width} = 2 \times 2.5 = 5 \text{ m}^2$

$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1$ m $\quad = 1000 \times 9.81 \times 5 \times 1$

= **49050 N. Ans.**

The force exerted by water on the end *BD* of the tank is zero as there is no water against the face *BD*

$\therefore$ $F_2 = 0$

$\therefore$ Difference of forces = 49050 − 0 = 49050 N

(*c*) *Difference of forces is equal to the force necessary to accelerate the mass of water in the tank.*

Volume of water in the tank = Area of *CED* × Width of tank

$$= \left(\frac{CE \times ED}{2}\right) \times 2.5 \qquad (\because \text{ Width of tank} = 2.5 \text{ m})$$

$$= \frac{2 \times 6}{2} \times 2.5 = 15 \text{ m}^3$$

$\therefore$ Force necessary to accelerate the mass of water in the tank

= Mass of water in tank × Acceleration

= (1000 × Volume of water) × 3.27

= 1000 × 15 × 3.27 = 49050 N

Difference of two forces is also = 49050 N

Hence difference between the forces on the two ends of the tank is equal to the force necessary to accelerate the mass of water in the tank.

(iii)* (*a*) *Horizontal acceleration when the bottom of the tank is exposed upto its mid-point.

Refer to Fig. 3.47 (*c*). In this case the free surface of water in the tank will be along CD^*, where D^* is the mid-point of ED.

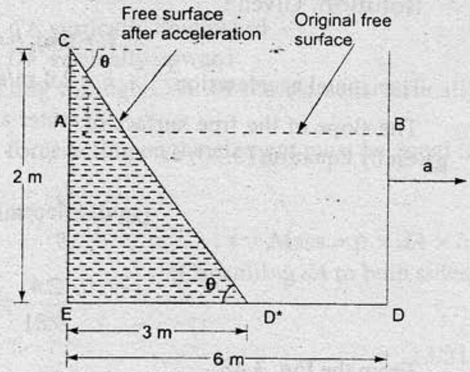

Fig. 3.47 (*c*)

Let a = required horizontal acceleration from Fig. 3.47 (*c*), it is clear that

$$\tan\theta = \frac{CE}{ED} = \frac{2}{3}$$

But from equation (3.20) numerically

$$\tan\theta = \frac{a}{g}$$

$$\therefore \qquad a = g \times \tan\theta = 9.81 \times \frac{2}{3} = \mathbf{6.54\ m/s^2}.\ \textbf{Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end CE of the tank is

$$F_1 = \rho \times g \times A_1 \times \bar{h}_1$$

where $A_1 = CE \times \text{Width} = 2 \times 2.5 = 5\ \text{m}^2$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1\ \text{m}$$

$$= 1000 \times 9.81 \times 5 \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end BD is zero as there is no water against the face BD.

$$\therefore \qquad F_2 = 0$$

$\therefore$ Difference of the forces $= F_1 - F_2 = 49050 - 0 = 49050$ M

(*c*) *Difference of the two forces is equal to the force necessary to accelerate the mass of water remaining in the tank.*

Volume of water in the tank = Area CED × Width of tank

$$= \frac{CE \times ED}{2} \times 2.5 = \frac{2 \times 3}{2} \times 2.5 = 7.5\ \text{m}^3$$

Force necessary to accelerate the mass of water in the tank

= Mass of water × Acceleration

$= \rho \times$ Volume of water $\times 6.54$ $\qquad (\because a = 6.54\ \text{m/s}^2)$

$= 1000 \times 7.5 \times 6.54$

$= 49050$ N

This is the same force as the difference of the two forces on the two ends of the tank.

Problem 3.36 *A rectangular tank of length 6 m, width 2.5 and height 2 m is completely filled with water when at rest. The tank is open at the top. The tank is subjected to a horizontal constant linear acceleration of 2.4 m/s² in the direction of its length. Find the volume of water spilled from the tank*

Solution. Given :

$L = 6$ m, $b = 2.5$ m and height, $H = 2$ m

Horizontal acceleration, $a = 2.4$ m/s^2.

The slope of the free surface of water after the tank is subjected to linear constant acceleration is given by equation (3.20) as

$$\tan\theta = \frac{a}{g} \text{ (Numerically)}$$

$$= \frac{2.4}{9.81} = 0.2446$$

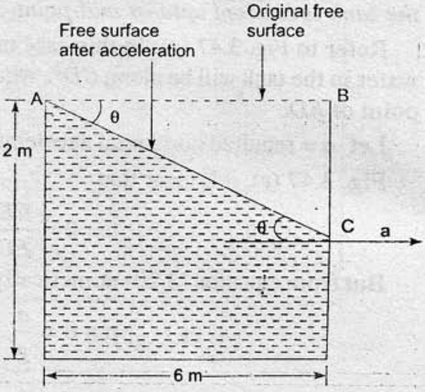

Fig. 3.48

From the Fig. 3.48,

$$\tan\theta = \frac{BC}{AB}$$

$\therefore$ $BC = AB \times \tan\theta$

$= 6 \times 0.2446$

$(\because AB = \text{Length} = 6 \text{ m} ;\ \tan\theta = 0.2446)$

$= 1.4676$ m

$\therefore$ Volume of water spilled = Area of $ABC \times$ Width of tank

$= \left(\frac{1}{2} \times AB \times BC\right) \times 2.5$ $(\because$ Width = 2.5 m)

$= \frac{1}{2} \times 6 \times 1.4676 \times 2.5$ $(\because BC = 1.4676$ m)

$=$ **11.007 m^3. Ans.**

3.8.2 Liquid Container Subjected to Constant Vertical Acceleration. Fig. 3.49 shows a tank containing a liquid and the tank is moving vertically upward with a constant acceleration. The liquid in the tank will be subjected to the same vertical acceleration. To obtain the expression for the pressure at any point in the liquid mass subjected to vertical upward acceleration, consider a vertical elementary prism of liquid *CDFE*.

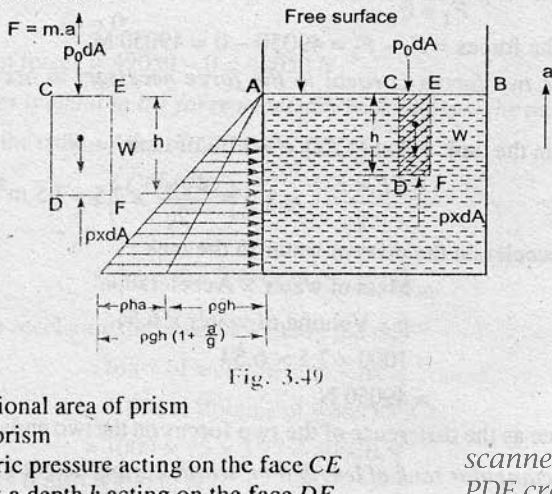

Fig. 3.49

Let dA = Cross-sectional area of prism

h = Height of prism

p_0 = Atmospheric pressure acting on the face CE

p = Pressure at a depth h acting on the face DF

The forces acting on the elementary prism are :

(*i*) Pressure force equal to $p_0 \times dA$ acting on the face *CE* vertically downward

(*ii*) Pressure force equal to $p_0 \times dA$ acting on the face *DF* vertically upward

(*iii*) Weight of the prism equal to $\rho \times g \times dA \times h$ acting through C.G. of the element vertically downward.

According to Newton's second law of motion, the net force acting on the element must be equal to mass multiplied by acceleration in the same direction.

$\therefore$ Net force in vertically upward direction = Mass × acceleration

$$p \times dA - p_0 \times dA - \rho g dA \,.\, h = (\rho \times dA \times h) \times a \qquad (\because \text{Mass} = \rho \times dA \times h)$$

or
$$p - p_0 - \rho g h = \rho h \times a \qquad \text{(Cancelling } dA \text{ to both sides)}$$

or
$$p - p_0 = \rho g h + \rho h a$$

$$= \rho g h \left[1 + \frac{a}{g}\right] \qquad \text{...(3.21)}$$

But $(p - p_0)$ is the gauge pressure. Hence gauge pressure at any point in the liquid mass subjected to a constant vertical upward acceleration, is given by

$$p_g = \rho g h \left[1 + \frac{a}{g}\right] \qquad \text{...(3.22)}$$

$$= \rho g h + \rho h a \qquad \text{...(3.22A)}$$

where $p_g = p - p_0$ = gauge pressure

In the equation (3.22) ρ, g and a are constant. Hence variation of guage pressure is linear. Also when $h = 0$, $p_g = 0$. This means $p - p_0 = 0$ or $p = p_0$. Hence when $h = 0$, the pressure is equal to atmospheric pressure. Hence free surface of liquid subjected to constant vertical acceleration will be horizontal.

From the equation (3.22*A*) it is also clear that the pressure at any point in the liquid mass is greater than the hydrostatic pressure (hydrostatic pressure is = $\rho g h$) by an amount of $\rho \times h \times a$.

The Fig. 3.49 shows the variation of pressure for the liquid mass subjected to a constant vertical upward acceleration.

If the tank containing liquid is moving vetically downward with a constant acceleration, then the gauge pressure at any point in the liquid at a depth of *h* from the free surface will be given by

$$(p - p_0) = \rho g h \left[1 - \frac{a}{g}\right] = \rho g h - \rho h a \qquad \text{...(3.23)}$$

The above equation shows that the pressure at any point in the liquid mass is less than the hydrostatic pressure by an amount of $\rho h a$. The Fig. 3.50 shows the variation of pressure for the liquid mass subjected to a constant vertical downward acceleration.

If the tank containing liquid is moving downward with a constant acceleration equal to *g* (*i.e.*, when $a = g$), then equation reduces to $p - p_0 = 0$ or $p = p_0$. This means the pressure at any point is the liquid is equal to surrounding atmospheric pressure. There will be no force on the walls or on the base of the tank.

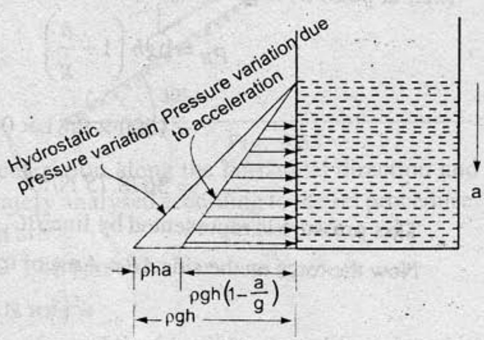

Fig. 3.50

Note. If a tank containing a liquid is subjected to a constant acceleration in the inclined direction, then the acceleration may be resolved along the horizontal direction and vertical directions. Then each of these cases may be separately analysed in accordance with the above procedure.

Problem 3.37 *A tank containing water upto a depth of 500 mm is moving vertically upward with a constant acceleration of 2.45 m/s². Find the force exerted by water on the side of the tank. Also calculate the force on the side of the tank when the width of tank is 2 m and*

(i) tank is moving vertically downward with a constant acceleration of 2.45 m/s², and

(ii) the tank is not moving at all.

Solution. Given :

Depth of water, $h = 500 \text{ mm} = 0.5 \text{ m}$

Vertical acceleration, $a = 2.45 \text{ m/s}^2$

Width of tank, $b = 2 \text{ m}$

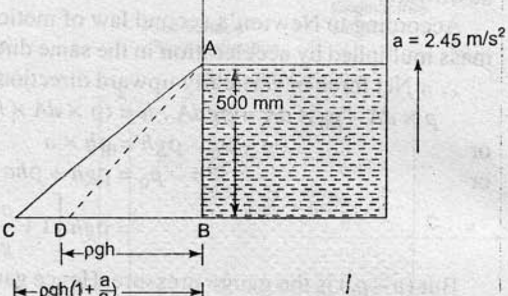

Fig. 3.51

To find the force exerted by water on the side of the tank when moving vertically upward, let us first find the pressure at the bottom of the tank.

The gauge pressure at the bottom (*i.e.*, at point B) for this case is given by equation as

$$p_B = \rho g h \left(1 + \frac{a}{g}\right)$$

$$= 1000 \times 9.81 \times 0.5 \left(1 + \frac{2.45}{9.81}\right) = 6131.25 \text{ N/m}^2$$

This pressure is represented by line BC.

Now the force on the side AB = Area of triangle $ABC \times$ Width of tank

$$= \left(\tfrac{1}{2} \times AB \times BC\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 6131.25\right) \times 2 \qquad (\because BC = 6131.25 \text{ and } b = 2 \text{ m})$$

$$= \mathbf{3065.6 \text{ N. Ans.}}$$

(*i*) **Force on the side of the tank, when tank is moving vertically downward.**

The pressure variation is shown in Fig. 3.52. For this case, the pressure at the bottom of the tank (*i.e.*, at point B) is given by equation (3.23) as

$$p_B = \rho g h \left(1 - \frac{a}{g}\right)$$

$$= 1000 \times 9.81 \times 0.5 \left(1 - \frac{2.45}{9.81}\right)$$

$$= 3678.75 \text{ N/m}^2.$$

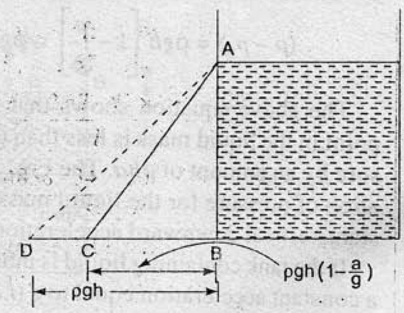

Fig. 3.52

This pressure is represented by line BC.

Now the force on the side AB = Area of triangle $ABC \times$ Width

$$= \left(\tfrac{1}{2} \times AB \times BC\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 3678.75\right) \times 2 \qquad (\because BC = 3678.75,\ b = 2)$$

$$= \mathbf{1839.37 \text{ N. Ans.}}$$

(*ii*) **Force on the side of the tank, when tank is stationary.**

The pressure at point B is given by,

$$p_B = \rho g h = 1000 \times 9.81 \times 0.5 = 4905 \text{ N/m}^2$$

This pressure is represented by line BD in Fig. 3.52

Force on the side AB = Area of triangle ABD × Width

$$= \left(\tfrac{1}{2} \times AB \times BD\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 4905\right) \times 2 \qquad (\because BD = 4905)$$

$$= \mathbf{2452.5 \text{ N. Ans.}}$$

For this case, the force on AB can also be obtained as

$$F_{AB} = \rho g A.\bar{h}$$

where $A = AB \times \text{Width} = 0.5 \times 2 = 1 \text{ m}^2$

$$\bar{h} = \frac{AB}{2} = \frac{0.5}{2} = 0.25 \text{ m} = 1000 \times 9.81 \times 1 \times 0.25$$

$$= \mathbf{2452.5 \text{ N. Ans.}}$$

Problem 3.38 *A tank contains water upto a depth of 1.5 m. The length and width of the tank are 4 m and 2 m respectively. The tank is moving up an inclined plane with a constant acceleration of 4 m/s². The inclination of the plane with the horizontal is 30° as shown in Fig. 3.53. Find,*

(i) the angle made by the free surface of water with the horizontal.

(ii) the pressure at the bottom of the tank at the front and rear ends.

Solution. Given :

Depth of water, $h = 1.5$ m ; Length, $L = 4$ m and Width, $b = 2$ m

Constant acceleration along the inclined plane,

$$a = 4 \text{ m/s}^2$$

Inclination of plane, $\alpha = 30°$

Let θ = Angle made by the free surface of water after the acceleration is imparted to the tank

p_A = Pressure at the bottom of the tank at the front end and

p_D = Pressure at the bottom of the tank at the rear end.

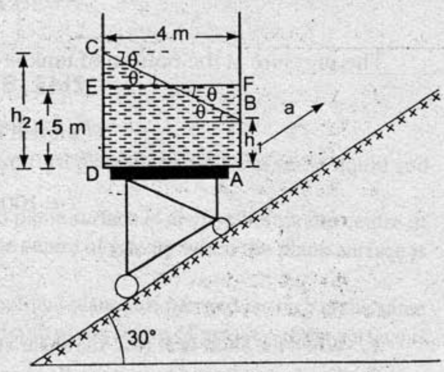

Fig. 3.53

This problem can be done by resolving the given acceleration along the horizontal direction and vertical directions. Then each of these cases may be separately analysed according to the set procedure.

Horizontal and vertical components of the acceleration are :

$$a_x = a \cos \alpha = 4 \cos 30° = 3.464 \text{ m/s}^2$$

$$a_y = a \sin \alpha = 4 \sin 30° = 2 \text{ m/s}^2$$

When the tank is stationary on the inclined plane, free surface of liquid will be along EF as shown in Fig. 3.53. But when the tank is moving upward along the inclined plane the free surface of liquid will be along BC. When the tank containing a liquid is moving up an inclined plan with a constant acceleration, the angle made by the free surface of the liquid with the horizontal is given by

$$\tan\theta = \frac{a_x}{a_y + g} = \frac{3.464}{2 + 9.81} = 0.2933$$

$$\therefore \quad \theta = \tan^{-1} 0.2933 = \mathbf{16.346^\circ} \quad \textbf{or} \quad \mathbf{16^\circ\ 20.8'}. \quad \textbf{Ans.}$$

Now let us first find the depth of liquid at the front and rear end of the tank.

Depth of liquid at front end = $h_1 = AB$

Depth of liquid at rear end = $h_2 = CD$

From the Fig. 5.21, in triangle COE, $\tan\theta = \dfrac{CE}{EO}$

or $\quad CE = EO \tan\theta = 2 \times 0.2933 \qquad (\because EO = 2 \text{ m}, \tan\theta = 0.2933)$

$= 0.5866$ m

$\therefore \quad CD = h_2 = ED + CE = 1.5 + 0.5866 = 2.0866$ m

Similarly $\quad h_1 = AB = AF - BF$

$= 1.5 - 0.5866 \qquad (\because AF = 1.5, BF = CE = 0.5866)$

$= 0.9134$ m

The pressure at the bottom of tank at the rear end is given by,

$$p_D = \rho g h_2 \left(1 + \frac{a_y}{g}\right)$$

$$= 1000 \times 9.81 \times 2.0866 \left(1 + \frac{2}{9.81}\right) = \mathbf{24642.7\ N/m^2}. \textbf{ Ans.}$$

The pressure at the bottom of tank at the front end is given by

$$p_A = \rho g h_1 \left(1 + \frac{a_y}{g}\right)$$

$$= 1000 \times 9.81 \times 0.9134 \left(1 + \frac{2}{9.81}\right) = \mathbf{10787.2\ N/m^2}. \textbf{ Ans.}$$

HIGHLIGHTS

1. When the fluid is at rest, the shear stress is zero.
2. The force exerted by a static fluid on a vertical, horizontal or an inclined plane immersed surface,

$$F = \rho g A \bar{h}$$

where ρ = Density of the liquid,

A = Area of the immersed surface, and

$\bar{h}$ = Depth of the centre of gravity of the immersed surface from free surface of the liquid.

3. Centre of pressure is defined as the point of application of the resultant pressure.
4. The depth of centre of pressure of an immersed surface from free surface of the liquid,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \text{for vertically immersed surface.}$$

$$= \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{for inclined immersed surface.}$$

5. The centre of pressure for a plane vertical surface lies at a depth of two-third the height of the immersed surface.

6. The total force on a curved surface is given by $F = \sqrt{F_x^2 + F_y^2}$

where F_x = Horizontal force on curved surface and is equal to total pressure force on the projected area of the curved surface on the vertical plane,

$= \rho g A \bar{h}$

and F_y = Vertical force on submerged curved surface and is equal to the weight of liquid actually or imaginary supported by the curved surface.

7. The inclination of the resultant force on curved surface with horizontal, $\tan \theta = \frac{F_y}{F_x}$.

8. The resultant force on a sluice gate, $F = F_1 - F_2$

where F_1 = Pressure force on the upstream side of the sluice gate and

F_2 = Pressure force on the downstream side of the sluice gate.

9. For a lock gate, the reaction between the two gates is equal to the reaction at the hinge, $R = P$.

Also the reaction between the two gates, $P = \frac{F}{2 \sin \theta}$

where F = Resultant water pressure on the lock gate $= F_1 - F_2$

and θ = Inclination of the gate with the normal to the side of the lock.

EXERCISE 3

(A) THEORETICAL PROBLEMS

1. What do you understand by 'Total Pressure' and 'Centre of Pressure' ?
2. Derive an expression for the force exerted on a sub-merged vertical plane surface by the static liquid and locate the position of centre of pressure.
3. Prove that the centre of pressure of a completely sub-merged plane surface is always below the centre of gravity of the sub-merged surface or at most coincide with the centre of gravity when the plane surface is horizontal.
4. Prove that the total pressure exerted by a static liquid on an inclined plane sub-merged surface is the same as the force exerted on a vertical plane surface as long as the depth of the centre of gravity of the surface is unaltered.
5. Derive an expression for the depth of centre of pressure from free surface of liquid of an inclined plane surface sub-merged in the liquid.
6. (*a*) How would you determine the horizontal and vertical components of the resultant pressure on a sub-merged curved surface ?

   (*b*) Explain the procedure of finding hydrostatic forces on curved surfaces.

   (Delhi University, Dec. 2002)
7. Explain how you would find the resultant pressure on a curved surface immersed in a liquid.

   (A.M.I.E., Summer 1981)
8. Why the resultant pressure on a curved sub-merged surface is determined by first finding horizontal and vertical forces on the curved surface ? Why is the same method not adopted for a plane inclined surface sub-merged in a liquid ?

9. Describe briefly with sketches the various methods used for measured pressure exerted by fluids. *(A.M.I.E., Summer 1980)*
10. Prove that the vertical component of the resultant pressure on a sub-merged curved surface is equal to the weight of the liquid supported by the curved surface.
11. What is the difference between sluice gates and lock gates ?
12. Prove that the reaction between the gates of a lock is equal to the reaction at the hinge.
13. Derive an expression for the reaction between the gates as $P = \dfrac{F}{2 \sin \theta}$
where F = Resultant water pressure on lock gate, θ = inclination of the gate with normal to the side of the lock.
14. When will centre of pressure and centre of gravity of an immersed plane surface coincide ? *(A.M.I.E., Summer 1990)*
15. Find an expression for the force exerted and centre of pressure for a completely sub-merged inclined plane surface. Can the same method be applied for finding the resultant force on a curved surface immersed in the liquid ? If not, why ? *(Delhi University, 1992)*
16. What do you understand by the hydrostatic equation ? With the help of this equation derive the expressions for the total thrust on a sub-merged plane area and the buoyant force acting on a sub-merged body. *(A.M.I.E., Summer 1990)*

(B) NUMERICAL PROBLEMS

1. Determine the total pressure and depth of centre of pressure on a plane rectangular surface of 1 m wide and 3 m deep when its upper edge is horizontal and (*a*) coincides with water surface (*b*) 2 m below the free water surface. **[Ans.** (*a*) 44145 N, 2.0 m, (*b*) 103005 N, 3.714 m**]**
2. Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that centre of plate is 2 m below the free surface of water. Find the position of centre of pressure also. **[Ans.** 34668.54 N, 2.07 m**]**
3. A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 6 m in length and depth of centroid of area is 8 m below the water surface. Prove that the depth of centre of pressure is given by 8.475 m.
4. A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate : (*i*) the force on the disc, and (*ii*) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 6 m. **[Ans.** (*i*) 416.05 kN (*ii*) 39005 Nm**]**
5. The pressure at the centre of a pipe of diameter 3 m is 29.43 N/cm^2. The pipe contains oil of sp. gr. 0.87 and is filled with a gate valve. Find the force exerted by the oil on the gate and position of centre of pressure. **[Ans.** 2.08 MN, .016 m below centre of pipe**]**
6. Determine the total pressure and centre of pressure on an isosceles triangular plate of base 5 m and altitude 5 m when the plate is immersed vertically is an oil of sp. gr. 0.8. The base of the plate is 1 m below the free surface of water. **[Ans.** 261927 N, 3.19 m**]**
7. The opening in a dam is 3 m wide and 2 m high. A vertical sluice gate is used to cover the opening. On the upstream of the gate, the liquid of sp. gr. 1.5, lies upto a height of 2.0 m above the top of the gate, whereas on the downstream side, the water is available upto a height of the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Assume that the gate is higher at the bottom. **[Ans.** 206010 N, 0.964 m above the hinge**]**

8. A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 12 m wide at the bottom and 8 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is 1 m below the top level of the caisson and dock is empty. *(A.M.I.E., Winter 1980)*
[**Ans.** 3.164 MN, 4.56 m below water surface]

9. A sliding gate 2 m wide and 1.5 m high lies in a vertical plane and has a co-efficient of friction of 0.2 between itself and guides. If the gate weighs one tonne, find the vertical force required to raise the gate if its upper edge is at a depth of 4 m from free surface of water. [**Ans.** 37768.5 N]

10. A tank contains water upto a height of 1 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1.5 m height. Calculate : (*i*) total pressure on one side of the tank, (*ii*) the position of centre of pressure for one side of the tank, which is 3 m wide. [**Ans.** 76518 N, 1.686 m from top]

11. A rectangular tank 4 m long, 1.5 m wide contains water upto a height of 2 m. Calculate the force due to water pressure on the base of the tank. Find also the depth of centre of pressure from free surface.
[**Ans.** 117720 N, 2 m from free surface]

12. A rectangular plane surface 1 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface. [**Ans.** 80932.5 N, 2.318 m]

13. A circular plate 3.0 m diameter is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface.
[**Ans.** 228.69 kN, 3.427 m from free surface]

14. A rectangular gate 6 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.54. To keep the gate in a stable position, a counter weight of 29430 N is attached at the upper end of the gate. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and also friction at the hinge and pulley. [**Ans.** 3.43 m]

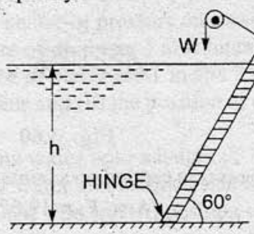

Fig. 3.54

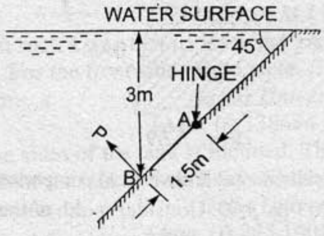

Fig. 3.55

15. An inclined rectangular gate of width 5 m and depth 1.5 m is installed to control the discharge of water as shown in Fig. 3.55. The end *A* is hinged. Determine the force normal to the gate applied at *B* to open it.
[**Ans.** 97435.8 N]

16. A gate supporting water is shown in Fig. 3.56. Find the height '*h*' of the water so that the gate begins to tip about the hinge. Take the width of the gate as unity. *(Delhi University, 1986)*
[**Ans.** $3 \times \sqrt{3}$ m]

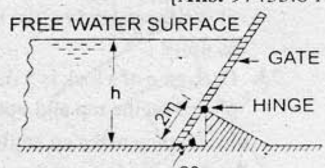

Fig. 3.56

17. Find the total pressure and depth of centre of pressure on a triangular plate of base 3 m and height 3 m which is immersed in water in such a way that plane of the plate makes an angle of 60° with the free surface. The base of the plate is parallel to water surface and at a depth of 2 m from water surface. [**Ans.** 126.52 kN, 2.996 m]

18. Find the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.57. Take the width of the gate 2 m. [**Ans.** $F_x = 117.72$ kN, $F_y = 140.114$ kN]

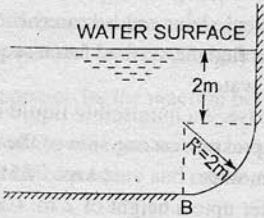

Fig. 3.57

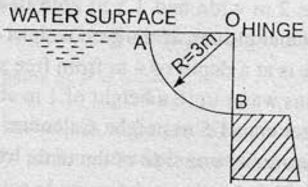

Fig. 3.58

19. Fig. 3.58 shows a gate having a quadrant shape of radius of 3 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act. [**Ans.** 82.201 kN, $\theta = 57° 31'$]

20. A roller gate is shown in Fig. 3.59. It is cylindrical form of 6.0 m diameter. It is placed on the dam. Find the magnitude and direction of the resultant force due to water acting on the gate when the water is just going to spill. The length of the gate is given 10 m. [**Ans.** 2.245 MN, $\theta = 38° 8'$]

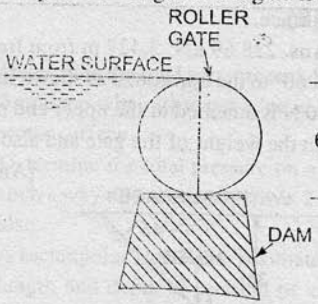

Fig. 3.59

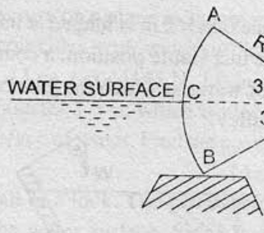

Fig. 3.60

21. Find the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 4 m as shown in Fig. 3.60. Consider width of the gate unity. [**Ans.** $F_x = 19.62$ kN, $F_y = 7102.44$ N]

22. Find the magnitude and direction of the resultant water pressure acting on a curved face of a dam which is shaped according to the relation $y = \frac{x^2}{6}$ as shown in Fig. 3.61. The height of water retained by the dam is 12 m. Take the width of dam as unity. [**Ans.** 970.74 kN, $\theta = 43° 19'$]

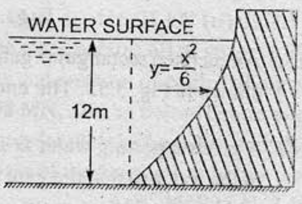

Fig. 3.61

23. Each gate of a lock is 5 m high and is supported by two hinges placed on the top and bottom of the gate. When the gates are closed, they make an angle of 120°. The width of the lock is 4 m. If the depths of water on the two sides of the gates are 4 m and 3 m respectively, determine : (*i*) the magnitude of resultant pressure on each gate, and (*ii*) magnitude of the hinge reactions. [**Ans.** (*i*) 79.279 kN (*ii*) $R_T = 27.924$ kN, $R_B = 51.355$ kN]

24. The end gates *ABC* of a lock are 8 m high and where closed make an angle of 120°. The width of lock is 10 m. Each gate is supported by two hinges located at 1 m and 5 m above the bottom of the lock. The depth of water on the upstream and downstream sides of the lock are 6 m and 4 m respectively. Find :

(*i*) Resultant water force on each gate.

(ii) Reaction between the gates AB and BC, and

(iii) Force on each hinge, considering the reaction of the gate acting in the same horizontal plane as resultant water pressure. [**Ans.** 566.33 kN, (ii) 566.33 kN and (iii) $R_T = 173.64$ kN, $R_B = 392.69$ kN]

25. A hollow circular plate of 2 m external and 1 m internal diameter is immersed vertically in water such that the centre of plate is 4 m deep from water surface. Find the total pressure and depth of centre of pressure. *(Punjab, 1972)* [**Ans.** 92.508 kN, 4.078 m]

26. A rectangular opening 2 m wide and 1 m deep in the vertical side of a tank is closed by a sluice gate of the same size. The gate can turn about the horizontal centroidal axis. Determine : (i) the total pressure on the sluice gate and (ii) the torque on the sluice gate. The head of water above the upper edge of the gate is 1.5 m. [**Ans.** (i) 39.24 kN and (ii) 1635 Nm]

27. Determine the total force and location of centre of pressure on one face of the plate shown in Fig. 3.62 immersed in a liquid of specific gravity 0.9. [**Ans.** 62.4 kN, 3.04 m]

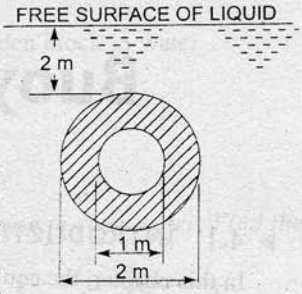

Fig. 3.62

28. A circular opening, 3 m diameter, in the vertical side of water tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter ? Calculate : (i) the force on the disc, and (ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m. [**Ans.** (i) 270 kN, and (ii) 38 kN m]

29. A penstock made up by a pipe of 2 m diameter contains a circular disc of same diameter to act as a valve which controls the discharge passing through it. It can rotate about a horizontal diameter. If the head of water above its centre is 20 m, find the total force acting on the disc and the torque required to maintain it in the vertical position. *(A.M.I.E., Summer 1990)*

30. A circular drum 1.8 m diameter and 1.2 m height is submerged with its axis vertical and its upper end at a depth of 1.8 m below water level. Determine :

(i) total pressure on top, bottom and curved surfaces of the drum,

(ii) resultant pressure on the whole surface, and

(iii) depth of centre of pressure on curved surface. *(A.M.I.E., Winter 1991)*

31. A circular plate of diameter 3 m is immersed in water in such a way that its least and greatest depth from the free surface of water are 1 m and 3 m respectively. For the front side of the plate, find (i) total force exerted by water and (ii) the position of centre of pressure. *(Delhi University, Dec. 1996)* [**Ans.** (i) 138684 N ; (ii) 2.125 m]

32. A tank contains water upto a height of 10 m. One of the sides of the tank is inclined. The angle between free surface of water and inclined side is 60°. The width of the tank is 5 m. Find : (i) the force exerted by water on inclined side and (ii) position of centre of pressure. *(Delhi University, June 1996)* [**Ans.** (i) 283.1901 kN (ii) 6.67 m]

33. A circular plate of 3 m diameter is under water with its plane making an angle of 30° with the water surface. If the top edge of the plate is 1 m below the water surface, find the force on one side of the plate and its location. *(J.N.T.U., Hyderabad S 2002)*

[**Hint.** $d = 3$ m, $\theta = 30°$, height of top edge = 1 m, $\bar{h} = 1 + 1.5 \times \sin 30° = 1.75$

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times \left(\frac{\pi}{4} \times 3^2\right) \times 1.75 = \textbf{121.35 kN. Ans.}$$

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64}(3^4) \times \frac{1}{4}}{\frac{\pi}{4}(3^2) \times 1.75} + 1.75 = 0.08 + 1.75 = 1.83 \text{ m. } \textbf{Ans.}]$$

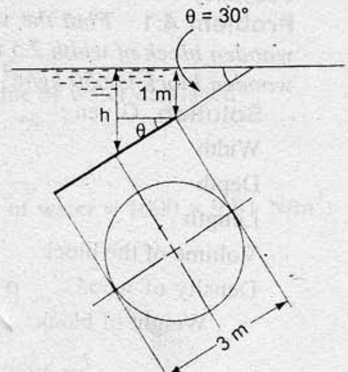

Fig. 3.63

4

CHAPTER

Buoyancy and Floatation

▶ 4.1 INTRODUCTION

In this chapter, the equilibrium of the floating and sub-merged bodies will be considered. Thus the chapter will include : 1. Buoyancy, 2. Centre of buoyancy, 3. Metacentre, 4. Metacentric height, 5. Analytical method for determining metacentric height, 6. Conditions of equilibrium of a floating and sub-merged body, and 7. Experimental method for metacentric height.

▶ 4.2 BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

▶ 4.3 CENTRE OF BUOYANCY

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

Problem 4.1 *Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length 6.0 m.*

Solution. Given :

Width $= 2.5$ m

Depth $= 1.5$ m

Length $= 6.0$ m

Volume of the block $= 2.5 \times 1.5 \times 6.0 = 22.50 \text{ m}^3$

Density of wood, $\rho = 650 \text{ kg/m}^3$

$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$

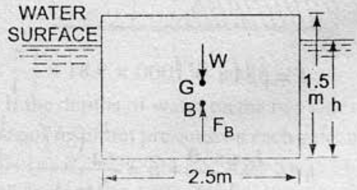

Fig. 4.1

For equilibrium the weight of water displaced = Weight of wooden block

$$= 143471 \text{ N}$$

∴ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625\ m^3.\ Ans.}$$

($\because$ Weight density of water = 1000×9.81 N/m^3)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

or $2.5 \times h \times 6.0 = 14.625$ m^3, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875\ m\ from\ base.\ Ans.}$$

Problem 4.2 *A wooden log of 0.6 m diameter and 5 m length is floating in river water. Find the depth of the wooden log in water when the sp. gravity of the log is 0.7.*

Solution. Given :

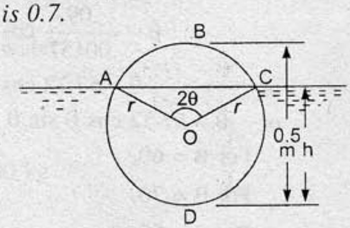

Fig. 4.2

Dia. of log = 0.6 m

Length, $L = 5$ m

Sp. gr., $S = 0.7$

∴ Density of log = $0.7 \times 1000 = 700$ kg/m^3

∴ Weight density of log, $w = \rho \times g$

$= 700 \times 9.81$ N/m^3

Find depth of immersion or h

Weight of wooden log = Weight density × Volume of log

$$= 700 \times 9.81 \times \frac{\pi}{4}(D)^2 \times L$$

$$= 700 \times 9.81 \times \frac{\pi}{4}(.6)^2 \times 5 \text{ N} = 989.6 \times 9.81 \text{ N}$$

For equilibrium,

Weight of wooden log = Weight of water displaced

= Weight density of water × Volume of water displaced

$$\therefore \quad \text{Volume of water displaced} = \frac{989.6 \times 9.81}{1000 \times 9.81} = 0.9896 \text{ m}^3$$

($\because$ Weight density of water = 1000×9.81 N/m^3)

Let h is the depth of immersion

∴ Volume of log inside water = Area of $ADCA$ × Length

= Area of $ADCA$ × 5.0

But volume of log inside water = Volume of water displaced = 0.9896 m^3

$\therefore$ $$0.9896 = \text{Area of } ADCA \times 5.0$$

$\therefore$ Area of $ADCA$ $$= \frac{0.9896}{5.0} = 0.1979 \text{ m}^2$$

But area of $ADCA$ = Area of curved surface $ADCOA$ + Area of ΔAOC

$$= \pi r^2 \left[\frac{360 - 2\theta}{360}\right] + \frac{1}{2} r \cos\theta \times 2r \sin\theta$$

$$= \pi r^2 \left[1 - \frac{\theta}{180}\right] + r^2 \cos\theta \sin\theta$$

$\therefore$ $$0.1979 = \pi (.3)^2 \left[1 - \frac{\theta}{180}\right] + (.3)^2 \cos\theta \sin\theta$$

$$0.1979 = .2827 - .00157\,\theta + 0.9 \cos\theta \sin\theta$$

or $$.00157\,\theta - .09 \cos\theta \sin\theta = .2827 - .1979 = 0.0848$$

$$\theta - \frac{.09}{.00157} \cos\theta \sin\theta = \frac{.0848}{.00157}$$

or $$\theta - 57.32 \cos\theta \sin\theta = 54.01.$$

or $$\theta - 57.32 \cos\theta \sin\theta - 54.01 = 0$$

For $\theta = 60$, $60 - 57.32 \times 0.5 \times .866 - 54.01 = 60 - 24.81 - 54.01 = -18.82$

For $\theta = 70$, $70 - 57.32 \times .342 \times 0.9396 - 54.01 = 70 - 18.4 - 54.01 = -2.41$

For $\theta = 72$, $72 - 57.32 \times .309 \times .951 - 54.01 = 72 - 16.84 - 54.01 = +1.14$

For $\theta = 71$, $71 - 57.32 \times .325 \times .9455 - 54.01 = 71 - 17.61 - 54.01 = -0.376$

$\therefore$ $\theta = 71.5°$, $71.5 - 57.32 \times .3173 \times .948 - 54.01 = 71.5 - 17.24 - 54.01 = +.248$

Then $$h = r + r \cos 71.5°$$

$$= 0.3 + 0.3 \times 0.3173 = \mathbf{0.395 \text{ m. Ans.}}$$

Problem 4.3 *A stone weighs 392.4 N in air and 196.2 N in water. Compute the volume of stone and its specific gravity.*

Solution. Given :

Weight of stone in air = 392.4 N

Weight of stone in water = 196.2 N

For equilibrium,

Weight in air – Weight of stone in water = Weight of water displaced

or $$392.4 - 196.2 = 196.2 = 1000 \times 9.81 \times \text{Volume of water displaced}$$

$\therefore$ Volume of water displaced

$$= \frac{196.2}{1000 \times 9.81} = \frac{1}{50} \text{ m}^3 = \frac{1}{50} \times 10^6 \text{ cm}^3 = \mathbf{2 \times 10^4 \text{ cm}^3. \text{ Ans.}}$$

= Volume of stone

$\therefore$ Volume of stone = $\mathbf{2 \times 10^4 \text{ cm}^3}$. **Ans.**

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Specific Gravity of Stone

Mass of stone $= \dfrac{\text{Weight in air}}{g} = \dfrac{392.4}{9.81} = 40$ kg

Density of stone $= \dfrac{\text{Mass in air}}{\text{Volume}} = \dfrac{40.0 \text{ kg}}{\frac{1}{50} \text{ m}^3} = 40 \times 50 = 2000 \dfrac{\text{kg}}{\text{m}^3}$

$\therefore$ Sp. gr. of stone $= \dfrac{\text{Density of stone}}{\text{Density of water}} = \dfrac{2000}{1000} =$ **2.0. Ans.**

Problem 4.4 *A body of dimensions 1.5 m × 1.0 m × 2 m, weighs 1962 N in water. Find its weight in air. What will be its specific gravity ?*

Solution. Given :

Volume of body $= 1.50 \times 1.0 \times 2.0 = 3.0 \text{ m}^3$

Weight of body in water $= 1962$ N

Volume of the water displaced = Volume of the body $= 3.0 \text{ m}^3$

$\therefore$ Weight of water displaced $= 1000 \times 9.81 \times 3.0 = 29430$ N

For the equilibrium of the body

Weight of body in air – Weight of water displaced = Weight in water

$\therefore \quad W_{air} - 29430 = 1962$

$W_{air} = 29430 + 1962 = 31392$ N

Mass of body $= \dfrac{\text{Weight in air}}{g} = \dfrac{31392}{9.81} = 3200$ kg

Density of the body $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{3200}{3.0} = 1066.67$

$\therefore$ Sp. gravity of the body $= \dfrac{1066.67}{1000} =$ **1.067. Ans.**

Problem 4.5 *Find the density of a metallic body which floats at the interface of mercury of sp. gr. 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.*

Solution. Let the volume of the body $= V \text{ m}^3$

Then volume of body sub-merged in mercury

$$= \frac{40}{100} V = 0.4 \; V \text{ m}^3$$

Volume of body sub-merged in water

$$= \frac{60}{100} \times V = 0.6 \; V \text{ m}^3$$

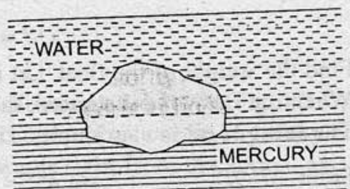

Fig. 4.3

For the equilibrium of the body

Total buoyant force (upward force) = Weight of the body

But total buoyant force = Force of buoyancy due to water + Force of buoyancy due to mercury

Force of buoyancy due to water = Weight of water displaced by body

= Density of water × g × Volume of water displaced

= 1000 × g × Volume of body in water

$= 1000 \times g \times 0.6 \times V$ N

and Force of buoyancy due to mercury $=$ Weight of mercury displaced by body

$= g \times$ Density of mercury $\times$ Volume of mercury displaced

$= g \times 13.6 \times 1000 \times$ Volume of body in mercury

$= g \times 13.6 \times 1000 \times 0.4\ V$ N

Weight of the body $=$ Density $\times g \times$ Volume of body $= \rho \times g \times V$

where ρ is the density of the body

$\therefore$ For equilibrium, we have

Total buoyant force $=$ Weight of the body

$$1000 \times g \times 0.6 \times V + 13.6 \times 1000 \times g \times .4\ V = \rho \times g \times V$$

or $$\rho = 600 + 13600 \times .4 = 600 + 54400 = 6040.00 \text{ kg/m}^3$$

$\therefore$ Density of the body $=$ **6040.00 kg/m³. Ans.**

Problem 4.6 *A float valve regulates the flow of oil of sp. gr. 0.8 into a cistern. The spherical float is 15 cm in diameter. AOB is a weightless link carrying the float at one end, and a valve at the other end which closes the pipe through which oil flows into the cistern. The link is mounted in a frictionless hinge at O and the angle AOB is 135°. The length of OA is 20 cm, and the distance between the centre of the float and the hinge is 50 cm. When the flow is stopped AO will be vertical. The valve is to be pressed on to the seat with a force of 9.81 N to completely stop the flow of oil into the cistern. It was observed that the flow of oil is stopped when the free surface of oil in the cistern is 35 cm below the hinge. Determine the weight of the float.* (U.P.S.C., Engg. Services, 1975)

Solution. Given :

Sp. gr. of oil $= 0.8$

$\therefore$ Density of oil, $\rho_0 = 0.8 \times 1000 = 800$ kg/m³

Dia. of float, $D = 15$ cm

$\angle AOB = 135°$

$OA = 20$ cm

Force, $P = 9.81$ N

$OB = 50$ cm

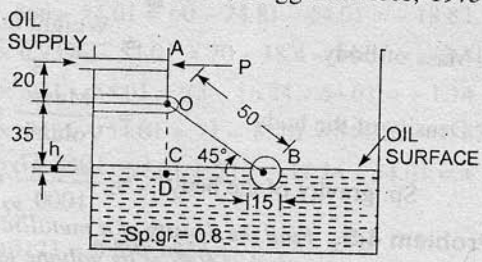

Fig. 4.4

Find the weight of the float. Let it is equal to W.

When the flow of oil stopped, the centre of float is shown in Fig. 4.4

The level of oil is also shown. The centre of float is below the level of oil, by a depth 'h'.

From ΔBOD, $$\sin 45° = \frac{OD}{OB} = \frac{OC + CD}{OB} = \frac{35 + h}{50}$$

$\therefore$ $$50 \times \sin 45° = 35 + h$$

or $$h = 50 \times \frac{1}{\sqrt{2}} - 35 = 35.355 - 35 = 0.355 \text{ cm} = .00355 \text{ m.}$$

The weight of float is acting through B, but the upward buoyant force is acting through the centre of weight of oil displaced.

Volume of oil displaced $= \frac{2}{3}\pi r^3 + h \times \pi r^2$ $\left\{ r = \frac{D}{2} = \frac{15}{2} = 7.5 \text{ cm} \right\}$

$$= \frac{2}{3} \times \pi \times (.075)^3 + .00355 \times \pi \times (.075)^2 = 0.000945 \text{ m}^3$$

$\therefore$ Buoyant force $=$ Weight of oil displaced

$$= \rho_0 \times g \times \text{Volume of oil}$$

$$= 800 \times 9.81 \times .000945 = 7.416 \text{ N}$$

The buoyant force and weight of the float passes through the same vertical line, passing through B. Let the weight of float is W. Then net vertical force on float

$$= \text{Buoyant force} - \text{Weight of float} = (7.416 - W)$$

Taking moments about the hinge O, we get

$$P \times 20 = (7.416 - W) \times BD = (7.416 - W) \times 50 \times \cos 45°$$

or

$$9.81 \times 20 = (7.416 - W) \times 35.355$$

$$\therefore \quad W = 7.416 - \frac{20 \times 9.81}{35.355} = 7.416 - 5.55 = \mathbf{1.866 \text{ N. Ans.}}$$

▶ 4.4 META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

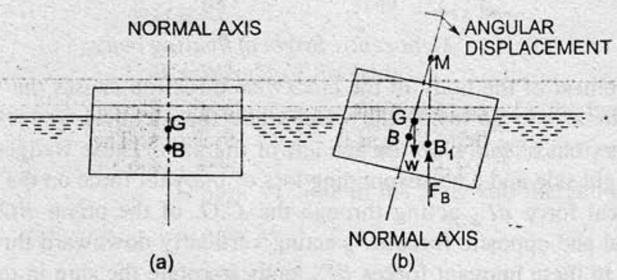

Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

▶ 4.5 META-CENTRIC HEIGHT

The distance MG, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

▶ 4.6 ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (*a*) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at *G* and *B*. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (*b*). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at *M*. Hence *M* is the meta-centre and *GM* is meta-centric height.

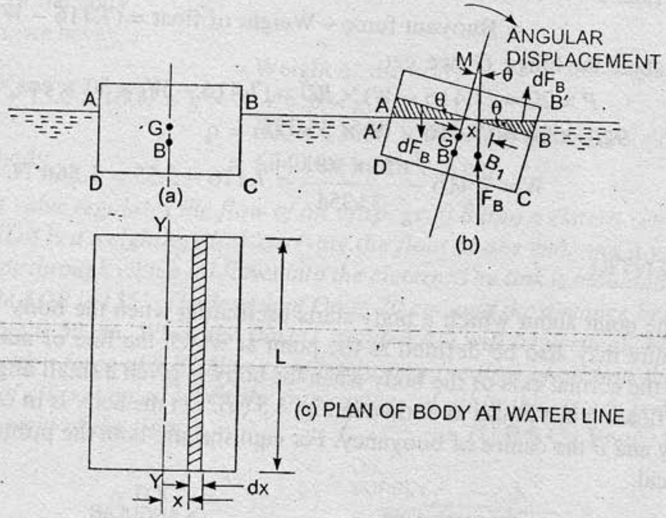

Fig. 4.6 *Meta-centre height of floating body.*

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism *BOB′* on the right of the axis to go inside the water while the identical wedge-shaped prism represented by *AOA′* emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the *C.G.* of the prism *BOB′* while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of *AOA′*. The couple due to these buoyant forces dF_B tends to rotate the ship in the counter clockwise direction. Also the moment caused by the displacement of the centre of buoyancy from *B* to B_1 is also in the counter clockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness *dx* at a distance *x* from *O* as shown in Fig. 4.5 (*b*). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{\because \angle BOB' = \angle AOA' = \angle BMB_1' = \theta\}$$

∴ Area of strip $= \text{Height} \times \text{Thickness} = x \times \theta \times dx$

If *L* is the length of the floating body, then

Volume of strip $= \text{Area} \times L$

$= x \times \theta \times L \times dx$

∴ Weight of strip $= \rho g \times \text{Volume} = \rho g x\, \theta L\, dx$

Similarly, if a small strip of thickness *dx* at a distance *x* from *O* towards the left of the axis is considered, the weight of strip will be $\rho g x \theta\, L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple $= \text{Weight of each strip} \times \text{Distance between these two weights}$

$$= \rho g x\ \theta L\ dx\ [x + x]$$

$$= \rho g x\ \theta L\ dx \times 2x = 2\rho g x^2\ \theta L\ dx$$

$\therefore$ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2\ \theta L\ dx \qquad \text{...(4.1)}$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$$= F_B \times BB_1$$

$$= F_B \times BM \times \theta \qquad \{\because BB_1 = BM \times \theta \text{ if } \theta \text{ is very small}\}$$

$$= W \times BM \times \theta \qquad \{\because F_B = W\} \text{ ...(4.2)}$$

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2\ \theta\ L dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$$

$$W \times BM = 2\rho g \int x^2 L dx$$

Now Ldx = Elemental area on the water line shown in Fig. 4.6 (c) and $= dA$

$$\therefore \qquad W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 (c) it is clear that $2 \int x^2\ dA$ is the second moment of area of the plan of the body at water surface about the axis y-y. Therefore

$$W \times BM = \rho g I \qquad \{\text{where } I = 2 \int x^2\ dA\}$$

$$\therefore \qquad BM = \frac{\rho g I}{W}$$

But W = Weight of the body

= Weight of the fluid displaced by the body

$= \rho g \times$ Volume of the fluid displaced by the body

$= \rho g \times$ Volume of the body sub-merged in water

$= \rho g \times \forall$

$$\therefore \qquad BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \qquad \text{...(4.3)}$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

$$\therefore \quad \text{Meta-centric height} = GM = \frac{I}{\forall} - BG. \qquad \text{...(4.4)}$$

Problem 4.7 *A rectangular pontoon is 5 m long, 3 m wide and 1.20 m high. The depth of immersion of the pontoon is 0.80 m in sea water. If the centre of gravity is 0.6 m above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m³.*

(Delhi University, 1992)

Solution. Given :

Dimension of pontoon $= 5 \text{ m} \times 3 \text{ m} \times 1.20 \text{ m}$

Depth of immersion $= 0.8$ m

Distance $AG = 0.6$ m

Distance $AB = \frac{1}{2} \times$ Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water $= 1025$ kg/m^3

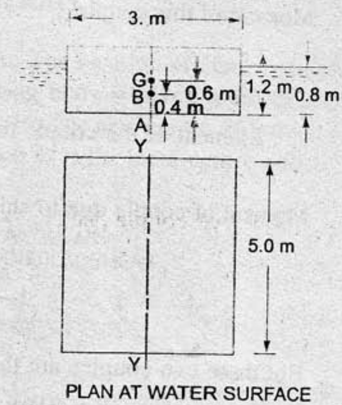

Fig. 4.7

Meta-centre height GM, given by Equation (4.4) is

$$GM = \frac{I}{\forall} - BG$$

where I = M.O. Inertia of the plan of the pontoon about y-y axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0$ m^3

$BG = AG - AB = 0.6 - 0.4 = 0.2$ m

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375 \text{ m.}} \quad \textbf{Ans.}$$

Problem 4.8 *A uniform body of size 3 m long × 2 m wide × 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.8 m ? Determine the meta-centric height also.*

Solution. Given :

Dimension of body $= 3 \times 2 \times 1$

Depth of immersion $= 0.8$ m

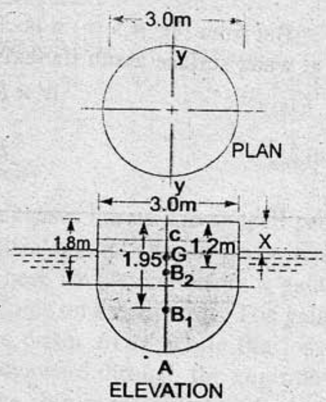

Fig. 4.8

Find (*i*) Weight of body, W

(*ii*) Meta-centric height, GM

(*i*) **Weight of Body, W**

= Weight of water displaced

= $\rho g \times$ Volume of water displaced

= $1000 \times 9.81 \times$ Volume of body in water

= $1000 \times 9.81 \times .3 \times 2 \times 0.8$ N

= **47088 N. Ans.**

(*ii*) **Meta-centric Height, GM**

Using equation (4.4), we get

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I about y-y axis of the plan of the body

$$= \frac{1}{12} \times 3 \times 2^3 = \frac{3 \times 2^3}{12} = 2.0 \text{ m}^4$$

$\forall$ = Volume of body in water

$= 3 \times 2 \times 0.8 = 4.8$ m^3

$$BG = AG - AB = \frac{1.0}{2} - \frac{0.8}{2} = 0.5 - 0.4 = 0.1$$

$$\therefore \quad GM = \frac{2.0}{4.8} - 0.1 = 0.4167 - 0.1 = \mathbf{0.3167 \text{ m.}} \quad \textbf{Ans.}$$

Problem 4.9 *A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m × 1 m × 0.8 m.*

Solution. Given :

Dimension of block = 2 × 1 × 0.8

Let depth of immersion = h m

Sp. gr. of wood = 0.7

Weight of wooden piece = Weight density of wood* × Volume

= 0.7 × 1000 × 9.81 × 2 × 1 × 0.8 N

Weight of water displaced = Weight density of water × Volume of the wood sub-merged in water

= 1000 × 9.81 × 2 × 1 × h N

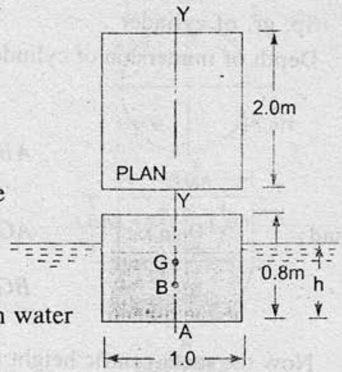

Fig. 4.9

For equilibrium,

Weight of wooden piece = Weight of water displaced

∴ 700 × 9.81 × 2 × 1 × 0.8 = 1000 × 9.81 × 2 × 1 × h

$$\therefore \quad h = \frac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1} = 0.7 \times 0.8 = 0.56 \text{ m}$$

∴ Distance of centre of Buoyancy from bottom, *i.e.*,

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 \text{ m}$$

and $$AG = 0.8/2.0 = 0.4 \text{ m}$$

$$\therefore \quad BG = AG - AB = 0.4 - 0.28 = 0.12 \text{ m}$$

The meta-centric height is given by equation (4.4) or

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{1}{12} \times 2 \times 1.0^3 = \frac{1}{6} \text{ m}^4$

$\forall$ = Volume of wood in water

= 2 × 1 × h = 2 × 1 × .56 = 1.12 m³

$$\therefore \quad GM = \frac{1}{6} \times \frac{1}{1.12} - 0.12 = 0.1488 - 0.12 = \mathbf{0.0288 \text{ m.}}\ \textbf{Ans.}$$

Problem 4.10 *A solid cylinder of diameter 4.0 m has a height of 3 metres. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder = 0.6.*

Solution. Given :

Dia. of cylinder, $D = 4.0$ m

Height of cylinder, $h = 3.0$ m

---

* Weight density of wood = $\rho \times g$, where ρ = density of wood

= 0.7 × 1000 = 700 kg/m³. Hence w for wood = 700 × 9.81 N/m³.

Sp. gr. of cylinder = 0.6

Depth of immersion of cylinder

$$= 0.6 \times 3.0 = 1.8 \text{ m}$$

$$\therefore \quad AB = \frac{1.8}{2} = 0.9 \text{ m}$$

and $$AG = \frac{3}{2} = 1.5 \text{ m}$$

$$\therefore \quad BG = AG - AB$$

$$= 1.5 - 0.9 = 0.6 \text{ m}$$

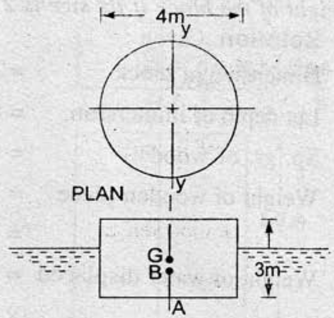

Fig. 4.10

Now the meta-centric height GM is given by equation (4.4)

$$GM = \frac{I}{\forall} - BG$$

But I = M.O.I. about y-y axis of the plan of the body

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

and $\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times \text{Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8 \text{ m}^3$$

$$\therefore \quad GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^2 \times 1.8} - 0.6$$

$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{1.8} - 0.6 = 0.55 - 0.6 = \mathbf{-\,0.05 \text{ m. Ans.}}$$

– ve sign means that meta-centre, (M) is below the centre of gravity (G).

Problem 4.11 *A body has the cylindrical upper portion of 3 m diameter and 1.8 m deep. The lower portion is a curved one, which displaces a volume of 0.6 m³ of water. The centre of buoyancy of the curved portion is at a distance of 1.95 m below the top of the cylinder. The centre of gravity of the whole body is 1.20 m below the top of the cylinder. The total displacement of water is 3.9 tonnes. Find the meta-centric height of the body.*

Solution. Given :

Dia. of body = 3.0 m

Depth of body = 1.8 m

Volume displaced by curved portion

$$= 0.6 \text{ m}^3 \text{ of water.}$$

Let B_1 is the centre of buoyancy of the curved surface and G is the centre of gravity of the whole body.

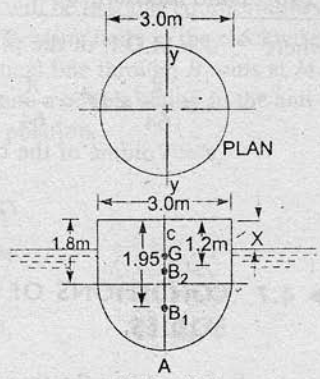

Fig. 4.11

Then $CB_1 = 1.95$ m

$CG = 1.20$ m

Total weight of water displaced by body = 3.9 tonnes

$= 3.9 \times 1000 = 3900$ kgf

$= 3900 \times 9.81$ N $= 38259$ N

Find **meta-centric** height of the body.

Let the height of the body above the water surface x m. Total weight of water displaced by body

= Weight density of water × [Volume of water displaced]

= 1000 × 9.81 × [Volume of the body in water]

= 9810 [Volume of cylindrical part in water + Volume of curved portion]

$$= 9810\left[\frac{\pi}{4} \times D^2 \times \text{Depth of cylindrical part in water} + \text{Volume displace by curved portion}\right]$$

or $$38259 = 9810\left[\frac{\pi}{4}(3)^2 \times (1.8 - x) + 0.6\right]$$

$\therefore$ $$\frac{\pi}{4}(3)^2 \times (1.8 - x) + 0.6 = \frac{38259}{9810} = 3.9$$

$\therefore$ $$\frac{\pi}{4} \times 3^2 \times (1.8 - x) = 3.9 - 0.6 = 3.3$$

or $$1.8 - x = \frac{3.3 \times 4}{\pi \times 3 \times 3} = 0.4668$$

$\therefore$ $$x = 1.8 - .4668 = 1.33 \text{ m}$$

Let B_2 is the centre of buoyancy of cylindrical part and B is the centre of buoyancy of the whole body.

Then depth of cylindrical part in water $= 1.8 - x = 0.467$ m

$\therefore$ $$CB_2 = x + \frac{.467}{2} = 1.33 + .2335 = 1.5635 \text{ m.}$$

The distance of the centre of buoyancy of the whole body from the top of the cylindrical part is given as

CB = (Volume of curved portion × CB_1 + Volume of cylindrical part in water × CB_2) ÷ (Total volume of water displaced)

$$= \frac{0.6 \times 1.95 + 3.3 \times 1.5635}{(0.6 + 3.3)} = \frac{1.17 + 5.159}{3.9} = 1.623 \text{ m.}$$

Then $BG = CB - CG = 1.623 - 1.20 = .423$ m.

Meta-centric height, GM, is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the plan of the body at water surface about y-y

$$= \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times 3^4 \text{ m}^4$$

$\forall$ = Volume of the body in water = 3.9 m^3

$$\therefore \quad GM = \frac{\pi}{64} \times \frac{3^4}{3.9} - .423 = 1.019 - .423 = \mathbf{0.596 \text{ m. Ans.}}$$

▶ 4.7 CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity (G) and centre of buoyancy (B_1) of a body determines the stability of a sub-merged body.

4.7.1 Stability of a Sub-merged Body. The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig. 4.12 (a). Let the weight of the balloon is W. The weight W is acting through G, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through B. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. 4.12 (a), then W and F_B constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig. 4.12 (a) is in stable equilibrium.

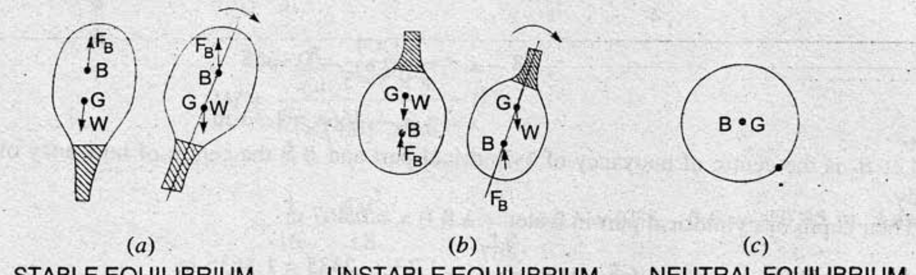

Fig. 4.12 *Stabilities of sub-merged bodies.*

(a) **Stable Equilibrium.** When $W = F_B$ and point B is above G, the body is said to be in stable equilibrium.

(b) **Unstable Equilibrium.** If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in Fig. 4.12 (b). A slight displacement to the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(c) **Neutral Equilibrium.** If $F_B = W$ and B and G are at the same point, as shown in Fig. 4.12 (c), the body is said to be in Neutral Equilibrium.

4.7.2 Stability of Floating Body. The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(*a*) **Stable Equilibrium.** If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.

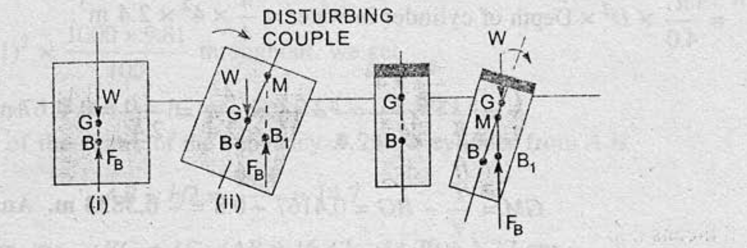

Fig. 4.13 *Stability of floating bodies.*

(*b*) **Unstable Equilibrium.** If the point M is below G, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Problem 4.12 *A solid cylinder of diameter 4.0 m has a height of 4.0 m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.*

Solution. Given : $D = 4$ m

Height, $h = 4$ m

Sp. gr. $= 0.6$

Depth of cylinder in water $=$ Sp. gr. $\times h$

$= 0.6 \times 4.0 = 2.4$ m

$\therefore$ Distance of centre of buoyancy (B) from A

or $$AB = \frac{2.4}{2} = 1.2 \text{ m}$$

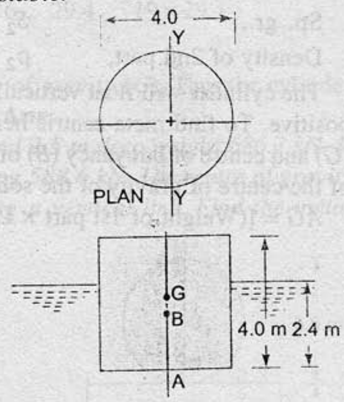

Fig. 4.14

Distance of centre of gravity (G) from A

or $$AG = \frac{h}{2} = \frac{4.0}{2} = 2.0 \text{ m}$$

$$\therefore \quad BG = AG - AB = 2.0 - 1.2 = 0.8 \text{ m}$$

Now the meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the plan of the body about y-y.

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4.0} \times D^2 \times \text{Depth of cylinder in water} = \frac{\pi}{4} \times 4^2 \times 2.4 \text{ m}^3$$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} \times 4^4}{\frac{\pi}{4} \times 4^2 \times 2.4} = \frac{1}{16} \times \frac{4^2}{2.4} = \frac{1}{2.4} = 0.4167 \text{ m}$$

$$\therefore \quad GM = \frac{I}{\forall} - BG = 0.4167 - 0.8 = -\mathbf{0.3833 \text{ m. Ans.}}$$

– ve sign means that the meta-centre (M) is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium. **Ans.**

Problem 4.13 *A solid cylinder of 10 cm diameter and 40 cm long, consists of two parts made of different materials. The first part at the base is 1.0 cm long and of specific gravity = 6.0. The other part of the cylinder is made of the material having specific gravity 0.6. State, if it can float vertically in water.*

Solution. Given : $D = 10$ cm

Length, $L = 40$ cm

Length of 1st part, $l_1 = 1.0$ cm

Sp. gr., $S_1 = 6.0$

Density of 1st part, $\rho_1 = 6 \times 1000 = 6000$ kg/m^3

Length of 2nd part, $l_2 = 40 - 1.0 = 39.0$ cm

Sp. gr., $S_2 = 0.6$

Density of 2nd part, $\rho_2 = 0.6 \times 1000 = 600$ kg/m^3

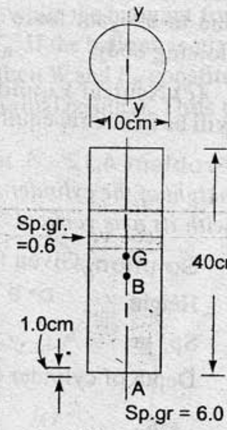

Fig. 4.15

The cylinder will float vertically in water if its meta-centric height GM is positive. To find meta-centric height, find the location of centre of gravity (G) and centre of buoyancy (B) of the combined solid cylinder. The distance of the centre of gravity of the solid cylinder from A is given as

AG = [(Weight of 1st part × Distance of C.G. of 1st part from A)
+ (Weight of 2nd part of cylinder
× Distance of C.G. of 2nd part from A)]
÷ [Weight of 1st part + weight of 2nd part]

$$= \frac{\left(\frac{\pi}{4} D^2 \times 1.0 \times 6.0 \times 0.5\right) + \left(\frac{\pi}{4} D^2 \times 39.0 \times 0.6 \times (1.0 \times 39/2)\right)}{\left(\frac{\pi}{4} D^2 \times 1.0 \times 6.0 + \frac{\pi}{4} D^2 \times 39 \times 0.6\right)}$$

$$= \frac{1.0 \times 6.0 \times 0.5 + 39.0 \times .6 \times (20.5)}{1.0 \times 6.0 + 39.0 \times 0.6}$$

Cancel $\frac{\pi}{4} D^2$ in the Numerator and Denominator $= \frac{3.0 + 479.7}{6.0 \times 23.4} = \frac{482.7}{29.4} = 16.42.$

To find the centre of buoyancy of the combined two parts or of the cylinder, determine the depth of immersion of the cylinder. Let the depth of immersion of the cylinder is h. Then

Weight of the cylinder = Weight of water displaced

$$\frac{\pi}{4} \times (.1)^2 \times \frac{39.0}{100} \times 600 \times 9.81 + \frac{\pi}{4} (.1)^2 \times \frac{1.0}{100} \times 6000 \times 9.81 = \frac{\pi}{4} (.1)^2 \times \frac{h}{100} \times 1000 \times 9.81$$

[∵ h is in cm]

or cancelling $\frac{\pi}{4} (.1)^2 \times \frac{1000 \times 9.81}{100}$ throughout, we get

$$39.0 \times 0.6 + 1.0 \times 6.0 = h \quad \text{or} \quad h = 23.4 + 6.0 = 29.4$$

∴ The distance of the centre of the buoyancy B, of the cylinder from A is

$$AB = h/2 = \frac{29.4}{2} = 14.7$$

∴ $$BG = AG - AB = 16.42 - 14.70 = 1.72 \text{ cm.}$$

Meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of plan of the body about y-y

$$= \frac{\pi}{64} D^4 = \frac{\pi}{.64} (10)^4 \text{ cm}^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times h = \frac{\pi}{4} (10)^2 \times 29.4 \text{ m}^3$$

∴ $$\frac{I}{\forall} = \frac{\pi}{64} (10)^4 \Big/ \frac{\pi}{4} (10)^2 \times 29.4 = \frac{1}{16} \times \frac{10^2}{29.4} = \frac{100}{19 \times 29.4} = 0.212$$

∴ $$GM = 0.212 - 1.72 = -1.508 \text{ cm.}$$

As GM – ve. It means that the Meta-centre M is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium and so it cannot float vertically in water. **Ans.**

Problem 4.14 *A rectangular pontoon 10.0 m long, 7 m broad and 2.5 m deep weighs 686.7 kN. It carries on its upper deek an empty boiler of 5.0 m diameter weighing 588.6 kN. The centre of gravity of the boiler and the pontoon are at their respective centres along a vertical line. Find the meta-centric height. Weight density of sea water is 10.104 kN/m³.*

Solution. Given : Dimension of pontoon = 10 × 7 × 2.5

Weight of pontoon, $W_1 = 686.7$ kN

Dia. of boiler, $D = 5.0$ m

Weight of boiler, $W_2 = 588.6$ kN

w for sea water $= 10.104$ kN/m³

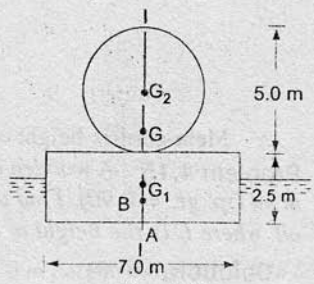

Fig. 4.16

To find the meta-centric height, first determine the common centre of gravity G and common centre of buoyancy B of the boiler and pontoon. Let G_1 and G_2 are the centre of gravities of pontoon and boiler respectively. Then

$$AG_1 = \frac{2.5}{2} = 1.25 \text{ m}$$

$$AG_2 = 2.5 + \frac{5.0}{2} = 2.5 + 2.5 = 5.0 \text{ m}$$

The distance of common centre of gravity G from A is given as

$$AG = \frac{W_1 \times AG_1 + W_2 \times AG_2}{W_1 + W_2}$$

$$= \frac{686.7 \times 1.25 + 588.6 \times 5.0}{(686.7 + 588.6)} = 2.98 \text{ m.}$$

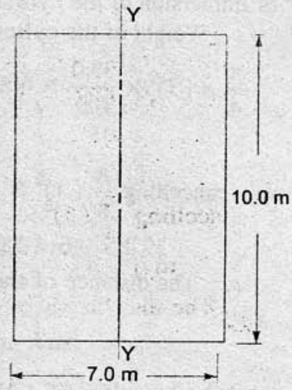

Fig. 4.17 *Plan of the body at water-line*

Let h is the depth of immersion. Then

Total weight of pontoon and boiler = Weight of sea water displaced

or $(686.7 + 588.6) = w \times$ Volume of the pontoon in water

$= 10.104 \times L \times b \times$ Depth of immersion

$\therefore$ $1275.3 = 10.104 \times 10 \times 7 \times h$

$$h = \frac{1275.3}{10 \times 7 \times 10.104} = 1.803 \text{ m}$$

$\therefore$ The distance of the common centre of buoyancy B from A is

$$AB = \frac{h}{2} = \frac{1.803}{2} = .9015 \text{ m}$$

$\therefore$ $BG = AG - AB = 2.98 - .9015 = 2.0785 \text{ m} \approx 2.078 \text{ m}$

Meta-centric height is given by $GM = \frac{I}{\forall} - BG$

where I = M.O.I. of the plan of the body at the water level along $y = y$

$$= \frac{1}{12} \times 10.0 \times 7^3 = \frac{10 \times 49 \times 7}{12} \text{ m}^4$$

$\forall$ = Volume of the body in water

$= L \times b \times h = 10.0 \times 7 \times 1.857$

$$\therefore \quad \frac{I}{\forall} = \frac{10 \times 49 \times 7}{12 \times 10 \times 7 \times 1.857} = \frac{49}{12 \times 1.857} = 2.198 \text{ m}$$

$$\therefore \quad GM = \frac{I}{\forall} - BG = 2.198 - 2.078 = 0.12 \text{ m.}$$

$\therefore$ Meta-centric height of both the pontoon and boiler = **0.12 m. Ans.**

Problem 4.15 *A wooden cylinder of sp. gr. = 0.6 and circular in cross-section is required to float in oil (sp. gr. = 0.90). Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameter.*

Solution. Given :

Dia. of cylinder $= D$

Height of cylinder $= L$

Sp. gr. of cylinder, $S_1 = 0.6$

Sp. gr. of oil $\quad S_2 = 0.9$

Let the depth of cylinder immersed in oil = h

For the principle of buoyancy

Weight of cylinder = wt. of oil displaced

$$\frac{\pi}{4} D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4} D^2 \times h \times 0.9 \times 1000 \times 9.81$$

or $$L \times 0.6 = h \times 0.9$$

$$\therefore \quad h = \frac{0.6 \times L}{0.9} = \frac{2}{3} L.$$

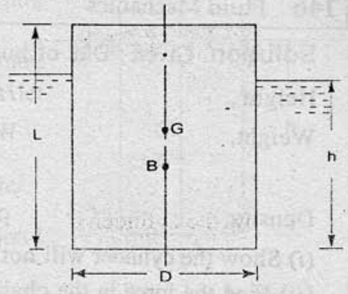

Fig. 4.18

The distance of centre of gravity C from A, $AG = \frac{L}{2}$

The distance of centre of buoyancy B from A,

$$AB = \frac{h}{2} = \frac{1}{2}\left[\frac{2}{3}L\right] = \frac{L}{3}$$

$$\therefore \quad BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{6}$$

The meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{\pi}{64} D^4$ and $\forall$ = volume of cylinder in oil = $\frac{\pi}{4} D^2 \times h$

$$\therefore \quad \frac{I}{\forall} = \left(\frac{\pi}{64} D^4 \Big/ \frac{\pi}{4} D^2 h\right) = \frac{1}{16} \frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L} \qquad \left\{\because \; h = \frac{2}{3} L\right\}$$

$$\therefore \quad GM = \frac{3D^3}{32L} - \frac{L}{6}.$$

For stable equilibrium, GM should be +ve or

$$GM > 0 \quad \text{or} \quad \frac{3D^2}{32L} - \frac{L}{6} > 0$$

or $$\frac{3D^2}{32L} > \frac{L}{6} \quad \text{or} \quad \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

or $$\frac{L^2}{D^2} < \frac{18}{32} \quad \text{or} \quad \frac{9}{16}$$

$$\therefore \quad \frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore$ **L/D < 3/4. Ans.**

Problem 4.16 *Show that a cylindrical buoy of 1 m diameter and 2.0 m height weighing 7.848 kN will not float vertically in sea water of density 1030 kg/m³. Find the force necessary in a vertical chain attached at the centre of base of the buoy that will keep it vertical.*

Solution. Given : Dia. of buoy, $D = 1$ m

Height, $H = 2.0$ m

Weight, $W = 7.848$ kN

$= 7.848 \times 1000 = 7848$ N

Density, $\rho = 1030$ kg/m^3

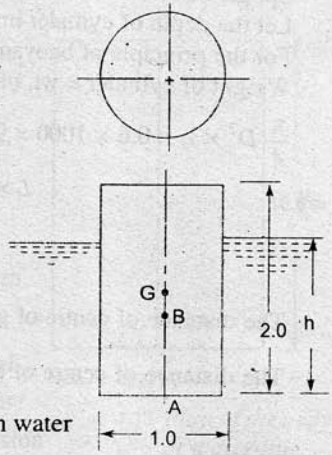

Fig. 4.19

(*i*) Show the cylinder will not float vertically.

(*ii*) Find the force in the chain.

Part I. The cylinder will not float if meta-centric height is – ve.

Let the depth of immersion be h

Then for equilibrium, Weight of cylinder

= Weight of water displaced

= Density × g × Volume of cylinder in water

$$\therefore \quad 7848 = 1030 \times 9.81 \times \frac{\pi}{4} D^2 \times h$$

$$= 10104.3 \times \frac{\pi}{4} (1)^2 \times h$$

$$\therefore \quad h = \frac{4 \times 7848}{10104.3 \times \pi} = 0.989 \text{ m.}$$

∴ The distance of centre of buoyancy B from A,

$$AB = \frac{h}{2} = \frac{0.989}{2} = 0.494 \text{ m.}$$

And the distance of centre of gravity G, from A is $AG = \frac{2.0}{2} = 1.0$ m

$$\therefore \quad BG = AG - AB = 1.0 - .494 = .506 \text{ m.}$$

Now meta-centric height GM is given by $GM = \frac{I}{\forall} - BG$

where $I = \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (1)^4$ m^4

and $\forall$ = Volume of cylinder in water $= \frac{\pi}{4} D^2 \times h = \frac{\pi}{4} 1^2 \times .989$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} \times 1^4}{\frac{\pi}{4} D^2 \times h} = \frac{\frac{\pi}{64} \times 1^4}{\frac{\pi}{4} \times 1^2 \times .989}$$

$$= \frac{1}{16} \times 1^2 \times \frac{1}{.989} = \frac{1}{16 \times .989} = 0.063 \text{ m}$$

$$\therefore \quad GM = .063 - .506 = \mathbf{-0.443 \text{ m. Ans.}}$$

As the meta-centric height is – ve, the point M lies below G and hence the cylinder will be in unstable equilibrium and hence cylinder will not float vertically.

Part II. Let the force applied in a vertical chain attached at the centre of the base of the buoy is T to keep the buoy vertical.

Now find the combined position of centre of gravity (G') and centre of buoyancy (B'). For the combined centre of buoyancy, let h' = depth of immersion when the force T is applied. Then

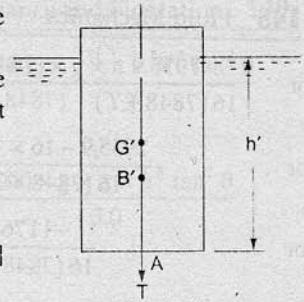

Fig. 4.20

Total downward force = Weight of water displaced

or $(7848 + T)$ = Density of water × g × Volume of cylinder in water

$$= 1030 \times 9.81 \times \frac{\pi}{4} D' \times h' \text{ [where } h' = \text{depth of immersion]}$$

$$\therefore \quad h' = \frac{7848 + T}{10104.3 \times \frac{\pi}{4} \times D^2} = \frac{7848 + T}{10104.3 \times \frac{\pi}{4} \times 1^2} = \frac{10104.3 + T}{7935.9} \text{ m}$$

$$\therefore \quad AB' = \frac{h'}{2} = \frac{1}{2}\left[\frac{7848 + T}{7935.9}\right] = \frac{7848 + T}{15871.8} \text{ m.}$$

The combined centre of gravity (G') due to weight of cylinder and due to tension T in the chain from A is

$$AG' = \text{[Wt. of cylinder} \times \text{Distance of C.G. of cylinder from } A$$
$$+ \, T \times \text{Distance of C.G. of } T \text{ from } A] + [\text{Weight of cylinder} + T]$$

$$= \left(7848 \times \frac{2}{2} + T \times 0\right) + [7848 + T] = \frac{7848}{7848 + T} \text{ m}$$

$$\therefore \quad B'G' = AG' - AB' = \frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}$$

The meta-centric height GM is given by $GM = \frac{I}{\forall} - B'G'$

where $\quad = \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times 1^4 = \frac{\pi}{64} \text{ m}^4$

and $\quad \forall = \frac{\pi}{4} D^2 \times h' = \frac{\pi}{4} \times 1^2 \times \frac{(7848 + T)}{7935.9} = \frac{\pi}{4} \times \frac{7848 + T}{7935.9}$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64}}{\frac{\pi}{4} \frac{(7848 + T)}{7935.9}} = \frac{1}{16} \times \frac{7935.9}{(7848 + T)}$$

$$\therefore \quad GM = \frac{7935.9}{16\,(7848 + T)} - \left[\frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}\right]$$

For stable equilibrium GM should be positive

or $$GM > 0$$

or $$\frac{7935.9}{16\,(7848 + T)} - \left[\frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}\right] > 0$$

or $$\frac{7935.9}{16(7848+T)} - \frac{7848}{(7848+T)} + \frac{7848+T}{15871.8} > 0$$

or $$\frac{7935.9 - 16 \times 7848}{16(7848+T)} + \frac{(7848+T)}{15871.8} > 0$$

or $$\frac{-117632}{16(7848+T)} + \frac{(7848+T)}{15871.8} > 0$$

or $$\frac{(7848+T)}{15871.8} > \frac{117632}{16(7848+T)}$$

or $$(7848+T)^2 > \frac{117632}{16.0} \times 15871.8$$

$$> 116689473.5$$

$$> (10802.3)^2$$

$\therefore$ $$7848 + T > 10802.3$$

$\therefore$ $$T > 10802.3 - 7848$$

$$> \mathbf{2954.3\ N.\ Ans.}$$

$\therefore$ The force in the chain must be at least 2954.3 N so that the cylindrical buoy can be kept in vertical position. **Ans.**

Problem 4.17 *A solid cone floats in water with its apex downwards. Determine the least apex angle of cone for stable equilibrium. The specific gravity of the material of the cone is given 0.8.*

Solution. Given :

Sp. gr. of cone $= 0.8$

Density of cone, $\rho = 0.8 \times 1000 = 800\ kg/m^3$

Let D = Dia. of the cone

d = Dia. of cone at water level

2θ = Apex angle of cone

H = Height of cone

h = Depth of cone in water

G = Centre of gravity of the cone

B = Centre of buoyancy of the cone

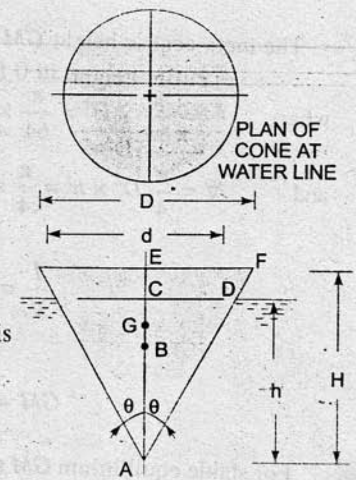

Fig. 4.21

For the cone, the distance of centre of gravity from the apex A is

$$AC = \tfrac{3}{4} \text{height of cone} = \tfrac{3}{4} H$$

also $$AB = \tfrac{3}{4} \text{depth of cone in water} = \tfrac{3}{4} h$$

Volume of water displaced $= \frac{1}{3} \pi r^2 \times h$

Volume of cone $= \frac{1}{3} \times \pi R^2 \times h$

$\therefore$ Weight of cone $= 800 \times g \times \frac{1}{3} \times \pi R^2 \times H$

Now from ΔAEF, $\tan\theta = \dfrac{EF}{EA} = \dfrac{R}{H}$

$\therefore$ $R = H \tan\theta$

Similarly, $r = h \tan\theta$

$\therefore$ Weight of cone $= 800 \times g \times \frac{1}{3} \times \pi \times (H \tan \theta)^2 \times H = \dfrac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3}$

$\therefore$ Weight of water displaced $= 1000 \times g \times \frac{1}{3} \times \pi r^2 \times h$

$$= 1000 \times g \times \tfrac{1}{3} \times \pi (h \tan \theta)^2 \times h = \frac{1000 \times g \times \pi \times h^3 \tan^2 \theta}{3.0}$$

For equilibrium

Weight of cone = Weight of water displaced

or $$\frac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3.0} = \frac{1000 \times 9.81 \times \pi \times h^3 \times \tan^2 \theta}{3.0}$$

or $$800 \times H^3 = 1000 \times h^3$$

$\therefore$ $$H^3 = \frac{1000}{800} \times h^3 \text{ or } \frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3}$$

For stable equilibrium, Meta-centric height GM should be positive. But GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of cone at water-line $= \dfrac{\pi}{64} d^4$

$\forall$ = Volume of cone in water $= \dfrac{1}{3} \dfrac{\pi}{4} d^2 \times h$

$\therefore$ $$\frac{I}{\forall} = \frac{\pi}{64} d^4 \Big/ \frac{1}{3} \times \frac{\pi}{4} d^2 \times h$$

$$= \frac{1 \times 3}{16} \times \frac{d^2}{h} = \frac{3d^2}{16h} = \frac{3}{16h} \times (2r)^2 = \frac{3}{4} \frac{r^2}{h}$$

$$= \frac{3}{4} \frac{(h \tan \theta)^2}{h} \qquad \{\because \quad r = h \tan \theta\}$$

$$= \tfrac{3}{4} h \tan^2 \theta$$

and $$BG = AG - AB = \tfrac{3}{4} H - \tfrac{3}{4} h = \tfrac{3}{4}(H - h)$$

$\therefore$ $$GM = \tfrac{3}{4} h \tan^2 \theta - \tfrac{3}{4}(H - h)$$

For stable equilibrium GM should be positive or

$$\tfrac{3}{4} h \tan^2 - \tfrac{3}{4}(H - h) > 0 \quad \text{or} \quad h \tan^2 \theta - (H - h) > 0$$

or $$h \tan^2 \theta > (H - h) \quad \text{or} \quad h \tan^2 \theta + h > H$$

or $$h[\tan^2 \theta + 1] > H \quad \text{or} \quad 1 + \tan^2 \theta > H/h \quad \text{or} \quad \sec^2 \theta > \frac{H}{h}$$

But $$\frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3} = 1.077$$

$\therefore$ $$\sec^2 \theta > 1.077 \text{ or } \cos^2 \theta > \frac{1}{1.077} = 0.9285$$

$\therefore$ $$\cos \theta > 0.9635$$

$\therefore$ $$\theta > 15° \, 30' \quad \text{or} \quad 2\theta > 31°$$

$\therefore$ Apex angle (2θ) should be at least **31°. Ans.**

Problem 4.18 *A cone of specific gravity S, is floating in water with its apex downwards. It has a diameter D and vertical height H. Show that for stable equilibrium of the cone* $H < \frac{1}{2}\left[\frac{D^2 . S^{1/3}}{2 - S^{1/3}}\right]^{1/2}$.

Solution. Given :

Dia. of cone $= D$

Height of cone $= H$

Sp. gr. of cone $= S$

Let G = Centre of gravity of cone

B = Centre of buoyancy

2θ = Apex angle and

A = Apex of the cone

h = Depth of immersion

d = Dia. of cone at water surface

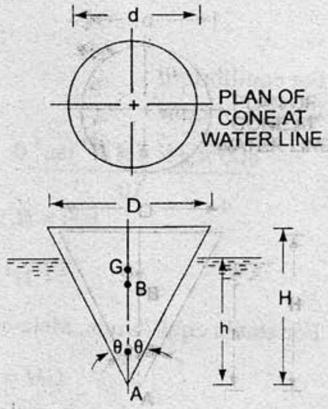

Fig. 4.22

Then $$AG = \frac{3}{4} H$$

$$AB = \frac{3}{4} h$$

Also weight of cone =Weight of water displaced.

$$1000\, S \times g \times \tfrac{1}{3}\, \pi R^2 \times \text{H} = 1000 \times g \times \frac{1}{3} \pi r^2 \times h \quad \text{or} \quad SR^2 H = r^2 h$$

$\therefore$ $$h = \frac{SR^2 H}{r^2}$$

But $$\tan\theta = \frac{R}{H} = \frac{r}{h}$$

$\therefore$ $$R = H \tan\theta,\ r = h \tan\theta$$

$\therefore$ $$h = \frac{S \times (H \tan\theta)^2 \times H}{(h \tan\theta)^2}$$

$$h = \frac{S \times H^2 \times \tan^2\theta \times H}{h^2 \tan^2\theta} = \frac{SH^3}{h^2} \quad \text{or} \quad h^3 = SH^3$$

or $$h = (SH^3)^{1/3} = S^{1/3} H \qquad \text{...(1)}$$

Distance, $$BG = AG - AB$$

$$= \frac{3}{4} H - \frac{3}{4} h = \frac{3}{4} (H - h) = \frac{3}{4} (H - S^{1/3} H) \qquad \{\because h = S^{1/3} H\}$$

$$= \frac{3}{4} H[1 - S^{1/3}] \qquad \text{...(2)}$$

Also I = M.O. Inertia of the plan of body at water surface

$$= \frac{\pi}{64} d^4$$

$$\forall = \text{Volume of cone in water} = \frac{1}{3} \times \frac{\pi}{4} \times d^2 \times h = \frac{1}{3} \frac{\pi}{4} d^2 [H S^{1/3}]$$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} d^4}{\frac{1}{3} \times \frac{\pi}{4} d^2 \ H.S^{1/3}} = \frac{3d^2}{16.H.S^{1/3}}$$

Now Meta-centric height GM is given as

$$GM = \frac{I}{\forall} - BG = \frac{3d^2}{16.H.S^{1/3}} - \frac{3H}{4}\,[1 - S^{1/3}]$$

GM should be +ve for stable equilibrium or $GM > 0$

or
$$\frac{3d^2}{16.H.S^{1/3}} - \frac{3H}{4}\,(1 - S^{1/3}) > 0$$

or
$$\frac{3d^2}{16.H.S^{1/3}} > \frac{3H}{4}\,(1 - S^{1/3}) \quad \ldots(3)$$

Also we know $R = H \tan \theta$ and $r = h \tan \theta$

$$\therefore \quad \frac{R}{r} = \frac{H}{h} = \frac{D}{d}$$

$$\therefore \quad d = \frac{Dh}{H} = \frac{D}{H} \times HS^{1/3} = DS^{1/3}$$

Substituting the value of d in equation (3), we get

$$\frac{3\left(DS^{1/3}\right)^2}{16.H.S^{1/3}} > \frac{3H}{4}\,(1 - S^{1/3}) \quad \text{or} \quad \frac{D^2 .S^{1/3}}{4.H} > H\,(1 - S^{1/3})$$

or
$$\frac{D^2 .S^{1/3}}{4\left(1 - S^{1/2}\right)} > H^2 \quad \text{or} \quad H^2 < \frac{D^2 .S^{1/3}}{4\left(1 - S^{1/3}\right)}$$

or
$$H < \frac{1}{2}\left[\frac{D^2 .S^{1/3}}{1 - S^{1/3}}\right]^{1/2} . \textbf{ Ans.}$$

▶ 4.8 EXPERIMENTAL METHOD OF DETERMINATION OF META-CENTRIC HEIGHT

The meta-centric height of a floating vessel can be determined, provided we know the centre of gravity of the floating vessel. Let w_1 is a known weight placed over the centre of the vessel as shown in Fig. 4.23 (*a*) and the vessel is floating.

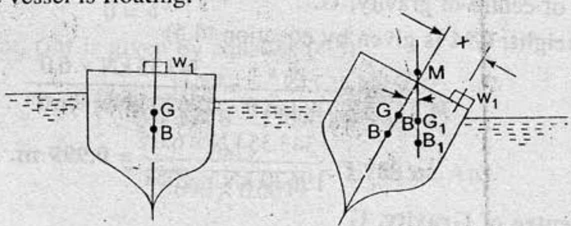

Fig. 4.23 *Meta-centric height.*

Let W = Weight of vessel including w_1

G = Centre of gravity of the vessel

B = Centre of buoyancy of the vessel

The weight w_1 is moved across the vessel towards right through a distance x as shown in Fig. 4.23 (*b*). The vessel will be tilted. The angle of heel θ is measured by means of a plumbline and a protractor attached on the vessel. The new centre of gravity of the vessel will shift to G_1 as the weight w_1 has been moved towards the right. Also the centre of buoyancy will change to B_1 as the vessel has tilted. Under equilibrium, the moment caused by the movement of the load w_1 through a distance x must be equal to the moment caused by the shift of the centre of gravity from G to G_1. Thus

The moment due to change of $G = GG_1 \times W = W \times GM \tan \theta$

The moment due to movement of $w_1 = w_1 \times x$

$\therefore \quad w_1 x = WGM \tan \theta$

Hence
$$GM = \frac{w_1 x}{W \tan \theta} \quad \ldots(4.5)$$

Problem 4.19 *A ship 70 m long and 10 m broad has a displacement of 19620 kN. A weight of 343.35 kN is moved across the deck through a distance of 6 m. The ship is tilted through 6°. The moment of inertia of the ship at water-line about its fore and aft axis is 75% of M.O.I. of the circumscribing rectangle. The centre of buoyancy is 2.25 m below water-line. Find the meta-centric height and position of centre of gravity of ship. Specific weight of sea water is 10104 N/m³.*

(Anna University, May, 1986)

Solution. Given :

Length of ship, $L = 70$ m

Breadth of ship, $b = 10$ m

Displacement, $W = 19620$ kN

Angle of heel, $\theta = 6°$

M.O.I. of ship at water-line = 75% of M.O.I. of circumscribing rectangle

w for sea-water = 10104 N/m^3 = 10.104 kN/m^3

Movable weight, $w_1 = 343.35$ kN

Distance moved by w_1, $x = 6$ m

Centre of buoyancy = 2.25 m below water surface

Find (*i*) Meta-centric height, GM

(*ii*) Position of centre of gravity, G.

(*i*) **Meta-centric height, GM** is given by equation (4.5)

$$\therefore \quad GM = \frac{w_1 x}{W \tan \theta} = \frac{343.35 \text{ kN} \times 6.0}{19620 \text{ kN} \times \tan 6°}$$

$$= \frac{343.35 \text{ kN} \times 6.0}{19620 \text{ kN} \times .1051} = \textbf{0.999 m. Ans.}$$

(*ii*) **Position of Centre of Gravity, G**

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the ship at water-line about y-y

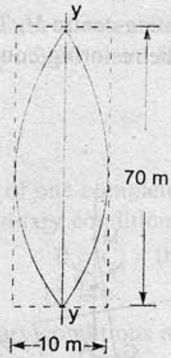

Fig. 4.24

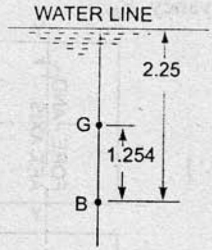

Fig. 4.25

$$= 75\% \text{ of } \frac{1}{12} \times 70 \times 10^3 = .75 \times \frac{1}{12} \times 70 \times 10^3 = 4375 \text{ m}^4$$

and $\forall$ = Volume of ship in water $= \dfrac{\text{Weight of ship}}{\text{Weight densit of water}} = \dfrac{19620}{10.104} = 1941.74 \text{ m}^3$

$$\therefore \quad \frac{I}{\forall} = \frac{4375}{1941.74} = 2.253 \text{ m}$$

$$\therefore \quad GM = 2.253 - BG \text{ or } .999 = 2.253 - BG$$

$$\therefore \quad BG = 2.253 - .999 = 1.254 \text{ m.}$$

From Fig. 4.25, it is clear that the distance of G from free surface of the water = distance of B from water surface − BG

$$= 2.25 - 1.254 = \mathbf{0.996 \text{ m. Ans.}}$$

Problem 4.20 *A pontoon of 15696 kN displacement is floating in water. A weight of 245.25 kN is moved through a distance of 8 m across the deck of pontoon, which tilts the pontoon through an angle 4°. Find meta-centric height of the pontoon.*

Solution. Given :

Weight of pontoon = Displacement

or $W = 15696$ kN

Movable weight, $w_1 = 245.25$ kN

Distance moved by weight w_1, $x = 8$ m

Angle of heel, $\theta = 4°$

The meta-centric height, GM is given by equation (4.5)

or
$$GM = \frac{w_1 x}{W \tan\theta} = \frac{245.25 \text{ kN} \times 8}{15696 \text{ kN} \times \tan 4°}$$

$$= \frac{1962}{15696 \times 0.0699} = \mathbf{1.788 \text{ m. Ans.}}$$

▶ 4.9 OSCILLATION (ROLLING) OF A FLOATING BODY

Consider a floating body, which is tilted through an angle by an overturning couple as shown in Fig. 4.26. Let the over-turning couple is suddenly removed. The body will start oscillating. Thus, the body will be in a state of oscillation as if suspended at the meta-centre M. This is similar to the case of a pendulum. The only force acting on the body is due to the restoring couple due to the weight W of the body force of buoyancy F_B.

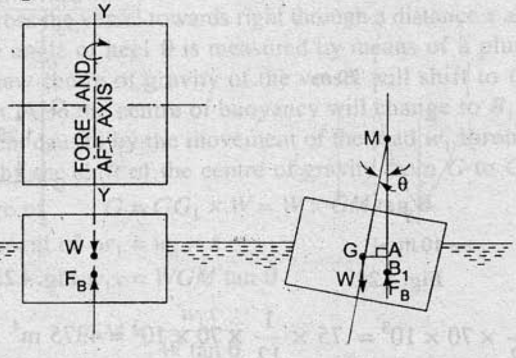

Fig. 4.26

$\therefore$ Restoring couple $= W \times$ Distance GA

$= W \times GM \sin\theta$ ...(*i*)

This couple tries to decrease the angle

Angular acceleration of the body, $\alpha = -\dfrac{d^2\theta}{dt^2}$.

– ve sign has been introduced as the restoring couple tries to decrease the angle θ.

Torque due to inertia $=$ Moment of Inertia about Y-$Y \times$ Angular acceleration

$$= I_{Y\text{-}Y} \times \left(-\frac{d^2\theta}{dt^2}\right)$$

But $I_{Y\text{-}Y} = \dfrac{W}{g} K^2$

where W = Weight of body, K = Radius of gyration about Y-Y

$$\therefore \quad \text{Inertia torque} = \frac{W}{g} K^2 \left(-\frac{d^2\theta}{dt^2}\right) = -\frac{W}{g} K^2 \frac{d^2\theta}{dt^2} \quad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$W \times GM \sin\theta = -\frac{W}{g} K^2 \frac{d^2\theta}{dt^2} \quad \text{or} \quad GM \sin\theta = -\frac{K^2}{g} \frac{d^2\theta}{dt^2}$$

For small angle θ. $\sin\theta \approx \theta$

$$\therefore \quad GM \times \theta = -\frac{K^2}{g} \frac{d^2\theta}{dt^2} \quad \text{or} \quad \frac{K^2}{g} \frac{d^2\theta}{dt^2} + GM \times \theta = 0$$

Dividing by $\dfrac{K^2}{g}$, we get $\dfrac{d^2\theta}{dt^2} + \dfrac{GM \times g \times \theta}{K^2} = 0$

The above equation is a differential equation of degree second. The solution is

$$\theta = C_1 \sin \sqrt{\frac{GM.g}{K^2}} \times t + C_2 \cos \sqrt{\frac{GM.g \times t}{K^2}} \quad ...(iii)$$

where C_1 and C_2 are constants of integration.

The values of C_1 and C_2 are obtained from boundary conditions which are

(*i*) at $t = 0, \theta = 0$

(*ii*) at $t = \frac{T}{2}, \theta = 0$

where T is the time period of one complete oscillation.

Substituting the 1st boundary condition in (*iii*), we get

$$0 = C_1 \times 0 + C_2 \times 1 \qquad \{\because \sin\theta = 0, \cos\theta = 1\}$$

$$\therefore \quad C_2 = 0$$

Substituting 2nd boundary conditions in (*iii*), we get

$$0 = C_1 \sin \sqrt{\frac{GM.g}{K^2}} \times \frac{T}{2}$$

But C_1 cannot be equal to zero and so the other alternative is

$$\sin \sqrt{\frac{GM.g}{K^2}} \times \frac{T}{2} = 0 = \sin\pi \qquad \{\because \sin\pi = 0\}$$

$$\therefore \quad \sqrt{\frac{GM.g}{K^2}} \times \frac{T}{2} = \pi \quad \text{or} \quad T = 2\pi\sqrt{\frac{K^2}{GM.g}} \qquad \ldots(4.6)$$

$\therefore$ Time period of oscillation is given by equation (4.6).

Problem 4.21 *The least radius of gyration of a ship is 8 m and meta-centric height 70 cm. Calculate the time period of oscillation of the ship.*

Solution. Given :

Least radius of gyration, $K = 8$ m

Meta-centric height, $GM = 70$ cm $= 0.70$ m

The time period of oscillation is given by equation (4.6).

$$T = 2\pi\sqrt{\frac{K^2}{GM.g}} = 2\pi\sqrt{\frac{8 \times 8}{0.7 \times 9.81}} = \mathbf{19.18 \text{ sec. Ans.}}$$

Problem 4.22 *The time period of rolling of a ship of weight 29430 kN in sea water is 10 seconds. The centre of buoyancy of the ship is 1.5 m below the centre of gravity. Find the radius of gyration of the ship if the moment of inertia of the ship at the water line about fore and aft. axis is 1000 m^4. Take specific weight of sea water as = 10100 N/m^3.*

Solution. Given :

Time period, $T = 10$ sec

Distance between centre of buoyancy and centre of gravity, $BG = 1.5$ m

Moment of Inertia, $I = 10000$ m^4

Weight, $W = 29430$ kN $= 29430 \times 1000$ N

Let the radius of gyration $= K$

First calculate the meta-centric height GM, which is given as

$$GM = BM - BG = \frac{I}{\forall} - BG$$

where I = M.O. Inertia

and $\forall$ = Volume of water displaced

$$= \frac{\text{Weight of ship}}{\text{Sp. weight of sea water}} = \frac{29430 \times 1000}{10104} = 2912.6 \text{ m}^3$$

$$\therefore \quad GM = \frac{10000}{2912.6} - 1.5 = 3.433 - 1.5 = 1.933 \text{ m.}$$

Using equation (4.6), we get $T = 2\pi \sqrt{\frac{K^2}{GM \times g}}$

or $$10 = 2\pi \sqrt{\frac{K^2}{1.933 \times 9.81}} = \frac{2\pi K}{\sqrt{1.933 \times 9.81}}$$

or $$K = \frac{10 \times \sqrt{1.933 \times 9.81}}{2\pi} = \mathbf{6.93 \text{ m. Ans.}}$$

HIGHLIGHTS

1. The upward force exerted by a liquid on a body when the body is immersed in the liquid is known buoyancy or force of buoyancy.
2. The point through which force of buoyancy is supposed to act is called centre of buoyancy.
3. The point about which a body starts oscillating when the body is tilted is known meta-centre.
4. The distance between the meta-centre and centre of gravity is known as meta-centric height.
5. The meta-centric height (GM) is given by $GM = \frac{I}{\forall} - BG$

   where I = Moment of Inertia of the floating body (in plan) at water surface about the axis Y-Y

   $\forall$ = Volume of the body sub-merged in water

   BG = Distance between centre of gravity and centre of buoyancy.
6. Conditions of equilibrium of a floating and submerged body are :

Equilibrium	*Floating Body*	*Sub-merged Body*
(*i*) Stable Equilibrium	M is above G	B is above G
(*ii*) Unstable Equilibrium	M is below G	B is below G
(*iii*) Neutral Equilibrium	M and G coincide	B and G coincide

7. The value of meta-centric height GM, experimentally is given as $GM = \frac{w_1 x}{W \tan \theta}$

   where w_1 = Movable weight

   x = Distance through which w_1 is moved

   W = Weight of the ship or floating body including w_1

   θ = Angle through the ship or floating body is tilted due to the movement of w_1.
8. The time period of oscillation or rolling of a floating body is given by $T = 2\pi \sqrt{\frac{K^2}{GM \times g}}$

   where K = Radius of gyration, GM = Meta-centric height

   T = Time of one complete oscillation.

EXERCISE 4

(A) THEORETICAL PROBLEMS

1. Define the terms 'buoyancy' and 'centre of buoyancy'.
2. Explain the terms 'meta-centre' and 'meta-centric height'.
3. Derive an expression for the meta-centric height of a floating body.
4. Show that the distance between the meta-centre and centre of buoyancy is given by $BM = \frac{I}{\forall}$

   where I = Moment of inertia of the plan of the floating body at water surface about longitudinal axis.

   $\forall$ = Volume of the body submerged in liquid.
5. What are the conditions of equilibrium of a floating body and a submerged body ? (*A.S.M.E., June 1992 ; Delhi University, 1982*)
6. How will you determine the meta-centric height of a floating body experimentally ? Explain with neat sketch.
7. Select the correct statement :
   (*a*) The buoyant force for a floating body passes through the
   (*i*) centre of gravity of the body (*ii*) centroid of volume of the body
   (*iii*) meta-centre of the body (*iv*) centre of gravity of the submerged part of the body
   (*v*) centroid of the displaced volume.
   (*b*) A body submerged in liquid is in equilibrium when :
   (*i*) its meta-centre is above the centre of gravity
   (*ii*) its meta-centre is above the centre of buoyancy
   (*iii*) its centre of gravity is above the centre of buoyancy
   (*iv*) its centre of buoyancy is above the centre of gravity
   (*v*) none of these. **[Ans.** 7 (*a*) (*v*), (*b*) (*iv*)**]**
8. Derive an expression for the time period of the oscillation of a floating body in terms of radius of gyration and meta-centric height of the floating body.
9. Define the terms : meta-centre, centre of buoyancy, meta-centric height, gauge pressure and absolute pressure. (*A.S.M.E., June 1992*)
10. What do you understand by the hydrostatic equation ? With the help of this equation, derive the expression for the buoyant force acting on a sub-merged body. (*A.M.I.E. S 1990*)
11. With neat sketches, explain the conditions of equilibrium for floating and sub-merged bodies. (*Delhi University, June 1996*)
12. Differentiate between :
    (*i*) Dynamic viscosity and kinematic viscoscity, (*ii*) Absolute and gauge pressure (*iii*) Simple and differential manometers (*iv*) Centre of gravity and centre of buoyancy. (*Delhi University, Dec. 2002*)

(B) NUMERICAL PROBLEMS

1. A wooden block of width 2 m, depth 1.5 m and length 4 m floats horizontally in water. Find the volume of water displaced and position of centre buoyancy. The specific gravity of the wooden block is 0.7. **[Ans.** $8.4\ m^3$, 0.525 m from the base**]**

2. A wooden log of 0.8 m diameter and 6 m length is floating in river water. Find the depth of wooden log in water when the sp. gr. of the wooden log is 0.7. [Ans. 0.54 m]
3. A stone weighs 490.5 N in air and 196.2 N in water. Determine the volume of stone and its specific gravity. [Ans. 0.03 m^3 or 3×10^4 cm^3, 1.67]
4. A body of dimensions 2.0 m × 1.0 m × 3.0 m weighs 3924 N in water. Find its weight in air. What will be its specific gravity ? [Ans. 62784 N, 1.0667]
5. A metallic body floats at the interface of mercury of sp. gr. 13.6 and water in such a way that 30% of its volume is submerged in mercury and 70% in water. Find the density of the metallic body. [Ans. 4780 kg/m^3]
6. A body of dimensions 0.5 m × 0.5 m × 1.0 m and of sp. gr. 3.0 is immersed in water. Determine the least force required to lift the body. [Ans. 4905 N]
7. A rectangular pontoon is 4 m long, 3 m wide and 1.40 m high. The depth of immersion of the pontoon is 1.0 m in sea-water. If the centre of gravity is 0.70 m above the bottom of the pontoon, determine the meta-centric height. Take the density of sea-water as 1030 kg/m^3. [Ans. 0.45 m]
8. A uniform body of size 4 m long × 2 m wide × 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.6 m ? Determine the meta-centric height also. [Ans. 47088 N, 0.355 m]
9. A block of wood of specific gravity 0.8 floats in water. Determine the meta-centric height of the block if its size is 3 m × 2 m × 1 m. [Ans. 0.316 m]
10. A solid cylinder of diameter 3.0 m has a height of 2 m. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder is 0.7. [Ans. 0.1017 m]
11. A body has the cylindrical upper portion of 4 m diameter and 2 m deep. The lower portion is a curved one, which displaces a volume of 0.9 m^3 of water. The centre of buoyancy of the curved portion is at a distance of 2.10 m below the top of the cylinder. The centre of gravity of the whole body is 1.50 m below the top of the cylinder. The total displacement of water is 4.5 tonnes. Find the meta-centric height of the body. [Ans. 2.387 m]
12. A solid cylinder of diameter 5.0 m has a height of 5.0 m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder is 0.7 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable. [Ans. – 0.304 m, Unstable Equilibrium]
13. A solid cylinder of 15 cm diameter and 60 cm long, consists of two parts made of different materials. The first part at the base is 1.20 cm long and of specific gravity = 5.0. The other parts of the cylinder is made of the material having specific gravity 0.6. State, if it can float vertically in water. [Ans. $GM = -5.26$, Unstable, Equilibrium]
14. A rectangular pontoon 8.0 m long, 7 m broad and 3.0 m deep weighs 588.6 kN. It carries on its upper deck an empty boiler of 4.0 m diameter weighing 392.4 kN. The centre of gravity of the boiler and the pontoon are at their respective centres along a vertical line. Find the meta-centric height. Weight density of sea-water is 10104 N/m^3. [Ans. 0.325 m]
15. A wooden cylinder of sp. gr. 0.6 and circular in cross-section is required to float in oil (sp. gr. 0.8). Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil where L is the height of cylinder and D is its diameter. [Ans. $(L/D) < 0.8164$]
16. Show that a cylindrical buoy of 1.5 m diameter and 3 m long weighing 2.5 tonnes will not float vertically in sea-water of density 1030 kg/m^3. Find the force necessary in a vertical chain attached at the centre of the base of the buoy that will keep it vertical. [Ans. 10609.5 N]
17. A solid cone floats in water its apex downwards. Determine the least apex angle of cone for stable equilibrium. The specific gravity of the material of the cone is given 0.7. [Ans. 39° 7′]
18. A ship 60 m long and 12 m broad has a displacement of 19620 kN. A weight of 294.3 kN is moved across the deck through a distance of 6.5 m. The ship is tilted through 5°. The moment of inertia of the ship at

water line about its force and aft. axis is 75% of moment of inertia the circumscribing rectangle. The centre of buoyancy is 2.75 m below water line. Find the meta-centric height and position of centre of gravity of ship. Take specific weight of sea water = 10104 N/m^3.

[Ans. 1.1145 m, 0.53 m below water surface]

19. A pontoon of 1500 tonnes displacement is floating in water. A weight of 20 tonnes is moved through a distance of 6 m across the deck of pontoon, which tilts the pontoon through an angle of 5°. Find meta-centric height of the pontoon. **[Ans.** 0.9145 m]

20. Find the time period of rolling of a solid circular cylinder of radius 2.5 m and 5.0 m long. The specific gravity of the cylinder is 0.9 and is floating in water with its axis vertical. **[Ans.** 0.35 sec]

6 CHAPTER

Dynamics of Fluid Flow

▶ 6.1 INTRODUCTION

In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

▶ 6.2 EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x \qquad ...(6.1)$$

In the fluid flow, the following forces are present :

(*i*) F_g, gravity force.
(*ii*) F_p, the pressure force.
(*iii*) F_v, force due to viscosity.
(*iv*) F_t, force due to turbulence.
(*v*) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

▶ 6.3 EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are :

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s \qquad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.

Now $\quad a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{dS}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho g\, dAds \cos\theta = \rho dAds \times \frac{v\partial v}{\partial s}$$

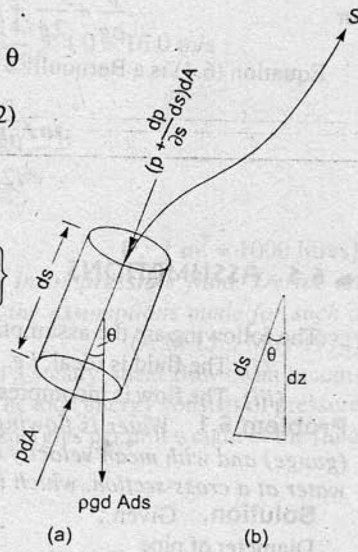

Fig. 6.1 *Forces on a fliiid element.*

Dividing by $\rho dsdA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $\quad \frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{\partial p}{\partial \rho} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + gdz + vdv = 0$$

or $$\frac{\partial p}{\rho} + gdz + vdv = 0 \qquad \text{...(6.3)}$$

Equation (6.3) is known as Euler's equation of motion.

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▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

Problem 6.1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe = 5 cm = 0.5 m

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0$ m/s

Datum head, $z = 5$ m

Total head = pressure head + kinetic head + datum head

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m} \qquad \left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

∴ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem 6.2 *A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.*

Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$
$$V_1 = 4.0 \text{ m/s}$$
$$D_2 = 0.1 \text{ m}$$

$\therefore$
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

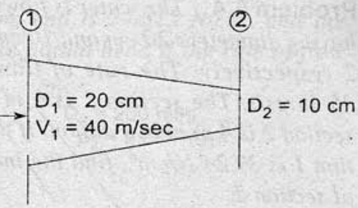

Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$\therefore$
$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2
$$= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$$

(*iii*) Rate of discharge
$$= A_1V_1 \quad \text{or} \quad A_2V_2$$
$$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$$
$$= \textbf{125.6 litres/s. Ans.} \qquad \{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$$

Problem 6.3 *State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation.* (A.M.I.E., May 1974)

Solution. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

$$\text{Pressure energy} = \frac{p}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{Constant.}$$

Derivation of Bernoulli's theorem. For derivation of Bernoulli's theorem, the Articles 6.3 and 6.4 should be written.

Assumptions are given in Article 6.5.

Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

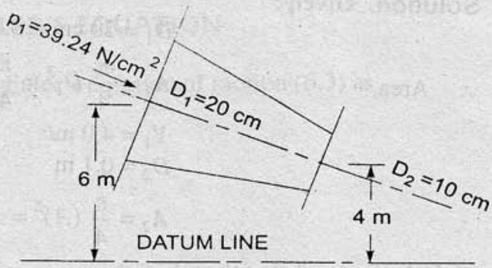

Fig. 6.3

Solution. Given :

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \dfrac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or

$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or

$$46.063 = \frac{p_2}{9810} + 5.012$$

$$\therefore \quad \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore \quad p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}. \text{ Ans.}$$

Problem 6.5 *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

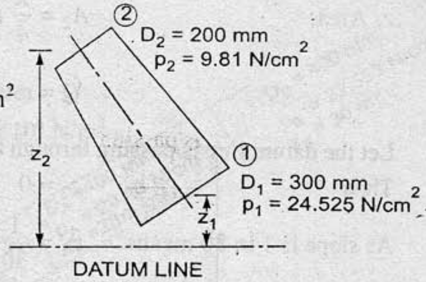

Fig. 6.4

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$$\therefore \quad V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$$\therefore \quad z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem 6.6 *The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².*

Solution. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$$

$$= 0.2827 \text{ m}^2$$

$$p_1 = \text{pressure at upper end}$$

$$= 19.62 \text{ N/cm}^2$$

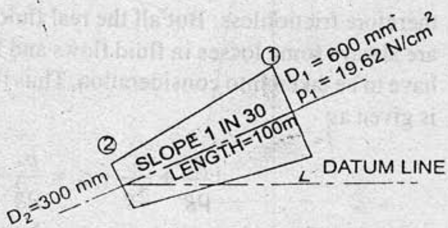

Fig. 6.5

$= 19.62 \times 10^4\ \text{N/m}^2$

Dia. at lower end, $D_2 = 300\ \text{mm} = 0.3\ \text{m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^{\,2} = \frac{\pi}{4}(.3)^2 = 0.07068\ \text{m}$

Q = rate of flow = 50 litres/s = $\frac{50}{1000} = 0.05\ \text{m}^3/\text{s}$

Let the datum line is passing through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}\ \text{m}$

Also we know $Q = A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768\ \text{m/sec} = 0.177\ \text{m/s}$$

and
$$V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074\ \text{m/sec} = 0.707\ \text{m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or
$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or
$$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or
$$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or
$$p_2 = 23.3 \times 9810\ \text{N/m}^2 = 228573\ \text{N/m}^2 = \mathbf{22.857\ N/cm^2.\ Ans.}$$

▶ 6.6 BERNOULLI'S EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \qquad \text{...(6.5)}$$

where h_L is loss of energy between points 1 and 2.

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

Solution. Given :

Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point A, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$

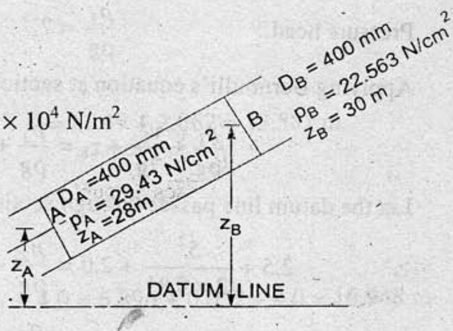

Fig. 6.6

$\therefore$ Total energy at A, $$E_A = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

At point B, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

$\therefore$ Total energy at B, $$E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 = $ **5.0 m. Ans.**

Problem 6.8 *A conical tube of length 2.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is* $\frac{0.35(v_1 - v_2)^2}{2g}$, *where* v_1 *is the velocity at the smaller end and* V_2 *at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.*

Solution. Let the smaller end is represented by (1) and lower end by (2)

Given :

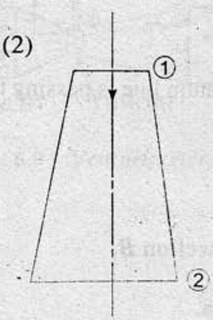

Fig. 6.7

Length of tube, $L = 2.0 \text{ m}$

$v_1 = 5 \text{ m/s}$

$p_1/\rho g = 2.5 \text{ m of liquid}$

$v_2 = 2 \text{ m/s}$

Loss of head $$= h_L = \frac{0.35(v_1 - v_2)^2}{2g}$$

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$$= \frac{0.35\,[5-2]^2}{2g} = \frac{0.35\times 9}{2\times 9.81} = 0.16 \text{ m}$$

Pressure head, $\dfrac{p_2}{\rho g} = ?$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2). Then $z_2 = 0$, $z_1 = 2.0$

$$\therefore \quad 2.5 + \frac{5^2}{2\times 9.81} + 2.0 = \frac{p_2}{\rho g} + \frac{2^2}{2\times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{p_2}{\rho g} + 0.203 + .16$$

or $$\frac{p_2}{\rho g} = (2.5 + 1.27 + 2.0) - (.203 + .16)$$

$$= 5.77 - .363 = \mathbf{5.407 \text{ m of fluid. Ans.}}$$

Problem 6.9 *A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 litres/s determine the loss of head and direction of flow.* (A.M.I.E., Summer 1976)

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3/\text{s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \dfrac{\text{kg}}{\text{m}^3}$

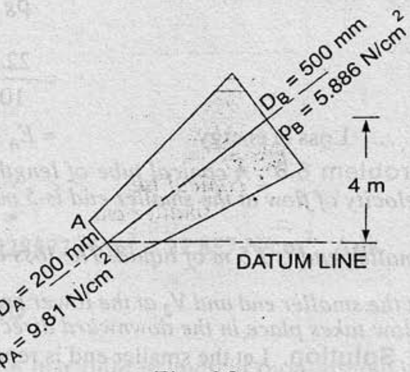

Fig. 6.8

Given : **At section *A*,** $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \dfrac{\pi}{4}(D_A)^2 = \dfrac{\pi}{4}(.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

If datum line is passing through *A*, then

$Z_A = 0$

$V_A = \dfrac{Q}{A_A} = \dfrac{0.2}{0.0314} = 6.369 \text{ m/s}$

At section *B*, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \dfrac{\pi}{4} D_B{}^2 = \dfrac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A $= E_A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B $= E_B = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

(*ii*) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

▶ 6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) $= \frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

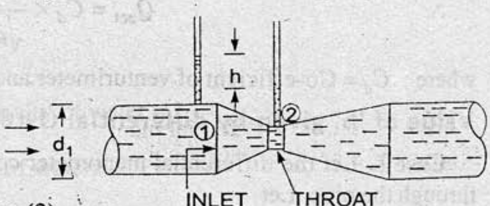

Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge,

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid
S_o = Sp. gravity of the liquid flowing through pipe
x = Difference of the heavier liquid column in U-tube

Then

$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad ...(6.10)$$

where S_1 = Sp. gr. of lighter liquid in U-tube

S_0 = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \quad ...(6.11)$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \quad ...(6.12)$$

Problem 6.10 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3\text{/s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

∴ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

∴ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3\text{/s}$$

$$= 70465 \text{ cm}^3\text{/s} = \mathbf{70.465 \text{ litres/s. Ans.}}$$

Problem 6.12 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Solution. Given : $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$$

Using the equation (6.8), $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$$\therefore \quad h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \quad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \quad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

∴ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

Problem 6.13 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm² and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$$

Dia. at throat, $d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$$

$$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$$

ρ for water $= 1000 \dfrac{\text{kg}}{\text{m}^3}$ and $\therefore \dfrac{p_1}{\rho g} = \dfrac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$$\frac{p_2}{\rho g} = -30 \text{ cm of mercury}$$

$$= -0.30 \text{ m of mercury} = -0.30 \times 13.6 = -4.08 \text{ m of water}$$

$\therefore$ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water $= 2208$ cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3\text{/s} = \textbf{165.555 lit/s. Ans.}$$

Problem 6.14 *The inlet and throat diameters of a horizontal venturimeter are 30 cm and 10 cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm² while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.* (A.M.I.E., Winter, 1980)

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} (10)^2 = 78.54 \text{ cm}^2$

Pressure, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $\frac{p_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14$ m of water

$\frac{p_2}{\rho g} = -37$ cm of mercury

$= \frac{-37 \times 13.6}{100}$ m of water $= -5.032$ m of water

Differential head, $h = p_1/\rho g - p_2/\rho g$

$= 14.0 - (-5.032) = 14.0 + 5.032$

$= 19.032$ m of water $= 1903.2$ cm

Head lost, $h_f = 4\%$ of $h = \frac{4}{100} \times 19.032 = 0.7613$ m

$\therefore$ $C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{19.032 - .7613}{19.032}} = 0.98$

$\therefore$ Discharge $= C_d \dfrac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$$= \frac{0.98 \times 706.85 \times 78.54 \times \sqrt{2 \times 981 \times 1903.2}}{\sqrt{(706.85)^2 - (78.54)^2}}$$

$$= \frac{105132247.8}{\sqrt{499636.9 - 6168}} = 149692.8 \text{ cm}^3\text{/s} = \mathbf{0.14969\ m^3/s.\ Ans.}$$

PROBLEMS ON INCLINED VENTURIMETER

Problem 6.15 *A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \dfrac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_l}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3\text{/s} = \mathbf{125.756\ lit/s.\ Ans.}$

Problem 6.16 *A 20 cm × 10 cm venturimeter is inserted in a vertical pipe carrying oil of sp. gr. 0.8, the flows of oil is in upward direction. The difference of levels between the throat and inlet section is 50 cm. The oil mercury differential manometer gives a reading of 30 cm of mercury. Find the discharge of oil. Neglect losses.*

Solution. Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

Differential manometer reading, $x = 30$ cm

$$\therefore \quad h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_g}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 30\,[17 - 1] = 30 \times 16 = 480 \text{ cm of oil}$$

$$C_d = 1.0$$

The discharge,
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{1.0 \times 314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 480} \text{ cm}^3/s$$

$$= \frac{23932630.7}{304} = 78725.75 \text{ cm}^3/\text{s} = \mathbf{78.725 \text{ litres/s. Ans.}}$$

Problem 6.17 *In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diametres are 16 cm and 8 cm respectively. A is 2 metres above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.* (A.M.I.E., Summer 1985)

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800 \frac{\text{kg}}{\text{m}^3}$

Dia. at A, $D_A = 16 \text{ cm} = 0.16 \text{ m}$

$\therefore$ Area at A, $A_1 = \frac{\pi}{4}(.16)^2 = 0.0201 \text{ m}^2$

Dia. at B, $D_B = 8 \text{ cm} = 0.08 \text{ m}$

$\therefore$ Area at B, $A_2 = \frac{\pi}{4}(.08)^2 = 0.005026 \text{ m}^2$

(*i*) Difference of pressures, $p_B - p_A = 0.981 \text{ N/cm}^2$

$$= 0.981 \times 10^4 \text{ N/m}^2 = \frac{9810\text{N}}{\text{m}^2}$$

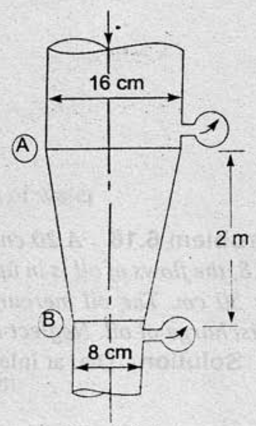

Fig. 6.9 (*a*)

Difference of pressure head

$$\therefore \quad \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25 \qquad (\because \rho = 800 \text{ kg/m}^3)$$

Applying Bernoulli's theorem at A and B and taking the reference line passing through section B, we get

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+Z_B$$

or $$\frac{p_A}{\rho g}+\frac{p_B}{p_g}+Z_A-Z_B=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or $$\left(\frac{p_A-p_B}{\rho g}\right)+2.0-0.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or $$-1.25+2.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \qquad \left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$$

$$0.75=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \qquad \ldots(i)$$

Now applying continuity equation at A and B, we get

$$V_A\times A_1=V_B\times A_2$$

or $$V_B=\frac{V_A\times A_1}{A_2}=\frac{V_A\times\frac{\pi}{4}(.16)^2}{\frac{\pi}{4}(.08)^2}=4V_A$$

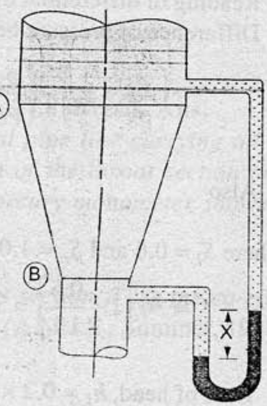

Fig. 6.9 (*b*)

Substituting the value of V_B in equation (i), we get

$$0.75=\frac{16V_A^2}{2g}-\frac{V_A^2}{2g}=\frac{15V_A^2}{2g}$$

$$\therefore \qquad V_A=\sqrt{\frac{0.75\times2\times9.81}{15}}=0.99 \text{ m/s}$$

$\therefore$ Rate of flow, $Q=V_A\times A_1$

$= 0.99\times0.0201=$ **0.01989 m³/s. Ans.**

(*ii*) Difference of level of mercury in the *U-tube*.

Let h = Difference of mercury level.

Then $$h=x\left(\frac{S_g}{S_o}-1\right)$$

where $$h=\left(\frac{p_A}{\rho g}+Z_A\right)-\left(\frac{p_B}{\rho g}+Z_B\right)=\frac{p_A-p_B}{\rho g}+Z_A-Z_B$$

$= -1.25 + 2.0 - 0$

$= 0.75 \qquad \left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$

$$\therefore \qquad 0.75=x\left[\frac{13.6}{0.8}-1\right]=x\times16$$

$\therefore \quad x = \frac{0.75}{16} = 0.04687 \text{ m} = \mathbf{4.687 \text{ cm. Ans.}}$

Problem 6.18 *Find the discharge of water flowing through a pipe 30 cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and throat is measured by a liquid of sp. gr. 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of the pipe.*

Solution. Dia. at inlet $= 30$ cm

$\therefore \quad d_1 = 30$ cm

$$a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$$

Dia. at throat, $d_2 = 15$ cm

$$\therefore \quad a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$$

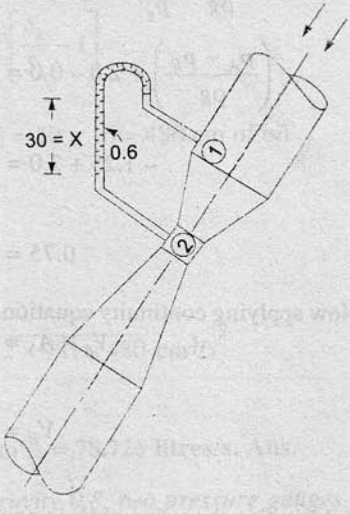

Fig. 6.10

Reading of differential manometer, $x = 30$ cm

Difference of pressure head, h is given by

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

Also
$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where $S_l = 0.6$ and $S_o = 1.0$

$$= 30\left[1 - \frac{0.6}{1.0}\right] = 30 \times .4 = 12.0 \text{ cm of water}$$

Loss of head, $h_L = 0.2 \times$ kinetic head of pipe $= 0.2 \times \frac{v_1^2}{2g}$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_L$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = 12.0 \text{ cm of water}$$

and
$$h_L = 0.2 \times v_1^2/2g$$

$$\therefore \quad 12.0 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.2 \times \frac{v_1^2}{2g}$$

$$\therefore \quad 12.0 + 0.8\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0 \quad \text{...(1)}$$

Applying continuity equation at (1) and (2), we get

$$a_1 v_1 = a_2 v_2$$

$$\therefore \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{\frac{\pi}{4}(15)^2 v_2}{\frac{\pi}{4}(30)^2} = \frac{v_2}{4}$$

Substituting this value of v_1 in equation (1), we get

$$12.0 + \frac{0.9}{2g}\left(\frac{v_2}{4}\right)^2 - \frac{v_2^2}{2g} = 0 \text{ or } 12.0 + \frac{v_2^2}{2g}\left[\frac{0.8}{16} - 1\right] = 0$$

or $$\frac{v_2^2}{2g}[.05 - 1] = -12.0 \text{ or } \frac{0.95\, v_2^2}{2g} = 12.0$$

$$\therefore \quad v_2 = \sqrt{\frac{2 \times 981 \times 12.0}{0.95}} = 157.4 \text{ cm/s}$$

$\therefore$ Discharge $= a_2 v_2$

$= 176.7 \times 157.4 \text{ cm}^3/\text{s} = 27800 \text{ cm}^3/\text{s} =$ **27.8 litres/s. Ans.**

Problem 6.19 *A 30 cm × 15 cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate :*

(i) the discharge of oil, and

(ii) the pressure difference between the entrance section and the throat section. Take the co-efficient of meter as 0.98 and specific gravity of mercury as 13.6. (A.M.I.E., Summer, 1978)

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ Area, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$

Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil.}$$

(*i*) The discharge, $\dot{Q}$ of oil $= C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

$$= \mathbf{148.79 \text{ litres/s. Ans.}}$$

(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

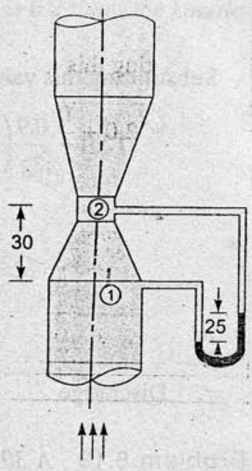

Fig. 6.11

or $\left(\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$

But $z_2 - z_1 = 30$ cm

$\therefore$ $\left(\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}\right) - 30 = 352.77$

$\therefore$ $\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g} = 352.77 + 30 = 382.77$ cm of oil = **3.8277 m of oil. Ans.**

or $(p_1 - p_2) = 3.8277 \times \rho g$

But density of oil = Sp. gr. of oil × 1000 kg/m^3

= 0.9 × 1000 = 900 kg/cm^3

$\therefore$ $(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \dfrac{\text{N}}{\text{m}^2}$

$= \dfrac{33795}{10^4}$ N/cm^2 = **3.3795 N/cm^2. Ans.**

Problem 6.20 *Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 litre per second through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm. The co-efficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tappings is 300 mm.*

(i) If two pressure gauges are connected at the tappings such that they are positioned at the levels of their corresponding tapping points, determine the difference of readings in N/cm^2 of the two pressure gauges.

(ii) If a mercury differential manometer is connected, in place of pressure gauges, to the tappings such that the connecting tube upto mercury are filled with oil, determine the difference in the level of the mercury column.
(A.M.I.E., Summer 1986)

Solution. Given :

Specific gravity of oil, $S_o = 0.85$

$\therefore$ Density, $\rho = 0.85 \times 1000 = 850\ \text{kg/m}^3$

Discharge, $Q = 60$ litre/s

$$= \frac{60}{1000} = 0.06\ \text{m}^3/\text{s}$$

Inlet dia, $d_1 = 200\ \text{mm} = 0.2\ \text{m}$

$\therefore$ Area, $a_1 = \frac{\pi}{4}(.2)^2 = 0.0314\ \text{m}^2$

Throat dia., $d_2 = 100\ \text{mm} = 0.1\ \text{m}$

$\therefore$ Area, $a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\ \text{m}^2$

Value of C_d $= 0.98$

Let section (1) represents inlet and section (2) represents throat. Then

$$z_2 - z_1 = 300\ \text{mm} = 0.3\ \text{m}$$

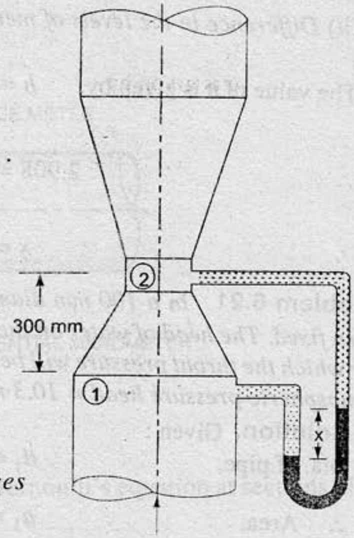

Fig. 6.11 (*a*)

(*i*) *Difference of readings in N/cm² of the two pressure gauges*

The discharge Q is given by,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or

$$0.06 = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$= \frac{0.98 \times 0.00024649}{0.0304} \times 4.429\ \sqrt{h}$$

$$\therefore \quad \sqrt{h} = \frac{0.06 \times 0.0304}{0.98 \times 0.00024649 \times 4.429} = 1.705$$

$$\therefore \quad h = 1.705^2 = 2.908\ \text{m}$$

But for a vertical venturimeter, $h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$

$$\therefore \quad 2.908 = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2$$

$$\frac{p_1 - p_2}{\rho g} = 2.908 + z_2 - z_1 = 2.908 + 0.3 \qquad (\because \quad z_2 - z_1 = 0.3\ \text{m})$$

$$= 3.208\ \text{m of oil}$$

$$\therefore \quad p_1 - p_2 = \rho g \times 3.208$$

$$= 850 \times 9.81 \times 3.208\ \text{N/m}^2 = \frac{850 \times 9.81 \times 3.208}{10^4}\ \text{N/cm}^2$$

$$= \mathbf{2.675\ N/cm^2}.\ \textbf{Ans.}$$

(*ii*) *Difference in the levels of mercury columns* (*i.e.*, *x*)

The value of *h* is given by, $h = x\left[\frac{S_g}{S_o} - 1\right]$

$$\therefore \quad 2.908 = x\left[\frac{13.6}{0.85} - 1\right] = x\,[16 - 1] = 15\,x$$

$$\therefore \quad x = \frac{2.908}{15} = 0.1938 \text{ m} = \mathbf{19.38 \text{ cm of oil. Ans.}}$$

Problem 6.21 *In a 100 mm diameter horizontal pipe a venturimeter of 0.5 contraction ratio has been fixed. The head of water on the metre when there is no flow is 3 m (gauge). Find the rate of flow for which the throat pressure will be 2 metres of water absolute. The co-efficient of meter is 0.97. Take atmospheric pressure head = 10.3 m of water.* **(A.M.I.E., Winter 1976)**

Solution. Given :

Dia. of pipe, $d_1 = 100 \text{ mm} = 10 \text{ cm}$

∴ Area, $a_1 = \frac{\pi}{4} d_1^{\,2} = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. at throat, $d_2 = 0.5 \times d_1 = 0.5 \times 10 = 5 \text{ cm}$

∴ Area, $a_2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ cm}^2$

Head of water for no flow $= \frac{p_1}{\rho g} = 3 \text{ m (gauge)} = 3 + 10.3 = 13.3 \text{ m (abs.)}$

Throat pressure head $= \frac{p_2}{\rho g} = 2 \text{ m of water absolute.}$

∴ Difference of pressure head, $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 13.3 - 2.0 = 11.3 \text{ m} = 1130 \text{ cm}$

∴ Rate of flow, Q is given by $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.97 \times \frac{78.54 \times 19.635}{\sqrt{(78.54)^2 - (19.635)^2}} \times \sqrt{2 \times 981 \times 1130}$$

$$= \frac{2227318.17}{76} = 29306.8 \text{ cm}^3/\text{s} = \mathbf{29.306 \text{ litres/s. Ans.}}$$

6.7.2 Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the down stream side from the orifice plate.

Let p_1 = pressure at section (1),

v_1 = velocity at section (1),

a_1 = area of pipe at section (1), and

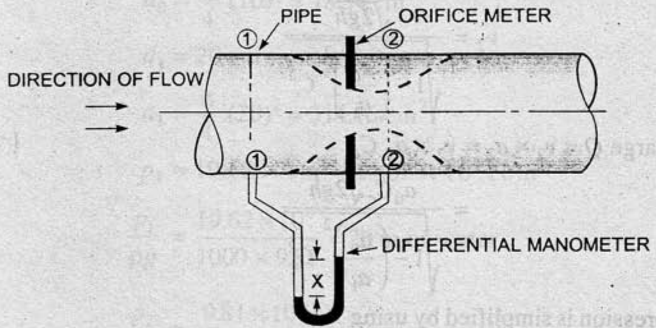

Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$ $$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = v_2^2 - v_1^2$$

or $$v_2 = \sqrt{2gh + v_1^2} \quad \ldots(i)$$

Now section (2) is at the vena contracta and a_2 represents the area at the vena contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$ $$a_2 = a_0 \times C_c \quad \ldots(ii)$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0\, C_c}{a_1} v_2 \quad \ldots(iii)$$

Substituting the value of v_1 in equation (*i*), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or
$$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 = \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2hg$$

$\therefore$
$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0 \, C_c$ $\{\because a_2 = a_0 C_c \text{ from } (ii)\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad \ldots(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$
$$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \quad \ldots(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Problem 6.22 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

$\therefore$ $$\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

Similarly $$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$$C_d = 0.6$$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = \textbf{68.21 litres/s. Ans.}$$

Problem 6.23 *An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of the meter = 0.64.*

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Sp. gr. of oil, $S_0 = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

$\therefore$ Differential head, $$h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil}$$

$$= 50 \times 14.11 = 705.5 \text{ cm of oil}$$

$$C_d = 0.64$$

∴ The rate of the flow, Q is given equation (6.13)

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3/\text{s} = \mathbf{137.414 \text{ litres/s. Ans.}}$$

6.7.3 Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig. 6.13.

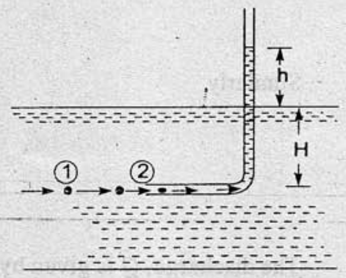

Fig. 6.13 *Pitot-tube.*

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_2^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $\quad v = C_v \sqrt{2gh}$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.
3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig. 6.16.

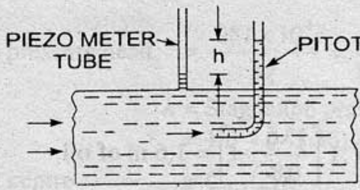

Fig. 6.14

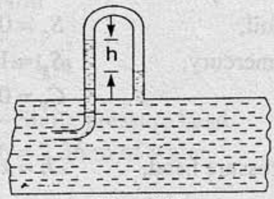

Fig. 6.15

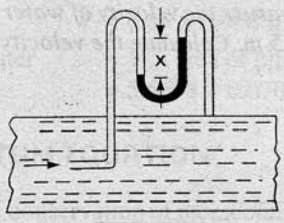

Fig. 6.16

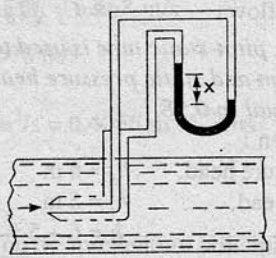

Fig. 6.17

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

Problem 6.24 *A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as C_v = 0.98.*

Solution. Given :

Dia. of pipe, $\quad d$ = 300 mm = 0.30 m

Diff. of pressure head, $\quad h$ = 60 mm of water = .06 m of water

$C_v = 0.98$

Mean velocity, $\quad \bar{V} = 0.80 \times$ Central velocity

Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore \quad \overline{V} = 0.80 \times 1.063 = 0.8504$ m/s

Discharge, $\quad Q =$ Area of pipe $\times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem 6.25 *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Solution. Given :

Diff. of mercury level, $\quad x = 100$ mm $= 0.1$ m

Sp. gr. of oil, $\quad S_o = 0.8$

Sp. gr. of mercury, $\quad S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $\quad h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6$ m of oil

$\therefore$ Velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} =$ **5.49 m/s. Ans.**

Problem 6.26 *A pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.* (A.M.I.E., Winter, 1979)

Solution. Given :

Stagnation pressure head, $\quad h_s = 6$ m

Static pressure head, $\quad h_t = 5$ m

$\therefore \quad h = 6 - 5 = 1$ m

Velocity of flow, $\quad V = C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1} =$ **4.34 m/s. Ans.**

Problem 6.27 *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.* (A.M.I.E., Winter, 1975)

Solution. Given :

Diff. of mercury level, $\quad x = 170$ mm $= 0.17$ m

Sp. gr. of mercury, $\quad S_g = 13.6$

Sp. gr. of sea-water, $\quad S_o = 1.026$

$$\therefore \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$$

$$\therefore \quad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$$

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01\ km/hr.\ Ans.}$$

Problem 6.28 *A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the pitot-*

tube is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take C_v = 0.98. (Converted to S.I. Units, A.M.I.E., Summer, 1987)

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

Static pressure head = 100 mm of mercury (vacuum)

$$= -\frac{100}{1000} \times 13.6 = -1.36 \text{ m of water}$$

Stagnation pressure $= .981 \text{ N/cm}^2 = .981 \times 10^4 \text{ N/m}^2$

∴ Stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1 \text{ m}$

∴ h = Stagnation pressure head – Static pressure head

$$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36 \text{ m of water}$$

∴ Velocity at centre $= C_v \sqrt{2gh}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668 \text{ m/s}$$

Mean velocity, $\overline{V} = 0.85 \times 6.668 = 5.6678 \text{ m/s}$

∴ Rate of flow of water $= \overline{V} \times$ area of pipe

$$= 5.6678 \times 0.07068 \text{ m}^3\text{/s} = \mathbf{0.4006 \text{ m}^3\text{/s}}. \textbf{ Ans.}$$

▶ 6.8 THE MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass '*m*' is given by the Newton's second law of motion,

$$F = m \times a$$

where *a* is the acceleration acting in the same direction as force *F*.

But $a = \frac{dv}{dt}$

∴ $F = m \frac{dv}{dt}$

$$= \frac{d(mv)}{dt} \quad \{m \text{ is constant and can be taken inside the differential}\}$$

$$\therefore \quad F = \frac{d(mv)}{dt} \qquad \text{...(6.15)}$$

Equation (6.15) is known as the momentum principle.

Equation (6.15) can be written as $F.dt = d(mv)$ ...(6.16)

which is known as the *impulse-momentum equation* and states that the impulse of a force *F* acting on a fluid of mass *m* in a short interval of time *dt* is equal to the change of momentum *d*(*mv*) in the direction of force.

Force exerted by a flowing fluid on a Pipe-Bend

The impulse-momentum equation (6.16) is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2), as shown in Fig. 6.18.

Let v_1 = velocity of flow at section (1),

p_1 = pressure intensity at section (1),

A_1 = area of cross-section of pipe at section (1) and

v_2, p_2, A_2 = corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x-and y-directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in the x-direction $= -F_x$ and in the direction of $y = -F_y$. The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x-direction is given by

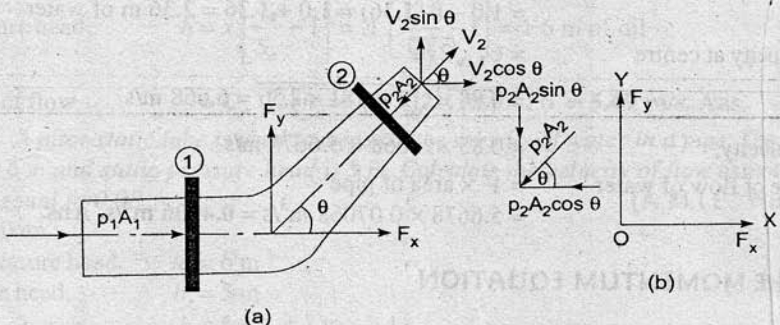

Fig. 6.18 *Forces on bend.*

Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$\therefore \quad p_1A_1 - p_2A_2 \cos\theta - F_x$ = (Mass per sec) (change of velocity)

$= \rho Q$ (Final velocity in the direction of x – Initial velocity in the direction of x)

$$= \rho Q\,(V_2 \cos\theta - V_1) \quad \text{...(6.17)}$$

$$\therefore \quad F_x = \rho Q\,(V_1 - V_2\cos\theta) + p_1A_1 - p_2A_2\cos\theta \quad \text{...(6.18)}$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2\sin\theta - F_y = \rho Q\,(V_2\sin\theta - 0) \quad \text{...(6.19)}$$

$$\therefore \quad F_y = \rho Q\,(-V_2\sin\theta) - p_2A_2\sin\theta \quad \text{...(6.20)}$$

Now the resultant force (F_R) acting on the bend

$$= \sqrt{F_x^2 + F_y^2} \quad \text{...(6.21)}$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x} \quad \text{...(6.22)}$$

Problem 6.29 *A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.*

Solution. Given :

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2$

$= 0.2827 \text{ m}^2$

Dia. at outlet, $D_2 = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (3)^2 = 0.07068 \text{ m}^2$

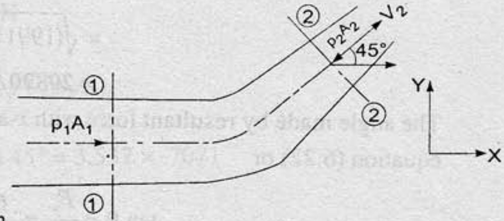

Fig. 6.19

Pressure at inlet, $p_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$

$Q = 600 \text{ lit/s} = 0.6 \text{ m}^3/\text{s}$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488 \text{ m/s.}$$

Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$\therefore$ $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$ or $\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$

$9 + .2295 = p_2/\rho g + 3.672$

$\therefore$ $\frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$

$\therefore$ $p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$

Forces on the bend in x- and y-directions are given by equations (6.18) and (6.20) as

$$F_x = \rho Q [v_1 - v_2 \cos \theta] + p_1A_1 - p_2A_2 \cos \theta$$

$= 1000 \times 0.6 [2.122 - 8.488 \cos 45°]$

$+ 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45°$

$= - 2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$

$= 19911.4 \text{ N}$

and $F_y = \rho Q [- V_2 \sin \theta] - p_2A_2 \sin \theta$

$= 1000 \times 0.6 [- 8.488 \sin 45°[- 5.45 \times 10^4 \times .07068 \times \sin 45°$

$= - 3601.1 - 2721.1 = - 6322.2 \text{ N}$

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$

$$= \mathbf{20890.9\ N.\ Ans.}$$

The angle made by resultant force with x-axis is given by equation (6.22) or

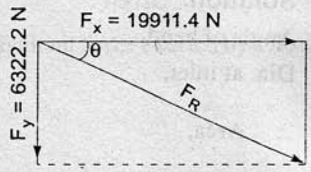

Fig. 6.20

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$$\therefore \qquad \theta = \tan^{-1} .3175 = \mathbf{17° 36'.\ Ans.}$$

Problem 6.30 *250 litres/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².*

(A.M.I.E., Winter, 1974)

Solution. Given :

Pressure, $p_1 = p_2 = 39.24\ N/cm^2 = 39.24 \times 10^4\ N/m^2$

Discharge, $Q = 250$ litres/s $= 0.25\ m^3/s$

Dia. of bend at inlet and outlet, $D_1 = D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068\ m^2$

Velocity of water at (1) and (2), $V = V_1 = V_2 = \frac{Q}{\text{Area}} = \frac{0.25}{.07068} = 3.537$ m/s.

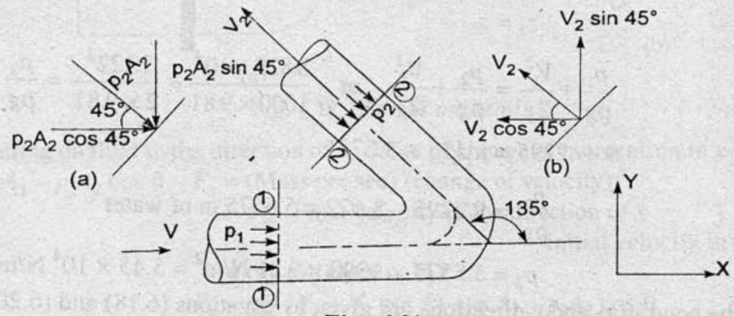

Fig. 6.21

Force along x-axis

$$= F_x = \rho Q[V_{1x} - V_{2x}] + p_{1x}A_1 + p_{2x}A_2$$

where V_{1x} = initial velocity in the direction of x = 3.537 m/s

V_{2x} = final velocity in the direction of $x = -V_2 \cos 45° = -3.537 \times .7071$

p_{1x} = pressure at (1) in x-direction

$= 39.24\ N/cm^2 = 39.24 \times 10^4\ N/m^2$

p_{2x} = pressure at (2) in x-direction

$= p_2 \cos 45° = 39.24 \times 10^4 \times .7071$

$$\therefore \quad F_x = 1000 \times .25[3.537 - (-3.537 \times .7071)] + 39.24 \times 10^4 \times .07068 + 39.24 \times 10^4 \times .07068 \times .7071$$

$$= 1000 \times .25[3.537 + 3.537 \times .7071] + 39.24 \times 10^4 \times .07068\,[1 + .7071]$$

$= 1509.4 + 47346 = 48855.4$ N

Force along y-axis

$$= F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$$

where V_{1y} = initial velocity in y-direction = 0

V_{2y} = final velocity in y-direction = $- V_2 \sin 45° = 3.537 \times .7071$

$(p_1A_1)_y$ = pressure force in y-direction = 0

$(p_2A_2)_y$ = pressure force at (2) in y-direction

$= - p_2A_2 \sin 45° = - 39.24 \times 10^4 \times .07068 \times .7071$

$$\therefore \quad F_y = 1000 \times .25[0 - 3.537 \times .7071] + 0 + (- 39.24 \times 10^4 \times .07068 \times .7071)$$

$$= - 625.2 - 19611.1 = - 20236.3 \text{ N}$$

– ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{48855.4^2 + 20236.3^2}$

$= \mathbf{52880.6}$ **N**

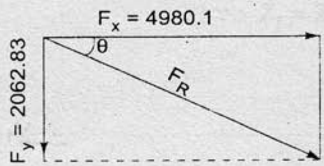

Fig. 6.22

The direction of the resultant force F_R, with the x-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$

$\therefore$ $\theta = \mathbf{22° \ 30'}$. **Ans.**

Problem 6.31 *A 300 mm diameter pipe carries water under a head of 20 metres with a velocity of 3.5 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.* (A.M.I.E., Summer, 1978)

Solution. Given :

Dia. of bend, $D = D_1 = D_2 = 300$ mm = 0.30 m

$\therefore$ Area, $A = A_1 = A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

Velocity, $V = V_1 = V_2 = 3.5$ m/s

$\theta = 45°$

Discharge, $Q = A \times V = 0.07068 \times 3.5 = 0.2475 \text{ m}^3\text{/s}$

Pressure head $= 20$ m of water or $\frac{p}{\rho g} = 20$ m of water

$\therefore$ $p = 20 \times \rho g = 20 \times 1000 \times 9.81 \text{ N/m}^2 = 196200 \text{ N/m}^2$

$\therefore$ Pressure intensity, $p = p_1 = p_2 = 196200 \text{ N/m}^2$

Now $V_{1x} = 3.5$ m/s, $V_{2x} = V_2 \cos 45° = 3.5 \times .7071$

$V_{1y} = 0$, $V_{2y} = V_2 \sin 45° = 3.5 \times .7071$

$(p_1A_1)_x = p_1A_1 = 196200 \times .07068$, $(p_1A_1)_y = 0$

$(p_2A_2)_x = - p_2A_2 \cos 45°$, $(p_2A_2)_y = - p_2A_2 \sin 45°$

Force along x-axis, $F_x = \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

$$= 1000 \times .2475[3.5 - 3.5 \times .7071] + 196200 \times .07068 - p_2A_2 \times \cos 45°$$

$$= 253.68 + 196200 \times .07068 - 196200 \times .07068 \times 0.7071$$
$$= 253.68 + 13871.34 - 9808.04 = 4316.98 \text{ N}$$

Force along y-axis, $F_y = \rho Q\,[v_1y - v_2y] + (p_1A_1)_y + (p_2A_2)_y$

$$= 1000 \times .2475[0 - 3.5 \times .7071] + 0 + [-p_2A_2 \sin 45°]$$
$$= -612.44 - 196200 \times .07068 \times .7071$$
$$= -612.44 - 9808 = -10420.44 \text{ N}$$

∴ Resultant force $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(4316.98)^2 + (10420.44)^2} = $ **11279 N. Ans.**

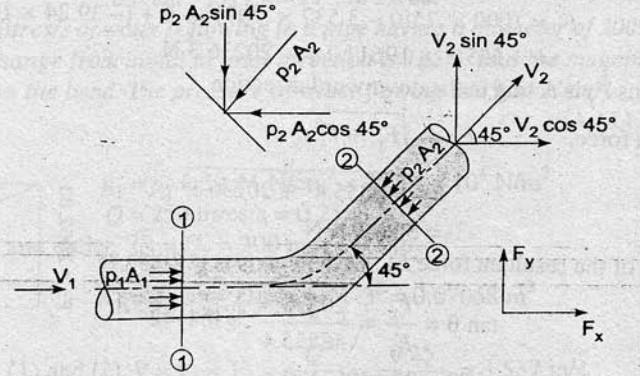

Fig. 6.23

The angle made by F_R with *x*-axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{10420.44}{4316.98} = 2.411$$

∴ $\theta = \tan^{-1} 2.411 = $ **67° 28′. Ans.**

Problem 6.32 *In a 45° bend a rectangular air duct of 1 m² cross-sectional area is gradually reduced to 0.5 m² area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m² section is 10 m/s, and pressure is 2.943 N/cm². Take density of air as 1.16 kg/m³.* (A.M.I.E., Winter, 1980)

Solution. Given :

Area at section (1), $A_1 = 1 \text{ m}^2$

Area at section (2), $A_2 = 0.5 \text{ m}^2$

Velocity at section (1), $V_1 = 10$ m/s

Pressure at section (1), $p_1 = 2.943 \text{ N/cm}^2 = 2.943 \times 10^4 \text{ N/m}^2 = 29430 \text{ N/m}^2$

Density of air, $\rho = 1.16 \text{ kg/m}^3$

Applying continuity equation at sections (1) and (2)

$$A_1V_1 = A_2V_2$$

∴ $$V_2 = \frac{A_1V_1}{A_2} = \frac{1}{0.5} \times 10 = 20 \text{ m/s}$$

Discharge $Q = A_1V_1 = 1 \times 10 = 10 \text{ m}^3/\text{s}$

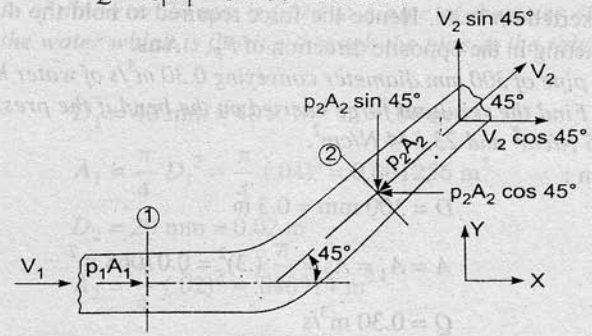

Fig. 6.24

Applying Bernoulli's equation at (1) and (2)

$$\therefore \quad \frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g} \qquad \{\because \quad Z_1 = Z_2\}$$

or
$$\frac{2.943\times10^4}{1.16\times9.81}+\frac{10^2}{2\times9.81}=\frac{p_2}{\rho g}+\frac{20^2}{2\times9.81}$$

$$\therefore \quad \frac{p_2}{\rho g}=\frac{2.943\times10^4}{1.16\times9.81}+\frac{10^2}{2\times9.81}-\frac{20^2}{2\times9.81}$$

$$= 2586.2 + 5.0968 - 20.387 = 2570.90 \text{ m}$$

$$\therefore \quad p_2 = 2570.90 \times 1.16 \times 9.81 = 29255.8 \text{ N}$$

Force along x-axis, $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $A_{1x} = 10$ m/s, $V_{2x} = V_2 \cos 45° = 20 \times .7071$,

$(p_1A_1)_x = p_1A_1 = 29430 \times 1 = 29430$ N

and $(p_2A_2)_x = -p_2A_2 \cos 45° = -p_2A_2 \cos 45° = -29255.8 \times 0.5 \times .7071$

$$\therefore \quad F_x = 1.16 \times 10[10 - 20 \times .7071] + 29430 \times 1 - 29255.8 \times .5 \times .7071$$

$$= -48.04 + 29430 - 10343.37 = 0 - 19038.59 \text{ N}$$

Similarly force along y-axis, $F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 \sin 45° = 20 \times .7071 = 14.142$

$(p_1A_1)_y$

$= 0$, $(p_2A_2)_y = -p_2A_2 \sin 45° = -29255.8 \times .5 \times .7071 = -10343.37$

$$F_y = 1.16 \times 10\,[0 - 14.142] + 0 - 10343.37$$

$$= -164.05 - 10343.37 = -10507.42 \text{ N}$$

$\therefore$ Resultant force. $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19038.6)^2 + (10507.42)^2} =$ **21746.6 N. Ans.**

The direction of F_R with x-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{10507.42}{19038.6} = 0.5519$$

$\therefore$ $\theta = \tan^{-1} .5519 =$ **28° 53′. Ans.**

F_R is the force exerted on bend. Hence the force required to hold the duct in position is equal to 21746.6 N but it is acting in the opposite direction of F_R. **Ans.**

Problem 6.33 *A pipe of 300 mm diameter conveying 0.30 m³/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm² and 23.544 N/cm².*

Solution. Given :

Dia. of bend, $D = 300 \text{ mm} = 0.3 \text{ m}$

∴ Area, $A = A_1 = A_2 = \frac{\pi}{4}(.3)^2 = 0.07068 \text{ m}^2$

∴ Discharge, $Q = 0.30 \text{ m}^3/\text{s}$

∴ Velocity, $V = V_1 = V_2 = \frac{Q}{A} = \frac{0.30}{.07068} = 4.244 \text{ m/s}$

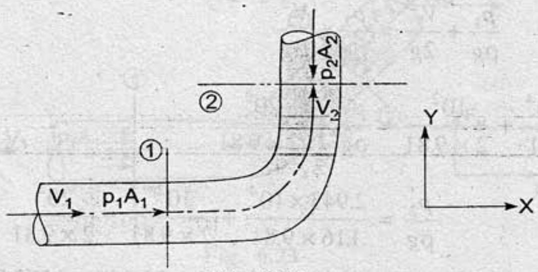

Fig. 6.25

Angle of bend, $\theta = 90°$

$$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2 = 245250 \text{ N/m}^2$$

$$p_2 = 23.544 \text{ N/cm}^2 = 23.544 \times 10^4 \text{ N/m}^2 = 235440 \text{ N/m}^2$$

Force of bend along x-axis $F_x = \rho Q [V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $\rho = 1000$, $V_{1x} = V_1 = 4.244$ m/s, $V_{2x} = 0$

$(p_1A_1)_x = p_1A_1 = 245250 \times .07068$

$(p_2A_2)_x = 0$

∴ $F_x = 1000 \times 0.30 [4.244 - 0] + 245250 \times .07068 + 0$

$= 1273.2 + 17334.3 = 18607.5$ N

Force on bend along y-axis, $F_y = \rho Q [V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 = 4.244$ m/s

$(p_1A_1)_y = 0$, $(p_2A_2)_y = - p_2A_2 = - 235440 \times .07068 = - 16640.9$

∴ $F_y = 1000 \times 0.30[0 - 4.244] + 0 - 16640.9$

$= - 1273.2 - 16640.9 = - 17914.1$ N

∴ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(18607.5)^2 + (17914.1)^2} = - 25829.3$ N

and $\tan\theta = \frac{F_y}{F_x} = \frac{17914.1}{18607.5} = 0.9627$

∴ $\theta =$ **43° 54′. Ans.**

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Problem 6.34 *A nozzle of diameter 20 mm is fitted to a pipe of diameter 40 mm. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 1.2 m³/minute.*

Solution. Given :

Dia. of pipe, $D_1 = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} = .04 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.04)^2 = 0.001256 \text{ m}^2$

Dia. of nozzle, $D_2 = 20 \text{ mm} = 0.02 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4} (.02)^2 = .000314 \text{ m}^2$

Discharge, $Q = 1.2 \text{ m}^3\text{/minute} = \frac{1.2}{60} \text{ m}^3\text{/s} = 0.02 \text{ m}^3\text{/s}$

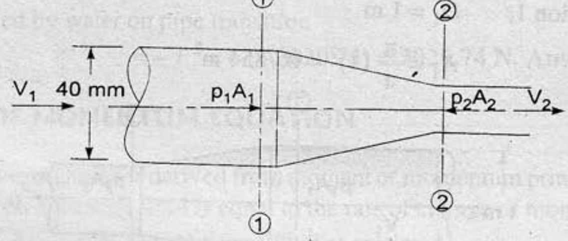

Fig. 6.26

Applying continuity equation at (1) and (2),

$$A_1V_1 = A_2V_2 = Q$$

∴ $$V_1 = \frac{Q}{A_1} = \frac{0.2}{.001256} = 15.92 \text{ m/s}$$

and $$V_2 = \frac{Q}{A_2} = \frac{0.2}{.000314} = 63.69 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Now $$z_1 = z_2, \frac{p_2}{\rho g} = \text{atmospheric pressure} = 0$$

∴ $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

∴ $$\frac{p_1}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{(63.69^2)}{2 \times 9.81} - \frac{(15.92^2)}{2 \times 9.81} = 206.749 - 12.917$$

$$= 193.83 \text{ m of water}$$

∴ $$p_1 = 193.83 \times 1000 \times 9.81 \frac{\text{N}}{\text{m}^2} = 1901472 \frac{\text{N}}{\text{m}^2}$$

Let the force exerted by the nozzle on water = F_x

Net force in the direction of x = rate of change of momentum in the direction of x

$$\therefore \quad p_1A_1 - p_2A_2 + F_x = \rho Q(V_2 - V_1)$$

where p_2 = atmospheric pressure = 0 and $\rho = 1000$

$$\therefore \quad 1901472 \times .001256 - 0 + F_x = 1000 \times 0.02(63.69 - 15.92) \text{ or } 2388.24 + F_x = 916.15$$

$$\therefore \quad F_x = -2388.24 + 916.15 = \mathbf{-1472.09.\ Ans.}$$

– ve sign indicates that the force exerted by the nozzle on water is acting from right to left.

Problem 6.35 *The diameter of a pipe gradually reduces from 1 m to 0.7 m as shown in Fig. 6.27. The pressure intensity at the centre-line of 1 m section 7.848 kN/m² and rate of flow of water through the pipe is 600 litres/s. Find the intensity of pressure at the centre-line of 0.7 m section. Also determine the force exerted by flowing water on transition of the pipe.*

Solution. Given :

Dia. of pipe at section 1, $D_1 = 1$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

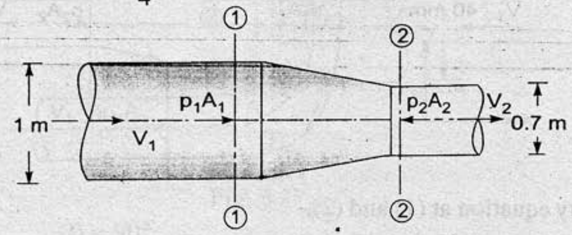

Fig. 6.27

Dia. of pipe at section 2, $D_2 = 0.7$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4}(0.7)^2 = 0.3848 \text{ m}^2$

Pressure at section 1, $p_1 = 7.848 \text{ kN/m}^2 = 7848 \text{ N/m}^2$

Discharge, $Q = 600 \text{ litres/s} = \frac{600}{1000} = 0.6 \text{ m}^3\text{/s}$

Applying continuity equation,

$$A_1V_1 = A_2V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_2} = \frac{0.6}{0.7854} = 0.764 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.3854} = 1.55 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad \{\because \text{ pipe is horizontal, } \therefore z_1 = z_2\}$$

or $$\frac{7848}{1000 \times 9.81} + \frac{(.764)^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{(1.55)^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = 0.8 + \frac{(.764)^2}{2 \times 9.81} - \frac{(1.55)^2}{2 \times 9.81}$$

$$= 0.8 + 0.0297 - 0.122 = 0.7077 \text{ m of water}$$

$$\therefore \quad p_2 = 0.7077 \times 9.81 \times 1000$$

$$= \mathbf{6942.54\ N/m^2\ or\ 6.942\ kN/m^2.\ Ans.}$$

Let F_x = the force exerted by pipe transition on the flowing water in the direction of flow

Then net force in the direction of flow = rate of change of momentum in the direction of flow

or $\quad p_1A_1 - p_2A_2 + F_x = \rho(V_2 - V_1)$

$\therefore \quad 7848 \times .7854 - 6942.54 \times .3848 + F_x = 1000 \times 0.6[1.55 - .764]$

or $\quad 6163.8 - 2671.5 + F_x = 471.56$

$\therefore \quad F_x = 471.56 - 6163.8 + 2671.5 = -3020.74$ N

$\therefore$ The force exerted by water on pipe transition

$$= -F_x = -(-3020.74) = \mathbf{3020.74\ N.\ Ans.}$$

▶ 6.9 MOMENT OF MOMENTUM EQUATION

Moment of momentum equation is derived from moment of momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

Let $\quad V_1$ = velocity of fluid at section 1

r_1 = radius of curvature at section 1,

Q = rate of flow of fluid,

ρ = density of fluid,

and $\quad V_2$ and r_2 = velocity and radius of curvature at section 2

Momentum of fluid at section 1 = mass × velocity = $\rho Q \times V_1/s$

$\therefore$ Moment of momentum per second at section 1,

$$= \rho Q \times V_1 \times r_1$$

Similarly moment of momentum per second of fluid at section 2

$$= \rho Q \times V_2 \times r_2$$

$\therefore$ Rate of change of moment of momentum

$$= \rho Q V_2 r_2 - \rho Q V_1 r_1 = \rho Q[V_2 r_2 - V_1 r_1]$$

According to moment of momentum principle

Resultant torque = rate of change of moment of momentum

or $$T = \rho Q[V_2 r_2 - V_1 r_1] \quad \text{...(6.23)}$$

Equation (6.23) is known as moment of momentum equation. This equation is applied :

1. For analysis flow problems in turbines and centrifugal pumps.
2. For finding torque exerted by water on sprinkler.

Problem 6.36 *A lawn sprinkler with two hozzles of diameter 4 mm each is connected across a top of water as shown in Fig. 6.28. The nozzles are at a distance of 30 cm and 20 cm from the centre of the tap. The rate of flow of water through tap is 120 cm³/s. The nozzles discharge water in the downward direction. Determine the angular speed at which the sprinkler will rotate free.*

Solution. Given :.

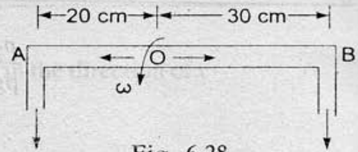

Fig. 6.28

Dia. of nozzles A and B,

$$D = D_A = D_B = 4 \text{ mm} = .004 \text{ m}$$

$\therefore$ Area, $A = \frac{\pi}{4}(.004)^2 = .00001256 \text{ m}^2$

Discharge $Q = 120 \text{ cm}^3/\text{s}$

Assuming the discharge to be equally divided between the two nozzles, we have

$$Q_A = Q_B = \frac{Q}{2} = \frac{120}{2} = 60 \text{ cm}^3/\text{s} = 60 \times 10^{-6} \text{ m}^3/\text{s}$$

$\therefore$ Velocity of water at the outlet of each nozzle,

$$V_A = V_B = \frac{Q_A}{A} = \frac{60 \times 10^{-6}}{.00001256} = 4.777 \text{ m/s.}$$

The jet of water coming out from nozzles A and B is having velocity 4.777 m/s. These jets of water will exert force in the opposite direction, *i.e.*, force exerted by the jets will be in the upward direction. The torque exerted will also be in the opposite direction. Hence torque at B will be in the anti-clockwise direction and at A in the clockwise direction. But torque at B is more than the torque at A and hence sprinkle, if free, will rotate in the anti-clockwise direction as shown in Fig. 6.28.

Let ω = angular velocity of the sprinkler.

Then absolute velocity of water at A,

$$V_1 = V_A + \omega \times r_A$$

where r_A = distance of nozzle A from the centre of top

= 20 cm = 0.2 m $\qquad \{\omega \times r_A = \text{tangential velocity due to rotation}\}$

$$V_1 = (4.777 + \omega \times 0.2) \text{ m/s}$$

Here $\omega \times r_A$ is added to V_A as V_A and tangential velocity due to rotation $(\omega \times r_A)$ are in the same direction as shown in Fig. 6.28.

Similarly absolute velocity of water at B,

$$V_2 = V_B - \text{tangential velocity due to rotation}$$

$$= 4.777 - \omega \times r_B \qquad \{\text{where } r_B = 30 \text{ cm} = 0.3 \text{ m}\}$$

$$= (4.777 - \omega \times 0.3)$$

Now applying equation (6.23), we get

$$T = \rho Q[V_2 r_2 - V_1 r_1] \qquad \text{Here } r_2 = r_B,\ r_1 = r_A$$

$$= \rho Q_A[V_2 r_B - V_1 r_A] \qquad Q = Q_A = Q_B$$

$$= 1000 \times 60 \times 10^{-6}\,[(4.777 \times 0.3\,\omega) \times .3 - (4.777 + 0.2\,\omega) \times .2]$$

The moment of momentum of the fluid entering sprinkler is given zero and also there is no external torque applied on the sprinkler. Hence resultant external torque is zero, *i.e.*, $T = 0$

$$\therefore \quad 1000 \times 60 \times 10^{-6}\,[(4.777 - 0.3\,\omega) \times .3 - (4.777 + 0.2\,\omega) \times .2] = 0$$

or $$(4.777 - 0.3\,\omega) \times 0.3 - (4.777 + 0.2\,\omega) \times .2 = 0$$

or $$4.777 \times .3 - .09\,\omega - 4.777 \times .2 - .04\,\omega = 0$$

or $$0.1 \times 4.777 = (.09 + .04)\omega = .13\,\omega$$

$$\therefore \quad \omega = \frac{.4777}{0.13} = \textbf{3.6746 rad/s. Ans.}$$

Problem 6.37 *A lawn sprinkler shown in Fig. 6.29 has 0.8 cm diameter nozzle at the end of a rotating arm and discharges water at the rate of 10 m/s velocity. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of the arm, if free to rotate.*

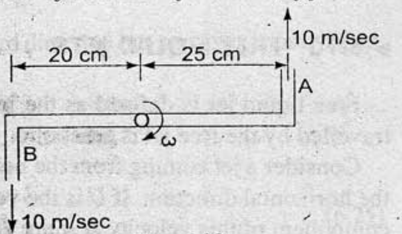

Fig. 6.29

Solution. Dia. of each nozzle $= 0.8 \text{ cm} = .008 \text{ m}$

$\therefore$ Area of each nozzle $= \frac{\pi}{4}(.008)^2 = .00005026 \text{ m}^2$

Velocity of flow at each nozzle $= 10$ m/s.

$\therefore$ Discharge through each nozzle,

$$Q = \text{Area} \times \text{Velocity}$$

$$= .00005026 \times 10 = .0005026 \text{ m}^3/\text{s}$$

Torque exerted by water coming through nozzle A on the sprinkler = moment of momentum of water through A

$$= r_A \times \rho \times Q \times V_A = 0.25 \times 1000 \times .0005026 \times 10 \text{ clockwise}$$

Torque exerted by water coming through nozzle B on the sprinkler

$$= r_B \times \rho \times Q \times V_B = 0.20 \times 1000 \times .0005026 \times 10 \text{ clockwise}$$

$\therefore$ Total torque exerted by water on sprinkler

$$= .25 \times 1000 \times .0005026 \times 10 + .20 \times 1000 \times .0005026 \times 10$$

$$= 1.2565 + 1.0052 = 2.26 \text{ Nm}$$

$\therefore$ Torque required to hold the rotating arm stationary = Torque exerted by water on sprinkler

$$= \mathbf{2.26 \text{ Nm. Ans.}}$$

Speed of rotation of arm, if free to rotate

Let ω = speed of rotation of the sprinkler

The absolute velocity of flow of water at the nozzles A and B are

$$V_1 = 10.0 - 0.25 \times \omega \text{ and } V_2 = 10.0 - 0.20 \times \omega$$

Torque exerted by water coming out at A, on sprinkler

$$= r_A \times \rho \times Q \times V_1 = 0.25 \times 1000 \times .0005026 \times (10 - 0.25\,\omega)$$

$$= 0.12565\,(10 - 0.25\,\omega)$$

Torque exerted by water coming out at B, on sprinkler

$$= r_B \times \rho \times Q \times V_2 = 0.20 \times 1000 \times .0005026 \times (10.0 - 0.2\,\omega)$$

$$= 0.10052\,(10.0 - 0.2\,\omega)$$

$\therefore$ Total torque exerted by water $= 0.12565\,(10.0 - 0.25\,\omega) + 0.10052\,(10.0 - 0.2\,\omega)$

Since moment of momentum of the flow entering is zero and no external torque is applied on sprinkler, so the resultant torque on the sprinkler must be zero.

$$\therefore \quad 0.12565\,(10.0 - 0.25\,\omega) + 0.10052(10.0 - 0.2\,\omega) = 0$$

$$1.2565 - 0.0314\,\omega + 1.0052 - 0.0201\,\omega = 0$$

$$1.2565 + 1.0052 = \omega\,(0.0314 + 0.0201)$$

$$2.2617 = 0.0515\,\omega$$

$$\therefore \quad \omega = \frac{2.2617}{0.0515} = \mathbf{43.9 \text{ rad/s. Ans.}}$$

and $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 43.9}{2\pi} = 419.2 \text{ r.p.m. Ans.}$$

▶ 6.10 FREE LIQUID JETS

Free liquid jet is defined as the jet of water coming out from the nozzle in atmosphere. The path travelled by the free jet is parabolic.

Consider a jet coming from the nozzle as shown in Fig. 6.30. Let the jet at A, makes an angle θ with the horizontal direction. If U is the velocity of jet of water, then the horizontal component and vertical component of this velocity at A are $U \cos \theta$ and $U \sin \theta$.

Consider another point $P(x, y)$ on the centre line of the jet. The co-ordinates of P from A are x and y. Let the velocity of jet at P in the x- and y-directions are u and v. Let a liquid particle takes time 't' to reach from A to P. Then the horizontal and vertical distances travelled by the liquid particle in time 't' are :

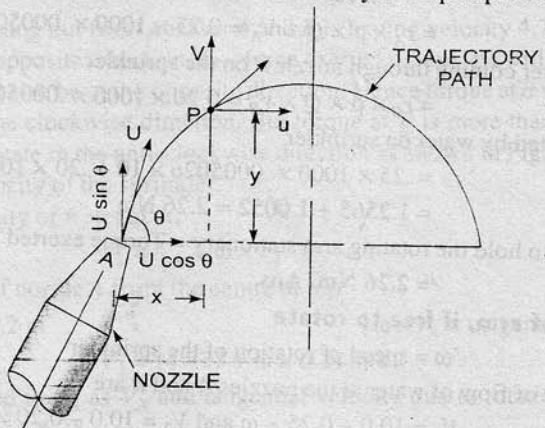

Fig. 6.30 *Free liquid jet.*

$$x = \text{velocity component in } x\text{-direction} \times t$$
$$= U \cos \theta \times t \quad \text{...}(i)$$

and $$y = (\text{vertical component in } y\text{-direction} \times \text{time} - \frac{1}{2} g t^2)$$
$$= U \sin \theta \times t - \frac{1}{2} g t^2 \quad \text{...}(ii)$$

{∵ Horizontal component of velocity is constant while the vertical distance is affected by gravity}

From equation (i), the value of t is given as $t = \frac{x}{U \cos \theta}$

Substituting this value in equation (ii)

$$y = U \sin \theta \times \frac{x}{U \cos \theta} - \frac{1}{2} \times g \times \left(\frac{x}{U \cos \theta}\right)^2 = x \frac{\sin \theta}{\cos \theta} - \frac{g x^2}{2U^2 \cos^2 \theta}$$

$$= x \tan \theta - \frac{g x^2}{2U^2} \sec^2 \theta \qquad \left\{\because \frac{1}{\cos^2 \theta} = \sec^2 \theta\right\} \quad \text{...(6.24)}$$

Equation (6.24) gives the variation of y with the square of x. Hence this is the equation of a parabola. Thus the path travelled by the free jet in atmosphere is parabolic.

(*i*) **Maximum height attained by the jet.** Using the relation $V_2^2 - V_1^2 = -2gS$, we get in this case $V_1 = 0$ at the highest point

$$V_1 = \text{Initial vertical component}$$
$$= U \sin \theta$$

– ve sign on right hand side is taken as g is acting in the downward direction but particles is moving up.

$$\therefore \quad 0 - (U \sin \theta)^2 = -2g \times S$$

where S is the maximum vertical height attained by the particle.

or
$$-U^2 \sin^2\theta = -2gS$$

$$\therefore \quad S = \frac{U^2 \sin^2 \theta}{2g} \qquad \text{...(6.25)}$$

(*ii*) **Time of flight.** It is the time taken by the fluid particle in reaching from A to B as shown in Fig. 6.30. Let T is the time of flight.

Using equation (*ii*), we have $y = U \sin \theta \times t - \frac{1}{2} gt^2$

when the particle reaches at B, $y = 0$ and $t = T$

∴ Above equation becomes as $0 = U \sin \theta \times T - \frac{1}{2} g \times T^2$

or
$$0 = U \sin \theta - \frac{1}{2} gT \qquad \{\text{Cancelling } T\}$$

or
$$T = \frac{2U \sin \theta}{g} \qquad \text{...(6.26)}$$

(*iii*) **Time to reach highest point.** The time to reach highest point is half the time of flight. Let T^* is the time to reach highest point, then

$$T^* = \frac{T}{2} = \frac{2U \sin\theta}{g \times 2} = \frac{U \sin\theta}{g} \qquad \text{...(6.27)}$$

(*iv*) **Horizontal range of the jet.** The total horizontal distance travelled by the fluid particle is called horizontal range of the jet, *i.e.*, the horizontal distance AB in Fig. 6.30 is called horizontal range of the jet. Let this range is denoted by x^*.

Then
$$x^* = \text{velocity component in } x\text{-direction} \times \text{time taken by the particle to reach from } A \text{ to } B$$
$$= U \cos \theta \times \text{Time of flight}$$
$$= U \cos \theta \times \frac{2U \sin \theta}{g} \qquad \left\{\because T = \frac{2U \sin \theta}{g}\right\}$$
$$= \frac{U^2}{g} 2 \cos \theta \sin \theta = \frac{U^2}{g} \sin 2\theta \qquad \text{...(6.28)}$$

(*v*) **Value of θ for maximum range.** The range x^* will be maximum for a given velocity of projection (U), when sin 2θ is maximum

or when
$$\sin 2\theta = 1 \text{ or } \sin 2\theta = \sin 90° = 1$$
$$\therefore \quad 2\theta = 90° \text{ or } \theta = 45°$$

Then maximum range,
$$x_{max^*} = \frac{U^2}{g} \sin^2 \theta = \frac{U^2}{g} \qquad \{\because \sin 90° = 1\} \text{ ...(6.29)}$$

Problem 6.38 *A vertical wall is of 8 m in height. A jet of water is coming out from a nozzle with a velocity of 20 m/s. The nozzle is situated at a distance of 20 m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water just clears the top of the wall.*

Solution. Given :

Height of wall $= 8$ m

Velocity of jet, $U = 20$ m/s

Distance of jet from wall, $x = 20$ m

Let the required angle $= \theta$

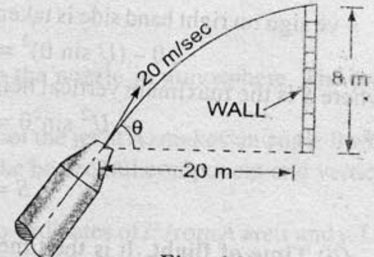

Fig. 6.31

Using equation (6.24), we have

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\rho$$

where $y = 8$ m, $x = 20$ m, $U = 20$ m/s

$$8 = 20\tan\theta - \frac{9.81\times 20^2}{2\times 20^2}\sec^2\theta$$

$$= 20\tan\theta - 4.905\sec^2\theta$$

$$= 20\tan\theta - 4.905\,[1+\tan^2\theta] \qquad \{\because\ \sec^2\theta = 1+\tan^2\theta\}$$

$$= 20\tan\theta - 4.905 - 4.905\tan^2\theta$$

or $\quad 4.905\tan^2\theta - 20\tan\theta + 8 + 4.905 = 0$

or $\quad 4.905\tan^2\theta - 20\tan\theta + 12.905 = 0$

$$\therefore \quad \tan\theta = \frac{20\pm\sqrt{20^2 - 4\times 12.905\times 4.905}}{2\times 4.905} = \frac{20\pm\sqrt{400-253.19}}{9.81}$$

$$= \frac{20\pm\sqrt{146.81}}{9.81} = \frac{20\pm 12.116}{9.81} = \frac{32.116}{9.81} \text{ or } \frac{7.889}{9.81}$$

$$\therefore \quad = 3.273 \text{ or } 0.8036$$

$$\therefore \quad \theta = \mathbf{73°\ 0.8' \text{ or } 38°\ 37'.\ Ans.}$$

Problem 6.39 *A fire-brigade man is holding a fire stream nozzle of 50 mm diameter as shown in Fig. 6.32. The jet issues out with a velocity of 13 m/s and strikes the window. Find the angle or angles of inclination with which the jet issues from the nozzle. What will be the amount of water falling on the window ?*

(A.M.I.E., Winter, 1975)

Solution. Given :

Dia. of nozzle, $d = 50$ mm $= .05$ m

$\therefore$ Area, $A = \frac{\pi}{4}(.05)^2 = 0.001963\ \text{m}^2$

Velocity of jet, $U = 13$ m/s.

The jet is coming out from nozzle at A. It strikes the window and let the angle made by the jet at A with horizontal is equal to θ.

The co-ordinates of window, with respect to origin at A.

$$x = 5 \text{ m},\ y = 7.5 - 1.5 = 6.0 \text{ m}$$

The equation of the jet is given by (6.24) as

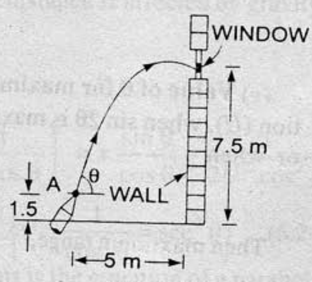

Fig. 6.32

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

or $$6.0 = 5 \times \tan\theta - \frac{9.81 \times 5}{2 \times 13^2}[1 + \tan^2\theta] \qquad \{\because \sec^2\theta = 1 + \tan^2\theta\}$$

or $$6.0 = 5\tan\theta - .7256\,(1 + \tan^2\theta)$$
$$= 5\tan\theta - .7256 - .7256\tan^2\theta$$

or $$0.7256\tan^2\theta - 5\tan\theta + 6 + .7256 = 0$$

or $$0.7256\tan^2\theta - 5\tan\theta + 6.7256 = 0$$

This is a quadratic equation in tan θ. Hence solution is

$$\tan\theta = \frac{5 \pm \sqrt{5^2 - 4 \times .7256 \times 6.7256}}{2 \times .7256}$$

$$= \frac{5 \pm \sqrt{25 - 19.52}}{1.4512} = \frac{5 + 2.341}{1.4512} = 5.058 \text{ or } 1.8322.$$

$\therefore$ $\theta = \tan^{-1} 5.058$ or $\tan^{-1} 1.8322 =$ **78.8° or 61.37°. Ans.**

Amount of water falling on window = Discharge from nozzle

= Area of nozzle × Velocity of jet at nozzle

$= 0.001963 \times U = 0.001963 \times 13.0 =$ **0.0255 m³/s. Ans.**

Problem 6.40 *A nozzle is situated at a distance of 1 m above the ground level and is inclined at an angle of 45° to the horizontal. The diameter of the nozzle is 50 mm and the jet of water from the nozzle strikes the ground at a horizontal distance of 4 m. Find the rate of flow of water.*

Solution. Given :

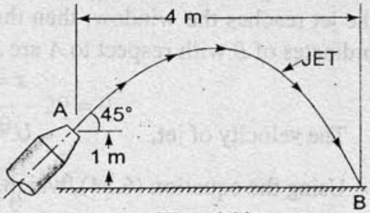

Fig. 6.33

Distance of nozzle above ground = 1 m

Angle of inclination. $\theta = 45°$

Dia. of nozzle, $d = 50 \text{ mm} = .05 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

The horizontal distance $x = 4$ m

The co-ordinates of the point *B*, which is on the centre-line of the jet of water and is situated on the ground, with respect to *A* (origin) are

$x = 4$ m and $y = -1.0$ m {From *A*, point *B* is vertically down by 1 m}

The equation of the jet is given by (6.24) as $y = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$

Substituting the known values as

$$-1.0 = 4\tan 45° - \frac{9.81 \times 4^2}{2U^2} \times \sec^2 45°$$

$$= 4 - \frac{78.48}{U^2} \times \left(\sqrt{2}\right)^2 \qquad \left\{\sec 45° \frac{1}{\cos 45°} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}\right\}$$

$$-1.0 = 4 - \frac{78.48 \times 2}{U^2} \quad \text{or} \quad \frac{78.48 \times 2}{U^2} = +4.0 + 1.0 = 5.0$$

$$\therefore \quad U^2 = \frac{78.48 \times 2.0}{5.0} = 31.39$$

$$\therefore \quad U = \sqrt{31.39} = 5.60 \text{ m/s}$$

Now the rate of flow of fluid $= \text{Area} \times \text{Velocity of jet}$

$$= A \times U = .001963 \times 5.6 \text{ m}^3/\text{sec}$$

$$= 0.01099 \approx \mathbf{.011 \ m^3/s.\ Ans.}$$

Problem 6.41 *A window, in a vertical wall, is at a distance of 30 m above the ground level. A jet of water, issuing from a nozzle of diameter 50 mm is to strike the window. The rate of flow of water through the nozzle is 3.5 m³/minute and nozzle is situated at a distance of 1 m above ground level. Find the greatest horizontal distance from the wall of the nozzle so that jet of water strikes the window.*

Solution. Given :

Distance of window from ground level = 30 m

Dia. of nozzle, $d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area $A = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

The discharge, $Q = 3.5 \text{ m}^3/\text{minute}$

$$= \frac{3.5}{60} = 0.0583 \text{ m}^3/\text{s}$$

Distance of nozzle from ground = 1 m.

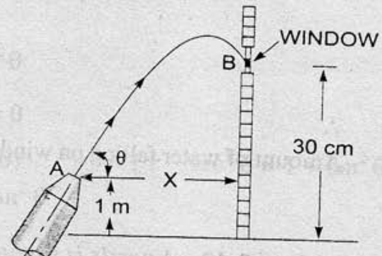

Fig. 6.34

Let the greatest horizontal distance of the nozzle from the wall = x and let angle of inclination = θ. If the jet reaches the window, then the point B on the window is on the centre-line of the jet. The co-ordinates of B with respect to A are .

$$x = x, \ y = 30 - 1.0 = 29 \text{ m}$$

The velocity of jet, $U = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A} = \frac{.0583}{.001963} = 29.69 \text{ m/sec}$

Using the equation (6.34), which is the equation of jet,

$$y = x \tan \theta - \frac{gx^2}{2U^2} \sec^2 \theta$$

or

$$29.0 = x \tan \theta - \frac{9.81x^2}{2 - (29.69)^2} \sec^2 \theta$$

$$= x \tan \theta - 0.0055 \sec^2 \theta \times x^2$$

$$= x \tan \theta - \frac{.0055\, x^2}{\cos^2 \theta}$$

$$x \tan \theta - .0055\, x^2/\cos^2\theta - 29 = 0 \quad \quad ...(i)$$

The maximum value of x with respect to θ is obtained, by differentiating the above equation w.r.t. θ and substituting the value of $\frac{dx}{d\theta} = 0$. Hence differentiating the equation (i) w.r.t. θ, we have

$$\left[x \sec^2 \theta + \tan \theta \times \frac{dx}{d\theta}\right]$$

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$$-0.0055\left[x^2\times\left(\frac{(-2)}{\cos^3\theta}\right)(-\sin\theta)+\frac{1}{\cos^2\theta}\times 2x\frac{dx}{d\theta}\right]$$

$$\left\{\because \frac{d}{d\theta}(x\sin\theta)=x\sec^2\theta+\tan\theta\frac{dx}{d\theta} \text{ and } \frac{d}{d\theta}\left(\frac{x^2}{\cos^2\theta}\right)=x^2\frac{d}{d\theta}\left(\frac{1}{\cos^2\theta}\right)+\frac{1}{\cos^2\theta}\frac{d}{d\theta}\left(x^3\right)\right\}$$

$$\therefore \quad x\sec^2\theta-\tan\theta\frac{dx}{d\theta}-.0055\left[\frac{2x^2\sin\theta}{\cos^3\theta}-\frac{2x}{\cos^2\theta}\frac{dx}{d\theta}\right]=0$$

For maximum value of x, w.r.t. θ, we have $\frac{dx}{d\theta}=0$

Substituting this value in the above equation, we have

$$x\sec^2\theta-.0055\left[\frac{2x^2\sin\theta}{\cos^3\theta}\right]=0$$

or $$\frac{x}{\cos^2\theta}=\frac{.0055\times 2x^2\sin\theta}{\cos^3\theta}=0 \text{ or } x-.011\times x^2\frac{\sin\theta}{\cos\theta}=0$$

or $$x-.011\,x^2\tan\theta=0 \text{ or } 1-.011\,x\tan\theta=0$$

or $$x\tan\theta=\frac{1}{.011}=90.9 \qquad ...(ii)$$

or $$x=\frac{90.9}{\tan\theta} \qquad ...(iii)$$

Substituting this value of x in equation (i), we get

$$\frac{90.9}{\tan\theta}\times\tan\theta-.0055\times\frac{(90.9)^2}{\tan^2\theta}\times\frac{1}{\cos^2\theta}-29=0$$

$$90.9-\frac{45.445}{\sin^2\theta}-29=0 \text{ or } 61.9-\frac{45.445}{\sin^2\theta}=0$$

or $$61.9=\frac{45.445}{\sin^2\theta} \text{ or } \sin^2\theta=\frac{45.445}{61.90}=0.7341$$

$$\therefore \quad \sin\theta=\sqrt{0.7341}=0.8568$$

$$\therefore \quad \theta=\tan^{-1}.8568=58°\,57.8'$$

Substituting the value of θ in equation (iii), we get

$$x=\frac{90.9}{\tan\theta}=\frac{90.9}{\tan 58°\,57.8'}=\frac{90.9}{\tan 58.95}=\frac{90.9}{1.66}=54.759 \text{ m}$$

$$= \mathbf{54.76 \text{ m. Ans.}}$$

HIGHLIGHTS

1. The study of fluid motion with the forces causing flow is called dynamics of fluid flow, which is analysed by the Newton's second law of motion.
2. Bernoulli's equation is obtained by integrating the Euler's equation of motion. Bernoulli's equation states "For a steady, ideal flow of an incompressible fluid, the total energy which consists of pressure energy, kinetic energy and datum energy, at any point of the fluid is constant". Mathematically,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

where $\frac{p_1}{\rho g}$ = pressure energy per unit weight = pressure head

$\frac{v_1^2}{2g}$ = kinetic energy per unit weight = kinetic head

z_1 = datum energy per unit weight = datum head.

3. Bernoulli's equation for real fluids

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

where h_L = loss of energy between sections 1 and 2.

4. The discharge, Q, through a venturimeter or an orifice meter is given by

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where a_1 = area at the inlet of venturimeter,
a_2 = area at the throat of venturimeter,
C_d = co-efficient of venturimeter,
h = difference of pressure head in terms of fluid head flowing through venturimeter.

5. The value of h is given by differential U-tube manometer $h = x\left[\frac{S_h}{S_o} - 1\right]$

when differential manometer contains heavier liquid $= x\left[1 - \frac{S_l}{S_o}\right]$

when differential manometer contains lighter liquid $= \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$

for inclined venturimeter and differential manometer contains heavier liquid

$$= \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

for inclined venturimeter and manometer contains lighter liquid.

where x = difference in the readings of differential manometer,
S_h = sp. gr. of heavier liquid
S_o = sp. gr. of fluid flowing through venturimeter
S_l = sp. gr. of lighter liquid.

6. Pitot-tube is used to find the velocity of a flowing fluid at any point in a pipe or a channel. The velocity is given by the relation

$$V = C_v \sqrt{2gh}$$

where C_v = co-efficient of Pitot-tube

h = rise of liquid in the tube above free surface of liquid – for channels

$= x\left[\frac{S_g}{S_o} - 1\right]$ – for pipes.

7. The momentum equation states that the net force acting on a fluid mass is equal to the change in momentum per second in that direction. This is given as $F = \frac{d}{dt}(mV)$

The impulse-momentum equation is given by $F \cdot dt = d(mV)$.

8. The force exerted by a fluid on a pipe bend in the directions of x and y are given by

$$F_x = \frac{\text{mass}}{\text{sec}} (\text{Initial velocity in the direction of } x - \text{Final velocity in } x\text{-direction})$$

$$+ \text{Initial pressure force in } x\text{-direction} + \text{Final pressure force in } x\text{-direction}$$

$$= \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$$

and $F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

and the direction of the resultant with horizontal is $\tan\theta = \frac{F_y}{F_x}$.

9.

The force exerted by the nozzle on the water is given by $F_x = \rho Q[V_{2x} - V_{1x}]$

and force exerted by the water on the nozzle is $= -F_x = \rho Q[V_{1x} - V_{2x}]$.

10. Moment of momentum equation states that the resultant torque acting on a rotating fluid is equal to the rate of change of moment of momentum. Mathematically, it is given by $T = \rho Q[V_2r_2 - V_1r_1]$.

11. Free liquid jet is the jet of water issuing from a nozzle in atmosphere. The path travelled by the free jet is parabolic. The equation of the jet is given by

$$y = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where x, y = are co-ordinates of any point on jet w.r.t. to the nozzle

U = velocity of jet of water issuing from nozzle

θ = inclination of jet issuing from nozzle with horizontal.

12. (*i*) Maximum height attained by jet $= \frac{U^2\sin^2\theta}{g}$

(*ii*) Time of flight, $T = \frac{2U\sin\theta}{g}$

(*iii*) Time to reach highest point, $T^* = \frac{T}{2} = \frac{U\sin\theta}{g}$

(*iv*) Horizontal range of the jet, $x^* = \frac{U^2}{g}\sin 2\theta$

(*v*) Value of θ for maximum range, $\theta = 45°$

(*vi*) Maximum range, $m^*_{max} = U^2/g$.

EXERCISE 6

(A) THEORETICAL PROBLEMS

1. Name the different forces present in a fluid flow. For the Euler's equation of motion, which forces are taken into consideration.
2. What is Euler's equation of motion ? How will you obtain Bernoulli's equation from it ?
3. Derive Bernoulli's equation for the flow of an incompressible frictionless fluid from consideration of momentum. *(A.M.I.E., Summer, 1988)*
4. State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's theorem from first principle and state the assumptions made for such a derivation. *(Delhi University, 1987)*
5. What is a venturimeter ? Derive an expression for the discharge through a venturimeter. *(A.S.M.E., June 1992 ; A.M.I.E., Winter, 1980)*
6. Explain the principle of venturimeter with a neat sketch. Derive the expression for the rate of flow of fluid through it.
7. Discuss the relative merits and demerits of venturimeter with respect to orifice-meter. *(Delhi University, Dec. 2002)*
8. Define an orifice-meter. Prove that the discharge through an orifice-meter is given by the relation

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

where a_1 = area of pipe in which orifice-meter is fitted
a_0 = area of orifice *(Technical University of M.P., S 2002)*

9. What is a pitot-tube ? How will you determine the velocity at any point with the help of pitot-tube ? *(Delhi University, Dec. 2002)*
10. What is the difference between pitot-tube and pitot-static tube ?
11. State the momentum equation. How will you apply momentum equation for determining the force exerted by a flowing liquid on a pipe bend ?
12. What is the difference between momentum equation and impulse momentum equation ?
13. Define moment of momentum equation. Where this equation is used ?
14. What is a free jet of liquid ? Derive an expression for the path travelled by free jet issuing from a nozzle.
15. Prove that the equation of the free jet of liquid is given by the expression,

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where x, y = co-ordinates of a point on the jet U = velocity of issuing jet
θ = inclination of the jet with horizontal.

16. Which of the following statement is correct in case of pipe flow :
    (*a*) flow takes place from higher pressure to lower pressure ;
    (*b*) flow takes place from higher velocity to lower velocity ;
    (*c*) flow takes place from higher elevation to lower elevation ;
    (*d*) flow takes place from higher energy to lower energy.
17. Derive Euler's equation of motion along a stream line for an ideal fluid stating clearly the assumptions. Explain how this is integrated to get Bernoulli's equation along a stream-line. *(A.M.I.E., Summer, 1984)*
18. State Bernoulli's theorem. Mention the assumptions made. How is it modified while applying in practice? List out its engineering application. *(A.M.I.E., Winter, 1987)*
19. Define continuity equation and Bernoulli's equation. *(Delhi University, 1992)*

20. What are the different forms of energy in a flowing fluid ? Represent schematically the Bernoulli's equation for flow through a tapering pipe and show the position of total energy line and the datum line. *(Osmania University, 1990)*

21. Write Euler's equation of motion long a streamline and integrate it to obtain Bernoulli's equation. State all assumptions made. *(A.M.I.E.. Winter, 1990)*

22. Describe with the help of sketch the construction, operation and use of Pitot-static tube. *(A.M.I.E.. Winter, 1988)*

23. Starting with Euler's equation of motion along a stream line, obtain Bernoulli's equation by its integration. List all the assumptions made. *(A.M.I.E.. Summer, 1991)*

24. State the different devices that one can use to measure the discharge through a pipe and also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help? *(A.M.I.E.. Summer, 1990)*

25. Derive Bernoulli's equation from fundamentals. *(J.N.T.U., Hyderabad, S 2002)*

(B) NUMERICAL PROBLEMS

1. Water is flowing through a pipe of 100 mm diameter under a pressure of 19.62 N/cm^2 (gauge) and with mean velocity of 3.0 m/s. Find the total head of the water at a cross-section, which is 8 m above the datum line. [Ans. 28.458 m]

2. A pipe, through which water is flowing is having diameters 40 cm and 20 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 5.0 m/s. Find the velocity head at the sections 1 and 2 and also rate of discharge. [Ans. 1.274 m : 20.387 m : 0.628 m^3/s]

3. The water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 litres/s. The section 1 is 6 m above datum line and section 2 is 3 m above the datum. If the pressure at section 1 is 29.43 N/cm^2, find the intensity of pressure at section 2. [Ans. 32.19 N/cm^2]

4. Water is flowing through a pipe having diameters 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 29.43 N/cm^2 and the pressure at the upper end is 14.715 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 50 lit/s. [Ans. 14.618 m]

5. The water is flowing through a taper pipe of length 50 m having diameters 40 cm at the upper end and 20 cm at the lower end, at the rate of 60 litres/s. The pipe has a slope of 1 in 40. Find the pressure at the lower end if the pressure at the higher level is 24.525 N/cm^2. [Ans. 25.58 N/cm^2]

6. A pipe of diameter 30 cm carries water at a velocity of 20 m/sec. The pressures at the points A and B are given as 34.335 N/cm^2 and 29.43 N/cm^2 respectively, while the datum head at A and B are 25 m and 28 m. Find the loss of head between A and B. [Ans. 2 m]

7. A conical tube of length 3.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 4 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.0 m of liquid. The loss of head in the tube is $0.95\,(v_1 - v_2)^2/2g$, where v_1 is the velocity at the smaller end and v_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in downward direction. [Ans. 5.56 m of fluid]

8. A pipe line carrying oil of specific gravity 0.8, changes in diameter from 300 mm at a position A to 500 mm diameter to a position B which is 5 m at a higher level. If the pressures at A and B are 19.62 N/cm^2 and 14.91 N/cm^2 respectively, and the discharge is 150 litres/s, determine the loss of head and direction of flow. [Ans. 1.45 m. Flow takes place from A to B]

9. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to inlet and throat is 10 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$. [Ans. 88.92 litres/s]

10. An oil of sp. gr. 0.9 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 20 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$. (*Delhi University, 1987*) [**Ans.** 59.15 litres/s]

11. A horizontal venturimeter with inlet diameter 30 cm and throat diameter 15 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 50 litres/s, find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$. [**Ans.** 2.489 cm]

12. A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 14.715 N/cm^2 and vacuum pressure at the throat is 40 cm of mercury. Find the discharge of water through venturimeter. [**Ans.** 162.539 lit./s]

13. A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury-manometer connected to the inlet and throat gives a reading of 30 cm. Find the discharge. Take $C_d = 0.98$. [**Ans.** 154.02 lit/s]

14. If in the problem 13. instead of water, oil of sp. gr. 0.8 is flowing through the venturimeter, determine the rate of flow of oil in litres/s. [**Ans.** 173.56 lit/s]

15. The water is flowing through a pipe of diameter 30 cm. The pipe is inclined and a venturimeter is inserted in the pipe. The diameter of venturimeter at throat is 15 cm. The difference of pressure between the inlet and throat of the venturimeter is measured by a liquid of sp. gr. 0.8 in an inverted *U*-tube which gives a reading of 40 cm. The loss of head between the inlet and throat is 0.3 times the kinetic head of the pipe. Find the discharge. [**Ans.** 22.64 lit./s]

16. A 20 × 10 cm venturimeter is provided in a vertical pipe line carrying oil of sp. gr. 0.8, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 50 cm. The differential *U*-tube mercury manometer shows a gauge deflection of 40 cm. Calculate : (*i*) the discharge of oil, and (*ii*) the pressure difference between the entrance section and the throat section. Take $C_d = 0.98$ and sp. gr. of mercury as 13.6. [**Ans.** (*i*) 89.132 lit/s, (*ii*) 5.415 N/cm^2]

17. In a 200 mm diameter horizontal pipe a venturimeter of 0.5 contraction ratio has been fixed. The head of water on the venturimeter when there is no flow is 4 m (gauge) Find the rate of flow for which the throat pressure will be 4 metres of water absolute. Take $C_d = 0.97$ and atmospheric pressure head = 10.3 m of water. [**Ans.** 111.92 lit/s]

18. An orifice-meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give readings of 14.715 N/cm^2 and 9.81 N/cm^2 respectively. Find the rate of flow of water through the pipe in litres/s. Take $C_d = 0.6$. [**Ans.** 108.434 lit/s]

19. If in problem 18, instead of water, oil of sp. gr. 0.8 is flowing through the orifice meter in which the pressure difference is measured by a mercury oil differential manometer on the two sides of the orifice meter, find the rate of flow of oil when the reading of manometer is 40 cm. [**Ans.** 122.68 lit/s]

20. The pressure difference measured by the two tappings of a pitotstatic tube, one tapping pointing upstream and other perpendicular to the flow, placed in the centre of a pipe line of diameter 40 cm is 10 cm of water. The mean velocity in the pipe is 0.75 times the central velocity. Find the discharge through the pipe. Take co-efficient of pitot-tube as 0.98. [**Ans.** 0.1293 m^3/s]

21. Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential *U*-tube manometer connected to the two tappings of the pitot-tube is 15 cm. Take sp. gr. of oil = 0.8 and co-efficient of pitot-tube as 0.98. [**Ans.** 6.72 m/s]

22. A sub-marine moves horizontally in sea and has its axis 20 m below the surface of water. A pitot-static tube placed in front of sub-marine and along its axis, is connected to the two limbs of a *U*-tube containing mercury. The difference of mercury level is found to be 20 cm. Find the speed of sub-marine. Take sp. gr. of mercury 13.6 and of sea-water 1.026. [**Ans.** 24.958 km/hr.]

23. A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 40 cm and 20 cm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of bend is 21.58 N/cm^2. The rate of flow of water is 500 litres/s. [**Ans.** 22696.5 N ; 20° 3.5′]

24. The discharge of water through a pipe of diameter 40 cm is 400 litres/s. If the pipe is bend by 135°, find the magnitude and direction of the resultant force on the bend. The pressure of flowing water is 29.43 N/cm^2.
[**Ans.** 7063.2 N, θ = 22° 29.9′ with *x*-axis clockwise]

25. A 30 cm diameter pipe carries water under a head of 15 metres with a velocity of 4 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.
[**Ans.** 8717.5 N, θ = 67° 30′]

26. A pipe of 20 cm diameter conveying 0.20 m^3/sec of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 22.563 N/cm^2 and 21.582 N/cm^2 respectively. [**Ans.** 11604.7 N, θ = 43° 54.2′]

27. A nozzle of diameter 30 mm is fitted to a pipe of 60 mm diameter. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 4.0 m^3/minute. [**Ans.** 7057.7 N]

28. A lawn sprinkler with two nozzles of diameters 3 mm each is connected across a tap of water. The nozzles are at a distance of 40 cm and 30 cm from the centre of the tap. The rate of water through tap is 100 cm^3/s. The nozzle discharges water in the downward directions. Determine the angular speed at which the sprinkler will rotate free. [**Ans.** 2.83 rad/s]

29. A lawn sprinkler has two nozzles of diameters 8 mm each at the end of a rotating arm and the velocity of flow of water from each nozzle is 12 m/s. One nozzle discharges water in the downward direction, while the other nozzle discharges water vertically up. The nozzles are at a distance of 40 cm from the centre of the rotating arm. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of arm, if it is free to rotate. [**Ans.** 5.78 Nm, 30 rad/s]

30. A vertical wall is of 10 m in height. A jet of water is issuing from a nozzle with a velocity of 25 m/s. The nozzle is situated at a horizontal distance of 20 m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water just clears the tap of the wall. [**Ans.** 79° 55′ or 36° 41′]

31. A fire-brigade man is holding a fire stream nozzle of 50 mm diameter at a distance of 1 m above the ground and 6 m from a vertical wall. The jet is coming out with a velocity of 15 m/s. This jet is to strike a window, situated at a distance of 10 m above ground in the vertical wall. Find the angle or angles of inclination with the horizontal made by the jet, coming out from the nozzle. What will be the amount of water falling on the window ? [**Ans.** 79° 16.7′ or 67° 3.7′ : 0.0294 m^3/s]

32. A window, in a vertical wall, is at a distance of 12 m above the ground level. A jet of water, issuing from a nozzle of diameter 50 mm, is to strike the window. The rate of flow of water through the nozzle is 40 litres/sec. The nozzle is situated at a distance of 1 m above ground level. Find the greatest horizontal distance from the wall of the nozzle so that jet of water strikes the window. [**Ans.** 29.38 m]

33. Explain in brief the working of a pitot-tube. Calculate the velocity of flow of water in a pipe of diameter 300 mm at a point, where the stagnation pressure head is 5 m and static pressure head is 4 m. Given the co-efficient of pitot-tube = 0.97. (*Delhi University, Nov. 1982*) [**Ans.** 4.3 m/sec]

34. Find the rate of flow of water through a venturimeter fitted in a pipeline of diameter 30 cm. The ratio of diameter of throat and inlet of the venturimeter is *. The pressure at the inlet of the venturimeter is 13.734 N/cm^2 (gauge) and vacuum in the throat is 37.5 cm of mercury. The co-efficient of venturimeter is given as 0.98. (*Delhi University, April, 1982*) [**Ans.** 0.15 m^3/s]

35. A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying an oil of sp. gr. 0.8, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 30 cm. The difference in the elevation of the throat section and inlet section is 50 cm. Find the rate of flow of oil. (*Delhi University, 1988*)

36. A venturimeter is used for measurement of discharge of water in horizontal pipe line. If the ratio of upstream pipe diameter to that of throat is 2 : 1, upstream diameter is 300 mm, the difference in pressure between the throat and upstream is equal to 3 m head of water and loss of head through meter is one-eighth of the throat velocity head, calculate the discharge in the pipe. (*A.M.I.E., Winter, 1987*)
[**Ans.** 0.107 m^3/s]

37. A liquid of specific gravity 0.8 is flowing upwards at the rate of 0.08 m^3/s, through a vertical venturimeter with an inlet diameter of 200 mm and throat diameter of 100 mm. The C_d = 0.98 and the vertical distance between pressure tappings is 300 mm. Find :

(*i*) the difference in readings of the two pressure gauges, which are connected to the two pressure tappings, and

(*ii*) the difference in the level of the mercury columns of the differential manometer which is connected to the tappings, in place of pressure gauges. (*Delhi University, 1992*)

[**Ans.** (*i*) 42.928 kN/m^2, (*ii*) 32.3 cm]

[**Hint.** $Q = 0.08$ m^3/s, $d_1 = 200$ mm $= 0.2$ m, $d_2 = 100$ mm $= 0.1$ m,

$C_d = 0.98$, $z_2 - z_1 = 300$ mm $= 0.3$ m, $a_1 = \frac{\pi}{4}(.2^2) = 0.0314$ m^2

$a_2 = \frac{\pi}{4}(.1^2) = 0.007854$ m^2. Using $Q = C_d \dfrac{a_1 \times a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

Find 'h'. This value of $h = 5.17$ m.

Now use $h = \left(\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}\right) + (z_1 - z_2)$, where $\rho = 800$ kg/m^3. Find $(p_1 - p_2)$.

Now use the formula $h = x\left[\dfrac{S_g}{S_f} - 1\right]$,

where $h = 5.17$ m, $S_g = 13.6$ and $S_f = 0.8$. Find the value of x which will be 32.3 cm.]

38. A venturimeter is installed in a 300 mm diameter horizontal pipe line. The throat pipe rates is 1/3. Water flows through the installation. The pressure in the pipe line is 13.783 N/cm^2 (gauge) and vacuum in the throat is 37.5 cm of mercury. Neglecting head loss in the venturimeter, determine the rate of flow in the pipe line. (*Osmania University, 1990*) [**Ans.** 0.153 m^3/sec)

[**Hint.** $d_1 = 300$ mm $= 0.3$ m, $d_2 = \frac{1}{3} \times 300 = 100$ mm $= 0.1$ m, $p_1 = 13.783$ N/cm^2 $= 13.783 \times 10^4$ N/m^2.

Hence $p_1/\rho \times g = 13.783 \times 10^4/1000 \times 9.81$

$= 14.05$ m, $p_2/\rho g = -37.5$ cm of Hg $= -0.375 \times 13.6$ m of water

$= -5.1$ m of water. Hence h $= 14.05 - (-5.1) = 19.15$ m of water.

Value of $C_d = 1.0$. Now use the formula $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$.

39. The maximum flow through a 300 mm diameter horizontal main pipe line is 18200 litre/minute. A venturimeter is introduced at a point of the pipe line where the pressure head is 4.6 m of water. Find the smallest dia. of throat so that the pressure at the throat is never negative. Assume co-efficient of meter as unity. (*A.M.I.E., Winter, 1989*), [**Ans.** $d_2 = 192.4$ mm]

[**Hint.** $d_1 = 300$ mm $= 0.3$ m, $Q = 18200$ litres/minute $= 18200/60 = 303.33$ litres/s $= 0.3033$ m^3/s, $p_1/\rho g = 4.6$ m, $p_2/\rho g = 0$. Hence $h = 4.6$ m, $C_d = 1$. $d_2 =$ dia. at throat. Use formula $Q = C_d \dfrac{a_1 \times a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$ and find the value of a_2. Then $a_2 = \frac{\pi}{4} d_2^2$ and find d_2.]

40. The following are the data given of a change in diameter effected in laying a water supply pipe. The change in diameter is gradual from 20 cm at A to 50 cm at B. Pressures at A and B are 7.848 N/cm^2 and 5.886 N/cm^2 respectively with the end B being 3 m higher than A. If the flow in the pipe line is 200 litre/s, find : (*i*) direction of flow, (*ii*) the head lost in friction between A and B.

(*Osmania University, 1990*) [**Ans.** (*i*) From A to B, (*ii*) 1.015 m]

[**Hint.** $D_A = 20$ cm $= 0.2$ m, $D_B = 50$ cm $= 0.5$ m, $p_A = 7.848$ N/cm^2 $= 7.848 \times 10^4$ N/m^2

$p_B = 5.886$ N/cm^2 $= 5.886 \times 10^4$ N/m^2, $Z_A = 0$, $Z_B = 3$ m, $Q = 0.2$ m^3/s

$V_A = 0.2/\frac{\pi}{4}(.2^2) = 6.369$ m/s, $V_B = 0.2/\frac{\pi}{4}(.5^2) = 1.018$ m/s

$$E_A = (p_A/\rho \times g) + \frac{V_A^2}{2g} + Z_A = (7.848 \times 10^4/1000 \times 9.81) + (6.369^2/2 \times 9.81) + 0 = 10.067 \text{ m}$$

$$E_B = (p_B/\rho \times g) + \frac{V_B^2}{2g} + Z_B = (5.886 \times 10^4/1000 \times 9.81) + (1.018^2/2 \times 9.81) + 3 = 9.052 \text{ m}]$$

41. A venturimeter of inlet diameter 300 mm and throat diameter 150 mm is fixed in a vertical pipe line. A liquid of sp. gr. 0.8 is flowing upward through the pipe line. A differential manometer containing mercury gives a reading of 100 mm when connected at inlet and throat. The vertical difference between inlet and throat is 500 mm. If $C_d = 0.98$, then find : (*i*) rate of flow of liquid in litre per second and (*ii*) difference of pressure between inlet and throat in N/m^2. *(Delhi University, 1988)*

[Ans. (*i*) 100 litre/s, (*ii*) 15980 N/m^2**]**

42. A venturimeter with a throat diameter of 7.5 cm is installed in a 15 cm diameter pipe. The pressure at the entrance to the meter is 70 kPa (gauge) and it is desired that the pressure at any point should not fall below 2.5 m of water absolute. Determine the maximum flow rate of water through the meter. Take $C_d = 0.97$ and atmospheric pressure as 100 kPa. *(J.N.T.U., Hyderabad S 2002)*

[Hint. The pressure at the throat will be minimum. Hence $\frac{p_2}{\rho g} = 2.5$ m (abs.)

Given : $d_1 = 15 \text{ cm} \therefore A_1 = \frac{\pi}{4}(15^2) = 176.7 \text{ cm}^2$

$$d_2 = 7.5 \text{ cm} \therefore A_2 = \frac{\pi}{4}(7.5^2) = 44.175 \text{ cm}^2$$

$$p_1 = 70 \text{ kPa} = 70 \times 10^3 \text{ N/m}^2 \text{ (gauge)}, p_{atm} = 100 \text{ kPa} = 100 \times 10^3 \text{ N/m}^2$$

$$\therefore \quad p_1 \text{ (abs.)} = 70 \times 10^3 + 100 \times 10^3 = 170 \times 10^3 \text{ N/m}^2 \text{ (abs.)}$$

$$\therefore \quad \frac{p_1}{\rho g} = \frac{170 \times 10^3}{1000 \times 9.81} = 17.33 \text{ m of water (abs.)}$$

$$\therefore \quad h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 17.33 - 2.5 = 14.83 \text{ m of water} = 1483 \text{ cm of water}$$

Now $$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = \frac{0.97 \times 176.7 \times 44.175 \times \sqrt{2 \times 981 \times 1483}}{\sqrt{176.7^2 - 44 \cdot 175^2}} = 75488 \text{ cm}^3/\text{s}$$

= 75.488 litre/s. Ans.]

43. Find the discharge of water flowing through a pipe 20 cm diameter placed in an inclined position, where a venturimeter is inserted, having a throat diameter of 10 cm. The difference of pressure between the main and throat is measured by a liquid of specific gravity 0.4 in an inverted *U*-tube, which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of pipe. *(Delhi University, Dec. 2002)*

[Hint. Given : $d_1 = 20 \text{ cm} \therefore a_1 = \frac{\pi}{4}(20^2) = 100\,\pi \text{ cm}^2$; $d_2 = 10 \text{ cm} \therefore a_2 = \frac{\pi}{4}(10^2) = 25\,\pi \text{ cm}^2$.

$$x = 30 \text{ cm}, h = x\left(1 - \frac{S_l}{S_o}\right) = 30\left(1 - \frac{0.4}{1.0}\right) = 18 \text{ cm} = 0.18 \text{ m}$$

But h is also $$= \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) \therefore \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 18 \text{ cm} = 0.18 \text{ m}$$

$$h_L = 0.2 \times \frac{V_1^2}{2g}$$

From Bernoulli's equation, $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$

or $\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$

or $0.18 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{0.2\,V_1^2}{2g}$ $\left(\because \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 0.18 \text{ m and } h_L = \frac{0.2\,V_1^2}{2g}\right)$

or $0.18 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.2\,V_1^2}{2g} = 0$ or $0.18 + \frac{0.8\,V_1^2}{2g} - \frac{V_2^2}{2g} = 0$

From continuity equation, $a_1V_1 = a_2V_2$ or $V_2 = \frac{a_1V_1}{a_2} = \frac{\frac{\pi}{4}(20^2)\,V_1}{\frac{\pi}{4}(10^2)} = 4V_1$

Now $0.18 + \frac{0.8\,V_1^2}{2g} - \frac{V_2^2}{2g} = 0$ or $0.18 + \frac{0.8\,V_1^2}{2g} - \frac{(4V_1)^2}{2g} = 0$

or $0.18 + \frac{0.8\,V_1^2}{2g} - \frac{16V_1^2}{2g} = 0$ or $0.18 = \frac{16V_1^2}{2g} - \frac{0.8\,V_1^2}{2g} = \frac{15.2V_1^2}{2g}$

$\therefore$ $V_1 = \sqrt{\frac{0.18 \times 2 \times 9.81}{15.2}} = 0.48 \text{ m/s} = 48 \text{ cm/s}$

$\therefore$ $Q = A_1V_1 = \frac{\pi}{4}(20^2) \times 48 = 15140 \text{ cm}^3\text{/s} =$ **15.14 litre/s. Ans.]**

7

CHAPTER

Orifices and Mouthpieces

▶ 7.1 INTRODUCTION

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

▶ 7.2 CLASSIFICATIONS OF ORIFICES

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The following are the important classifications :

1. The orifices are classified as **small orifice** or **large orifice** depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.
2. The orifices are classified as (*i*) Circular orifice, (*ii*) Triangular orifice, (*iii*) Rectangular orifice and (*iv*) Square orifice depending upon their cross-sectional areas.
3. The orifices are classified as (*i*) Sharp-edged orifice and (*ii*) Bell-mouthed orifice depending upon the shape of upstream edge of the orifices.
4. The orifices are classified as (*i*) Free discharging orifices and (*ii*) Drowned or sub-merged orifices depending upon the nature of discharge.

The sub-merged orifices are further classified as (*a*) Fully sub-merged orifices and (*b*) Partially sub-merged orifices.

▶ 7.3 FLOW THROUGH AN ORIFICE

Consider a tank fitted with a circular orifice in one of its sides as shown in Fig. 7.1. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section CC, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called **Vena-contracta**. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

Consider two points 1 and 2 as shown in Fig. 7.1. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 and 2.

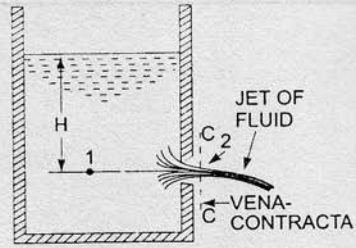

Fig. 7.1 *Tank with an orifice.*

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $\qquad z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $\qquad \frac{p_1}{\rho g} = H$

$$\frac{p_2}{\rho g} = 0 \text{ (atmospheric pressure)}$$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$$\therefore \quad H + 0 = 0 + \frac{v_2^2}{2g}$$

$$\therefore \quad v_2 = \sqrt{2gH} \qquad \text{...(7.1)}$$

This is theoretical velocity. Actual velocity will be less than this value.

▶ 7.4 HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients are

1. Co-efficient of velocity, C_v
2. Co-efficient of contraction, C_c
3. Co-efficient of discharge, C_d.

7.4.1 Co-efficient of Velocity (C_v). It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, C_v is given as

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \text{ where } V = \text{actual velocity}, \sqrt{2gH} = \text{Theoretical velocity} \qquad \text{...(7.2)}$$

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of $C_v = 0.98$ is taken for sharp-edged orifices.

7.4.2 Co-efficient of Contraction (C_c). It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice and

a_c = area of jet at vena-contracta.

Then
$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a} \quad \text{...(7.3)}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken 0.64.

7.4.3 Co-efficient of Discharge (C_d). It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$\therefore \quad C_d = C_v \times C_c \quad \text{...(7.4)}$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

Problem 7.1 *The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.*

Solution. Given :

Head, $H = 10$ cm

Dia. of orifice, $d = 40$ mm $= 0.04$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.04)^2 = .001256 \text{ m}^2$

$C_d = 0.6$

$C_v = 0.98$

(*i*) $$\frac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6$$

But Theoretical discharge = V_{th} × Area of orifice

V_{th} = Theoretical velocity, where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14$ m/s

$\therefore$ Theoretical discharge $= 14 \times .001256 = 0.01758 \ \frac{\text{m}^2}{\text{s}}$

$\therefore$ Actual discharge $= 0.6 \times$ Theoretical discharge

$= 0.6 \times .01758 =$ **0.01054 m³/s. Ans.**

(ii) $\dfrac{\text{Actual velocity}}{\text{Theoretical velocity}} = C_v = 0.98$

$\therefore$ Actual velocity $= 0.98 \times$ Theoretical velocity

$= 0.98 \times 14 =$ **13.72 m/s. Ans.**

Problem 7.2 *The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litre/s. Find the co-efficient of discharge.*

Solution. Given :

Dia. of orifice, $d = 20 \text{ mm} = .02 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.02)^2 = .000314 \text{ m}^2$

Head, $H = 1 \text{ m}$

Actual discharge, $Q = 0.85 \text{ litre/s} = .00085 \text{ m}^3\text{/s}$

Theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429 \text{ m/s}$

$\therefore$ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice

$= 4.429 \times .000314 = 0.00139 \text{ m}^3\text{/s}$

$\therefore$ Co-efficient of discharge $= \dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = \dfrac{0.00085}{0.00139} =$ **0.61. Ans.**

▶ 7.5 EXPERIMENTAL DETERMINATION OF HYDRAULIC CO-EFFICIENTS

7.5.1 Determination of C_d. The water is allowed to flow through an orifice fitted to a tank under a constant head, H as shown in Fig. 7.2. The water is collected in a measuring tank for a known time, t. The height of water in the measuring tank is noted down. Then actual discharge through orifice,

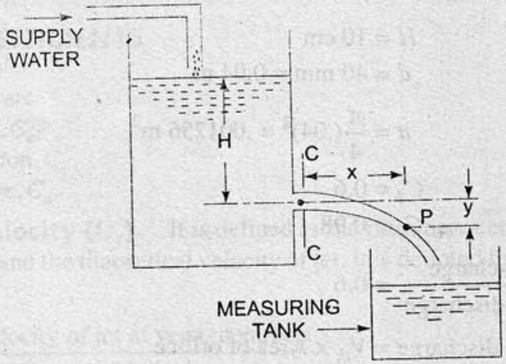

Fig. 7.2 *Value of C_d.*

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time } (t)}$$

and theoretical discharge = area of orifice $\times \sqrt{2gH}$

$$C_d = \frac{Q}{a \times \sqrt{2gH}} \qquad \text{...(7.5)}$$

7.5.2 Determination of Co-efficient of Velocity (C_v). Let *C-C* represents the vena-contracta of a jet of water coming out from an orifice under constant head *H* as shown in Fig. 7.2. Consider a liquid particle which is at vena-contracta at any time and takes the position at *P* along the jet in time '*t*'.

Let x = horizontal distance travelled by the particle in time '*t*'
y = vertical distance between *P* and *C-C*
V = actual velocity of jet at vena-contracta.

Then horizontal distance, $x = V \times t$ ...(*i*)

and vertical distance, $y = \frac{1}{2} gt^2$ ...(*ii*)

From equation (*i*), $t = \frac{x}{V}$

Substituting this value of '*t*' in (*ii*), we get

$$y = \frac{1}{2} g \times \frac{x^2}{V^2}$$

$$V^2 = \frac{gx^2}{2y}$$

$$\therefore \quad V = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity,

$$V_{th} = \sqrt{2gH}$$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

$$= \frac{x}{\sqrt{4yH}}. \quad \text{...(7.6)}$$

7.5.3 Determination of Co-efficient of Contraction (C_c). The co-efficient of contraction is determined from the equation (7.4) as

$$C_d = C_v \times C_c$$

$$\therefore \quad C_c = \frac{C_d}{C_v} \quad \text{...(7.7)}$$

Problem 7.3 *A jet of water, issuing from a sharp-edged vertical orifice under a constant head of 10.0 cm, at a certain point, has the horizontal and vertical co-ordinates measured from the vena-contracta as 20.0 cm and 10.5 cm respectively. Find the value of C_v. Also find the value of C_c if $C_d = 0.60$.*

Solution. Given :

Head, $H = 10.0$ cm
Horizontal distance, $x = 20.0$ cm
Vertical distance, $y = 10.5$ cm
$C_d = 0.6$

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{20.0}{\sqrt{4 \times 10.5 \times 10.0}} = \frac{20}{20.493} = 0.9759 = \mathbf{0.976.\ Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.976} = 0.6147 = \mathbf{0.615.\ Ans.}$$

Problem 7.4 *The head of water over an orifice of diameter 100 mm is 10 m. The water coming out from orifice is collected in a circular tank of diameter 1.5 m. The rise of water level in this tank is 1.0 m in 25 seconds. Also the co-ordinates of a point on the jet, measured from vena-contracta are 4.3 m horizontal and 0.5 m vertical. Find the co-efficients, C_d, C_v and C_c.*

Solution. Given :

Head, $H = 10$ m

Dia. of orifice, $d = 100$ mm $= 0.1$ m

$\therefore$ Area of orifice, $a = \frac{\pi}{4}(.1)^2 = 0.007853\ m^2$

Dia. of measuring tank, $D = 1.5$ m

$\therefore$ Area, $A = \frac{\pi}{4}(1.5)^2 = 1.767\ m^2$

Rise of water, $h = 1$ m

in time, $t = 25$ seconds

Horizontal distance, $x = 4.3$ m

Vertical distance, $y = 0.5$ m

Now theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14.0$ m/s

$\therefore$ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice $= 14.0 \times .007854 = 0.1099\ m^3/s$

Actual discharge, $$Q = \frac{A \times h}{t} = \frac{1.767 \times 1.0}{25} = 0.07068$$

$\therefore$ $$C_d = \frac{Q}{Q_{th}} = \frac{.07068}{.1099} = \mathbf{0.643.\ Ans.}$$

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.3}{\sqrt{4 \times 0.5 \times 10}} = \frac{4.3}{4.472} = \mathbf{0.96.\ Ans.}$$

C_c is given by equation (7.7) as $C_c = \frac{C_d}{C_v} = \frac{0.643}{0.96} = \mathbf{0.669.\ Ans.}$

Problem 7.5 *Water discharge at the rate of 98.2 litres/s through a 120 mm diameter vertical sharp-edged orifice placed under a constant head of 10 metres. A point, on the jet, measured from the vena-contracta of the jet has co-ordinates 4.5 metres horizontal and 0.54 metres vertical. Find the co-efficient C_v, C_c and C_d of the orifice.* (A.M.I.E., Winter, 1983)

Solution. Given :

Discharge, $Q = 98.2 \text{ lit/s} = 0.0982 \text{ m}^3\text{/s}$

Dia. of orifice, $d = 120 \text{ mm} = 0.12 \text{ m}$

$\therefore$ Area of orifice, $a = \frac{\pi}{4}(.12)^2 = 0.01131 \text{ m}^2$

Head, $H = 10 \text{ m}$

Horizontal distance of a point on the jet from vena-contracta, $x = 4.5$ m and vertical distance, $y = 0.54$ m

Now theoretical velocity, $V_{th} = \sqrt{2g \times H} = \sqrt{2 \times 9.81 \times 10} = 14.0 \text{ m/s}$

Theoretical discharge, $Q_{th} = V_{th} \times \text{Area of orifice}$

$= 14.0 \times .01131 = 0.1583 \text{ m}^3\text{/s}$

The value of C_d is given by $C_d = \frac{\text{Actual discharge}}{\text{The oretical discharge}} = \frac{Q}{Q_{th}} = \frac{0.0982}{0.1583} = $ **0.62. Ans.**

The value of C_c is given by equation (7.6),

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = \textbf{0.968. Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.62}{0.968} = \textbf{0.64. Ans.}$$

Problem 7.6 *A 25 mm diameter nozzle discharges 0.76 m³ of water per minute when the head is 60 m. The diameter of the jet is 22.5 mm. Determine : (i) the values of co-efficients C_c, C_v and C_d and (ii) the loss of head due to fluid resistance.* (A.M.I.E., Summer 1988)

Solution. Given :

Dia. of nozzle, $D = 25 \text{ mm} = 0.025 \text{ m}$

Actual discharge, $Q_{act} = 0.76 \text{ m}^3\text{/minute} = \frac{0.76}{60} = 0.01267 \text{ m}^3\text{/s}$

Head, $H = 60 \text{ m}$

Dia. of jet, $d = 22.5 \text{ mm} = 0.0225 \text{ m}.$

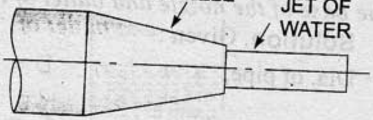

Fig. 7.3

(i) Values of co-efficients :

Co-efficient of contraction (C_c) is given by,

$$C_c = \frac{\text{Area of jet}}{\text{Area of nozzle}}$$

$$= \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} = \frac{d^2}{D^2} = \frac{0.0225^2}{0.025^2} = \textbf{0.81. Ans.}$$

Co-efficient of discharge (C_d) is given by,

$$C_d = \frac{\text{Actual discharge}}{\text{Theorctical discharge}}$$

$$= \frac{0.01267}{\text{Theoretical velocity} \times \text{Area of nozzle}}$$

$$= \frac{0.01267}{\sqrt{2gH} \times \frac{\pi}{4} D^2} = \frac{0.01267}{\sqrt{2 \times 9.81 \times 60} \times \frac{\pi}{4} (0.025)^2}$$

$$= \mathbf{0.752.\ Ans.}$$

Co-efficient of velocity (C_v) is given by,

$$C_v = \frac{C_d}{C_c} = \frac{0.752}{0.81} = \mathbf{0.928.\ Ans.}$$

(ii) Loss of head due to fluid resistance :

Applying Bernoulli's equation at the outlet of nozzle and to the jet of water, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head}$$

But $\quad \frac{p_1}{\rho g} = \frac{p_2}{\rho g} = \text{Atmospheric pressure head}$

$$z_1 = z_2,\ V_1 = \sqrt{2gH},\ V_2 = \text{Actual velocity of jet} = C_v \sqrt{2gH}$$

$$\therefore \quad \frac{\left(\sqrt{2gH}\right)^2}{2g} = \frac{\left(C_v\sqrt{2gH}\right)^2}{2g} + \text{Loss of head}$$

or $\quad H = C_v^2 \times H + \text{Loss of head}$

$\therefore$ Loss of head $\quad = H - C_v^2 \times H = H(1 - C_v^2)$

$$= 60(1 - 0.928^2) = 60 \times 0.1388 = \mathbf{8.328\ m.\ Ans.}$$

Problem 7.7 *A pipe, 100 mm in diameter, has a nozzle attached to it at the discharge end, the diameter of the nozzle is 50 mm. The rate of discharge of water through the nozzle is 20 litres/s and the pressure at the base of the nozzle of 5.886 N/cm². Calculate the co-efficient of discharge. Assume that the base of the nozzle and outlet of the nozzle are at the same elevation.* (A.M.I.E., Winter, 1977)

Solution. Given :

Dia. of pipe, $\quad D = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore \quad A_1 = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Dia. of nozzle, $\quad d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore \quad A_2 = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

Actual discharge, $\quad Q = 20 \text{ lit/s} = 0.02 \text{ m}^3\text{/s}$

Pressure at the base, $\quad p_1 = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \frac{\text{N}}{\text{m}^2}$

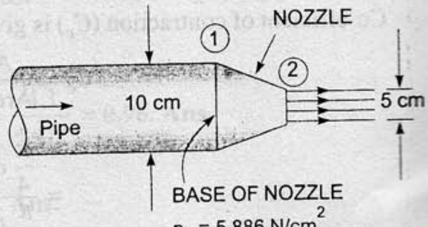

Fig. 7.4

From continuity equation, $A_1 V_1 = A_2 V_2$

or $\quad .007854\ V_1 = .001963\ V_2$

$$\therefore \qquad V_1 = \frac{.001963 V_2}{.007854} = \frac{V_2}{4}$$

where V_1 and V_2 are theoretical velocity at (1) and (2).

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $\qquad z_1 = z_2$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

or $$\frac{5.886 \times 10^4}{1000 \times 9.81} + \frac{\left(\frac{V_2}{4}\right)^2}{2g} = 0 + \frac{V_2^2}{2g} \qquad \left\{\because \frac{p_2}{\rho g} = \text{Atmospheric pressure} = 0\right\}$$

$$6.0 + \frac{V_2^2}{2g \times 16} = \frac{V_2^2}{2g}$$

or $$\frac{V_2^2}{2g}\left[1 - \frac{1}{16}\right] = 6.0 \text{ or } \frac{V_2^2}{2g}\left[\frac{15}{16}\right] = 6.0$$

$$\therefore \qquad V_2 = \sqrt{6.0 \times 2 \times 9.81 \times \frac{16}{15}} = 11.205 \text{ m/sec}$$

$\therefore$ Theoretical discharge $\qquad = V_2 \times A_2 = 11.205 \times .001963 = 0.022 \text{ m}^3\text{/s}$

$$\therefore \qquad C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.02}{0.022} = \mathbf{0.909.\ Ans.}$$

Problem 7.8 *A tank has two identical orifices in one of its vertical sides. The upper orifice is 3 m below the water surface and lower one is 5 m below the water surface. If the value of C_v for each orifice is 0.96, find the point of intersection of the two jets.*

Solution. Given :

Height of water from orifice (1), $H_1 = 3$ m

From orifice (2), $\qquad H_2 = 5$ m

C_v for both $= 0.96$

Let P is the point of intersection of the two jets coming from orifices (1) and (2), such that

x = horizontal distance of P

y_1 = vertical distance of P from orifice (1)

y_2 = vertical distance of P from orifice (2)

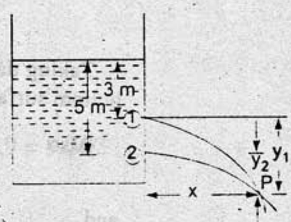

Fig. 7.5

Then $\qquad y_1 = y_2 + (5 - 3) = y_2 + 2$ m

The value of C_v is given by equation (7.6) as

For orifice (1), $C_{v_1} = \dfrac{x}{\sqrt{4y_1 H_1}} = \dfrac{x}{\sqrt{4y_1 \times 3.0}}$

For orifice (2), $C_{v_2} = \dfrac{x}{\sqrt{4y_2 H_2}} = \dfrac{x}{\sqrt{4 \times y_2 \times 5.0}}$

As both the orifices are identical

$$\therefore \quad C_{v_1} = C_{v_2}$$

or $$\frac{x}{\sqrt{4y_1 \times 3.0}} = \frac{x}{\sqrt{4y_2 \times 5.0}} \text{ or } 3y_1 = 5y_2$$

But $$y_1 = y_2 + 2.0$$

$$\therefore \quad 3(y_2 + 2.0) = 5y_2$$

$$\therefore \quad 2y_2 = 6.0$$

From (*ii*), $$C_{v_2} = \frac{x}{\sqrt{4y_2 \times H_2}}$$

or $$0.96 = \frac{x}{\sqrt{4 \times 3.0 \times 5.0}}$$

$$\therefore \quad x = 0.96 \times \sqrt{4 \times 3.0 \times 5.0} = 7.436 \text{ m. Ans.}$$

Problem 7.9 *A closed vessel contains water upto a height of 1.5 m and over the water surface there is air having pressure 7.848 N/cm² (0.8 kgf/cm²) above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 100 mm. Find the rate of flow of water from orifice. Take $C_d = 0.6$.*

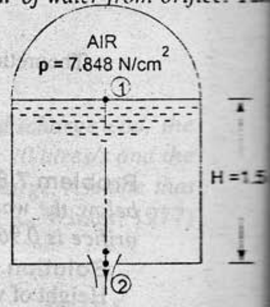

Fig. 7.6

Solution. Given :

Dia. of orifice, $d = 100 \text{ mm} = 0.1 \text{ m}$

$C_d = 0.6$

Height of water, $H = 1.5 \text{ m}$

Air pressure, $p = 7.848 \text{ N/cm}^2 = 7.848 \times 10^4 \text{ N/m}^2$

Applying Bernoulli's equation at (1) (water surface) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Taking datum line passing through (2) which is very close to the bottom surface of the tank. Then $z_2 = 0$, $z_1 = 1.5$ m

Also $$\frac{p_2}{\rho g} = 0 \text{ (atmospheric pressure)}$$

and $$\frac{p_1}{\rho g} = \frac{7.848 \times 10^4}{1000 \times 9.81} = 8 \text{ m of water}$$

$$\therefore \quad 8 + 0 + 1.5 = 0 + \frac{V_2^2}{2g} + 0 \qquad \{V_1 \text{ is negligible}\}$$

$$\therefore \quad 9.5 = \frac{V_2^2}{2g}$$

$\therefore \quad V_2 = \sqrt{2 \times 9.81 \times 9.5} = 13.652$ m/s

$\therefore$ Rate of flow of water $= C_d \times a_2 \times V_2$

$$= 0.6 \times \frac{\pi}{4} (.1)^2 \times 13.652 \text{ m}^3\text{/s} = 0.0643 \text{ m}^3\text{/s. } \textbf{Ans.}$$

Problem 7.10 *A closed tank partially filled with water upto a height of 0.9 m having an orifice of diameter 15 mm at the bottom of the tank. The air is pumped into the upper part of the tank. Determine the pressure required for a discharge of 1.5 litres/s through the orifice. Take $C_d = 0.62$.*

Solution. Given :

Height of water above orifice, $H = 0.9$ m

Dia. of orifice, $d = 15 \text{ mm} = 0.015$ m

$\therefore$ Area, $a = \frac{\pi}{4}[d^2] = \frac{\pi}{4}(.015)^2 = 0.0001767 \text{ m}^2$

Discharge, $Q = 1.5 \text{ litres/s} = .0015 \text{ m}^3\text{/s}$

$C_d = 0.62$

Let p is intensity of pressure required above water surface in N/cm².

Then pressure head of air $= \frac{p}{\rho g} = \frac{p \times 10^4}{1000 \times 9.81} = \frac{10p}{9.81}$ m of water.

If V_2 is the velocity at outlet of orifice, then

$$V_2 = \sqrt{2g\left(H + \frac{p}{\rho g}\right)} = \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)}$$

$\therefore$ Discharge $Q = C_d \times a \times \sqrt{2g(H + p/\rho g)}$

$$.0015 = 0.6 \times .0001767 \times \sqrt{2 \times 9.81(0.9 + p/\rho g)}$$

$$\therefore \quad \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)} = \frac{.0015}{0.6 \times .0001767} = 14.148$$

or $$2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right) = 14.148 \times 14.148$$

$$\therefore \quad \frac{10p}{9.81} = \frac{14.148 \times 14.148}{2 \times 9.81} - 0.9 = 10.202 - 0.9 = 9.302$$

$$\therefore \quad p = \frac{9.302 \times 9.81}{10} = \mathbf{9.125 \text{ N/cm}^2\text{. Ans.}}$$

▶ 7.6 FLOW THROUGH LARGE ORIFICES

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by $Q = C_d \times a \times \sqrt{2gh}$. But in case of a large orifice, the velocity is not constant over the entire cross-section of the jet and hence Q cannot be calculated by $Q = C_d \times a \times \sqrt{2gh}$.

7.6.1 Discharge Through Large Rectangular Orifice. Consider a large rectangular orifice in one side of the tank discharging freely into atmosphere under a constant head, H as shown in Fig. 7.7.

Let H_1 = height of liquid above top edge of orifice

H_2 = height of liquid above bottom edge of orifice

b = breadth of orifice

d = depth of orifice = $H_2 - H_1$

C_d = co-efficient of discharge.

Consider an elementary horizontal strip of depth 'dh' at a depth of 'h' below the free surface of the liquid in the tank as shown in Fig. 7.7 (b).

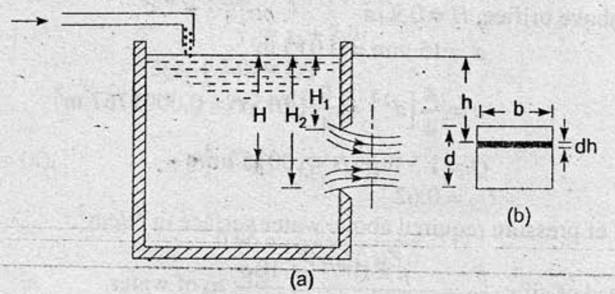

Fig. 7.7 *Large rectangular orifice.*

$\therefore$ Area of strip = $b \times dh$

and theoretical velocity of water through strip = $\sqrt{2gh}$.

$\therefore$ Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity}$$

$$= C_d \times b \times dh \times \sqrt{2gh} = C_d b \times \sqrt{2gh}\ dh$$

By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained

$$\therefore \quad Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\ dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\ dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b\ \sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right] \quad \text{...(7.8)}$$

Problem 7.11 *Find the discharge through a rectangular orifice 2.0 m wide and 1.5 m deep fitted to a water tank. The water level in the tank is 3.0 m above the top edge of the orifice. Take C_d = 0.62.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1.5$ m

Height of water above top of the orifice, $H_1 = 3$ m

Height of water above bottom edge of the orifice,

$$\therefore \quad H_2 = H_1 + d = 3 + 1.5 = 4.5 \text{ m}$$
$$C_d = 0.62$$

Discharge Q is given by equation (7.8) as

$$Q = \frac{2}{3} C_d \times b \times \sqrt{2g} \; [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 2.0 \times \sqrt{2 + 9.81} \, [4.5^{1.5} - 3^{1.5}] \text{ m}^3\text{/s}$$

$$= 3.66[9.545 - 5.196] \text{ m}^3\text{/s} = \mathbf{15.917 \text{ m}^3\text{/s. Ans.}}$$

Problem 7.12 *A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take the co-efficient of discharging for the orifice = 0.6.*

Solution. Given :

Width of orifice, $b = 1.5$ m

Depth of orifice, $d = 1.0$ m

$$H_1 = 3.0 \text{ m}$$
$$H_2 = H_1 + d = 3.0 + 1.0 = 4.0 \text{ m}$$
$$C_d = 0.6$$

Discharge, Q is given by the equation (7.8) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \; [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 1.5 \times \sqrt{2 + 9.81} \, [4.0^{1.5} - 3.0^{1.5}] \text{ m}^3\text{/s}$$

$$= 2.657 \, [8.0 - 5.196] \text{ m}^3\text{/s} = \mathbf{7.45 \text{ m}^3\text{/s. Ans.}}$$

Problem 7.13 *A rectangular orifice 0.9 m wide and 1.2 m deep is discharging water from a vessel. The top edge of the orifice is 0.6 m below the water surface in the vessel. Calculate the discharge through the orifice if $C_d = 0.6$ and percentage error if the orifice is treated as a small orifice.*

Solution. Given :

Width of orifice, $b = 0.9$ m

Depth of orifice, $d = 1.2$ m

$$H_2 = 0.6 \text{ m}$$
$$H_2 = H_1 + d = 0.6 + 1.2 = 1.8 \text{ m}$$
$$C_d = 0.6$$

Discharge Q is given as $Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times [H_2^{3/2} - H_1^{3/2}]$

$$= \frac{2}{3} \times 0.6 \times 2.9 \times \sqrt{2 \times 9.81} \; [1.8^{3/2} - 0.6^{3/2}] \text{ m}^3\text{/s}$$

$$= 1.5946 \, [2.4149 - .4647] = \mathbf{3.1097 \text{ m}^3\text{/s. Ans.}}$$

Discharging for a small orifice

$$Q_1 = C_d \times a \times \sqrt{2gh}$$

where $h = H_1 + \frac{d}{2} = 0.6 + \frac{1.2}{2} = 1.2$ m and $a = b \times d = 0.9 \times 1.2$

$$Q_1 = 0.6 \times .9 \times 1.2 \times \sqrt{2 \times 9.81 \times 1.2} = 3.1442 \text{ m}^3/\text{s}$$

$$\% \text{ error} = \frac{Q_1 - Q}{Q} = \frac{3.1442 - 3.1097}{3.1097} = \mathbf{0.01109 \text{ or } 1.109\%. \text{ Ans.}}$$

▶ 7.7 DISCHARGE THROUGH FULLY SUB-MERGED ORIFICE

Fully sub-merged orifice is one which has its whole of the outlet side sub-merged under liquid so that it discharges a jet of liquid into the liquid of the same kind. It is also called totally drowned orifice. Fig. 7.8 shows the fully sub-merged orifice. Consider two points (1) and (2), point 1 being in the reservoir on the upstream side of the orifice and point 2 being at the vena-contracta as shown in Fig. 7.8.

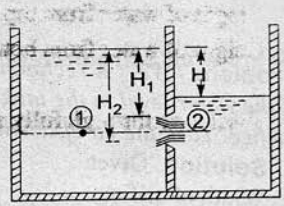

Fig. 7.8 *Fully sub-merged orifice.*

Let H_1 = Height of water above the top of the orifice on the upstream side.

H_2 = Height of water above the bottom of the orifice

H = Difference in water level

b = Width of orifice

C_d = Co-efficient of discharge.

Height of water above the centre of orifice on upstream side

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2} \quad \text{...(1)}$$

Height of water above the centre of orifice on downstream side

$$= \frac{H_1 + H_2}{2} - H \quad \text{...(2)}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad [\because z_1 = z_2]$$

Now $\frac{p_1}{\rho g} = \frac{H_1 + H_2}{2}$, $\frac{p_2}{\rho g} = \frac{H_1 + H_2}{2} - H$ and V_1 is negligible

$$\therefore \quad \frac{H_1 + H_2}{2} + 0 = \frac{H_1 + H_2}{2} - H + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = H$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

Area of orifice $= b \times (H_2 - H_1)$

$\therefore$ Discharge through orifice $= C_d \times \text{Area} \times \text{Velocity}$

$$= C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}$$

$$\therefore \quad Q = C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}. \quad \text{...(7.9)}$$

Problem 7.14 *Find the discharging through a fully sub-merged orifice of width 2 m if the difference of water levels on both sides of the orifice be 50 cm. The height of water from top and bottom of the orifice are 2.5 m and 2.75 m respectively. Take $C_d = 0.6$.*

Solution. Given :

Width of orifice, $b = 2$ m

Difference of water level, $H = 50$ cm $= 0.5$ m

Height of water from top of orifice, $H_1 = 2.5$ m

Height of water from bottom of orifice, $H_2 = 2.5$ m

$C_d = 0.6$

Discharge through fully sub-merged orifice is given by equation (7.9)

or
$$Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$$
$$= 0.6 \times 2.0 \times (2.75 - 2.5) \times \sqrt{2 \times 9.81 \times 0.5} \text{ m}^3/\text{s}$$
$$= \mathbf{0.9396 \text{ m}^3/\text{s. Ans.}}$$

Problem 7.15 *Find the discharge through a totally drowned orifice 2.0 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 3 m. Take $C_d = 0.62$.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1$ m.

Difference of water level on both the sides

$H = 3$ m

$C_d = 0.62$

Discharge through orifice is $Q = C_d \times \text{Area} \times \sqrt{2gH}$
$$= 0.62 \times b \times d \times \sqrt{2gH}$$
$$= 0.62 \times 2.0 \times 1.0 \times \sqrt{2 \times 9.81 \times 3} \text{ m}^3/\text{s} = \mathbf{9.513 \text{ m}^3/\text{s. Ans.}}$$

▶ 7.8 DISCHARGE THROUGH PARTIALLY SUB-MERGED ORIFICE

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. 7.9. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.

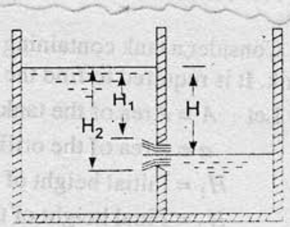

Fig. 7.9 *Partially sub-merged orifice.*

Discharge through the sub-merged portion is given by equation (7.9)
$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

Discharge through the free portion is given by equation (7.8) as

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$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g} \; [H^{3/2} - H_1^{3/2}]$$

$\therefore$ Total discharge
$$Q = Q_1 + Q_2$$
$$= C_d \times b \times (H_2 - H) \times \sqrt{2gH} + \frac{2}{3} C_d \times b \times \sqrt{2g} \; [H^{3/2} - H^{3/2}]. \quad ...(7.10)$$

Problem 7.16 *A rectangular orifice of 2 m width and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 3 m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.5 m below its top edge. Calculate the discharge through the orifice if $C_d = 0.64$.*

Solution. Given : Width of orifice, $b = 2$ m

Depth of orifice, $d = 1.2$ m

Height of water from top edge of orifice, $H_1 = 3$ m

Difference of water level on both sides, $H = 3 + 0.5 = 3.5$ m

Height of water from the bottom edge of orifice, $H_2 = H_1 + d = 3 + 1.2 = 4.2$ m

The orifice is partially sub-merged. The discharge through sub-merged portion,

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$
$$= 0.64 \times 2.0 \times (4.2 - 3.5) \times \sqrt{2 \times 9.81 \times 3.5} = 7.4249 \text{ m}^3/\text{s}$$

The discharge through free portion is

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g} \, [H^{3/2} - H_1^{3/2}]$$
$$= \frac{2}{3} \times 0.64 \times 2.0 \times \sqrt{2 \times 9.81} \; [3.5^{3/2} - 3.0^{3/2}]$$
$$= 3.779 \, [6.5479 - 5.1961] = 5.108 \text{ m}^3/\text{s}$$

$\therefore$ Total discharge through the orifice is

$$Q = Q_1 + Q_2 = 7.4249 + 5.108 = \mathbf{12.5329 \text{ m}^3/\text{s}. \text{ Ans.}}$$

7.9 TIME OF EMPTYING A TANK THROUGH AN ORIFICE AT ITS BOTTOM

Consider a tank containing some liquid upto a height of H_1. Let an orifice is fitted at the bottom of the tank. It is required to find the time for the liquid surface to fall from the height H_1 to a height H_2.

Let A = Area of the tank

a = Area of the orifice

H_1 = Initial height of the liquid

H_2 = Final height of the liquid

T = Time in seconds for the liquid to fall from H_1 to H_2.

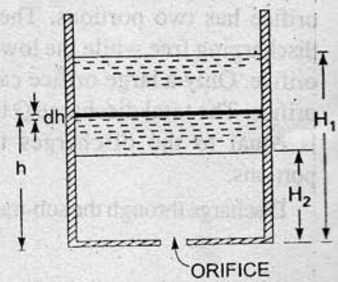

Fig. 7.9. (*d*)

Let at any time, the height of liquid from orifice is h and let the liquid surface fall by a small height dh in time dT. Then

Volume of liquid leaving the tank in time, $dT = A \times dh$

Also the theoretical velocity through orifice, $V = \sqrt{2gh}$

$\therefore$ Discharge through orifice/sec,

$$dQ = C_d \times \text{Area of orifice} \times \text{Theoretical velocity} = C_d \,.\, a \,.\, \sqrt{2gh}$$

$\therefore$ Discharge through orifice in time interval

$$dT = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

As the volume of liquid leaving the tank is equal to the volume of liquid flowing through orifice in time dT, we have

$$A(-dh) = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

– ve sign is inserted because with the increase of time, head on orifice decreases.

$$\therefore \quad -Adh = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT \text{ or } dT = \frac{-A\,dh}{C_d\,.\,a\,.\,\sqrt{2gh}} = \frac{-A(h)^{-1/2}}{C_d\,.\,a\,.\,\sqrt{2g}}dh$$

By integrating the above equation between the limits H_1 to H_2, the total time, T is obtained as

$$\int_0^T dT = \int_{H_2}^{H_1} \frac{-Ah^{-1/2}\,dh}{C_d\,.\,a\,.\,\sqrt{2g}} = \frac{-A}{C_d\,.\,a\,\sqrt{2g}}\int_{H_1}^{H_2} h^{-1/2}\,dh$$

or

$$T = \frac{-A}{C_d\,.a\,.\sqrt{2g}}\left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_{H_1}^{H_2} = \frac{-A}{C_d\,.a\,.\sqrt{2g}}\left[\frac{\sqrt{h}}{\frac{1}{2}}\right]_{H_1}^{H_2}$$

$$= \frac{-2A}{C_d\,.\,a\,.\sqrt{2g}}\left[\sqrt{H_2}-\sqrt{H_1}\right] = \frac{2A\left[\sqrt{H_1}-\sqrt{H_2}\right]}{C_d\,.\,a\,.\sqrt{2g}} \qquad \text{...(7.11)}$$

For emptying the tank completely, H_2 becomes = 0 and hence

$$T = \frac{2A\sqrt{H_1}}{C_d\,.\,a\,.\sqrt{2g}}. \qquad \text{...(7.12)}$$

Problem 7.17 *A circular tank of diameter 4 m contains water upto a height of 5 m. The tank is provided with an orifice of diameter 0.5 m at the bottom. Find the time taken by water (i) to fall from 5 m to 2 m (ii) for completely emptying to tank. Take $C_d = 0.6$.*

Solution. Given :

Dia. of tank, $D = 4$ m

$\therefore$ Area, $A = \frac{\pi}{4}(4)^2 = 12.566 \text{ m}^2$

Dia. of orifice, $d = 0.5$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

Initial height of water, $H_1 = 5$ m

Final height of water, (*i*) $H_2 = 2$ m (*ii*) $H_2 = 0$

First Case. When $H_2 = 2$ m

Using equation (7.11), we have $T = \frac{2A}{C_d\,.\,a\,.\sqrt{2g}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$

$$= \frac{2 \times 12.566}{0.6 \times .1963 \times \sqrt{2 \times 9.81}} \left[\sqrt{5} - \sqrt{2.0}\right] \text{ seconds}$$

$$= \frac{20.653}{0.521} = \textbf{39.58 seconds. Ans.}$$

Second Case. When $H_2 = 0$

$$T = \frac{2A}{C_d \,.\, a \,.\, \sqrt{2g}} \sqrt{H_1} = \frac{2 \times 12.566 \times \sqrt{5}}{0.6 \times .1963 \times \sqrt{2 \times 9.81}}$$

$$= \textbf{107.7 seconds. Ans.}$$

Problem 7.18 *A circular tank of diameter 1.25 m contains water upto a height of 5 m. An orifice of 50 mm diameter is provided at its bottom. If $C_d = 0.62$, find the height of water above the orifice after 1.5 minutes.*

Solution. Given :

Dia. of tank, $D = 1.25$ m

$\therefore$ Area, $A = \frac{\pi}{4}(1.25)^2 = 1.227 \text{ m}^2$

Dia. of orifice, $d = 50 \text{ mm} = .05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

$C_d = 0.62$

Initial height of water, $H_1 = 5$ m

Time in seconds, $T = 1.5 \times 60 = 90$ seconds

Let the height of water after 90 seconds $= H_2$

Using equation (7.11), we have $T = \dfrac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \,.\, a \,.\, \sqrt{2g}}$

or
$$90 = \frac{2 \times 1.227\left[\sqrt{5} - \sqrt{H_2}\right]}{0.62 \times 0.001963 \times \sqrt{2 \times 9.81}} = 455.215\left[2.236 - \sqrt{H_2}\right]$$

$\therefore$
$$\sqrt{H_2} = 2.236 - \frac{90}{455.215} = 236 - 0.1977 = 2.0383$$

$\therefore$
$$H_2 = 2.0383 \times 2.0383 = \textbf{4.154 m. Ans.}$$

▶ 7.10 TIME OF EMPTYING A HEMISPHERICAL TANK

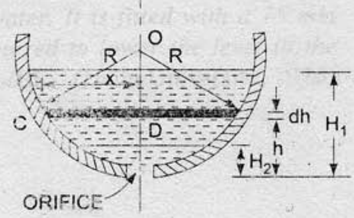

Fig. 7.10 *Hemispherical tank.*

Consider a hemispherical tank of Radius R fitted with an orifice of area 'a' at its bottom as shown in Fig. 7.10. The tank contains some liquid whose initial height is H_1 and in time T, the height of liquid falls to H_2. It is required to find the time T.

Let at any instant of time, the head of liquid over the orifice is h and at this instant let x be the radius of the liquid surface. Then

Area of liquid surface, $A = \pi x^2$

and theoretical velocity of liquid $= \sqrt{2gh}$.

Let the liquid level falls down by an amount of dh in time dT.

$\therefore$ Volume of liquid leaving tank in time $dT = A \times dh$

$$= \pi x^2 \times dh \quad ...(i)$$

Also volume of liquid flowing through orifice

$$= C_d \times \text{area of orifice} \times \text{velocity} = C_d . a . \sqrt{2gh} \text{ second}$$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d . a . \sqrt{2gh} \times dT \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$\pi x^2 (-dh) = C_d . a . \sqrt{2gh} . dT$$

– ve sign is introduced, because with the increase of T, h will decrease

$$\therefore \quad -\pi x^2 \, dh = C_d . a . \sqrt{2gh} . dT \quad ...(iii)$$

But from Fig. 7.10, for ΔOCD, we have $OC = R$

$$DO = R - h$$

$$\therefore \quad CD = x = \sqrt{OC^2 - OD^2} = \sqrt{R^2 - (R-h)^2}$$

$$\therefore \quad x^2 = R^2 - (R-h)^2 = R^2 - (R^2 + h^2 - 2Rh) = 2Rh - h^2$$

Substituting x^2 in equation (*iii*), we get

$$-\pi(2Rh - h^2)dh = C_d . a . \sqrt{2gh} . dT$$

or

$$dT = \frac{-\pi\left(2Rh - h^2\right)dh}{C_d . a . \sqrt{2gh}} = \frac{-\pi}{C_d . a . \sqrt{2g}}\left(2Rh - h^2\right)h^{-1/2} \; dh$$

$$= \frac{-\pi}{C_d . a . \sqrt{2g}}(2Rh^{1/2} - h^{3/2})dh$$

The total time T required to bring the liquid level from H_1 to H_2 is obtained by integrating the above equation between the limits H_1 to H_2.

$$\therefore \quad T = \int_{H_2}^{H_1} \frac{-\pi}{C_d . a . \sqrt{2g}} (2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d . a . \sqrt{2g}} \int_{H_1}^{H_2} (2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2R \frac{h^{1/2+1}}{\frac{1}{2}+1} - \frac{h^{3/2}+1}{\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2 \times \frac{2}{3} Rh^{3/2} - \frac{2}{5} h^{5/2} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_2^{3/2} - H_1^{3/2} \right) - \frac{2}{5} \left(H_2^{5/2} - H_1^{5/2} \right) \right]$$

$$= \frac{\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_1^{3/2} - H_2^{3/2} \right) - \frac{2}{5} \left(H_1^{5/2} - H_2^{5/2} \right) \right] \quad ...(7.13)$$

For completely emptying the tank, $H_2 = 0$ and hence

$$T = \frac{\pi}{C_d \, . \, a \, . \, \sqrt{2g}} \left[\frac{4}{3} RH_1^{3/2} - \frac{2}{5} H_1^{5/2} \right]. \quad ...(7.14)$$

Problem 7.19 *A hemispherical tank of diameter 4 m contains water upto a height of 1.5 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water (i) to fall from 1.5 m to 1.0 m (ii) for completely emptying the tank. Tank $C_d = 0.6$.*

Solution. Given :

Dia. of hemispherical tank, $D = 4$ m

$\therefore$ Radius, $R = 2.0$ m

Dia. of orifice, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

Initial height of water, $H_1 = 1.5$ m

$C_d = 0.6$

First case. $H_2 = 1.0$

Time T is given by equation (7.13)

$$\therefore \quad T = \frac{\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_1^{3/2} - H_2^{3/2} \right) - \frac{2}{5} \left(H_1^{5/2} - H_2^{5/2} \right) \right]$$

$$= \frac{\pi}{0.6 \times .001963 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 2.0 \left(1.5^{3/2} - 1.0^{3/2} \right) - \frac{2}{5} \left(1.5^{5/2} - 1.0^{5/2} \right) \right]$$

$= 602.189 \, [2.2323 - 0.7022] = 921.4$ second

= 15 min 21.4 sec. Ans.

Second case. $H_2 = 0$ and hence time T is given by equation (7.14)

$$\therefore \quad T = \frac{\pi}{C_d \, . \, a \, . \, \sqrt{2g}} \left[\frac{4}{3} RH_1^{3/2} - \frac{2}{5} H_1^{5/2} \right]$$

$$= \frac{\pi}{0.6 \times .001963 \sqrt{2 \times 9.81}} \left[\frac{4}{3} \times 2.0 \times 1.5^{3/2} - \frac{2}{5} \times 1.5^{5/2} \right]$$

$= 602.189\ [4.8989 - 1.1022]$ sec $= 2286.33$ sec

= 38 min 6.33 sec. Ans.

Problem 7.20 *A hemispherical cistern of 6 m radius is full of water. It is fitted with a 75 mm diameter sharp edged orifice at the bottom. Calculate the time required to lower the level in the cistern by 2 metres. Assume co-efficient of discharge for the orifice is 0.6.* (Delhi University, 1976)

Solution. Given :

Radius of hemispherical cistern, $R = 6$ m

Initial height of water, $H_1 = 6$ m

Dia. of orifice, $d = 75$ mm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004418\ \text{m}^2$

Fall of height of water $= 2$ m

$\therefore$ Final height of water, $H_2 = 6 - 2 = 4$ m

$C_d = 0.6$

The time T is given by equation (7.31)

$$T = \frac{\pi}{C_d \times a \times \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

$$= \frac{\pi}{0.6 \times .004418 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 6\left(6.0^{3/2} - 4.0^{3/2}\right) - \frac{2}{5}\left(6.0^{5/2} - 4.0^{5/2}\right)\right]$$

$= 267.56\ [8(14.6969 - 8.0) - 0.4\ (88.18 - 32.0)]$

$= 267.56\ [53.575 - 22.472]$ sec

$= 8321.9$ sec $=$ **2hrs 18 min 42 sec. Ans.**

Problem 7.21 *A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 5 m and diameter is 4 m. At the bottom of this tank an orifice of diameter 200 mm is fitted. Find the time required to completely emptying the tank. Take $C_d = 0.6$.*

Solution. Given :

Height of cylindrical portion (II) $= 5$ m

Dia. of tank $= 4.0$ m

$\therefore$ Area, $A = \frac{\pi}{4}(4)^2 = 12.566\ \text{m}^2$

Dia. of orifice, $d = 200$ mm $= 0.2$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.2)^2 = 0.0314\ \text{m}^2$

$C_d = 0.6$

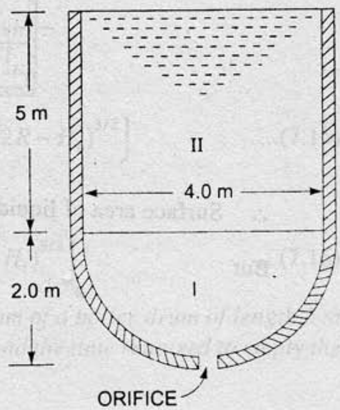

Fig. 7.11

The tank is splitted in two portions. First portion is a hemispherical tank and second portion is cylindrical tank.

Let T_1 = time for emptying hemispherical portion I.

T_2 = time for emptying cylindrical portion II.

Then total time $T = T_1 + T_2$.

For Portion I. $H_1 = 2.0$ m, $H_2 = 0$. Then T_1 is given by equation (7.14) as

$$T_1 = \frac{\pi}{C_d \times a \times \sqrt{2g}}\left[\frac{4}{3}RH_1^{3/2} - \frac{2}{5}H_1^{5/2}\right]$$

$$= \frac{\pi}{0.6 \times .0314 \times \sqrt{2 \times 9.81}}\left[\frac{4}{3} \times 2.0 \times 2.0^{3/2} - \frac{2}{5} \times 2.0^{5/2}\right]$$

$$= 37.646\,[7.5424 - 2.262] \text{ sec} = \mathbf{198.78 \text{ sec.}}$$

For Portion II. $H_1 = 2.0 + 5.0 = 7.0$ m, $H_2 = 2.0$. Then T_2 is given by equation (7.11) as

$$T_2 = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \times a \times \sqrt{2g}} = \frac{2 \times 12.566\left[\sqrt{7} - \sqrt{2.0}\right]}{0.6 \times .0314 \times \sqrt{2 \times 9.81}} \text{ sec} = 370.92 \text{ sec}$$

$\therefore$ Total time, $\quad T = T_1 + T_2 = 198.78 + 370.92 = 569.7$ sec

$= \mathbf{9 \text{ min } 29 \text{ sec.}}$ **Ans.**

▸ 7.11 TIME OF EMPTYING A CIRCULAR HORIZONTAL TANK

Consider a circular horizontal tank of length L and radius R, containing liquid upto a height of H_1. Let an orifice of area 'a' is fitted at the bottom of the tank. Then the time required to bring the liquid level from H_1 to H_2 is obtained as :

Let at any time, the height of liquid over orifice is 'h' and in time dT, let the height falls by an height of 'dh'. Let at this time, the width of liquid surface = AC as shown in Fig. 7.12.

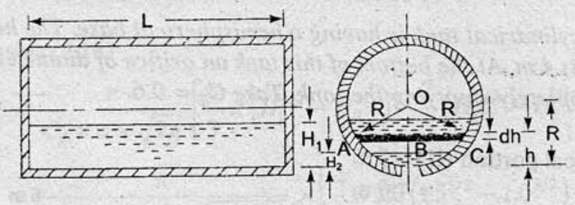

Fig. 7.12

$\therefore$ Surface area of liquid = $L \times AC$

But $\quad AC = 2 \times AB = 2\left[\sqrt{AO^2 - OB^2}\right] = 2\left[\sqrt{R^2 - (R-h)^2}\right]$

$$= 2\sqrt{R^2 - \left(R^2 + h^2 - 2Rh\right)} = 2\sqrt{2Rh - h^2}$$

$\therefore$ Surface area, $\quad A = L \times 2\sqrt{2Rh - h^2}$

$\therefore$ Volume of liquid leaving tank in time dT

$$= A \times dh = 2L\sqrt{2Rh - h^2} \times dh \quad \text{...(i)}$$

Also the volume of liquid flowing through orifice in time dT

$$= C_d \times \text{Area of orifice} \times \text{Velocity} \times dT$$

But the velocity of liquid at the time considered $= \sqrt{2gh}$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d \times a \times \sqrt{2gh} \times dT \quad \text{...(ii)}$$

Equating (*i*) and (*ii*), we get

$$2L\sqrt{2Rh - h^2} \times (-dh) = C_d \times a \times \sqrt{2gh \times dT}$$

– ve sign is introduced as with the increase of T, the height h decreases,

$$\therefore \quad dT = \frac{-2L\sqrt{2Rh - h^2}\, dh}{C_d \times a \times \sqrt{2gh}} = \frac{-2L\sqrt{(2R - h)}\, dh}{C_d \times a \times \sqrt{2g}}$$

[Taking $\sqrt{h}$ common]

$$\therefore \quad \text{Total time, } T = \int_{H_1}^{H_2} \frac{-2L(2R - h)^{1/2}\, dh}{C_d \times a \times \sqrt{g}}$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \int_{H_1}^{H_2} [2R - h]^{1/2}\, dh \; [2R - h]^{1/2}\, dh$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \left[\frac{(2R - h)^{1/2+1}}{\frac{1}{2} + 1} \times (-1) \right]_{H_1}^{H_2}$$

$$= \frac{2L}{C_d \times a \times \sqrt{2g}} \times \frac{2}{3} \times \left[(2R - h)^{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2} \right] \quad \text{...(7.15)}$$

For completely emptying the tank, H_2O and hence

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2} \right]. \quad \text{...(7.16)}$$

Problem 7.22 *An orifice of diameter 100 mm is fitted at the bottom of a boiler drum of length 5 m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler, given the value of* $C_d = 0.6$.

Solution. Given :

Dia. of orifice, $d = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Length, $L = 5 \text{ m}$

Dia. of drum, $D = 2 \text{ m}$

$\therefore$ Radius, $R = 1 \text{ m}$

Initial height of water, $H_1 = 1 \text{ m}$

Final height of water, $H_2 = 0$

$C_d = 0.6$

For completely emptying the tank, T is given by equation (7.16)

$$\therefore \quad T = \frac{4L}{3 \times C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]$$

$$= \frac{4 \times 5.0}{3 \times .06 \times .007854 \times \sqrt{2 \times 9.81}} \left[(2 \times 1)^{3/2} - (2 \times 1 - 1)^{3/2}\right]$$

$= 319.39\ [2.8284 - 1.0] = 583.98 \text{ sec} =$ **9 min 44 sec. Ans.**

Problem 7.23 *An orifice of diameter 150 mm is fitted at the bottom of a boiler drum of length 8 m and of diameter 3 metres. The drum is horizontal and contains water upto a height of 2.4 m. Find the time required to empty the boiler. Take $C_d = 0.6$.*

Solution. Given :

Dia. of orifice, $d = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Length, $L = 8.0 \text{ m}$

Dia. of boiler, $D = 3.0 \text{ m}$

$\therefore$ Radius, $R = 1.5 \text{ m}$

Initial height of water, $H_1 = 2.4 \text{ m}$

Find height of water, $H_2 = 0$

$C_d = 0.6.$

For completely emptying the tank, T is given by equation (7.16) as

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]$$

$$= \frac{4 \times 8.0}{3 \times .6 \times .01767 \times \sqrt{2 \times 9.81}} \left[(2 \times 1.5)^{3/2} - (2 \times 1.5 - 2.4)^{3/2}\right]$$

$= 227.14\ [5.196 - 0.4647] = 1074.66 \text{ sec}$

$=$ **17 min 54.66 sec. Ans.**

▶ 7.12 CLASSIFICATION OF MOUTHPIECES

1. The mouthpieces are classified as (*i*) External mouthpiece or (*ii*) Internal mouthpiece depending upon their position with respect to the tank or vessel to which they are fitted.

2. The mouthpiece are classified as (*i*) Cylindrical mouthpiece or (*ii*) Convergent mouthpiece or (*iii*) Convergent-divergent mouthpiece depending upon their shapes.

3. The mouthpieces are classified as (*i*) Mouthpieces running full or (*ii*) Mouthpieces running free, depending upon the nature of discharge at the outlet of the mouthpiece. This classification is only for internal mouthpieces which are known Borda's or Re-entrant mouthpieces. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

▶ 7.13 FLOW THROUGH AN EXTERNAL CYLINDRICAL MOUTHPIECE

A mouthpiece is a short length of a pipe which is two or three times its diameter in length. If this pipe is fitted externally to the orifice, the mouthpiece is called external cylindrical mouthpiece and the discharge through orifice increases.

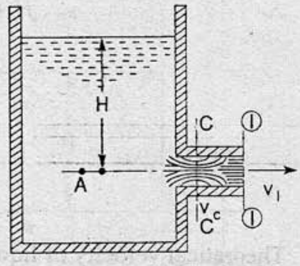

Fig. 7.13 *External cylindrical mouthpieces.*

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1, attached to one of its sides as shown in Fig. 7.13. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section *C-C*. Beyond this section, the jet again expands and fill the mouthpiece completely.

Let H = Height of liquid above the centre of mouthpiece
v_c = Velocity of liquid at *C-C* section
a_c = Area of flow at vena-contracta
v_1 = Velocity of liquid at outlet
a_1 = Area of mouthpiece at outlet
C_c = Co-efficient of contraction.

Applying continuity equation at *C-C* and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$\therefore \quad v_c = \frac{a_1 v_1}{a_1} = \frac{v_1}{a_c / a_1}$$

But $\quad \dfrac{a_c}{a_1} = C_c$ = Co-efficient of contraction

Taking $C_d = 0.62$, we get $\dfrac{a_c}{a_1} = 0.62$

$$\therefore \quad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section *C-C* suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \dfrac{(v_c - v_1)^2}{2g}$

* Please refer Art. 11.4.1 for loss of head due to sudden enlargement.

But $$v_c = \frac{v_1}{0.62} = \frac{\left(\frac{v_1}{0.62} - v_1\right)^2}{2g} = \frac{v_1^2}{2g}\left[\frac{1}{0.62} - 1\right]^2 = \frac{0.375\, v_1^2}{2g}$$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\frac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad H + O = O + \frac{v_1^2}{2g} + .375\frac{v_1^2}{2g}$$

$$\therefore \quad H = 1.375\frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus $$C_d = C_c \times C_v = 1.0 \times .855 = 0.855$$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so discharge through mouthpiece will be more.

Problem 7.24 *Find the discharge from a 100 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 4 metres.* (A.M.I.E., Summer, 1977)

Solution. Given :

Dia. of mouthpiece = 100 m= 0.1 m

$\therefore$ Area, $$a = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$$

Head, $H = 4.0$ m

C_d for mouthpiece = 0.855

$\therefore$ Discharge $= C_d \times \text{Area} \times \text{Velocity} = 0.855 \times a \times \sqrt{2gH}$

$= .855 \times .007854 \times \sqrt{2 \times 9.81 \times 4.0}$ = **.05948 m³/s. Ans.**

Problem 7.25 *An external cylindrical mouthpiece of diameter 150 mm is discharging water under a constant head of 6 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take C_d = 0.855 and C_c for vena-contracta = 0.62. Atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Dia. of mouthpiece, $d = 150 \text{ mm} = 0.15 \text{ cm}$

$\therefore$ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Head, $H = 6.0 \text{ m}$

$C_d = 0.855$

C_c at vena-contracta = 0.62

Atmospheric pressure head, $H_a = 10.3$ m

$\therefore$ Discharge $= C_d \times a \times \sqrt{2gH}$

$= 0.855 \times .01767 \times \sqrt{2 \times 9.81 \times 6.0} =$ **0.1639 m³/s. Ans.**

Pressure head at vena-contracta

Applying Bernoulli's equation at A and C-C, we get

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

But $\frac{p_A}{\rho g} = H_a + H,\ v_A = 0,$

$z_A = z_c$

$$\therefore \quad H_a + H + O = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} = H_c + \frac{v_c^2}{2g}$$

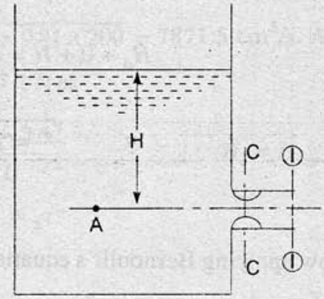

Fig. 7.14

$$\therefore \quad H_c = H_a + H - \frac{v_c^2}{2g}$$

But $v_c = \frac{v_1}{0.62}$

$$\therefore \quad H_c = H_a + H\left(\frac{v_1}{.62}\right)^2 \times \frac{1}{2g} = H_a + H - \frac{v_1^2}{2g} \times \frac{1}{(.62)^2}$$

But $\text{H} = 1.375 \ \frac{v_1^2}{2g}$

$$\therefore \quad \frac{v_1^2}{2g} = \frac{H}{1.375} = 0.7272\ H$$

$$\therefore \quad H_c = H_a + H - .7272\ H \times \frac{1}{(.62)^2}$$

$= H_a + H - 1.89\ H = H_a - .89\ H$

$= 10.3 - .89 \times 6.0$ $\{\because H_a = 10.3 \text{ and } H = 6.0\}$

$= 10.3 - 5.34 =$ **4.96 m (Absolute). Ans.**

▶ 7.14 FLOW THROUGH A CONVERGENT-DIVERGENT MOUTHPIECE

If a mouthpiece converges upto vena-contracta and then diverges as shown in Fig. 7.15 that type of mouthpiece is called Convergent-Divergent Mouthpiece. As in this mouthpiece there is no sudden enlargement of the jet, the loss of energy due to sudden enlargement is eliminated. The co-efficient of discharge for this mouthpiece is unity. Let H is the head of liquid over the mouthpiece.

Applying Bernoulli's equation to the free surface of water in tank and section C-C, we have

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

Taking datum passing through the centre of orifice, we get

$$\frac{p}{\rho g} = H_a,\ v = 0,\ z = H,\ \frac{p_c}{\rho g} = H_c,\ z_c = 0$$

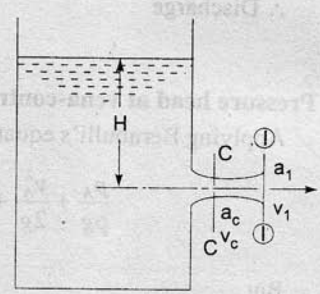

Fig. 7.15 *Convergent-divergent mouthpiece.*

$$\therefore \quad H_a + 0 + H = H_c + \frac{v_c^2}{2g} + 0 \qquad ...(i)$$

$$\therefore \quad \frac{v_c^2}{2g} = H_a + H - H_c \qquad ...(ii)$$

or $$v_c = \sqrt{2g(H_a + H - H_c)}$$

Now applying Bernoulli's equation at sections C-C and (1)-(1)

$$\frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

But $$z_c = z_1 \text{ and } \frac{p_1}{\rho g} = H_a$$

$$\therefore \quad H_c + \frac{v_c^2}{2g} = H_a + \frac{v_1^2}{2g}$$

Also from (*i*), $H_c + v_c^2/2g = H + H_a$

$\therefore \quad H_a + v_1^2/2g = H + H_a$

$$\therefore \quad v_1 = \sqrt{2gH} \qquad ...(iii)$$

Now by continuity equation, $a_c v_c = v_1 \times a_1$

$$\therefore \quad \frac{a_1}{a_c} = \frac{v_c}{v_1} = \frac{\sqrt{2g(H_a + H - H_c)}}{\sqrt{2gH}} = \sqrt{\frac{H_a}{H} + 1 - \frac{H_c}{H}}$$

$$= \sqrt{1 + \frac{H_a - H_c}{H}} \qquad ...(7.17)$$

The discharge, Q is given as $Q = a_c \times \sqrt{2gH}$ ...(7.18)

where a_c = area at vena-contracta.

Problem 7.26 *A convergent-divergent mouthpiece having throat diameter of 4.0 cm is discharging water under a constant head of 2.0 m determine the maximum outer diameter for maximum discharge. Find maximum discharge also. Take H_a = 10.3 m of water and H_{sep} = 2.5 m of water (absolute).*

Solution. Given :

Dia. of throat, $d_c = 4.0$ cm

$\therefore$ Area, $a_c = \frac{\pi}{4}(4)^2 = 12.566 \text{ cm}^2$

Constant head, $H = 2.0$ m

Find max. dia. at outlet, d_1 and Q_{max}

$H_a = 10.3$ m

$H_{sep} = 2.5$ m (absolute)

The discharge, Q in convergent-divergent mouthpiece depends on the area at throat.

$$\therefore \quad Q_{max} = a_c \times \sqrt{2gH} = 12.566 \times \sqrt{2 \times 9.81 \times 200} = \mathbf{7871.5 \text{ cm}^3/\text{s. Ans.}}$$

Now ratio of areas at outlet and throat is given by equation (7.17) as

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} = \sqrt{1 + \frac{10.3 - 2.5}{2.0}} \quad \{\because H_c = H_{sep} = 2.5\}$$

$$= 2.2135$$

$$\frac{\pi}{4} d_1^2 \Big/ \frac{\pi}{4} d_c^2 = 2.2135 \text{ or } \left(\frac{d_1}{d_c}\right)^2 = 2.2135$$

$$\therefore \quad \frac{d_1}{d_c} = \sqrt{2.2135} = 1.4877$$

$$\therefore \quad d_1 = 1.4877 \times d_c = 1.4877 \times 4.0 = \mathbf{5.95 \text{ cm. Ans.}}$$

Problem 7.27 *The throat and exit diameters of Convergent-Divergent mouthpiece are 5 cm and 10 cm respectively. It is fitted to the vertical side of a tank, containing water. Find the maximum head of a water for steady flow. The maximum vacuum pressure is 8 m of water and take atmospheric pressure = 10.3 m water.*

Solution. Given :

Dia. at throat, $d_c = 5$ cm

Dia. at exit, $d_1 = 10$ cm

Atmospheric pressure head, $H_a = 10.3$ m

The maximum vacuum pressure will be at a throat only

$\therefore$ Pressure head at throat = 8 m (vacuum)

or $H_c = H_a - 8.0$ (absolute)

$= 10.3 - 8.0 = 2.3$ m (abs.)

Let maximum head of water over mouthpiece = H m of water.

The ratio of areas at outlet and throat of a convergent-divergent mouthpiece is given by equation (7.17).

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} \text{ or } \frac{\frac{\pi}{4}(d)^2}{\frac{\pi}{4}(d^2)^2} = \sqrt{1 + \frac{10.3 - 2.3}{H}}$$

or
$$\frac{10^2}{5^5} = 4 = \sqrt{1 + \frac{8}{H}} \text{ or } 16 = 1 + \frac{8}{H} \text{ or } 15 = \frac{8}{H}$$

$$\therefore \quad H = \frac{8}{15} = 0.5333 \text{ m of water}$$

∴ Maximum head of water = **0.533̇ m. Ans.**

Problem 7.28 *A convergent-divergent mouthpiece is fitted to the side of a tank. The discharge through mouthpiece under a constant head of 1.5 m is 5 litres/s. The head loss in the divergent portion is 0.10 times the kinetic head at outlet. Find the throat and exit diameters, if separation pressure is 2.5 m and atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Constant head, $H = 1.5$ m

Discharge, $Q = 5$ litres $= .005 \text{ m}^3/\text{s}$

h_L or Head loss in divergent = 0.1 × kinetic head at outlet

H_c or $H_{sep} = 2.5$ (abs.)

$H_a = 10.3$ m of water

Find (*i*) Dia. at throat, d_c

(*ii*) Dia. at outlet, d_1

(*i*) **Dia. at Throat (d_c).** Applying Bernoulli's equation to the free water surface and throat section, we get (See Fig. 7.15).

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

Taking the centre line of mouthpiece as datum, we get

$$H_a + 0 + H = H_c + \frac{v_c^2}{2g}$$

$$\therefore \quad \frac{v_c^2}{2g} = H_a + H - H_c = 10.3 + 1.5 - 2.5 = 9.3 \text{ m of water}$$

$$\therefore \quad v_c = \sqrt{2 \times 9.81 \times 9.3} = 13.508 \text{ m/s}$$

Now
$$Q = a_c \times v_c \text{ or } .005 = \frac{\pi}{4} d_c^2 \times 13.508$$

$$\therefore \quad d_c = \sqrt{\frac{.005 \times 4}{\pi \times 13.508}} = \sqrt{.00047} = .0217 \text{ m} = \mathbf{2.17 \text{ cm. Ans.}}$$

(*ii*) **Dia. at outlet (d_1).** Applying Bernoulli's equation to the free water surface and outlet of mouthpiece (See Fig. 7.15), we get

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_2^2}{2g} + z_1 + h_L$$

$$H_a + 0 + H = H_a + \frac{v_1^2}{2g} + 0 + 0.1 \times \frac{v_1^2}{2g} \qquad \left\{\because \frac{p_1}{w} = H_a\right\}$$

$$\therefore \quad H = \frac{v_1^2}{2g} + .1 \times \frac{v_1^2}{2g} = 1.1\ \frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.1}} = \sqrt{\frac{2 \times 9.81 \times 1.5}{1.1}} = 5.1724$$

Now $Q = A_1 v_1$ or $.005 = \frac{\pi}{4} d_1^2 \times v_1$

$$\therefore \quad d_1 = \sqrt{\frac{4 \times .005}{\pi \times v_1}} = \sqrt{\frac{4 \times .005}{\pi \times 5.1724}} = 0.035 \text{ m} = \textbf{3.5 cm. Ans.}$$

7.15 FLOW THROUGH INTERNAL OR RE-ENTRANT ON BORDA'S MOUTHPIECE

A short cylindrical tube attached to an orifice in such a way that the tube projects inwardly to a tank, is called an internal mouthpiece. It is also called Re-entrant or Borda's mouthpiece. If the length of the tube is equal to its diameter, the jet of liquid comes out from mouthpiece without touching the sides of the tube as shown in Fig. 7.16. The mouthpiece is known as *running free*. But if the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to the diameter of mouthpiece at outlet as shown in Fig. 7.17. The mouthpiece is said to be *running full*.

(*i*) Borda's Mouthpiece Running Free. Fig. 7.16 shows the Borda's mouthpiece running free.

Let H = height of liquid above the mouthpiece,

a = area of mouthpiece,

a_c = area of contracted jet in the mouthpiece,

v_c = velocity through mouthpiece.

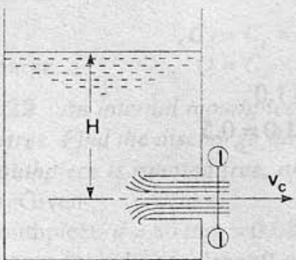

Fig. 7.16

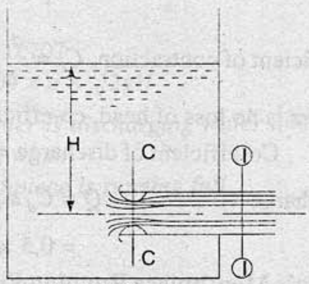

Fig. 7.17

The flow of fluid through mouthpiece is taking place due to the pressure force exerted by the fluid on the entrance section of the mouthpiece. As the area of the mouthpiece is 'a' hence total pressure force on entrance

$$= \rho g \cdot a \cdot h$$

where h = distance of C.G. of area 'a' from free surface = H.

$$= \rho g \cdot a \cdot H \qquad \ldots(i)$$

According to Newton's second law of motion, the net force is equal to the rate of change of momentum.

Now mass of liquid flowing/sec = $\rho \times a_c \times v_c$

The liquid is initially at rest and hence initial velocity is zero but final velocity of fluid is v_c.

$\therefore$ Rate of change of momentum = mass of liquid flowing/sec × [final velocity – initial velocity]

$$= \rho a_c \times v_c[v_c - 0] = \rho a_c v^2 \qquad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$\rho g .a.H. = \rho a_c. v_c^2 \qquad ...(iii)$$

Applying Bernoulli's equation to free surface of liquid and section (1)-(1) of (Fig. 7.16)

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

Taking the centre line of mouthpiece as datum, we have

$$z = H, z_1 = 0, \frac{p}{\rho g} = \frac{p_1}{\rho g} = p_{atmosp.} = 0,$$

$$v_1 = v_c, \qquad v = 0$$

$$\therefore \quad 0 + 0 + H = 0 + \frac{v_c^2}{2g} + 0 \text{ or } H = \frac{v_c^2}{2g}$$

$$\therefore \quad v_c = \sqrt{2gH}$$

Substituting the value of v_c in (*iii*), we get

$$\rho g \,.\, a \,.\, H. = \rho \,.\, a_c \,.\, 2g \,.\, H$$

or

$$a = 2a_c \text{ or } \frac{a_c}{a} = \frac{1}{2} = 0.5$$

$\therefore$ Co-efficient of contraction, $C_c = \frac{a_c}{a} = 0.5$

Since there is no loss of head, co-efficient velocity, $C_v = 1.0$

$\therefore$ Co-efficient of discharge = $C_c \times C_v = 0.5 \times 1.0 = 0.5$

$\therefore$ Discharge

$$Q = C_d a\sqrt{2gH} \qquad ...(7.19)$$

$$= 0.5 \times a\sqrt{2gH}$$

(*ii*) **Borda's Mouthpiece Running Full.** Fig. 7.17 shows Borda's mouthpiece running full.

Let H = height of liquid above the mouthpiece,

v_1 = velocity at outlet or at (1)-(1) of mouthpiece,

a = area of mouthpiece,

a_c = area of the flow at *C-C*,

v_c = velocity of liquid at vena-contracta or at *C-C*.

The jet of liquid after passing through *C-C*, suddenly enlarges at section (1)-(1). Thus there will be a loss of head due to sudden enlargement.

$$\therefore \quad h_L = \frac{(v_c - v_1)^2}{2g} \qquad ...(i)$$

Now from continuity, we have $a_c \times v_c = a_1 \times v_1$

$$\therefore \quad v_c = \frac{a_1}{a_c} \times v_1 = \frac{v_1}{a_c / a_1} = \frac{v_1}{C_c} = \frac{v_1}{0.5} \quad \{\because C_c = 0.5\}$$

or

$$v_c = 2v_1$$

Substituting this value of v_c in (*i*), we get $h_L = \frac{(2v_1 - v_1)^2}{2g} = \frac{v_1^2}{2g}$

Applying Bernoulli's equation to free surface of water in tank and section (1)-(1), we get

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

Taking datum line passing through the centre line of mouthpiece

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$$\therefore \quad H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g} = \frac{v_1^2}{g}$$

$$\therefore \quad v_1 = \sqrt{gH}$$

Here v_1 is actual velocity as losses have been taken into consideration,

But theoretical velocity, $v_{th} = \sqrt{2gH}$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{v_1}{v_{th}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}} = 0.707$$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence co-efficient of contraction = 1

$$\therefore \quad C_d = C_c \times C_v = 1.0 \times .707 = 0.707$$

$$\therefore \quad \text{Discharge,} \quad Q = C_d \times a \times \sqrt{2gH} = 0.707 \times a \times \sqrt{2gH} \quad ...(7.20)$$

Problem 7.29 *An internal mouthpiece of 80 mm diameter is discharging water under a constant head of 8 metres. Find the discharge through mouthpiece, when*

(i) The mouthpiece is running free, and (ii) The mouthpiece is running full.

Solution. Given :

Dia. of mouthpiece, $d = 80$ mm = 0.08 m

$$\therefore \text{ Area,} \quad a = \frac{\pi}{4}(.08)^2 = .005026 \text{ m}^2$$

Constant head, $H = 4$ m.

(*i*) **Mouthpiece running free.** The discharge, Q is given by equation (7.19) as

$$Q = 0.5 \times a \times \sqrt{2gH}$$

$$= 0.5 \times .005026 \times \sqrt{2 \times 9.81 \times 4.0}$$

$$= 0.02226 \text{ m}^3\text{/s} = \textbf{22.26 litres/s. Ans.}$$

(*ii*) **Mouthpiece running full.** The discharge, Q is given by equation (7.20) as

$$Q = 0.707 \times a \times \sqrt{2gH}$$
$$= 0.707 \times .005026 \times \sqrt{2 \times 9.81 \times 4.0}$$
$$= 0.03147 \text{ m}^3\text{/s} = \mathbf{31.47 \text{ litre/s. Ans.}}$$

HIGHLIGHTS

1. Orifice is a small opening on the side or at the bottom of a tank while mouthpiece is a short length of pipe which is two or three times its diameter in length.
2. Orifices as well as mouthpieces are used for measuring the rate of flow of liquid.
3. Theoretical velocity of jet of water from orifice is given by

   $V = \sqrt{2gH}$, where H = Height of water from the centre of orifice.
4. There are three hydraulic co-efficients namely :

   (a) Co-efficient of velocity, $C_v = \dfrac{\text{Actual velocity at vena - contracta}}{\text{Theoretical velocity}} = \dfrac{x}{\sqrt{4yH}}$

   (b) Co-efficient of contraction, $C_c = \dfrac{\text{Area of jet at vena - contracta}}{\text{Area of orifice}}$

   (c) Co-efficient of discharge, $C_d = \dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = C_v \times C_c$

   where x and y are the co-ordinates of any point of jet of water from vena-contracta.
5. A large orifice is one, where the head of liquid above the centre of orifice is less than 5 times the depth of orifice. The discharge through a large rectangular orifice is

   $$Q = \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H_2^{3/2} - H_1^{3/2}]$$

   where b = Width of orifice,
   C_d = Co-efficient of discharge for orifice,
   H_1 = Height of liquid above top edge of orifice, and
   H_2 = Height of liquid above bottom edge of orifice.
6. The discharge through fully sub-merged orifice, $Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$

   where b = Width of orifice,
   C_d = Co-efficient of discharge for orifice,
   H_2 = Height of liquid above bottom edge of orifice on upstream side,
   H_1 = Height of liquid above top edge orifice of upstream side,
   H = Difference of liquid levels on both sides of the orifice.
7. Discharge through partially sub-merged orifice,

   $$Q = Q_1 + Q_2$$
   $$= C_d b\,(H_2 - H) \times \sqrt{2gH} + 2/3\, C_d b \times \sqrt{2g}\,[H^{3/2} - H_1^{3/2}]$$

   where b = Width of orifice
   C_d, H_1, H_2 and H are having their usual meaning.
8. Time of emptying a tank through an orifice at its bottom is given by,

   $$T = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \cdot a \cdot \sqrt{2g}}$$

   where H_1 = Initial height of liquid in tank,
   H_2 = Final height of liquid in tank,
   A = Area of tank,

a = Area of orifice,
C_d = Co-efficient of discharge.

If the tank is to be completely emptied, then time T,

$$T = \frac{2A\sqrt{H}}{C_d \cdot a \cdot \sqrt{2g}}.$$

9. Time of emptying a hemispherical tank by an orifice fitted at its bottom,

$$T = \frac{\pi}{C_d \cdot a \cdot \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{6}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

and for completely emptying the tank, $T = \frac{\pi}{C_d \cdot a \cdot \sqrt{2g}}\left[\frac{4}{3}RH_1^{3/2} - \frac{5}{2}H_1^{5/2}\right]$

where R = Radius of the hemispherical tank,
H_1 = Initial height of liquid,
H_2 = Final height of liquid,
a = Area of orifice, and
C_d = Co-efficient of discharge.

10. Time of emptying a circular horizontal tank by an orifice at the bottom of the tank,

$$T = \frac{4L}{3C_d \cdot a \cdot \sqrt{2g}}\left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}\right]$$

and for completely emptying the tank, $T = \frac{4L}{3C_d \cdot a \cdot \sqrt{2g}}\left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]$

where L = Length of horizontal tank.

11. Co-efficient of discharge for,

(*i*) External mouthpiece, $C_d = 0.855$
(*ii*) Internal mouthpiece, running full, $C_d = 0.707$
(*iii*) Internal mouthpiece running free, $C_d = 0.50$
(*iv*) Convergent or convergent-divergent, $C_d = 1.0$.

12. For an external mouthpiece, absolute pressure head at vena-contracta

$$H_c = H_a - 0.89\,H$$

where H_a = atmospheric pressure head = 10.3 m of water
H = head of liquid above the mouthpiece.

13. For a convergent-divergent mouthpiece, the ratio of area's at outlet and at vena-contracta is

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}}$$

where a_1 = Area of mouthpiece at outlet
a_c = Area of mouthpiece at vena-contracta
H_a = Atmospheric pressure head
H_c = Absolute pressure head at vena-contracta
H = Height of liquid above mouthpiece.

14. In case of internal mouthpieces, if the jet of liquid comes out from mouthpiece without touching its sides it is known as running free. But if the jet touches the sides of the mouthpiece, it is known as running full.

EXERCISE 7

(A) THEORETICAL PROBLEMS

1. Define an orifice and a mouthpiece. What is the difference between the two ?
2. Explain the classification of orifices and mouthpieces based on their shape, size and sharpness ?
3. What are hydraulic co-efficients ? Name them.
4. Define the following co-efficients : (*i*) Co-efficient of velocity, (*ii*) Co-efficient of contraction and (*iii*) Co-efficient of discharge.
5. Derive the expression $C_d = C_v \times C_c$.
6. Define vena-contracta.
7. Differentiate between a large and a small orifice. Obtain an expression for discharge through a large rectangular orifice.
8. What do you understand by the terms wholly sub-merged orifice and partially sub-merged orifice ?
9. Prove that the expression for discharge through an external mouthpiece is given by

$$Q = .855 \times a \times v$$

where a = Area of mouthpiece at outlet and
v = Velocity of jet of water at outlet.

10. Distinguish between : (*i*) External mouthpiece and internal mouthpiece, (*ii*) Mouthpiece running free and mouthpiece running full.
11. Obtain an expression for absolute pressure head at vena-contracta for an external mouthpiece.
12. What is a convergent-divergent mouthpiece ? Obtain an expression for the ratio of diameters at outlet and at vena-contracta for a convergent-divergent 'mouthpiece' in terms of absolute pressure head at vena-contracta, head of liquid above mouthpiece and atmospheric pressure head.
13. The length of the divergent outlet part in a venturimeter is usually made longer compared with that of the converging inlet part. Why ?
14. Justify the statement, "In a convergent-divergent mouthpiece the loss of head is practically eliminated".

(B) NUMERICAL PROBLEMS

1. The head of water over an orifice of diameter 50 mm is 12 m. Find the actual discharge and actual velocity of jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$. **[Ans.** .018 m^3/s ; 15.04 m/s]
2. The head of water over the centre of an orifice of diameter 30 mm is 1.5 m. The actual discharge through the orifice is 2.35 litres/sec. Find the co-efficient of discharge. **[Ans.** 0.613]
3. A jet of water, issuing from a sharp edged vertical orifice under a constant head of 60 cm, has the horizontal and vertical co-ordinates measured from the vena-contracta at a certain point as 10.0 cm and 0.45 cm respectively. Find the value of C_v. Also find the value of C_c if $C_d = 0.60$. **[Ans.** 0.962, 0.623]
4. The head of water over an orifice of diameter 100 mm is 5 m. The water coming out from orifice is collected in a circular tank of diameter 2 m. The rise of water level in circular tank is .45 m in 30 seconds. Also the co-ordinates of a certain point on the jet, measured from vena-contracta are 100 cm horizontal and 5.2 cm vertical. Find the hydraulic co-efficients C_d, C_v and C_c. **[Ans.** 0.605, 0.98, 0.617]
5. A tank has two identical orifices in one of its vertical sides. The upper orifice is 4 m below the water surface and lower one 6 m below the water surface. If the value of C_v for each orifice is 0.98, find the point of inter-section of the two jets. **[Ans.** At a horizontal distance of 9.60 cm]
6. A closed vessel contains water upto a height of 2.0 m and over the water surface there is air having pressure 8.829 N/cm^2 above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 15 cm. Find the rate of flow of water from orifice. Take $C_d = 0.6$. **[Ans.** 0.15575 m^3/s]

7. A closed tank partially filled with water upto a height of 1 m, having an orifice of diameter 20 mm at the bottom of the tank. Determine the pressure required for a discharge of 3.0 litres/s through the orifice. Take $C_d = 0.62$. **[Ans.** 10.88 N/cm^2]
8. Find the discharge through a rectangular orifice 3.0 m wide and 2 m deep fitted to a water tank. The water level in the tank is 4 m above the top edge of the orifice. Take $C_d = 0.62$ **[Ans.** 36.77 m^3/s]
9. A rectangular orifice, 2.0 m wide and 1.5 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take $C_d = 0.6$. **[Ans.** 15.40 m^3/s]
10. A rectangular orifice, 1.0 m wide and 1.5 m deep is discharging water from a vessel. The top edge of the orifice is 0.8 m below the water surface in the vessel. Calculate the discharge through the orifice if $C_d = 0.6$. Also calculate the percentage error if the orifice is treated as a small orifice. **[Ans.** 1.058%]
11. Find the discharge through a fully sub-merged orifice of width 2 m if the difference of water levels on both the sides of the orifice be 800 mm. The height of water from top and bottom of the orifice are 2.5 m and 3 m respectively. Take $C_d = 0.6$. **[Ans.** 2.377 m^3/s]
12. Find the discharge through a totally drowned orifice 1.5 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 2.5 m. Take $C_d = 0.62$. **[Ans.** 6.513 m^3/s]
13. A rectangular orifice of 1.5 m wide and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 2 m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.4 m below its top edge. Calculate the discharge through the orifice if $C_d = 0.62$. **[Ans.** 7.549 m^3/s]
14. A circular tank of diameter 3 m contains water upto a height of 4 m. The tank is provided with an orifice of diameter 0.4 m at the bottom. Find the time taken by water : (*i*) to fall from 4 m to 2 m and (*ii*) for completely emptying the tank. Take $C_d = 0.6$. **[Ans.** (*i*) 24.8 s, (*ii*) 84.7 s]
15. A circular tank of diameter 1.5 m contains water upto a height of 4 m. An orifice of 40 mm diameter is provided at its bottom. If $C_d = 0.62$, find the height of water above the orifice after 10 minutes. **[Ans.** 2 m]
16. A hemispherical tank of diameter 4 m contains water upto a height of 2.0 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water (*i*) to fall from 2.0 m to 1.0 m (*ii*) for completely emptying the tank. Take $C_d = 0.6$ **[Ans.** (*i*) 30 min 14.34 s, (*ii*) 52 min 59 s]
17. A hemispherical cistern of 4 m radius is full of water. It is fitted with a 60 mm diameter sharp edged orifice at the bottom. Calculate the time required to lower the level in the cistern by 2 metres. Take $C_d = 0.6$. **[Ans.** 1 hr 58 min 45.9 s]
18. A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 4 m and diameter is 3 m. At the bottom of this tank an orifice of diameter 300 mm is fitted. Find the time required to completely emptying the tank. Take $C_d = 0.6$. **[Ans.** 2 min 7.37 s]
19. An orifice of diameter 200 mm is fitted at the bottom of a boiler drum of length 6 m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler, given the value of $C_d = 0.6$ **[Ans.** 2 min 55.20 s]
20. An orifice of diameter 150 mm is fitted at the bottom of a boiler drum of length 6 m and of diameter 2 m. The drum is horizontal and contains water upto a height of 1.8 m. Find the time required to empty the boiler. Take $C_d = 0.6$. **[Ans.** 7 min 46.64 s]
21. Find the discharge from a 80 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 6 m. **[Ans.** 0.0466 m^3/s]
22. An external cylindrical mouthpiece of diameter 100 mm is discharging water under a constant head of 8 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take $C_d = 0.855$ and C_c for vena-contracta = 0.62. Take atmospheric pressure head = 10.3 m of water. **[Ans.** 0.084 m^3/s ; 3.18 m]
23. A convergent-divergent mouthpiece having throat diameter of 60 mm is discharging water under a constant head of 3.0 m. Determine the maximum outlet diameter for maximum discharge. Find maximum discharge also. Take atmospheric pressure head = 10.3 m of water and separation pressure head = 2.5 m of water absolute. **[Ans.** 6.88 cm, $Q_{max} = 0.01506$ m^3/s]

24. The throat and exit diameter of a convergent-divergent mouthpiece are 40 mm and 80 mm respectively. It is fitted to the vertical side of a tank, containing water. Find the maximum head of water for steady flow. The maximum vacuum pressure is 8 m of water. Take atmospheric pressure head = 10.3 m of water. **[Ans.** 0.533 m]

25. The discharge through a convergent-divergent mouthpiece fitted to the side of a tank under a constant head of 2 m is 7 litres/s. The head loss in the divergent portion is 0.10 times the kinetic head at outlet. Find the throat and exit diameters, if separation pressure head = 2.5 m and atmospheric pressure head = 10.3 m of water. **[Ans.** 25.3 mm ; 38.6 mm]

26. An internal mouthpiece of 100 mm diameter is discharging water under a constant head of 5 m. Find the discharge through mouthpiece, when
(*i*) the mouthpiece is running free, and (*ii*) the mouthpiece is running full.
[Ans. (*i*) 38.8 litres/s, (*ii*) 54.86 litres/s]

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8

CHAPTER

Notches and Weirs

▶ 8.1 INTRODUCTION

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

▶ 8.2 CLASSIFICATION OF NOTCHES AND WEIRS

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :
   (*i*) Rectangular weir, (*ii*) Triangular weir, and
   (*iii*) Trapezoidal weir (Cippoletti weir)

(*b*) According to the shape of the crest :
   (*i*) Sharp-crested weir, (*ii*) Broad-crested weir.
   (*iii*) Narrow-crested weir, and (*iv*) Ogee-shaped weir.

(c) According to the effect of sides on the emerging nappe :
(i) Weir with end contraction, and (ii) Weir without end contraction.

▶ 8.3 DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.

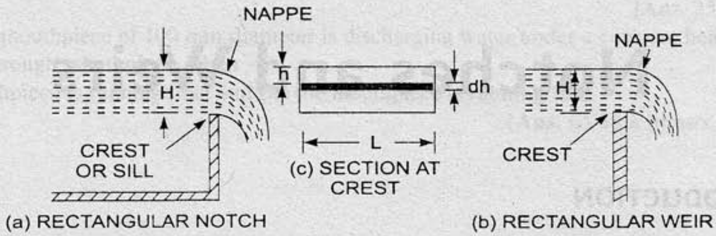

Fig. 8.1 *Rectangular notch and weir.*

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h form the free surface of water as shown in Fig. 8.1(c).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad \text{...(i)}$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [H]^{3/2}. \qquad \text{...(8.1)}$$

Problem 8.1 *Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.*

Solution. Given :

Length of the notch, $L = 2.0$ m

Head over notch, $H = 300 \text{ m} = 0.30 \text{ m}$

$C_d = 0.60$

Discharge,
$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\left[H^{3/2}\right]$$

$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times [.30]^{1.5} \text{ m}^3/\text{s}$$

$$= 3.5435 \times 0.1643 = \mathbf{0.582\ m^3/s.\ Ans.}$$

Problem 8.2 *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.*

Solution. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000 \text{ lit/s} = 2 \text{ m}^3/\text{s}$

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir (Fig. 8.2)

The discharge over the weir is given by the equation (8.1) as

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

or
$$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\ H^{3/2}$$

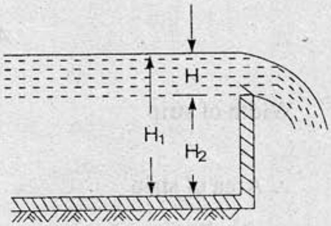

Fig. 8.2

$$\therefore \quad H^{3/2} = \frac{2.0}{10.623}$$

$$\therefore \quad H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

$\therefore$ Height of weir, $H_2 = H_1 - H$

= Depth of water on upstream side $- H$

$= 1.8 - .328 = \mathbf{1.472\ m.\ Ans.}$

Problem 8.3 *The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/s. Find the length of the notch, when $C_d = 0.62$.*

Solution. Given :

Head over notch, $H = 90 \text{ cm} = 0.9 \text{ m}$

Discharge, $Q = 300 \text{ lit/s} = 0.3 \text{ m}^3/\text{s}$

$C_d = 0.62$

Let length of notch $= L$

Using equation (8.1), we have

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$$= 1.83 \times L \times 0.8538$$

$\therefore$ $$L = \frac{0.3}{1.83 \times .8538} = .192 \text{ m} = \textbf{192 mm. Ans.}$$

▶ 8.4 DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (*b*), we have

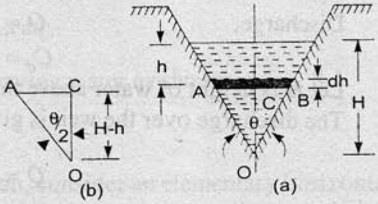

Fig. 8.3 *The triangular notch.*

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$\therefore$ $$AC = (H-h)\tan\frac{\theta}{2}$$

Width of strip $$= AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$$

$\therefore$ Area of strip $$= 2\,(H-h)\tan\frac{\theta}{2} \times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times gh$$

$\therefore$ Total discharge, Q is $$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H.H^{3/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15} H^{5/2}\right]$$

$$= \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \quad \ldots(8.2)$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge
$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \quad \ldots(8.3)$$

$$= 1.417\ H^{5/2}.$$

Problem 8.4 *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a V-notch is given by equation (8.2)

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan\frac{60}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.8182 \times 0.0493 = \mathbf{0.040\ m^3/s.\ Ans.}$$

Problem 8.5 *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

(Osmania University, 1990 ; A.M.I.E., Winter, 1975)

Solution. Given :

For rectangular weir. length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir. $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2} \text{ m}^3/\text{s} = 0.10635 \text{ m}^3/\text{s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation (8.2) for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan\frac{90}{2} \times \sqrt{2g} \times H_1^{5/2} \qquad \{\because \theta = 90° \text{ and } H = H_1\}$$

$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times H_1^{5/2} = 1.3936\, H_1^{5/2}$$

$$\therefore \quad H_1^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = \mathbf{0.3572 \text{ m. Ans.}}$$

Problem 8.5A *Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge co-efficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360 mm, find the depth of water over the rectangular weir.* (A.M.I.E., Summer, 1990)

Solution. Given :

For triangular weir : $\theta = 90°$, $C_d = 0.6$, $H = 360$ mm $= 0.36$ m

For rectangular weir : $L = 1$ m, $C_d = 0.7$, $H = ?$

The discharge for a triangular weir is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan\left(\frac{90}{2}\right) \times \sqrt{2 \times 9.81} \times (0.36)^{5/2} = 0.1102 \text{ m}^3/\text{s}$$

The same discharge is passing through the rectangular weir. But discharge for a rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or
$$0.1102 = \frac{2}{3} \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times H^{3/2} = 2.067\, H^{3/2}$$

or
$$H^{3/2} = \frac{0.1102}{2.067} = 0.0533$$

$$\therefore \quad H = (0.0533)^{2/3} = 0.1415 \text{ m} = \mathbf{141.5 \text{ mm. Ans.}}$$

Problem 8.6 *A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take $C_d = 0.62$.*

Solution. Given :

Width of rectangular channel, $L = 2.0$ m

Discharge, $Q = 250$ lit/s $= 0.25$ m^3/s

Depth of water in channel $= 1.3$ m

Let the height of water over V-notch $= H$

The rate of flow through V-notch is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} \times H^{5/2}$$

where $C_d = 0.62$, $\theta = 90°$

$$\therefore \quad Q = \frac{8}{15} \times .62 \times \sqrt{2 \times 9.81} \times \tan\frac{90}{2} \times H^{5/2}$$

or
$$0.25 = \frac{8}{15} \times .62 \times 4.429 \times 1 \times H^{5/2}$$

or
$$H^{5/2} = \frac{.25 \times 15}{8 \times .62 \times 4.429} = 0.1707$$

$$\therefore \quad H = (.1707)^{2/5} = (.1707)^{0.4} = 0.493 \text{ m}$$

Position of apex of the notch from the bed of channel

= depth of water in channel-height of water over V-notch

= 1.3 – .493 = **0.807 m. Ans.**

▶ 8.5 ADVANTAGES OF TRIANGULAR NOTCH OR WEIR OVER RECTANGULAR NOTCH OR WEIR

A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons :

1. The expression for discharge for a right-angled V-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.
3. In case of triangular notch, only one reading, *i.e.*, (H) is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

▶ 8.6 DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

As shown in Fig. 8.4, a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = Height of water over the notch

L = Length of the crest of the notch

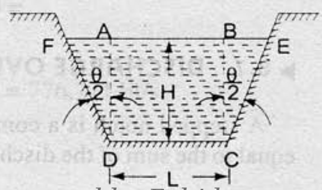

Fig. 8.4 *The trapezoidal notch.*

C_{d_1} = Co-efficient or discharge for rectangular portion *ABCD* of Fig. 8.4.

C_{d_2} = Co-efficient of discharge for triangular portion [*FAD* and *BCE*]

The discharge through rectangular portion *ABCD* is given by (8.1)

or
$$Q_1 = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches *FDA* and *BCE* is equal to the discharge through a single triangular notch of angle θ and it is given by equation (8.2) as

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir $FDCEF = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}. \quad \text{...(8.4)}$$

Problem 8.7 *Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion as = 0.62 while for triangular portion = 0.60.*

Solution. Given :

Top width, $AE = 1$ m

Base width, $CD = L = 0.4$ m

Head of water, $H = 0.20$ m

For rectangular portion, $C_{d_1} = 0.62$

For triangular portion, $C_{d_2} = 0.60$

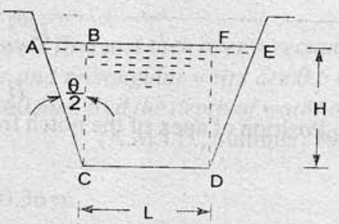

Fig. 8.5

From ΔABC, we have

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{H}$$

$$= \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

Discharge through trapezoidal notch is given by equation (8.4)

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times .60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$= 0.06549 + 0.02535 = 0.09084 \text{ m}^3/\text{s}$ = **90.84 litres/s. Ans.**

▶ 8.7 DISCHARGE OVER A STEPPED NOTCH

A stepped notch is a combination of rectangular notches. The discharge through stepped notch is equal to the sum of the discharges through the different rectangular notches.

Consider a stepped notch as shown in Fig. 8.6.

Let H_1 = Height of water above the crest of notch (1),

L_1 = Length of notch 1,

H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

C_d = Co-efficient of discharge for all notches

$\therefore$ Total discharge $Q = Q_1 + Q_2 + Q_3$

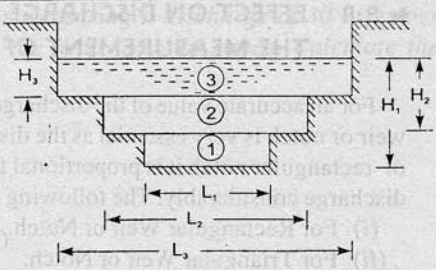

Fig. 8.6 *The stepped notch.*

or
$$Q = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$
$$+ \frac{2}{3}\ C_d \times L_2 \times \sqrt{2g}\ [H_2^{3/2} - H_3^{3/2}] + \frac{2}{3}\ C_d \times L_3 \times \sqrt{2g}\ \times H_3^{3/2}. \quad ...(8.5)$$

Problem 8.8 *Fig. 8.7 shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62.*

Solution. Given :

$L_1 = 40$ cm, $L_2 = 80$ cm,

$L_3 = 120$ cm

$H_1 = 50 + 30 + 15 = 95$ cm,

$H_2 = 80$ cm, $H_3 = 50$ cm,

$C_d = 0.62$

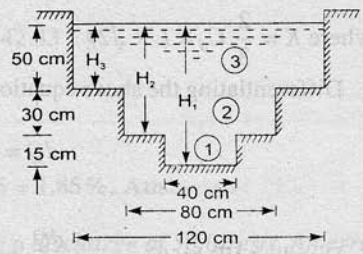

Fig. 8.7

Total discharge, $Q = Q_1 + Q_2 + Q_3$

where $Q_1 = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$

$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 981} \times [95^{3/2} - 80^{3/2}]$

$= 732.26[925.94 - 715.54] = 154067\ cm^3/s = 154.067$ lit/s

$Q_2 = \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \times [H_2^{3/2} - H_3^{3/2}]$

$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 981} \times [80^{3/2} - 50^{3/2}]$

$= 1464.52[715.54 - 353.55]\ cm^3/s = 530141\ cm^3/s = 530.144$ lit/s

and $Q_3 = \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$

$= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 981} \times 50^{3/2} = 776771\ cm^3/s = 776.771$ lit/s

$\therefore$ $Q = Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771$

= 1460.98 lit/s. Ans.

▶ 8.8 EFFECT ON DISCHARGE OVER A NOTCH OR WEIR DUE TO ERROR IN THE MEASUREMENT OF HEAD

For an accurate value of the discharge over a weir or notch, an accurate measurement of head over the weir or notch is very essential as the discharge over a triangular notch is proportional to $H^{5/2}$ and in case of rectangular notch it is proportional to $H^{3/2}$. A small error in the measurement of head, will affect the discharge considerably. The following cases of error in the measurement of head will be considered :

(*i*) For Rectangular Weir or Notch.

(*ii*) For Triangular Weir or Notch.

8.8.1 For Rectangular Weir or Notch. The discharge for a rectangular weir or notch is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= KH^{3/2} \qquad ...(i)$$

where $K = \frac{2}{3} C_d \times L \times \sqrt{2g}$

Differentiating the above equation, we get

$$dQ = K \times \frac{3}{2} H^{1/2} \, dH \qquad ...(ii)$$

Dividing (*ii*) by (*i*), $$\frac{dQ}{Q} = \frac{K \times \frac{3}{2} \times H^{1/2} dH}{KH^{3/2}} = \frac{3}{2} \frac{dH}{H} \qquad ...(8.6)$$

Equation (8.6) shows that an error of 1% in measuring H will produce 1.5% error in discharge over a rectangular weir or notch.

8.8.2 For Triangular Weir or Notch. The discharge over a triangular weir or notch is given by equation (8.2) as

$$Q = \frac{8}{15} C_d \cdot \tan \frac{\theta}{2} \sqrt{2g} \times H^{5/2}$$

$$= KH^{5/2} \qquad ...(iii)$$

where $K = \frac{8}{15} C_d \cdot \tan \frac{\theta}{2} \sqrt{2g}$

Differentiating equation (*iii*), we get

$$dQ = K \frac{5}{2} H^{3/2} \times dH \qquad ...(iv)$$

Dividing (*iv*) by (*iii*), we get $$\frac{dQ}{Q} = \frac{K \frac{5}{2} H^{3/2} \, dH}{KH^{5/2}} = \frac{5}{2} \frac{dH}{H} \qquad ...(8.7)$$

Equation (8.7) shows that an error of 1% in measuring H will produce 2.5% error in discharge over a triangular weir or notch.

Problem 8.9 *A rectangular notch 40 cm long is used for measuring a discharge of 30 litres per second. An error of 1.5 mm wars made, while measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.60$.*

Solution. Given :

Length of notch, $L = 40$ cm

Discharge, $Q = 30$ lit/s $= 30{,}000$ cm^3/s

Error in head, $dH = 1.5$ mm $= 0.15$ cm

$C_d = 0.60$

Let the height of water over rectangular notch $= H$

The discharge through a rectangular notch is given by (8.1)

or $$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$30{,}000 = \frac{2}{3} \times 0.60 \times 40 \times \sqrt{2 \times 981} \times H^{3/2}$$

or $$H^{3/2} = \frac{3 \times 30000}{2 \times .60 \times 40 \times \sqrt{2 \times 981}} = 42.33$$

$$\therefore \quad H = (42.33)^{2/3} = 12.16 \text{ cm}$$

Using equation (8.6), we get

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H} = \frac{3}{2} \times \frac{0.15}{12.16} = 0.0185 = \mathbf{1.85\%.\ Ans.}$$

Problem 8.10 *A right-angled V-notch is used for measuring a discharge of 30 litres/s. An error of 1.5 mm was made while measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.62$.*

Solution. Given :

Angle of *V*-notch, $\theta = 90°$

Discharge, $Q = 30$ lit/s $= 30000$ cm^3/s

Error in head, $dH = 1.5$ mm $= 0.15$ cm

$C_d = 0.62$

Let the head over the *V*-notch $= H$

The discharge Q through a triangular notch is given by equation (8.2)

$$Q = \frac{8}{15} C_d \,.\, \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

or $$30000 = \frac{8}{15} \times 0.62 \times \tan\left(\frac{90°}{2}\right) \times \sqrt{2 \times 981} \times H^{5/2}$$

$$= \frac{8}{15} \times .62 \times 1 \times 44.29 \times H^{5/2}$$

$$\therefore \quad H^{5/2} = \frac{30000 \times 15}{8 \times .62 \times 44.29} = 2048.44$$

$$\therefore \quad H = (2048.44) = 21.11 \text{ cm}$$

Using equation (8.7), we get

$$\frac{dQ}{Q} = \frac{5}{2}\frac{dH}{H} = 2.5 \times \frac{0.15}{21.11} = 0.01776 = \mathbf{1.77\%}. \quad \textbf{Ans.}$$

Problem 8.11 *The head of water over a triangular notch of angle 60° is 50 cm and co-efficient of discharge is 0.62. The flow measured by it is to be within an accuracy of 1.5% up or down. Find the limiting values of the head.*

Solution. Given :

Angle of *V*-notch $\theta = 60°$

Head of water, $H = 50$ cm

$C_d = 0.62$

$$\frac{dQ}{Q} = \pm 1.5\% = \pm 0.015$$

The discharge Q over a triangular notch is

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} . H^{5/2}$$

$$= \frac{8}{15} \times 0.62 \times \sqrt{2 \times 981} \times \tan\frac{60°}{2} \times (50)^{5/2}$$

$$= 14.64 \times 0.5773 \times 17677.67 = 149405.86 \text{ cm}^3/\text{s}$$

Now applying equation (8.7), we get

$$\frac{dQ}{Q} = \frac{5}{2}\frac{dH}{H} \text{ or } \pm .015 = 2.5\frac{dH}{H} \text{ or } \frac{dH}{H} = \pm\frac{.015}{2.5}$$

$$\therefore \quad dH = \pm\frac{.015}{2.5} \times H = \pm\frac{.015}{2.5} \times 50 = \pm 0.3$$

$\therefore$ The limiting values of the head

$$= H \pm dH = 50 \pm 0.3 = 50.3 \text{ cm, } 49.7 \text{ cm}$$

$$= \textbf{50.3 cm and 49.7 cm. Ans.}$$

▶ 8.9. (a) TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A RECTANGULAR WEIR OR NOTCH

Consider a reservoir or tank of uniform cross-sectional area A. A rectangular weir or notch is provided in one of its sides.

Let L = Length of crest of the weir or notch

C_d = Co-efficient of discharge

H_1 = Initial height of liquid above the crest of notch

H_2 = Final height of liquid above the crest of notch

T = Time required in seconds to lower the height of liquid from H_1 to H_2.

Let at any instant, the height of liquid surface above the crest of weir or notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then,

$$-A dh = Q \times dT$$

–ve sign is taken as with the increase of T, h decreases.

But $$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}$$

$$\therefore \quad -Adh = \frac{2}{3} C_d \times L \times \sqrt{2g} \, . \, h^{3/2} \times dT \text{ or } dT = \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

The total time T is obtained by integrating the above equation between the limits H_1 to H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

or $$T = \frac{-A}{\frac{2}{3} C_d \times L \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-3/2} \, dh = \frac{-3A}{2 \, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-3/2+1}}{-\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-3A}{2 \, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-1/2}}{-\frac{1}{2}} \right]_{H_1}^{H_2} = \frac{-3A}{2 \, C_d \times L \times \sqrt{2g}} \left(-\frac{2}{1} \right) \left[\frac{1}{\sqrt{h}} \right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] \quad \text{...(8.8)}$$

(b) TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A TRIANGULAR WEIR OR NOTCH

Consider a reservoir or tank of uniform cross-sectional area A, having a triangular weir or notch in one of its sides.

Let θ = Angle of the notch

C_d = Co-efficient of discharge

H_1 = Initial height of liquid above the apex of notch

H_2 = Final height of liquid above the apex of notch

T = Time required in seconds, to lower the height from H_1 to H_2 above the apex of the notch.

Let at any instant, the height of liquid surface above the apex of weir or notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then

$$-Adh = Q \times dT$$

–ve sign is taken, as with the increase of T, h decreases.

And Q for a triangular notch is

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \sqrt{2g} \times h^{5/2}$$

$$\therefore \quad -Adh = \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times h^{5/2} \times dT$$

$$\therefore \quad dT = \frac{Adh}{\frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2}}$$

The total time T is obtained by integrating the above equation between the limits H_1 to H_2.

$$\therefore \quad \int_0^T dT = \int_{H_1}^{H_2} \frac{-Ah}{\frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g}\, h^{5/2}}$$

or
$$T = \frac{-A}{\frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-5/2}\, dh$$

$$= \frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{h^{-3/2}}{-\frac{3}{2}}\right]_{H_1}^{H_2}$$

$$= \frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \times \left(-\frac{2}{3}\right)\left[\frac{1}{h^{3/2}}\right]_{H_1}^{H_2}$$

$$= \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]. \quad \text{...(8.9)}$$

Problem 8.12 *Find the time required to lower the water level from 3 m to 2 m in a reservoir of dimension 80 m × 80 m, by a rectangular notch of length 1.5 m. Take $C_d = 0.62$.*

Solution. Given :

Initial height of water, $H_1 = 3$ m

Final height of water, $H_2 = 2$ m

Dimension of reservoir = 80 m × 80 m

or Area, $A = 80 \times 80 = 6400 \text{ m}^2$

Length of notch, $L = 1.5$ m, $C_d = 0.62$

Using the relation given by the equation (8.8)

$$T = \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

$$= \frac{3 \times 6400}{0.62 \times 1.5 \times \sqrt{2 \times 9.81}} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right]$$

= 4661.35 [0.7071 − 0.5773] seconds

= 605.04 seconds = **10 min 5 sec. Ans.**

Problem 8.13 *If in problem 8.12, instead of a rectangular notch, a right-angled V-notch is used, find the time required. Take all other data same.*

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Solution. Given :

Angle of notch, $\theta = 90°$

Initial height of water, $H_1 = 3$ m

Final height of water, $H_2 = 2$ m

Area of reservoir, $A = 80 \times 80 = 6400$ m^2

$C_d = 0.62$

Using the relation given by equation (8.9)

$$T = \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$= \frac{5 \times 6400}{4 \times .62 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81}}\left[\frac{1}{2^{1.5}} - \frac{1}{3^{1.5}}\right] \qquad \left\{\because \tan\frac{90}{2} = 1\right\}$$

$$= 2913.34 \times \left[\frac{1}{2.8284} - \frac{1}{5.1961}\right]$$

$$= 2913.34\ [0.3535 - 0.1924] \text{ seconds}$$

$$= 469.33 \text{ seconds} = \mathbf{7\ min\ 49.33\ sec.\ Ans.}$$

Problem 8.14 *A right-angled V-notch is inserted in the side of a tank of length 4 m and width 2.5 m. Initial height of water above the apex of the notch is 30 cm. Find the height of water above the apex if the time required to lower the head in tank from 30 cm to final height is 3 minutes. Take $C_d = 0.60$.*

Solution. Given :

Angle of notch, $\theta = 90°$

Area of tank, $A = \text{Length} \times \text{width} = 4 \times 2.5 = 10.0$ m^2

Initial height of water, $H_1 = 30$ cm $= 0.3$ m

Time, $T = 3$ min $= 3 \times 60 = 180$ seconds

$C_d = 0.60$

Let the final height of water above the apex of notch $= H_2$

Using the relation given by equation (8.9)

$$T = \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$180 = \frac{5 \times 10}{4 \times .60 \times \tan\left(\frac{90}{2}\right) \times \sqrt{2 \times 9.81}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{(0.3)^{3/2}}\right]$$

$$= \frac{50}{4 \times .60 \times 1 \times 4.429}\left[\frac{1}{H_2^{3/2}} - \frac{1}{(0.3)^{3/2}}\right]$$

or $$\frac{1}{H_2^{1.5}} - \frac{1}{0.3^{1.5}} = \frac{180 \times 4 \times 0.60 \times 4.429}{50} = 38.266.$$

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or $$\frac{1}{H_2^{1.5}} - 6.0858 = 38.266$$

$$\therefore \quad \frac{1}{H_2^{1.5}} = 36.266 + 6.0858 = 42.35 \text{ or } H_2^{1.5} = \frac{1}{42.35} = 0.0236$$

$$\therefore \quad H_2 = (.0236)^{1/1.5} = (0.0236)^{.6667} = 0.0822 \text{ m} = \mathbf{8.22 \text{ cm. Ans.}}$$

▶ 8.10 VELOCITY OF APPROACH

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it. Thus if V_a is the velocity of approach, then an additional head h_a equal to $\frac{V_a^2}{2g}$ due to velocity of approach, is acting on the water flowing over the notch. Then initial height of water over the notch becomes $(H + h_a)$ and final height becomes equal to h_a. Then all the formulae are changed taking into consideration of velocity of approach.

The velocity of approach, V_a is determined by finding the discharge over the notch or weir neglecting velocity of approach. Then dividing the discharge by the cross-sectional area of the channel on the upstream side of the weir or notch, the velocity of approach is obtained. Mathematically,

$$V_a = \frac{Q}{\text{Area of channel}}$$

This velocity of approach is used to find an additional head $\left(h_a = \frac{V_a^2}{2g}\right)$. Again the discharge is calculated and above process is repeated for more accurate discharge.

Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} C_d \times L \times \sqrt{2g} \; [(H_1 + h_a)^{3/2} - h_a^{3/2}] \qquad \text{...(8.10)}$$

Problem 8.15 *Water is flowing in a rectangular channel of 1 m wide and 0.75 m deep. Find the discharge over a rectangular weir of crest length 60 cm if the head of water over the crest of weir is 20 cm and water from channel flows over the weir. Take $C_d = 0.62$. Neglect end contractions. Take velocity of approach into consideration.*

Solution. Given :

Area of channel, $A = \text{Width} \times \text{depth} = 1.0 \times 0.75 = 0.75 \text{ m}^2$

Length of weir, $L = 60 \text{ cm} = 0.6 \text{ m}$

Head of water, $H_1 = 20 \text{ cm} = 0.2 \text{ m}$

$C_d = 0.62$

Discharge over a rectangular weir without velocity of approach is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H_1^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} \text{ m}^3/\text{s}$$

$= 1.098 \times 0.0894 = 0.0982 \text{ m}^3\text{/s}$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{.0982}{0.75} = 0.1309 \text{ m/s}$

$\therefore$ Additional head, $h_a = \frac{V_a^2}{2g} = (.1309)^2/2 \times 9.81 = .0008733 \text{ m}$

Then discharge with velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \; [(H_1 + h_a)^{3/2} - h_a^{\;3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \; [(0.2 + .00087)^{3/2} - (.00087)^{3/2}]$$

$$= 1.098 \; [0.09002 - .00002566]$$

$$= 1.098 \times 0.09017 = \mathbf{.09881 \text{ m}^3\text{/s. Ans.}}$$

Problem 8.16 *Find the discharge over a rectangular weir of length 100 m. The head of water over the weir is 1.5 m. The velocity of approach is given as 0.5 m/s. Take $C_d = 0.60$.*

Solution. Given :

Length of weir, $L = 100$ m

Head of water, $H_1 = 1.5$ m

Velocity of approach, $V_a = 0.5$ m/s

$C_d = 0.60$

$\therefore$ Additional head, $h_a = \frac{V_a^{\;2}}{2g} = \frac{0.5 \times 0.5}{2 \times 9.81} = 0.0127 \text{ m}$

The discharge, Q over a rectangular weir due to velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \; [(H_1 + h_a)^{3/2} - h_a^{\;3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 100 \times \sqrt{2 \times 9.81} \; [(1.5 + .0127)^{3/2} - .0127^{3/2}]$$

$$= 177.16 \; [\, 1.5127^{3/2} - .0127^{3/2}]$$

$$= 177.16 \; [1.8605 - .00143] = \mathbf{329.35 \text{ m}^3\text{/s. Ans.}}$$

Problem 8.17 *A rectangular weir of crest length 50 cm is used to measure the rate of flow of water in a rectangular channel of 80 cm wide and 70 cm deep. Determine the discharge in the channel if the water level is 80 mm above the crest of weir. Take velocity of approach into consideration and value of $C_d = 0.62$.*

Solution. Given :

Length of weir, $L = 50 \text{ cm} = 0.5 \text{ m}$

Area of channel, $A = \text{Width} \times \text{depth} = 80 \text{ cm} \times 70 \text{ cm} = 0.80 \times 0.70 = 0.56 \text{ m}^2$

Head over weir, $H = 80 \text{ mm} = 0.08 \text{ m}$

$C_d = 0.62$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81} \times (0.08)^{3/2} \text{ m}^3/\text{s}$$

$$= 0.9153 \times .0226 = .0207 \text{ m}^3/\text{s}$$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{.0207}{0.56} = .0369$ m/s

$\therefore$ Head due to V_a, $h_a = V_a^2/2g = \frac{(.0369)^2}{2 \times 9.81} = .0000697$ m

Discharge with velocity of approach is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81}\ [(.08 + .0000697)^{3/2} - .0000697^{3/2}]$$

$$= 0.9153 \times [.0800697^{1.5} - .0000697^{1.5}]$$

$$= .9153\ [.02265 - .000000582] = \mathbf{0.2073\ m^3/s.\ Ans.}$$

Problem 8.18 *A suppressed rectangular weir is constructed across a channel of 0.77 m width with a head of 0.39 m and the crest 0.6 m above the bed of the channel. Estimate the discharge over it. Consider velocity of approach and assume $C_d = 0.623$.* (A.M.I.E., Summer, 1988)

Solution. Given :

Width of channel, $b = 0.77$ m

Head over weir, $H = 0.39$ m

Height of crest from bed of channel = 0.6 m

$\therefore$ Depth of channel = 0.6 + 0.39 = 0.99

Value of $C_d = 0.623$

Suppressed weir means that the width of channel is equal to width of weir *i.e.*, there is no end contraction.

$\therefore$ Width of channel = Width of weir = 0.77 m

Now area of channel, A = Width of channel × Depth of channel

= 0.77 × 0.99

The discharge over a rectangular weir without velocity of approach is given by equation (8.1).

$$\therefore \quad Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times H^{3/2} \quad (\because \text{ Here } b = L)$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times \sqrt{2 \times 9.81} \times 0.39^{3/2} = 0.345 \text{ m}^3/\text{s}$$

Now velocity of approach, $V_a = \frac{Q}{\text{Area of channel}} = \frac{0.345}{0.77 \times 0.99} = 0.4526$ m/s

Head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{0.4526^2}{2 \times 9.81} = 0.0104 \text{ m}$$

Now the discharge with velocity of approach is given by,

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \; [(H + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times \sqrt{2 \times 9.81} \; [(0.39 + 0.0104)^{3/2} - (0.0104)^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times 4.43 \; [0.2533 - 0.00106]$$

$$= \mathbf{0.3573 \text{ m}^3\text{/s. Ans.}}$$

Problem 8.19 *A sharp crested rectangular weir of 1 m height extends across a rectangular channel of 3 m width. If the head of water over the weir is 0.45 m, calculate the discharge. Consider velocity of approach and assume $C_d = 0.623$.* (A.M.I.E., Winter, 1987)

Solution. Given :

Width of channel, $b = 3$ m

Height of weir $= 1$ m

Head of water over weir, $H = 0.45$ m

∴ Depth of channel = Height of weir + Head of water over weir

$= 1 + 0.45 = 1.45$ m

Value of $C_d = 0.623$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.623 \times 3 \times \sqrt{2 \times 9.81} \times 0.45^{3/2} = 1.665 \text{ m}^3\text{/s}$$

Now velocity of approach is given by

$$V_a = \frac{Q}{\text{Area of channel}}$$

$$= \frac{1.665}{\text{Width of channel} \times \text{Depth of channel}} = \frac{1.665}{3 \times 1.45} = 0.382 \text{ m/s}$$

Head due to velocity of approach is given by,

$$h_a = \frac{V_a^2}{2g} = \frac{0.382^2}{2 \times 9.81} = 0.0074 \text{ m}$$

Now the discharge with velocity of approach is given by.

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \; [(H + h_a)^{3/2} - (h_a)^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 3 \times \sqrt{2 \times 9.81} \; [(0.45 + 0.0074)^{3/2} - (0.0074)^{3/2}]$$

$$= \mathbf{1.703 \text{ m}^3\text{/s. Ans.}}$$

▶ 8.11 EMPIRICAL FORMULAE FOR DISCHARGE OVER RECTANGULAR WEIR

The discharge over a rectangular weir is given by

$$Q = \frac{2}{3} C_d \sqrt{2g} \times L \times [H^{3/2}] \text{ without velocity of approach} \quad ...(i)$$

$$= \frac{2}{3} C_d \sqrt{2g} \times L \times [(H + h_a)^{3/2} - h_a^{3/2}] \text{ with velocity of approach} \quad ...(ii)$$

Equation (*i*) and (*ii*) are applicable to the weir of notch for which the crest length is equal to the width of the channel. This type of weir is called *Suppressed weir*. But if the weir is not suppressed, the effect of end contraction will be taken into account.

(*a*) **Francis's Formula.** Francis on the basis of his experiments established that end contraction decreases the effective length of the crest of weir and hence decreases the discharge. Each end contraction reduces the crest length by $0.1 \times H$, where H is the head over the weir. For a rectangular weir there are two end contractions only and hence effective length

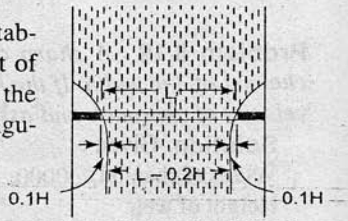

Fig. 8.8

$$L = (L - 0.2\,H)$$

and
$$Q = \frac{2}{3} \times C_d \times [L - 0.2 \times H] \times \sqrt{2g}\; H^{3/2}$$

If
$$C_d = 0.623,\ g = 9.81 \text{ m/s}^2, \text{ then}$$

$$Q = \frac{2}{3} \times .623 \times \sqrt{2 \times 9.81} \times [L - 0.2 \times H] \times H^{3/2}$$

$$= 1.84\,[L - 0.2 \times H] H^{3/2} \quad ...(8.11)$$

If end contractions are suppressed, then

$$H = 1.84\; LH^{3/2} \quad ...(8.12)$$

If velocity of approach is considered, then

$$Q = 1.84\; L\,[(H + h_a)^{3/2} - h_a^{3/2}] \quad ...(8.13)$$

(*b*) **Bazin's Formula.** On the basis of results of a series of experiments, Bazin's proposed the following formula for the discharge over a rectangular weir as

$$Q = m \times L \sqrt{2g} \times H^{3/2} \quad ...(8.14)$$

where $m = \frac{2}{3} \times C_d = 0.405 + \frac{.003}{H}$

H = height of water over the weir

If velocity of approach is considered, then

$$Q = m_1 \times L \times \sqrt{2g}\; [(H + h_a)^{3/2}] \quad ...(8.15)$$

where $m_1 = 0.405 + \frac{.003}{(H + h_a)}$.

Problem 8.20 *The head of water over a rectangular weir is 40 cm. The length of the crest of the weir with end contraction suppressed is 1.5 m. Find the discharge using the following formulae : (i) Francis's Formula and (ii) Bazin's Formula.*

Solution. Given :

Head of water, $H = 40 \text{ cm} = 0.40 \text{ m}$

Length of weir, $L = 1.5 \text{ m}$

(*i*) Francis's Formula for end contraction suppressed is given by equation (8.12).

$$Q = 1.84\, L \times H^{3/2} = 1.84 \times 1.5 \times (.40)^{3/2}$$

$$= 0.6982 \text{ m}^3\text{/s}$$

(*ii*) Bazin's Formula is given by equation (8.14)

$$Q = m \times L \times \sqrt{2g} \times H^{3/2}$$

where $m = 0.405 + \dfrac{.003}{H} = 0.405 + \dfrac{.003}{.40} = 0.4125$

$$\therefore \quad Q = .4125 \times 1.5 \times \sqrt{2 \times 9.81} \times (.4)^{3/2}$$

$$= \mathbf{0.6932\ m^3/s.\ Ans.}$$

Problem 8.21 *A weir 36 metres long is divided into 12 equal bays by vertical posts, each 60 cm wide. Determine the discharge over the weir if the head over the crest is 1.20 m and velocity of approach is 2 metres per second.* (A.M.I.E., Summer, 1978)

Solution. Given :

Length of weir, $L_1 = 36 \text{ m}$

Number of bays, $= 12$

For 12 bays, no. of vertical post = 11

Width of each post $= 60 \text{ cm} = 0.6 \text{ m}$

∴ Effective length, $L = L_1 - 11 \times 0.6 = 36 - 6.6 = 29.4 \text{ m}$

Head on weir, $H = 1.20 \text{ m}$

Velocity of approach, $V_a = 2 \text{ m/s}$

∴ Head due to V_a, $h_a = \dfrac{V_a^2}{2g} = \dfrac{2^2}{2 \times 9.81} = 0.2038 \text{ m}$

Number of end contraction, $n = 2 \times 12$ {Each bay has two end contractions}

$= 24$

∴ Discharge by Francis Formula with end contraction and velocity of approach is

$$Q = 1.84\ [L - 0.1 \times n(H + h_a)][(H + h_a)^{3/2} - h_a^{3/2}]$$

$$= 1.84[29.4 - 0.1 \times 24(1.20 + .2038)] \times [(1.2 + .2038)^{1.5} - .2038^{1.5}]$$

$$= 1.84[29.4 - 3.369][1.663 - .092]$$

$$= \mathbf{75.246\ m^3/s.\ Ans.}$$

Problem 8.22 *A discharge of 2000 m³/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 10 m. If the velocity of approach is 4 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 2 m.*

Solution. Given :

Total discharge, $Q = 2000 \text{ m}^3\text{/s}$

Length of each opening, $L = 10$

Velocity of approach, $V_a = 4 \text{ m/s}$

Head over weir, $H = 2$ m

Let number of openings = N

Head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{4 \times 4}{2 \times 9.81} = 0.8155 \text{ m}$$

For each opening, number of end contractions are two. Hence discharge for each opening considering velocity of approach is given by Francis Formula

i.e.,
$$Q = 1.84[L - 0.1 \times 2 \times (H + h_a)][(H + h_a)^{3/2} - h_a^{3/2}]$$
$$= 1.84[10.0 - 0.2 \times (2 + .8155)][2.8155^{1.5} - .8155^{1.5}]$$
$$= 17.363[4.7242 - 0.7364] = \mathbf{69.24\ m^3/s}$$

$\therefore$ Number of opening $= \dfrac{\text{Total discharge}}{\text{Discharg for one opening}} = \dfrac{2000}{69.24}$

= 28.88 (say 29) = **29. Ans.**

8.12 CIPOLLETTI WEIR OR NOTCH

Cipolletti weir is a trapezoidal weir, which has side slopes of 1 horizontal to 4 vertical as shown in Fig. 8.9. Thus in ΔABC,

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/4}{H} = \frac{1}{4}$$

$$\therefore \quad \frac{\theta}{2} = \tan\frac{1}{4} = 14° \, 2'.$$

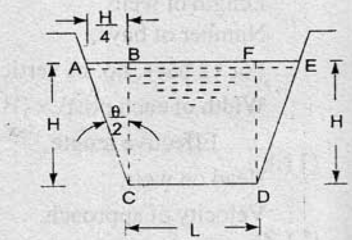

Fig. 8.9 *The cipolletti weir.*

By giving this slopes to the sides, an increase in discharge through the triangular portions *ABC* and *DEF* of the weir is obtained. If this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease. Thus in case of cipolletti weir, the factor of end contraction is not required which is shown below.

The discharge through a rectangular weir with two end contractions is

$$Q = \frac{2}{3} \times C_d \times (L - 0.2\,H)\,\sqrt{2g} \times H^{3/2}$$
$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\, H^{3/2} - \frac{2}{15}\, C_d \times \sqrt{2g} \times H^{5/2}$$

Thus due to end contraction, the discharge decreases by $\frac{2}{15}\, C_d \times \sqrt{2g} \times H^{5/2}$. This decrease in discharge can be compensated by giving such a slope to the sides that the discharge through two triangular portions is equal to $\frac{2}{15}\, C_d \times \sqrt{2g} \times H^{5/2}$. Let the slope is given by θ/2. The discharge through a *V*-notch of angle θ is given by

$$= \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2}\, H^{5/2}$$

Thus $$\frac{8}{15} \times C_d \times \sqrt{2g} \times \tan \frac{\theta}{2} H^{5/2} = \frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$$

$\therefore$ $$\tan \frac{\theta}{2} = \frac{2}{15} \times \frac{15}{8} = \frac{1}{4} \quad \text{or} \quad \theta/2 = \tan^{-1} \frac{1}{4} = 14° 2'.$$

Thus discharge through cipolletti weir is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ H^{3/2} \qquad ...(8.16)$$

If velocity of approach, V_a is to be taken into consideration,

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H + h_a)^{3/2} - h_a^{3/2}] \qquad ...(8.17)$$

Problem 8.23 *Find the discharge over a cipolletti weir of length 2.0 m when the head over the weir is 1 m. Take $C_d = 0.62$.*

Solution. Given :

Length of weir, $L = 20$ m

Head over weir, $H = 1.0$ m

$C_d = 0.62$

Using equation (8.16), the discharge is given as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 2.0 \times \sqrt{2 \times 9.81} \times (1)^{3/2} = \mathbf{3.661\ m^3/s.\ Ans.}$$

Problem 8.24 *A cipolletti weir of crest length 60 cm discharges water. The head of water over the weir is 360 mm. Find the discharge over the weir if the channel is 80 cm wide and 50 cm deep. Take $C_d = 0.60$.*

Solution. Given :

$C_d = 0.60$

Length of weir, $L = 60$ cm $= 0.60$ m

Head of water, $H = 360$ mm $= 0.36$ m

Channel width $= 80$ cm $= 0.80$ m

Channel depth $= 50$ cm $= 0.50$ m

A = cross-sectional area of channel $= 0.8 \times 0.5 = 0.4\ m^2$

To find velocity of approach, first determine discharge over the weir as

$$Q = \frac{2}{3}\ C_d \times L \times \sqrt{2g} \times H^{3/2}$$

The velocity of approach, $V_a = \frac{Q}{A}$

$\therefore$ $$Q = \frac{2}{3} \times 0.60 \times 0.60 \times \sqrt{2 \times 9.81} \times (0.36)^{3/2}\ m^3/s = 0.2296\ m^3/s$$

$\therefore$ $$V_a = \frac{.2296}{0.40} = 0.574\ m/s$$

Head due to velocity of approach,

$$h_a = V_a^2/2g = \frac{(0.574)^2}{2 \times 9.81} = 0.0168 \text{ m}$$

Thus the discharge is given by equation (8.17) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H + h_a)^{1.5} - h_a^{1.5}]$$

$$= \frac{2}{3} \times 0.60 \times .6 \times \sqrt{2 \times 9.81}\ [(.36 + .0168)^{1.5} - (.0168)^{1.5}]$$

$$= 1.06296 \times [.2313 - .002177] = \mathbf{0.2435\ m^3/s.\ Ans.}$$

▶ 8.13 DISCHARGE OVER A BROAD-CRESTED WEIR

A weir having a wide crest is known as broad-crested weir.

Let H = height of water, above the crest

L = length of the crest

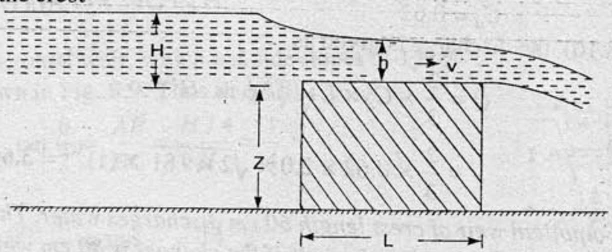

Fig. 8.10 *Broad-crested weir.*

If $2L > H$, the weir is called broad crested weir

If $2L < H$, the weir is called a narrow crested weir

Fig. 8.10 shows a broad-crested weir.

Let h = head of water at the middle of weir which is constant

v = velocity of flow over the weir

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir,

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$$\therefore \quad \frac{v^2}{2g} = H - h$$

$$\therefore \quad v = \sqrt{2g(H - h)}$$

∴ The discharge over weir $Q = C_d \times$ Area of flow × Velocity

$$= C_d \times L \times h \times \sqrt{2g(H - h)}$$

$$= C_d \times L \times \sqrt{2g\left(Hh^2 - h^3\right)} \qquad \text{...(8.18)}$$

The discharge will be maximum, if $(Hh^2 - h^2)$ is maximum

or $$\frac{d}{dh}(Hh^2 - h^3) = 0 \text{ or } 2h \times H - 3h^3 = 0 \text{ or } 2H = 3h$$

$\therefore$ $$h = \frac{2}{3}H$$

Q_{max} will be obtained by substituting this value of h in equation (8.18) as

$$Q_{max} = C_d \times L \times \sqrt{2g\left[H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3\right]}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{H \times \frac{4}{9} \times H^2 - \frac{8}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d \times L \times \sqrt{2g}\sqrt{\frac{(12-8)H^3}{27}}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{27}H^3} = C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= .3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2} = 1.7047 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times C_d \times L \times H^{3/2}. \qquad \text{...(8.19)}$$

▶ 8.14 DISCHARGE OVER A NARROW-CRESTED WEIR

For a narrow-crested weir, $2L < H$. It is similar to a rectangular weir or notch hence, Q is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.20)}$$

▶ 8.15 DISCHARGE OVER AN OGEE WEIR

Fig. 8.15 shows an Ogee weir, in which the crest of the weir rises up to maximum height of $0.115 \times H$ (where H is the height of water above inlet of the weir) and then falls as shown in Fig. 8.11. The discharge for an Ogee weir is the same as that of a rectangular weir, and it is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.21)}$$

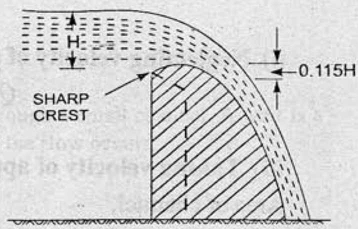

Fig. 8.11 *An Ogee weir.*

▶ 8.16 DISCHARGE OVER SUBMERGED OR DROWNED WEIR

When the water level on the downstream side of a weir is above the crest of the weir, then the weir is called to be a submerged or drowned weir. Fig. 8.12 shows a submerged weir. The total discharge, over the weir is obtained by dividing the weir into two parts. The portion between upstream and downstream water surface may be treated as free weir and portion between downstream water surface and crest of weir as a drowned weir.

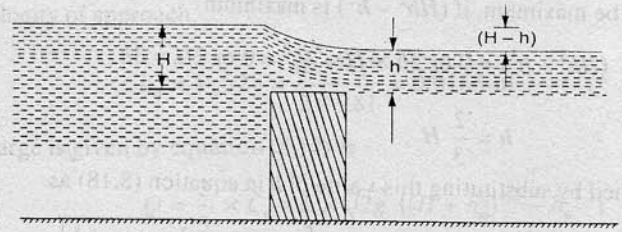

Fig. 8.12 *Submerged weir.*

Let H = height of water on the upstream of the weir

h = height of water on the downstream side of the weir

Then Q_1 = discharge over upper portion

$$= \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2}$$

Q_2 = discharge through drowned portion

$$= C_{d_2} \times \text{Area of flow} \times \text{Velocity of flow}$$

$$= C_{d_2} \times L \times h \times \sqrt{2g(H-h)}$$

∴ Total discharge, $Q = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2} + C_{d_2} \times L \times h \times \sqrt{2g(H-h)} \quad \text{....(8.22)}$$

Problem 8.25 *(a) A broad crested weir of 50 m length, has 50 cm height of water above its crest. Find the maximum discharge. Take C_d = 0.60. Neglect velocity of approach. (b) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 50 m² on the upstream side.*

Solution. Given :

Length of weir, $L = 50$ m

Head of water, $H = 50$ cm $= 0.5$ m

$C_d = 0.60$

(*i*) **Neglecting velocity of approach.** Maximum discharge is given by equation (8.19) as

$$Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times 0.60 \times 50 \times (.5)^{3/2} = \mathbf{18.084\ m^3/s.\ Ans.}$$

(*ii*) **Taking velocity of approach into consideration**

Area of channel, $A = 50\ \text{m}^2$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{18.084}{50} = 0.36$ m/s

∴ Head due to V_a, $h_a = \frac{V_a^2}{2g} = \frac{0.36 \times .36}{2 \times 9.81} = .0066$ m

Maximum discharge, Q_{max} is given by

$$Q_{max} = 1.705 \times C_d \times L \times [(H + h_a)^{3/2} - h_a^{\ 3/2}]$$

$$= 1.705 \times 0.6 \times 50 \times [(.50 + .0066)^{1.5} - (.0066)^{1.5}]$$

$= 51.15[0.3605 - .000536] = \mathbf{18.412\ m^3/s.\ Ans.}$

Problem 8.26 *An Ogee weir 5 metres long has a head of 40 cm of water. If $C_d = 0.6$, find the discharge over the weir.*

Solution. Given :

Length of weir, $L = 5$ m

Head of water, $H = 40$ cm $= 0.40$ m

$C_d = 0.6$

Discharge over Ogee weir is given by equation (8.21) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times 5.0 \times \sqrt{2 \times 9.81} \times (0.4)^{3/2} = \mathbf{2.2409\ m^3/s.\ Ans.}$$

Problem 8.27 *The heights of water on the upstream and downstream side of a submerged weir of 3 m length are 20 cm and 10 cm respectively. If C_d for free and drowned portions are 0.6 and 0.8 respectively, find the discharge over the weir.*

Solution. Given :

Height of water on upstream side, $H = 20$ cm $= 0.20$ m

Height of water on downstream side, $h = 10$ cm $= 0.10$ m

Length of weir, $L = 3$ m

$C_{d_1} = 0.6$

$C_{d_2} = 0.8$

Total discharge Q is the sum of discharge through free portion and discharge through the drowned portion. This is given by equation (8.22) as

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2} + C_{d_2} \times L \times h \times \sqrt{2g\,(H - h)}$$

$$= \frac{2}{3} \times 0.6 \times 3 \times \sqrt{2 \times 9.81}\ [.20 - .10]^{1.5} + 0.8 \times 3 \times .10 \times \sqrt{2 \times 9.81\,(.2 - .1)}$$

$$= 0.168 + 0.336 = \mathbf{0.504\ m^3/s.\ Ans.}$$

HIGHLIGHTS

1. A notch is a device used for measuring the rate of flow of a liquid through a small channel. A weir is a concrete or masonary structure placed in the open channel over which the flow occurs.
2. The discharge through a rectangular notch or weir is given by

$$Q = \frac{2}{3} C_d \times L \times H^{3/2}$$

where C_d = Co-efficient of discharge,
L = Length of notch or weir,
H = Head of water over the notch or weir.

3. The discharge over a triangular notch or weir is given by

$$Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{3/2}$$

where θ = total angle of triangular notch.

4. The discharge through a trapezoidal notch or weir is equal to the sum of discharge through a rectangular notch and the discharge through a triangular notch. It is given as

$$Q = \frac{2}{3} C_{d_1} \times L \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

where C_{d_1} = co-efficient of discharge for rectangular notch,

C_{d_2} = co-efficient of discharge for triangular notch,

$\theta/2$ = slope of the side of trapezoidal notch.

5. The error in discharge due to the error in the measurement of head over a rectangular and triangular notch or weir is given by

$$\frac{dQ}{Q} = \frac{3}{2} \frac{dH}{H} \text{ ...... For a rectangular weir or notch}$$

$$= \frac{5}{2} \frac{dH}{H} \text{ ...... For a triangular weir or notch}$$

where Q = discharge through rectangular or triangular notch or weir

H = head over the notch or weir.

6. The time required to empty a reservoir or a tank by a rectangular or a triangular notch is given by

$$H = \frac{3A}{C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] \text{ ... By a rectangular notch}$$

$$= \frac{5A}{4C_d \tan \frac{\theta}{2} \times \sqrt{2g}} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}} \right] \text{ ... By a triangular notch}$$

where A = cross-sectional area of a tank or a reservoir

H_1 = initial height of liquid above the crest or apex of notch

H_2 = final height of liquid above the crest or apex of notch.

7. The velocity with which the water approaches the weir or notch is called the velocity of approach. It is denoted by V_a and is given by

$$V_a = \frac{\text{Discharge over the notch or weir}}{\text{Cross - sectiona area of channel}}.$$

8. The head due to velocity of approach is given by $h_a = \frac{V_a^2}{2g}$.

9. Discharge over a rectangular weir, with velocity of approach,

$$Q = \frac{2}{3} C_d L \sqrt{2g} \, [(H_1 + h_a)^{3/2} - h_a^{3/2}].$$

10. Francis's Formula for a rectangular weir is given by

$$Q = 1.84[L - 0.2 H] H^{3/2} \text{ ... For two end contractions}$$

$$= 1.84 L H^{3/2} \text{ ... If end contractions are suppressed}$$

$$= 1.84 L [(H + h_a)^{3/2} - h_a^{3/2}] \text{ ... If velocity of approach is considered}$$

where L = length of weir,

H = height of water above the crest of the weir,

h_a = head due to velocity of approach.

11. Bazin's Formula for discharge over a rectangular weir,

$$Q = m L \sqrt{2gH^{3/2}}$$ ... without velocity of approach

$$= m L \sqrt{2g}\ [(H + h_d)^{3/2}]$$ ... with velocity of approach

where $m = \frac{2}{3} C_d = 0.405 + \frac{.003}{H}$ ... without velocity of approach

$$= 0.405 + \frac{.003}{(H + h_a)}$$ ... with velocity of approach.

12. *A* trapezoidal weir, with side slope or 1 horizontal to 4 vertical, is called Cipolletti weir. The discharge through Cipolletti weir is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$ ... without velocity of approach

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [(H + h_a)^{3/2} - h_a^{3/2}]$$ ... with velocity of approach.

13. The discharge over a Broad-crested weir is given by,

$$Q = C_d L \sqrt{2g\left(Hh^2 - h^3\right)}$$

where H = height of water, above the crest

h = head of water at the middle of the weir which is constant

L = length of the weir.

14. The condition for maximum discharge over a Board-crested weir is $h = \frac{2}{3} H$ and maximum discharge is given by $Q_{max} = 1.705\ C_d\ L\ H^{3/2}$.

15. The discharge over an Ogee weir is given by $Q = \frac{2}{3} C_d L \times \sqrt{2g} \times H^{3/2}$.

16. The discharge over sub-merged or drowned weir is given by

Q = discharge over upper portion + discharge through downed portion

$$= \frac{2}{3} C_{d_1}\ Lh \times \sqrt{2g}\ (H - h)^{3/2} + C_{d_2}\ Lh \times \sqrt{2g\,(H - h)}$$

where H = height of water on the upstream side of the weir,

h = height of water on thedownstream side of the weir.

EXERCISE 8

(A) THEORETICAL PROBLEMS

1. Define the terms : notch, weir, nappe and crest.
2. How are the weirs and notches classified ?
3. Find an expression for the discharge over a rectangular weir in terms of head of water over the crest of the weir.
4. Prove that the discharge through a triangular notch or weir is given by

$$Q = \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\ H^{3/2}$$

where H = head of water over the notch or weir

θ = angle of notch or weir.

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5. What are the advantages of triangular notch or weir over rectangular notch ?
6. Prove that the error in discharge due to the error in the measurement of head over a rectangular notch is given by

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H}$$

where Q = discharge through rectangular notch
and H = head over the rectangular notch.
7. Find an expression for the time required to empty a tank of area of cross-section A, with a rectangular notch.
8. What do you understand by 'Velocity of Approach' ? Find an expression for the discharge over a rectangular weir with velocity of approach.
9. Define 'end contraction' of a weir. What is the effect of end contraction on the discharge through a weir ?
10. What is a Cipolletti Weir ? Prove that the discharge through Cipolletti weir is given by

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

where L = length of weir, and H = head of water over weir.
11. Differentiate between Broad-crested weir and Narrow-crested weir. Find the condition for maximum discharge over a Broad-crested weir and hence derive an expression for maximum discharge over a broad-crested weir.
12. What do you mean by a drowned weir ? How will you determine the discharge for the downed weir ?
13. Discuss 'end contraction' of a weir. *(A.M.I.E., Summer, 1987)*
14. State the different devices that can be used to measure the discharge through a pipe also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help. *(A.M.I.E., Summer, 1992)*
15. What is the difference between a notch and a weir ?
16. Define velocity of approach. How does the velocity of approach affect the discharge over a weir ?

(B) NUMERICAL PROBLEMS

1. Find the discharge of water flowing over rectangular notch of 3 m length when the constant head of water over the notch is 40 cm. Take $C_d = 0.6$. **[Ans.** 1.344 m^3/s]
2. Determine the height of a rectangular weir of length 5 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.5 m and discharge is 1.5 m^3 per second. Take $C_d = 0.6$ and neglect end contractions. **[Ans.** 1.194 m]
3. Find the discharge over a triangular notch of angle 60° when the head over the triangular notch is 0.20 m. Take $C_d = 0.6$. **[Ans.** 0.0164 m^3/s]
4. A rectangular channel 1.5 m wide has a discharge of 200 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not be exceed 1 m. Take $C_d = 0.62$. **[Ans.** .549 m]
5. Find the discharge through a trapezoidal notch which is 1.2 m wide at the top and 0.50 m at the bottom and is 40 cm in height. The head of water on the notch is 30 cm. Assume C_d for rectangular portion as 0.62 while for triangular portion = 0.60. **[Ans.** 0.22 m^3/s]
6. A rectangular notch 50 cm long is used for measuring a discharge of 40 litres per second. An error of 2 mm was made in measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.6$. **[Ans.** 2.37%]
7. A right-angled V-notch is used for measuring a discharge of 30 litres/s. An error of 2 mm was made in measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.62$. **[Ans.** 2.37%]

8. Find the time required to lower the water level from 3 m to 1.5 m in a reservoir of dimension 70 m × 70 m, by a rectangular notch of length 2.0 m. Take $C_d = 0.60$. [Ans. 11 min 1 s]
9. If in the problem 8, instead of a rectangular notch, a right angled *V*-notch is used, find the time required. Take all other data same. [Ans. 13 min 31 s]
10. Water is flowing in a rectangular channel of 1.2 m wide and 0.8 m deep. Find the discharge over a rectangular weir of crest length 70 cm if the head of water over the crest of weir is 25 cm and water from channel flows over the weir. Take $C_d = 0.60$. Neglect end contractions but consider velocity of approach. [Ans. 0.1557 m^3/s]
11. Find the discharge over a rectangular weir of length 80 m. The head of water over the weir is 1.2 m. The velocity of approach is given as 1.5 m/s. Take $C_d = 3.6$. [Ans. 208.11 m^3/s]
12. The head of water over a rectangular weir is 50 cm. The length of the crest of the weir with end contraction suppressed is 1.4 m. Find the discharge using following formulae : (*i*) Francis's Formula and (*ii*) Bazin's Formula. [Ans. (*i*) 0.91 m^3/s, (*ii*) .901 m^3/s]
13. A discharge of 1500 m^3/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 7.5 m. If the velocity of approach is 3 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 1.8. [Ans. 37.5 say 38]
14. Find the discharge over a cipolletti weir of length 1.8 m when the head over the weir is 1.2 m. Take $C_d = 0.62$ [Ans. 4.331 m^3/s]
15. (*a*) A broad-crested weir of length 40 m, has 400 mm height of water above its crest. Find the maximum discharge. Take $C_d = 0.6$. Neglect velocity of approach. [Ans. 10.352 m^3/s]
    (*b*) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 40 m^2 on the upstream side. [Ans. 10.475 m^3/s]
16. An Ogee weir 4 m long has a head of 500 mm of water. If $C_d = 0.6$, find the discharge over the weir. [Ans. 2.505 m^3/s]
17. The heights of water on the upstream and downstream side of a submerged weir of length 3.5 m are 300 mm and 150 mm respectively. If C_d for free and drowned portion are 0.6 and 0.8 respectively, find the discharge over the weir. [Ans. 1.0807 m^3/s]