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FUNDAMENTALS OF SURVEYING

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by S.K. Roy

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In Memory of My Parents

Contents

Preface

Modern surveying involves use of sophisticated scientific instruments, mathematical methods and computational techniques. In writing this book on surveying, I have tried therefore to explain comprehensively the principles of surveying instruments and derivation of mathematical formulae. Separate chapters have been written on 'Underground Surveys' and 'Computer Programs in Surveying' to incorporate the recent developments in this field.

I acknowledge my gratefulness to all the authors listed in the Bibliography as their works form the background of this book. I am also deeply indebted to Mr. John Garner and Dr. John Uren of the University of Leads, Dr. R. Baker of the University of Salford, Engineering Council (UK), the Institution of Civil Engineers (UK) and other UK universities for permitting me to use their questions in this book. I have also utilized the questions from the examinations conducted by the Institution of Engineers (India) and some figures and tables of the standards prepared by the Bureau of Indian Standards, for which I express my sincere thanks to them. I am grateful to my colleague in the Civil Engineering Department, Dr. K.K. Bhar, who rendered immense help by collaborating with me in writing the chapter on 'Computer Programs in Surveying'.

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I wish to express my appreciation to my wife, Subrata and my sons and daughters-in-law, Santayan and Riya, Saptak and Barnali, for their encouragement and support throughout the course of writing the manuscript.

Finally, I express my deep appreciation to my publishers Prentice-Hall of India for their excellent work in editing the book thoroughly.

S.K. Roy

1

Introduction

1.1 DEFINITION

Surveying is basic to engineering. Before any engineering work can be started we must prepare a plan or map of the area showing topographical details. This involves both horizontal and vertical measurements.

Engineering surveying is defined as those activities involved in the planning and execution of surveys for the location, design, construction, operation and maintenance of civil and other engineering projects. The surveying activities are:

1. Preparation of surveys and related mapping specifications.

2. Execution of photogrammetric and field surveys for the collection of required data including topographic and hydrographic data.

3. Calculation, reduction and plotting of survey data for use in engineering design.

4. Design and provision of horizontal and vertical control survey networks.

5. Provision of line and grade and other layout work for construction and mining activities.

Thus the scope of surveying is very wide and inter-disciplinary in character. Basically it involves accurate measurements and accurate computations. In surveying we use modern sophisticated instruments, e.g. electronic instruments for measurements and modern computational tools, e.g. computers for accurate mathematical computations. Hence thorough knowledge of basic science—say, physics and mathematics—is required in grasping modern surveying.

1.2 CLASSIFICATION OF SURVEYING

Surveying is a very old profession and can be classified in many different ways.

Classification Based on Accuracy of Work. Two general classifications of surveys are *geodetic* and *plane*. In geodetic surveying the curvature of the earth is taken into account. Surveys are conducted with a high degree of accuracy. However in plane surveying, except for levelling, the reference base for field work and computations is assumed to be a flat horizontal surface. The error caused by assuming the earth to be a plane area is not serious if the area measured is small say, within 250 km^2.

Classification Based on Use or Purpose of Resulting Maps. This can be classified as follows:

(a) Control surveys establish a network of horizontal and vertical points that serve as a reference framework for other surveys.

(b) Topographic surveys show the natural features of a country such as rivers, streams, lakes, forests, hills, etc.

(c) Land, boundary or cadastral surveys establish property lines and corners.

(d) Hydrographic surveys define the shore lines and depth of water bodies, e.g. oceans, reservoirs and lakes.

(e) Route surveys are done as a preliminary to construction of roads and railways.

(f) Mine surveys are done above and below the ground to guide mining operations under ground.

Classification Based on Equipments Used. In chain, theodolite, plane table, tacheometric surveys, the equipment named is the major equipment used in survey work. In photogrammetric surveying, major equipment is a photogrammetric camera.

Classification Based on Position of Instruments. When measurement is done on the ground by say chain, tape or electronic distance measuring equipments it is *ground* survey; when photographic observations are taken from air; it is *aerial* survey.

1.3 HISTORY OF SURVEYING

The earliest preserved writings on surveying are those of Heron the Elder, a Greek who lived in Alexandria about 150–100 B.C. His writings include a treatise, Dioptra (Surveyor's Transit); a geometry book, Measurement; and an optical work, Mirrors. In Measurement, he describes the method used in determining the area of a triangle from the lengths of three sides. The dioptra could be used for measuring angles and levelling (Fig. 1.1).

In contrast to the Greeks, the Romans were more interested in practical applications of mathematics and surveying for civil and military works. To layout a route for a road the Roman surveyors used a few simple instruments for establishing horizontal lines and right angles. For laying out right angles, they used a *groma* adapted from an Egyptian device. For long distance measurement between cities, the Romans had an ingenious invention, the *hodometer*. With the fall of the Roman empire, the ancient civilized world came to an end. All technical disciplines, including surveying were no longer needed when even the basic laws protecting life and property could not be enforced.

During the Dark ages, the art of surveying was almost forgotten. It was not until the beginning of Renaissance that a revival in exploration and trade created

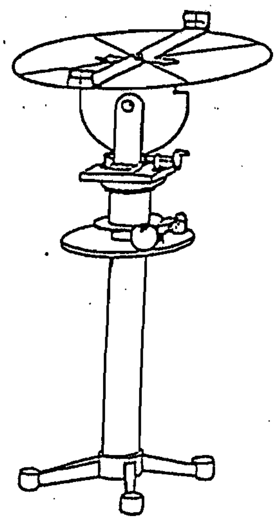


Fig. 1.1 Heron's dioptra.

new interest in western world in navigation, astronomy, cartography and surveying. During the thirteenth century, the magnetic compass was invented by Neckam, an Englishman as an aid to navigation.

In 1571 Thomas Digges an English mathematician known as the father of modern surveying published a book describing a new "topographical instrument" developed from the quadrant which became known as the "theodelitus". This simple instrument had all the essential features of modern theodolite except for the telescope.

The plane table was described almost in its present form by Jean Practorius in 1590.

Development of the telescope in the late sixteenth century greatly increased the speed and accuracy of surveying. Although several scientists share credit for this discovery, it was Galileo Galilei who perfected the instrument in 1609.

The first man who attempted to tie established points together by triangulation was a young Dutch mathematics professor Willebrod Snellvan Roijen (1581–1626).

By the end of the eighteenth century many instruments and tools used by modern surveyors had been developed. *The Construction and Principal Uses of Mathematical Instruments* published in 1723 by French writer Nicholas Bion showed sketches of rulers, compasses, dividers, protractors, and pantograph. Also shown were ropes, rods, chains and pins for surveying plus angle and level instruments mounted on tripods. Advances of eighteenth century left nineteenth century engineers and surveyors a remarkable heritage in tools and instruments.

Surveying methods and instruments used at the beginning of the twentieth century were basically the same as those used in the nineteenth century. However, new light weight metals and more advanced callibration techniques resulted in development of lighter and more accurate instruments needed for the precise layout requirements of high speed railroads and highways.

Use of aerial photography for mapping began in the 1920s, and advanced rapidly during the following decades. By 1950 photogrammetric methods had revolutionized survey procedures, especially in route surveying and site selection.

1.4 MODERN TRENDS IN SURVEYING

Recent developments in photogrammetric and surveying equipment have been closely associated with advances in electronic and computer technologies. Electronic distance measuring instruments for ground surveying now are capable of printing output data in machine-readable language for computer input and/or combining distance and angle measurements for direct readout of horizontal and vertical distances to the nearest 0.001 of a centimetre. The incorporation of data collectors, and electronic field books with interfaces to computer, printer, and plotter devices has resulted in the era of total station surveying.

The recent refinement in global positioning systems and techniques developed for military navigation has led to yet another dramatic change in surveying instrumentation. Inertial surveying, with its miniaturized packaging of accelerometers and gyroscopes and satellite radio surveying have already revolutionized geodetic control surveying and promises to impact all phases of the surveying process.

The principal change in levelling instruments has been widespread adoption of the automatic level, in which the main level bubble has been replaced with a pendulum device which after the instrument has been rough levelled, automatically levels the line of sight. Lasers are being used for acquisition of vertical control data in photogrammetry and for providing line and grade in construction related surveying.

As a result of the technological breakthroughs in surveying and mapping the survey engineer of 1990s must be better trained in a much broader field of science than the surveyor of even a decade ago. A background in higher mathematics, computer technology, photogrammetry, geodetic science and electronics is necessary for today's survey engineer to compete in this rapidly expanding discipline.

1.5 THE SHAPE AND SIZE OF THE EARTH

Since in surveying we are mainly concerned with measurements on the surface of the earth, it is necessary to know as fully as possible the shape and size of the earth. The surface of the earth is not of a regular shape because of presence of mountains in some parts and oceans in other. This surface is the topographical surface. The force and direction of gravity at each point varies with the shape of the topographical surface. The surface which is normal to the direction of gravity is defined as a geoid. It is the surface to which the waters of the oceans would tend to conform if allowed to flow into very narrow and shallow canals cut through the land. Geoid is very irregular and to help in mathematical computation a spheroid (which is obtained by rotating an ellipse about its minor axis) is assumed which nearly fits the shape of the earth. Different countries have their own reference spheroid because they base their computations on the spheroid which fits the geoid with part of earth's surface in their respective countries.

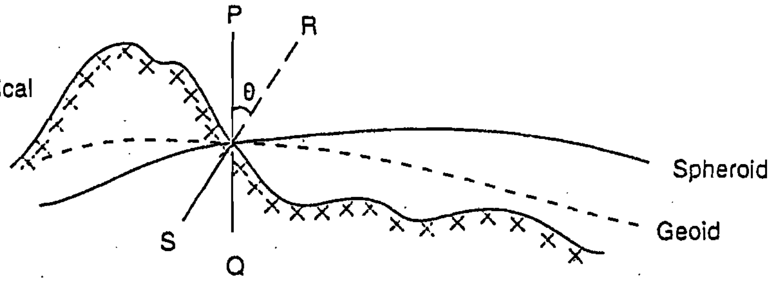
Figure 1.2 shows the three surfaces. The angle between normal to geoid and normal to the spheroid is known as *deflection of the vertical* or *station error.*

The standard reference spheroid has the following dimensions:

Semi-major axis a = 6378388 m

Semi-minor axis b = 6356911.946 m

$$\text{Flattening } f = \frac{a-b}{a} = .0033670034$$



PQ = Normal to spheroid; RS = Normal to geoid: θ = Deflection of the vertical.

Fig. 1.2 Approximate shape of the earth.

1.6 HORIZONTAL AND LEVEL DISTANCES

A horizontal plane is perpendicular to the plumb line at a point but a level surface is at all points perpendicular to the local plumb line. The two surfaces are coincident at the instrument station but diverge with increasing distance from it due to the earth's curvature. Hence there is a technical difference between a horizontal distance (HD) and a level distance (LD). Figure 1.3 shows how horizontal distance is measured in plane surveying and this distance is independent of elevation. Thus HD (1) is the right triangle component of the slope distance.

However, if precision of a long and/or steep distance measurement is to be preserved, then convergence of the plumb lines becomes important and horizontal

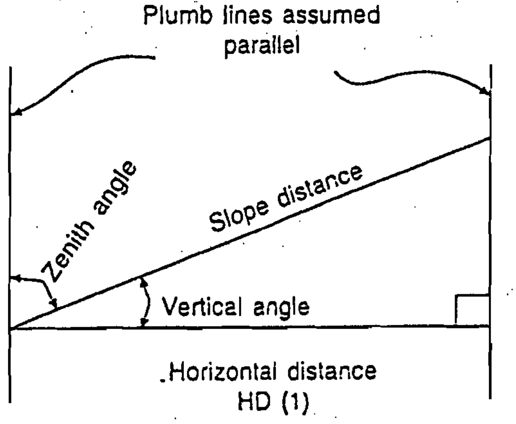


Fig. 1.3 Right triangle horizontal distance.

distance becomes elevation dependent. Figure 1.4 shows how horizontal distance between two points can be variously defined when curvature of the earth of various approximations are taken into account.

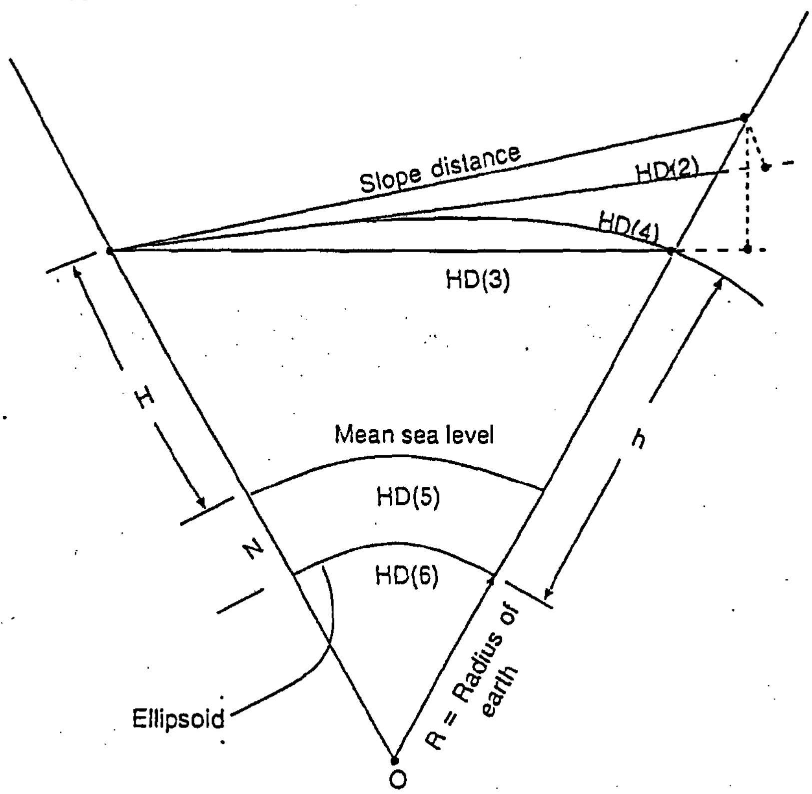


Fig. 1.4 Horizontal distance between two points. HD(2)—The distance between two plumb lines in a plane *tangent* to the earth at the instrument station. HD(3)—The chord distance between two plumb lines. The two end points have the same elevation and the chord is perpendicular to the vertical (plumb line) only at the chord mid-point. HD(4)—The arc distance along some level surface between two plumb lines. HD(5)—the arc distance at mean sea level between two plumb lines. HD(6)—the distance along the geodesic on the ellipsoid surface between two plumb lines. O = Centre of Earth.

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2

Errors in Measurement

2.1 INTRODUCTION

Surveying is based on measurements and whenever we take measurements, say of a length or of an angle, we make errors. These errors are due to (i) Natural causes (ii) Instrumental imperfections and (iii) Personal limitations.

Examples of natural causes are variation in speed of wind, temperature, humidity, refraction, gravity and magnetic declination. A tape or chain which is normally of 30 m length does not remain so if the temperature changes and as a result error in the measurement of length occurs. However sophisticated an instrument may be, it is never perfect. Graduations of the horizontal circle of a theodolite or on a levelling rod may not be perfectly spaced and this may lead to errors. Finally there are limitations of the human senses of sight and touch. However much we may try it is difficult to bisect exactly a rod while taking measurements of an angle.

2.2 TYPES OF ERRORS

Very broadly errors are of two types:

(a) Systematic or cumulative.
(b) Accidental, random or compensating.

The third type, i.e. mistake or blunder cannot be classified under any category of error because they are due to carelessness or callousness on the part of the observer. Mistake can be corrected only if discovered. Comparing several measurements of the same quantity is one of the best ways of isolating mistakes.

Systematic errors can always be corrected because their magnitude and sign can both be determined. For example, if a chain is of standard length under a particular pull and temperature and if the pull or temperature changes, we can compute its effect on the length of the chain, i.e. whether it will increase or decrease and by how much and then apply suitable corrections.

Accidental, random or compensating errors on the other hand, are subject to chance and hence follow the laws of probability. The magnitude and sign of errors are not definitely known. They are sometimes positive, sometimes negative, sometimes of small magnitude, sometimes of large magnitude and hence cannot be computed or eliminated. However, by taking a large number of observations we can make an estimate of magnitude of the error likely to be involved.

2.3 ACCURACY AND PRECISION OF MEASUREMENTS

We have already said that whenever we take measurements we make errors. Hence the true value of a measured quantity is never known. *Accuracy* is the closeness or nearness of the measurements to the "true" or "actual" value of the quantity being measured. The term precision (or repeatability) refers to the closeness with which the measurements agree with each other. Figure. 2.1 explains the four possibilities of measurements.

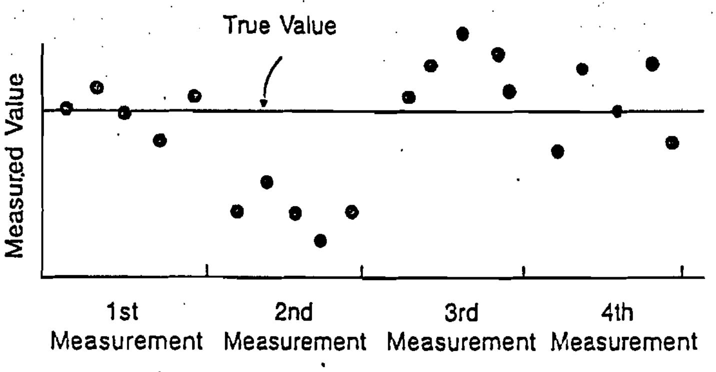


Fig. 2.1 Accuracy and precision of measurements. 1st Measurement—accurate and precise. 2nd Measurement—not accurate but precise. 3rd Measurement—not accurate, not precise. 4th Measurement—accurate but not precise.

2.4 NATURE OF RANDOM ERRORS

Random errors or accidental errors are unpredictable both as regard to size and algebraic signs. They are truly accidental and cannot be avoided. There are, however, three basic characteristics of random errors: (i) small errors are more frequent than large errors; (ii) very large errors do not occur at all; and (iii) positive and negative errors of the same size occur with equal frequency.

The above three characteristics can be graphically represented by means of a bell shaped curve called the *probability curve* or the *normal error distribution curve*. Such a curve is shown in Fig. 2.2. The equation of the curve is

$$y = ke^{-h^2x^2} \tag{2.1}$$

in which y is the relative frequency of occurrence of an error of a given size, x is the size of the error, k and h are constants that determine the shape of the curve, and e is the base of the natural logarithms. In practice true error is never known as true value of a quantity cannot be determined. Instead, we find *most probable value* and *residual error*. Most probable value is that value of a quantity that has the most frequent chance of occurrence or that has the maximum probability of

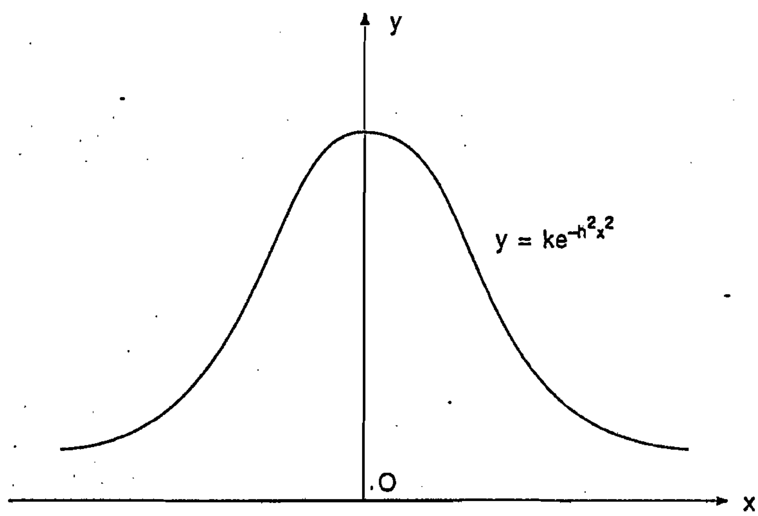


Fig. 2.2 Probability curve.

occurrence. The difference between the observed value and the most probable value is known as residual error. A residual error is treated as a random error in every respect. It follows the laws of probability and can be expressed in the form

$$y = ke^{-h^2 v^2} \tag{2.2}$$

where v is the residual error. Figure 2.3 shows the probability curve of residual error.

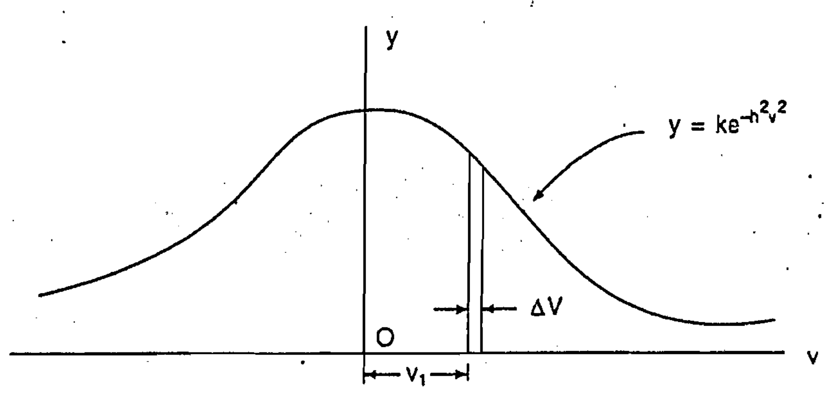


Fig. 2.3 Probability curve of residual error.

The probability of residual error v_1 is the area of the probability curve at this value. Area is equal to the ordinate of the probability curve at v_1 multiplied by an arbitrary increment ΔV. Hence

$$Pv_1 = y_1 \Delta V = ke^{-h^2 v_1^2}\ \Delta V$$

$$Pv_2 = y_2 \Delta V = ke^{-h^2 v_2^2}\ \Delta V$$

... ...

... ...

$$Pv_n = y_n \Delta V = ke^{-h^2 v_n^2} \Delta V$$

According to the laws of probability; the probability that a set of events will occur simultaneously is the product of their separate probabilities. Therefore the probability of residual error $v_1, v_2, \ldots, v_n$ occurring simultaneously is obtained by the product of

$$Pv_1, Pv_2, \ldots, Pv_n$$

Therefore

$$P(v_1, v_2, \ldots, v_n) = (ke^{-h^2 v_1^2} \Delta V)(ke^{-h^2 v_2^2} \Delta V) \ldots (ke^{-h^2 v_n^2} \cdot \Delta V)$$

$$= k^n \Delta V^n \cdot e^{-h^2 (v_1^2 + v_2^2 + \ldots + v_n^2)} \quad (2.3)$$

The expression is maximum, i.e. probability is maximum when $v_1^2 + v_2^2 + \ldots + v_n^2$ is minimum. This gives us the theory of *least squares* which says that the most probable value or the value of a quantity which has the maximum probability of occurrence is obtained when sum of the squares of the residuals is minimum.

Let a quantity be measured n number of times and let its value be $M_1, M_2, \ldots, M_n$. Let M be the most probable value. Then

$$v_1 = M_1 - M \qquad v_1^2 = (M_1 - M)^2$$

$$v_2 = M_2 - M \qquad v_2^2 = (M_2 - M)^2$$

... ...

... ...

$$v_n = M_n - M \qquad v_n^2 = (M_n - M)^2$$

Then from the theory of least squares $\Sigma\, v^2$, i.e. $(M_1 - M)^2 + (M_2 - M)^2 + \ldots + (M_n - M)^2$ should be minimum. Thus

$$\frac{d}{dM} \Sigma\, v^2 \text{ should be zero}$$

and

$$\frac{d^2}{dM^2} \Sigma\, v^2 \text{ should be positive}$$

Putting

$$\frac{d}{dM}(\Sigma\, v^2) = -2(M_1 - M) - 2(M_2 - M) + \ldots + -2(M_n - M)$$

$$= 0$$

we get

$$M = \frac{M_1 + M_2 + M_3 + \dots + M_n}{n}$$

which is nothing but the arithmetic mean of the observed values:

$$\frac{d^2}{dM^2}\,\Sigma\, v^2 = 2 + 2 + 2 + \dots = \text{a positive quantity indicating a minimum.}$$

In practice the number of observations being limited, we get the most probable value by taking mean of the observed values and instead of true errors we get residuals by obtaining deviation from the mean.

2.5 MEASURES OF PRECISION

Though the shape of the curve more or less remains the same, the spread or dispersion changes with the values of h and k in the equation $y = ke^{-h^2v^2}$. In Fig. 2.4(a) most of the data are close to the mean value and hence the measurements are more precise than in Fig. 2.4(b) where there is considerable scatter.

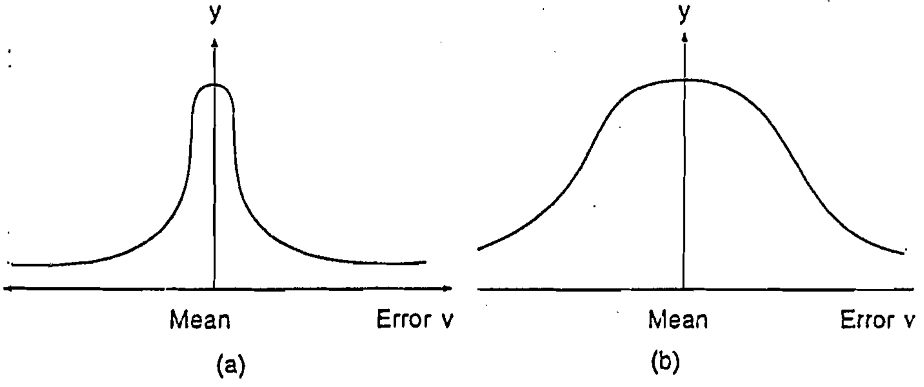


Fig. 2.4 Dispersion of error.

Statistically, precision can be measured by means of a quantity σ known as standard deviation or *standard error* and is given by

$$\sigma = \sqrt{\frac{\Sigma\, v^2}{n-1}} \tag{2.4}$$

where $\Sigma\, v^2$ = sum of the squares of the residuals
n = no. of measurements.

The smaller σ becomes the greater is the precision. The term $(n - 1)$ in Eq. 2.4 represents the *degree of freedom*, i.e. number of extra measurements taken to determine a value. σ is also known as estimated standard value as the deviations are measured not from the true value but from the mean value. In terms of actual measurements when the estimated standard deviation is known the equation $y = ke^{-h^2v^2}$ can be written as:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{(-1/2\sigma^2)v^2} \tag{2.5}$$

in which y is the relative frequency of occurrence of a residual of given size. This curve is known as the *normal distribution curve*. It is noted that $y = ke^{-h^2v^2}$ is the probability curve and Eq. (2.5) is the curve representing the distribution of errors derived from an actual set of measurements. We then have

$$k = \frac{1}{\sigma\sqrt{2\pi}}, \quad h^2 = \frac{1}{2\sigma^2}$$

As k and h increase, the precision of the measurements also increases. The quantity h is known as the precision modulus of the measurements.

Example 2.1 An angle was measured six times, the observed values being 49°23′00″, 49°23′20″, 49°22′40″, 49°22′20″, 49°23′40″ and 49°24′00″. Calculate the most probable value of the angle and the standard error of the measurement.

Solution

Angle	Residual v	v^2
49°23′00″	− 10″	100
49°23′20″	+ 10″	100
49°22′40″	− 30″	900
49°22′20″	− 50″	2500
49°23′40″	+ 30″	900
49°24′00″	+ 50″	2500
Σ 6 × 49° + 139′	Mean = 49°23′10″	Σ v^2 = 7000

Standard deviation $\sigma = \pm\sqrt{\frac{v^2}{n-1}}$

$= \pm\sqrt{\frac{7000}{6-1}}$

$= \pm\ 37.41''$

From the study of the residuals it can be observed that four residuals are within ± 37.41″. This is a characteristic of standard error. Approximately 2/3rd of the residuals will lie within standard error.

2.6 THE E_{50}, E_{90} AND E_{95} ERRORS

Similarly we can find out the limit within which 50, 90 and 95% of the errors will lie. They bear a relation with standard error and are given by the following.

$$E_{50} = 0.6745\ \sigma$$

$$E_{90} = 1.6449\ \sigma$$

$$E_{95} = 1.9599\ \sigma$$

E_{50} or 50% error was previously known as probable error i.e. the limit within which 50% of the error will lie. This can also be interpreted as stating that an error has equal chances of lying within this limit as outside it, i.e. the probability of lying within the limit is 1/2. The term 'probable error' is not used now a days as it is a misnomer. It does not indicate that this error is more probable than any other value. All these are explained in Fig. 2.5.

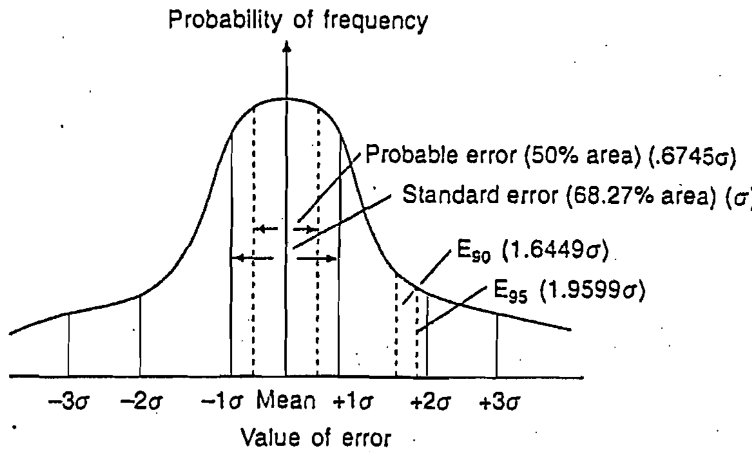


Fig. 2.5 Typical probability curve showing E_{50}, $E_{63.27}$, E_{90} and E_{95}.

2.7 PROPAGATION OF RANDOM ERRORS

Suppose a length is measured in three parts whose random errors are say E_x, E_y and E_z respectively. Then the random error of the sum is given by.

$$E_{sum} = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

This can be derived as follows (Fig. 2.6):

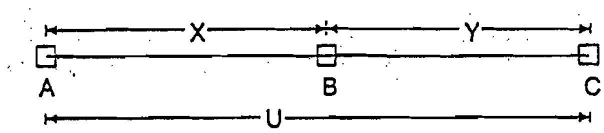


Fig. 2.6 Propagation of random errors.

The length AC is measured in two parts AB and BC. Let

$$AB = X$$
$$BC = Y$$
$$AC = U$$

Then

$$U = X + Y$$

If $\bar{X}$ is the mean value of X, X_i is any measured value and x_i corresponding residual. Then we can write,

$$X_1 = \bar{X} + x_1$$

$$X_2 = \bar{X} + x_2$$

$$\dots \quad \dots$$

$$X_i = \bar{X} + x_i$$

$$X_n = \bar{X} + x_n$$

Similarly

$$Y_1 = \bar{Y} + y_1$$

$$Y_2 = \bar{Y} + y_2$$

$$\dots \quad \dots$$

$$Y_n = \bar{Y} + y_n$$

If $\bar{U}$ denotes the most probable value of the sum of the two distances obtained by adding $\bar{X} + \bar{Y}$ and if u_i denotes the differences between $\bar{U}$ and the value U_i obtained by adding the measurements $X_i + Y_i$, then

$$U_1 = \bar{U} + u_1 = X_1 + Y_1 = \bar{X} + x_1 + \bar{Y} + y_1$$

$$U_2 = \bar{U} + u_2 = X_2 + Y_2 = \bar{X} + x_2 + \bar{Y} + y_2$$

$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$

$$U_n = \bar{U} + u_n = X_n + Y_n = \bar{X} + x_n + \bar{Y} + y_n$$

But

$$\bar{U} = \bar{X} + \bar{Y}$$

Hence

$$u_1 = x_1 + y_1$$

$$u_2 = x_2 + y_2$$

$$\dots \quad \dots$$

$$u_n = x_n + y_n$$

Squaring both sides and adding gives

$$u_1^2 = x_1^2 + 2x_1y_1 + y_1^2$$

$$u_2^2 = x_2^2 + 2x_2y_2 + y_2^2$$

$$\dots \quad \dots \quad \dots$$

$$u_n^2 = x_n^2 + 2x_ny_n + y_n^2$$

$$\Sigma u^2 = \Sigma x^2 + 2\,\Sigma xy + \Sigma y^2 \qquad (2.6)$$

The measurements X_i for AB and the measurements Y_i for BC are independent and uncorrelated and hence their residuals also are independent and uncorrelated. In such a case the term Σxy tend to zero. Hence Eq. (2.6) can be written as

$$\Sigma u^2 = \Sigma x^2 + \Sigma y^2 \qquad (2.7)$$

Dividing both sides by $n - 1$

$$\frac{\sum u^2}{n-1} = \frac{\sum x^2}{n-1} + \frac{\sum y^2}{n-1}$$

or
$$\sigma_u^2 = \sigma_x^2 + \sigma_y^2$$

and
$$\sigma_u = \sqrt{\sigma_x^2 + \sigma_y^2}$$

i.e.
$$E_{\text{sum}} = \sqrt{E_x^2 + E_y^2} \tag{2.8}$$

The above deviations can be still more generalized with the help of calculus. Let

$$U = f(X, Y, Z, \ldots, Q)$$

when each independent variable is changed by a small quantity dX, dY, dZ and dQ the total change dU in U is given by

$$dU = \frac{\partial U}{\partial X} \cdot dx + \frac{\partial U}{\partial Y} \cdot dy + \ldots + \frac{\partial U}{\partial Q} \cdot dq$$

Taking dX as x_i, dY as y_i, …, dQ as q_i and dU as u_i and putting 1, 2, 3 etc. in place of i, we get

$$\left.\begin{aligned} u_1 &= \frac{\partial U}{\partial X} x_1 + \frac{\partial U}{\partial Y} y_1 + \ldots + \frac{\partial U}{\partial Q} q_1 \\ u_2 &= \frac{\partial U}{\partial X} x_2 + \frac{\partial U}{\partial Y} y_2 + \ldots + \frac{\partial U}{\partial Q} q_2 \\ \ldots & \quad \ldots \quad \ldots \quad \ldots \\ u_n &= \frac{\partial U}{\partial X} x_n + \frac{\partial U}{\partial Y} y_n + \ldots + \frac{\partial U}{\partial Q} q_n \end{aligned}\right\} \tag{2.9}$$

Squaring both sides and adding

$$u_1^2 = \left(\frac{\partial U}{\partial X}\right)^2 x_1^2 + 2\left(\frac{\partial U}{\partial X}\right)\left(\frac{\partial U}{\partial Y}\right) \cdot x_1 y_1 + \ldots + \left(\frac{\partial U}{\partial Y}\right)^2 y_1^2 + \left(\frac{\partial U}{\partial Q}\right)^2 q_1^2 + \ldots$$

$$u_2^2 = \left(\frac{\partial U}{\partial X}\right)^2 x_2^2 + 2\left(\frac{\partial U}{\partial X}\right)\left(\frac{\partial U}{\partial Y}\right) x_2 y_2 + \ldots + \left(\frac{\partial U}{\partial Y}\right)^2 y_2^2 + \left(\frac{\partial U}{\partial Q}\right)^2 q_2^2 + \ldots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$u_n^2 = \left(\frac{\partial U}{\partial X}\right)^2 x_n^2 + 2\left(\frac{\partial U}{\partial X}\right)\left(\frac{\partial U}{\partial Y}\right) x_n y_n + \left(\frac{\partial U}{\partial Y}\right)^2 y_n^2 + \left(\frac{\partial U}{\partial Q}\right)^2 q_n^2 + \ldots$$

We can rewrite this in the form

$$\Sigma u^2 = \left(\frac{\partial U}{\partial X}\right)^2 \Sigma x^2 + 2\left(\frac{\partial U}{\partial X}\right)\left(\frac{\partial U}{\partial Y}\right) \Sigma xy + \left(\frac{\partial U}{\partial Y}\right)^2 \Sigma y^2 + \ldots + \left(\frac{\partial U}{\partial Q}\right)^2 \Sigma q^2 \qquad (2.10)$$

+ other square terms + cross product terms

If the measured quantities are independent and are not correlated with one another, the cross products tend to zero. Then

$$\Sigma u^2 = \left(\frac{\partial U}{\partial X}\right)^2 \Sigma x^2 + \left(\frac{\partial U}{\partial Y}\right)^2 \Sigma y^2 + \ldots + \left(\frac{\partial U}{\partial Q}\right)^2 \Sigma q^2$$

Dividing both sides by $(n - 1)$, we get

$$\frac{\Sigma u^2}{n-1} = \left(\frac{\partial U}{\partial X}\right)^2 \cdot \frac{\Sigma x^2}{n-1} + \left(\frac{\partial U}{\partial Y}\right)^2 \cdot \frac{\Sigma y^2}{n-1} + \ldots + \left(\frac{\partial U}{\partial Q}\right)^2 \cdot \frac{\Sigma q^2}{n-1}$$

or $$\sigma_u^2 = \left(\frac{\partial U}{\partial X}\right)^2 \cdot \sigma_x^2 + \left(\frac{\partial U}{\partial Y}\right)^2 \cdot \sigma_y^2 + \ldots + \left(\frac{\partial U}{\partial Q}\right)^2 \cdot \sigma_q^2 \qquad (2.11)$$

Now if $U = X + Y + Z$

$$\left(\frac{\partial U}{\partial X}\right)^2 = \left(\frac{\partial U}{\partial Y}\right)^2 = \left(\frac{\partial U}{\partial Z}\right)^2 = 1$$

Hence $$\sigma_u^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$$

or $$\sigma_u = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

If $U = X - Y$, $$\sigma_u^2 = \sigma_x^2 + \sigma_y^2$$

or $$\sigma_u = \sqrt{\sigma_x^2 + \sigma_y^2} \qquad (2.12)$$

If $U = XY$ $$\frac{\partial U}{\partial X} = Y \quad \text{and} \quad \left(\frac{\partial U}{\partial X}\right)^2 = Y^2$$

Similarly $$\frac{\partial U}{\partial Y} = X \quad \text{and} \quad \left(\frac{\partial U}{\partial Y}\right)^2 = X^2$$

Then $$\sigma_u^2 = Y^2 \cdot \sigma_x^2 + X^2 \cdot \sigma_y^2$$

or $$\sigma_u = \sqrt{Y^2 \cdot \sigma_x^2 + X^2 \cdot \sigma_y^2} \qquad (2.13)$$

Finally if $U = AX$, where A is a constant

$$\frac{\partial U}{\partial X} = A \quad \text{and} \quad \left(\frac{\partial U}{\partial X}\right)^2 = A^2$$

$$\sigma_u^2 = A^2 \cdot \sigma_x^2$$

or

$$\sigma_u = A\ \sigma_x \tag{2.14}$$

Example 2.2 A line AD is measured in three sections, AB, BC and CD with length and standard errors as given below:

$$AB = 125.85 \text{ m} \pm .021 \text{ m.}$$
$$BC = 205.72 \text{ m} \pm .290 \text{ m.}$$
$$CD = 246.205 \text{ m} \pm .025 \text{ m.}$$

What is the standard error in the total length AD?

Solution Here

$$AD = AB + BC + CD$$

and is of the form

$$U = X + Y + Z$$

Applying Eq. (2.8), we have

$$\sigma_u = \pm \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$
$$= \pm \sqrt{(.021)^2 + (.290)^2 + (.025)^2}$$
$$= \pm .292 \text{ m.}$$

Example 2.3 What is the area of the rectangular field and its error for the following data? sides 85.45 ± 0.012 m by 145.05 ± 0.020 m

Solution Area = Length × Breadth

= 85.45 × 145.05

= 12394.523 m²

This is of the form $U = X \cdot Y$.

Hence applying Eq. (2.13)

$$\sigma_u = \pm \sqrt{(.012)^2\,(145.05)^2 + (.020)^2\,(85.45)^2}$$
$$= \pm 2.439 \text{ m}^2$$

Example 2.4 Two sides and the included angle of a triangle were measured with the following results a = 155.25 m and $\sigma_a = \pm .03$ m, b = 71.25 m and $\sigma_b = \pm .02$ m, $C = 40°20'$ and $\sigma_c = \pm 20''$. Compute the area of the triangle and standard error of the area.

Solution

Area of triangle $A = \frac{1}{2}\, ab \sin C.$

$$= \frac{1}{2} \times 155.25 \times 71.25 \sin 40°20'$$

$$= 3579.6817 \text{ m}^2.$$

The standard error of the area from Eq. (2.11), is

$$\sigma_A = \sqrt{\left(\frac{\partial A}{\partial a} \cdot \sigma_a\right)^2 + \left(\frac{\partial A}{\partial b} \cdot \sigma_b\right)^2 + \left(\frac{\partial A}{\partial c} \cdot \sigma_c\right)^2}$$

Here $$\frac{\partial A}{\partial a} = \frac{1}{2}\, b \sin C = \frac{1}{2} \times 71.25 \times \sin 40°20'$$

$$= 23.05 \text{ m}^2\text{/m}.$$

$$\frac{\partial A}{\partial b} = \frac{1}{2}\, a \sin C = \frac{1}{2} \times 155.25 \times \sin 40°20'$$

$$= 50.241 \text{ m}^2\text{/m}.$$

$$\frac{\partial A}{\partial C} = \frac{1}{2}\, ab \cdot \cos C = \frac{1}{2} \times 71.25 \times 155.25 \times \cos 40°20'$$

$$= 4216.09 \text{ m}^2\text{/rad}.$$

$\sigma_c = 20''$, expressed in radian

$= 20 \times 0.00000485$

$= 0.000097$ rad.

$$\sigma_A = \sqrt{(23.05 \times .03)^2 + (50.241 \times .02)^2 + (4216.09 \times 0.000097)^2}$$

$$= \pm\ 1.286 \text{ m}^2$$

Example 2.5 One side and two adjacent angles of a triangle are measured in order to determine the lengths of the other two sides because the vertex opposite the measured side is inaccessible (Fig. 2.7). The side c measures 320 ± .02 m, angle A measures 70°30′ ± 20″ angle B measures 60°10′ ± 40″. Compute angle C, side a and side b. Compute the standard error of each quantity.

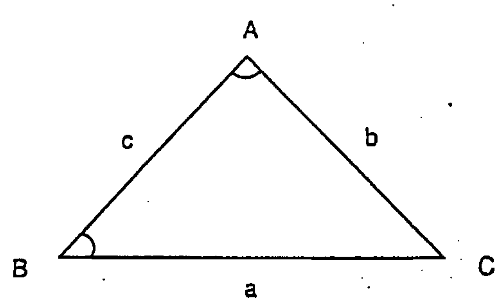


Fig. 2.7 Example 2.5

Solution

(i)
$$C = 180° - (A + B) = 49°20'$$
$$\sigma_C^2 = \left(\frac{\partial C}{\partial A}\right)^2 \cdot \sigma_A^2 + \left(\frac{\partial C}{\partial B}\right)^2 \cdot \sigma_B^2$$
$$= (-1)^2 \cdot \sigma_A^2 + (-1)^2 \cdot \sigma_B^2$$
$$\sigma_C = \sqrt{20^2 + 40^2}$$
$$= \pm 44.72''$$

(ii)
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$a = \frac{\sin A}{\sin C} \cdot c$$
$$\sigma_a^2 = \left(\frac{\partial a}{\partial A}\right)^2 \cdot (\sigma_A)^2 + \left(\frac{\partial a}{\partial C}\right)^2 (\sigma_C)^2 + \left(\frac{\partial a}{\partial c}\right)^2 (\sigma_c)^2$$
$$\frac{\partial a}{\partial A} = \frac{\cos A \cdot c}{\sin C} = \frac{\cos 70°30'}{\sin 49°20'} \cdot 320$$
$$= 140.83 \text{ m/radian.}$$
$$\frac{\partial a}{\partial C} = -\sin A \cdot c \cdot \frac{1}{(\sin C)^2} \cdot \cos C$$
$$= \frac{-\sin 70°30' \times (320)}{(\sin 49°20')^2} \cdot \cos 49°20'$$
$$= -341.71 \text{ m/rad.}$$
$$\frac{\partial a}{\partial c} = \frac{\sin A}{\sin C} = \frac{\sin 70°30'}{\sin 49°20'} = 1.24 \text{ m/m}$$
$$\sigma_a^2 = (140.83)^2 \times \{20 \times (\cdot 00000\ 485)\}^2 + (341.71)^2$$
$$\times (44.72 \times \cdot 00000485)^2 + 1.24^2 \times (\cdot 02)^2$$
$$= .0001866095 + \cdot 0054929095 + \cdot 00061504$$
$$\sigma_a = \pm \cdot 0793378 \text{ m}$$

(iii)
$$b = \frac{c \sin B}{\sin C}$$
$$\frac{\partial b}{\partial C} = -c \cdot \sin B \cdot \frac{1}{(\sin C)^2} \cdot \cos C$$
$$= -(320) \sin 60°10' \frac{1}{(\sin 49°20')^2} \cos 49°20'$$
$$= -314.47 \text{ m/radian}$$

$$\frac{\partial b}{\partial B} = \frac{c \cos B \cdot}{\sin C}$$

$$= \frac{320 \cos 60°10'}{\sin 49°20'}$$

$$= 209.88 \text{ m/radian}$$

$$\frac{\partial b}{\partial c} = \frac{\sin B}{\sin C} = \frac{\sin 60°10'}{\sin 49°20'}$$

$$= 1.144 \text{ m/radian}$$

$$\sigma_b^2 = \left(\frac{\partial b}{\partial C}\right)^2 \cdot \sigma_C^2 + \left(\frac{\partial b}{\partial B}\right)^2 \cdot \sigma_B^2 + \left(\frac{\partial b}{\partial c}\right)^2 \cdot \sigma_c^2$$

$$= (314.47)^2 \cdot (44.72)^2 (.00000485)^2$$

$$+ (209.88)^2 \cdot (40 \times .00000485)^2 + (1.144)^2 \cdot (.02)^2$$

$$= .006833$$

$$\sigma_b = \pm .08266 \text{ m}$$

2.8 ERROR OF A SERIES

Sometimes a series of similar quantities such as the length of a line are measured a number of times with each measurement being in error by about the same amount. The total error in the total series of measurement is called the error of the series and is designated as E_{series}. If the error in each measurement is E and no. of such measurement is n then

$$E_{series} = \sqrt{E^2 + E^2 + E^2 + \ldots + \text{upto } n \text{ terms}}$$

$$= \sqrt{nE^2} = \pm E\sqrt{n} \qquad (2.15)$$

The above equation shows that when the same error is repeated n times, the error is proportional to square root of the number of observations.

Example 2.6 The standard error in a tape of 30 m tape length is ± .008 m. A distance of 1200 m is to be taped. What is the expected 90% error in 1200 m?

Solution No of 30 m tape in 1200 m $= \frac{1200}{30} = 40$

$$90\% \text{ error} = \pm 1.6449\sigma$$

$$= \pm 1.6449 \times (.008)$$

$$= \pm 0.013159$$

$$E_{series} = E_{90} \sqrt{n}$$

$$= 0.013159 \sqrt{40}$$

$$= \pm 0.083226$$

2.9 ERROR OF A MEAN

When a number of like measurements are taken, the error of the sum = $E\sqrt{n}$, where E is the standard error of an individual measurement and n is equal to number of measurements. Now mean is sum divided by number of measurements. Hence the standard error of mean

$$E_m = \frac{E\sqrt{n}}{n} = \frac{E}{\sqrt{n}}$$

where $$E = \sqrt{\frac{\sum v^2}{n-1}}$$

therefore $$E_m = \sqrt{\frac{\sum v^2}{n(n-1)}} \tag{2.16}$$

similarly $$(E_{50})_m = 0.6745\sqrt{\frac{\sum v^2}{n(n-1)}} \tag{2.17}$$

$$(E_{90})_m = 1.6449\sqrt{\frac{\sum v^2}{n(n-1)}} \tag{2.18}$$

These equations show that the error of the mean varies inversely as the square root of the number of repetitions. Thus to double the accuracy or reduce the error by one half four times as many measurements should be made.

Example 2.7 In Example 2.1 what is the standard error of the mean?

Solution Standard error of a single observation = ± 37.41″
There are six observations

$$\text{Standard error of the mean} = \pm\frac{37.41}{\sqrt{6}}$$

$$= \pm 15.27''$$

Example 2.8 Specifications for measuring angles of an n-sided figure limit the total angular closure to E. How accurately must each angle be measured for the following values of n and E?

(i) $n = 4$, $E = 20$ sec
(ii) $n = 10$, $E = 1$ min

Solution

(i) Here $$E_{series} = 20 \text{ sec}$$

$$n = 4$$

$$E_{\text{individual angle}} = \pm\frac{E_{series}}{\sqrt{n}}$$

$$= \pm \frac{20}{\sqrt{4}} = 10 \text{ sec}$$

(ii) Here

$$E_{series} = 60 \text{ sec}$$
$$n = 10$$
$$E_{individual\ angle} = \pm \frac{60}{\sqrt{10}}$$
$$= \pm 18.97 \text{ sec}$$

2.10 WEIGHTS OF MEASUREMENTS

Sometimes it is obvious to a surveyor that one measurement is more precise than another. There may be many reasons for this. It may be that one equipment is more sophisticated than the other or the field conditions during one measurement may be much superior. In such a case we can take this variation into account by assigning different relative weights to different measurements. We already know that precision is indicated by standard deviation σ. Square of standard deviation σ^2 is variance and weight is taken as inversely proportional to variance and directly proportional to h^2 which is known as *precision modulus*.

2.11 THEORY OF LEAST SQUARES APPLIED TO OBSERVATIONS OF UNEQUAL WEIGHTS

Let $M_1, M_2, \ldots, M_n$ be a set of measurements with varying weights $p_1, p_2, \ldots, p_n$ and the corresponding residuals $v_1, v_2, \ldots, v_n$. The probability P that $v_1, v_2, \ldots, v_n$ will occur in the set is as follows.

$$Pv_1 = y_1 \Delta V = k_1 \exp(-h_1^2 v_1^2)\, \Delta V$$
$$Pv_2 = y_2 \Delta V = k_2 \exp(-h_2^2 v_2^2)\, \Delta V$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$Pv_n = y_n \Delta V = k_n \exp(-h_n^2 v_n^2)\, \Delta V$$

Probability that residuals will occur simultaneously is equal to the product of their separate probabilities

$$P(v_1, v_2, \ldots, v_n) = (k_1 \exp(-h_1^2 v_1^2)\, \Delta V)\,(k_2 \exp(-h_2^2 v_2^2)\, \Delta V)\ldots (k_n \exp(-h_n^2 v_n^2)\, \Delta V)$$

or

$$P(v_1, v_2, \ldots, v_n) = (k_1, k_2, \ldots, k_n)\, (\Delta V)^n \exp\,[-(h_1^2 v_1^2 + h_2^2 v_2^2 + \ldots + h_n^2 v_n^2)]$$

Most probable value of the quantity is obtained when P is maximum. In such a case, the negative exponent of e must be minimum, i.e. $h_1^2 v_1^2 + h_2^2 v_2^2 + \ldots + h_n^2 v_n^2$ = minimum. Now weight p is proportional to h^2, therefore

$$p_1v_1^2 + p_2v_2^2 + \ldots + p_nv_n^2 = \text{minimum}$$

or

$$\Sigma pv^2 = \text{minimum}$$

Let $\overline{M}$ be the most probable value of a quantity whose observed values are:

$$M_1, M_2, \ldots, M_n$$

Then

$$v_1 = M_1 - \overline{M}$$

$$v_2 = M_2 - \overline{M}$$

$$\ldots \quad \ldots$$

$$v_n = M_n - \overline{M}$$

From the theory of least squares,

$$p_1(M_1 - \overline{M})^2 + p_2(M_2 - \overline{M})^2 + \ldots + p_n(M_n - \overline{M})^2 = \text{Minimum}$$

Differentiating with respect to $\overline{M}$

$$2p_1(M_1 - \overline{M}) + 2p_2(M_2 - \overline{M}) + \ldots + 2p_n(M_n - \overline{M}) = 0$$

$$\overline{M} = \frac{p_1M_1 + p_2M_2 + \ldots + p_nM_n}{p_1 + p_2 + \ldots + p_n}$$

$$= \frac{\Sigma pM}{\Sigma p} \tag{2.19}$$

This is the weighted mean of the observations and the most probable value of a quantity with unequal weights.

In line with observations of equal weight the following formulae can be derived for observations of unequal weight

1. Standard error of single observation of unit weight $= \sqrt{\dfrac{\Sigma pv^2}{n-1}}$ (2.20)

2. Standard error of the weighted mean $= \sqrt{\dfrac{\Sigma pv^2}{\Sigma p(n-1)}}$ (2.21)

2.12 CALCULATING WEIGHTS AND CORRECTIONS TO FIELD OBSERVATIONS

The following are the relevant rules for calculating weights and applying corrections to field observations:

1. The weight of a measurement varies directly as the number of observations made for that measurement.

2. Weight is inversely proportional to standard error as shown before.

3. In the case of levels, weight varies inversely as the length of the route.

4. Corrections to be applied to various observed quantities are in inverse proportion to their weights. Hence correction to an observation is directly proportional to standard error. Correction for a line of level is directly proportional to the length of the line.

Example 2.9 Find (i) the probable error of a single observation which when repeated on the same angle gave values of 43°20′ plus: 10″, 30″, 10″, 20″, 00″, 40″, 00″, 10″, 30″, 00″, 10″.

(ii) The best value of a quantity which measured four times by a method having a probable error of 3 units, gave an average value of 1800 units and measured nine times by a method having a probable error of 6 units gave an average value of 1808 units of (L.U.)

Solution

(i) Observed value	Deviation from mean	Squares of deviation
43°20′10″	− 04.5″	20.25
30″	+ 15.5″	240.25
10″	− 04.5″	20.25
20″	+ 04.5″	20.25
00″	− 14.5″	210.25
40″	+ 25.5″	650.25
00″	− 14.5″	210.25
10″	− 04.5″	20.25
30″	+ 15.5″	240.25
00″	− 14.5″	210.25
10″	− 4.5″	20.25
Mean Value 43°20′14.5″	Σ = 0	Σ 1862.75

$$\text{p.e. of a single observation} = \pm 0.6745\sqrt{\frac{1862.75}{10}}$$

$$= \pm 9.20''$$

(ii) Probable error of mean by 1st method

$$= \pm \frac{3}{\sqrt{4}} = \pm \frac{3}{2} \text{ units}$$

Probable error of mean by 2nd method

$$= \pm \frac{6}{\sqrt{9}} = \pm 2 \text{ units}$$

Weights are inversely proportional to squares of probable error

$$w_1 : w_2 = \frac{4}{9} : \frac{1}{4}$$

Most probable value = weighted mean

$$= \frac{\frac{4}{9} \times 1800 + \frac{1}{4} \times 1808}{4/9 + 1/4}$$

$$= 1802.88 \text{ units}$$

Example 2.10 The following are the observed values of angles in a triangle of a triangulation survey. Adjust the angles.

$A = 87°35'11.1''$ weight 1
$B = 43°15'17.0''$ weight 2
$C = 49°09'34.1''$ weight 3

[AMIE A.S. Winter 1985]

Solution The angles of a plane triangle should sum 180°. Here

$$A + B + C = 180°00'2.2''$$

Hence there is a total error of 2.2″ and correction – 2.2″

As per rule 4 of Sec. 2.12 corrections are to be distributed inversely to weights of observations.

Therefore $$C_A : C_B : C_C :: \frac{1}{1} : \frac{1}{2} : \frac{1}{3} :: 6 : 3 : 2$$

Hence $$C_A = -\ 1.2''$$

$$C_B = -\ 0.6''$$

$$C_C = -\ 0.4''$$

Example 2.11 A, B, C, D form a round of angles at a triangulation station, their observed values taken with a theodolite are:

$A = 110°20'48''$ weight 4
$B = 92°30'12''$ weight 1
$C = 56°12'00''$ weight 2
$D = 100°57'04''$ weight 3

Adjust the angles A, B, C, and D closing the horizon. [AMIE A.S. Summer 1989]

Solution The sum of four angles closing the horizon = 360°

Here the observed angles add to 360°00′04″

The error is 4″ and the correction – 4″ which should be distributed inversely to weightage. Therefore,

$$C_A : C_B : C_C : C_D = \frac{1}{4} : \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 3 : 12 : 6 : 4$$

$$C_A = -\frac{4}{25} \times 3'' = -\ 0.48''$$

$$C_B = -\frac{4}{25} \times 12'' = -1.92''$$

$$C_C = -\frac{4}{25} \times 6'' = -0.96''$$

$$C_D = -\frac{4}{25} \times 4'' = \underline{-0.64''}$$
$$\overline{-4.00''}$$

Example 2.12 The following are the observed values of an angle:

Angle	Weight
40°20′20″	2
40°20′18″	2
40°20′19″	3

Find

(i) Probable error of single observation of unit weight.
(ii) Probable error of weighted arithmetic mean.

Solution Weighted arithmetic mean

$$= 40°20' + \frac{2 \times 20 + 2 \times 18 + 3 \times 19}{7}$$

$$= 40°20'19''$$

Then

v	v^2	pv^2
+ 1	1	2
− 1	1	2
0	0	0
		$\Sigma pv^2 = 4$

Probable error of a single measurement $= \pm 0.6745 \sqrt{\frac{\Sigma pv^2}{n-1}}$

$$= \pm 0.6745 \sqrt{\frac{4}{2}}$$

$$= \pm 0.9538''$$

Probable error of weighted mean

$$= \pm 0.6745 \sqrt{\frac{\Sigma pv^2}{\Sigma p(n-1)}}$$

$$= \pm 0.6745 \sqrt{\frac{4}{7(2)}}$$

$$= \pm 0.3605$$

PROBLEMS

2.1 What causes errors in measurements?

2.2 What are the different types of errors?

2.3 Distinguish between accuracy and precision?

2.4 Derive the theory of least squares. How does it change when measurements are weighted?

2.5 What are the rules for adjustments of weighted field observations?

2.6 The following are six equally reliable and direct measurements of a base line in meter.

702.0; 701.4; 701.8; 701.6; 701.5 and 701.9.

Calculate the most probable value and its probable error.

[AMIE A.S. Winter 1978]

2.7 (a) Explain the terms: Residual of an observation, most probable value.
(b) Following observations were recorded for a plane triangle *ABC*

$\angle A = 77°14'20''$ wt 4

$\angle B = 44°40'35''$ wt 3

$\angle C = 53°04'52''$ wt 2

Compute the adjusted value of the angles. [AMIE A.S. Winter 1979]

2.8 (a) Explain the terms:

(i) Systematic errors, (ii) Accidental errors, (iii) Mistakes.

(b) The following angles were measured at a station 0 so as to close the horizon.

$a = 83°42'\,28.8''$ weight 3

$b = 102°15'43.3''$ weight 2

$c = 94°38'27.3''$ weight 4

$d = 79°23'23.6''$ weight 2

Find the most probable value of angles. [AMIE A.S. Winter 1980]

2.9 Define the principle of least squares and explain how this law can be applied to obtain the most probable value of a quantity. [AMIE A.S. Summer 1982]

2.10 Explain the following:

(i) Mistakes, Systematic error, Accidental error.
(ii) Normal distributions.
(iii) Probable error.
(iv) Least square method.

[AMIE A.S. Summer 1983]

2.11 What is the weight of an observation? What considerations weigh in deciding it? [AMIE A.S. Summer 1986]

2.12 Explain clearly the theory of least squares as applied to the adjustment of survey measurements. Are there any assumptions involved in the method? [AMIE A.S. Winter 1987]

2.13 Explain the following terms:

(i) Standard deviation,
(ii) Normal distribution,
(iii) Most probable value,
(iv) Least square method,
(v) Weight of an observation.

[AMIE A.S. Winter 1990]

3

Measurement of Horizontal Distances

3.1 INTRODUCTION

One of the most important operations in surveying is measurement of horizontal distance between two points. If the points are at different elevations, the distance is the horizontal length between plumb lines at the points.

3.2 METHODS OF MEASURING HORIZONTAL DISTANCES

Depending on the accuracy desired and time available for measurement, there are many methods of measuring horizontal distances. They are: (i) Pacing, (ii) Odometer readings, (iii) Tacheometry, (iv) Electronic distance measurement, (v) Chaining, and (vi) Taping. While chaining and taping are most common in our country, electronic distance measurements (EDM) are gradually being increasingly used.

3.2.1 PACING

Pacing is an approximate method of measuring distance. Initially the surveyor must walk a known distance a number of times in his own natural way so that his natural pace is known. To count the number of paces a pedometer or a passometer may be used.

3.2.2 ODOMETER

An odometer converts the number of revolutions of a wheel of a known circumference to a distance. This method can often be used to advantage on preliminary surveys where precise distances are not necessary. Odometer distances should be converted to horizontal distance when the slope of the ground is steep.

3.2.3 TACHEOMETRY

Here distance is measured not directly but indirectly with the help of an optical instrument called tacheometer. A theodolite with three cross hairs can also be used with the intercept on a levelling staff between the top and bottom cross hairs multiplied by a constant giving the horizontal distance. In subtense bar method, the angle subtended at the end of a line by a known horizontal base at the other end is measured and the horizontal length is geometrically obtained.

3.2.4 ELECTRONIC DISTANCE MEASUREMENT (EDM)

This is a modern development in surveying where electromagnetic waves are utilized to measure distance. They are basically of two types: (i) Electro optical instruments which use light waves for measurement of distances such as geodimeter, mekometer and range master. (ii) Microwave equipment, which transmits microwaves with frequencies in the range of 3 to 35 GHz corresponding to wavelengths of about 1 dm to 8.6 mm.

3.2.5 CHAINS

Chains are used to measure distances when very great precision is not required. In our country it is frequently used though in other countries it is being gradually replaced by tapes. The chain is robust, easily read and easily repaired in the field if broken. It does not, however, give correct length owing to wear on the metal to metal surfaces, bending of the links, mud between the bearing surfaces, etc. Also the weight is a disadvantage when the chain has to be suspended.

In India link type surveying chains of 30 m lengths are frequently used in land measurement. Nomenclatures and dimensions of different parts of a chain are given in Fig. 3.1. Details of a 30 m chain are given in Fig. 3.2. For 5 and 10 m chains the shape of tallies and the corresponding distances are shown in Fig. 3.3. There is also 30 m chain with 100 links (instead of 150) so that each link is 0.3 m. There are tallies at every 3 m.

3.2.6 TAPES

Tapes are used for accurate work and may be of (i) cloth or linen, (ii) metal, (iii) steel, (iv) invar.

Tapes used for surveying are 30 m in length and graduated in meter, decimeter and centimeter. Cloth or metallic tapes are made of high grade linen with fine copper wires running length-wise to give additional strength and prevent excessive elongation. They come in enclosed reels and are not suitable for precise work. Steel tape is superior to metal tape, is usually 6 to 10 mm wide and is more accurately graduated. It cannot, however, withstand rough usage. If the tape gets wet, it should be wiped with a dry cloth and then with an oily rag. Invar tape is used for very precise work. It is made of 35% nickel and 65% steel. The coefficient of thermal expansion is very small, about 1/30 to 1/60 of that of an ordinary steel tape. The invar tape is soft and also very expensive.

3.3 CHAINING AND TAPING ACCESSORIES

The small instruments and accessories used with chain or tape are (i) Arrows, (ii) Pegs, (iii) Ranging rods, (iv) Offset rods, (v) Plumb bobs.

Arrows or chain pins are used to mark the position of the ends of the chain on the ground. Details are shown in Fig. 3.4

Wooden pegs are used to mark the positions of the survey stations or the end

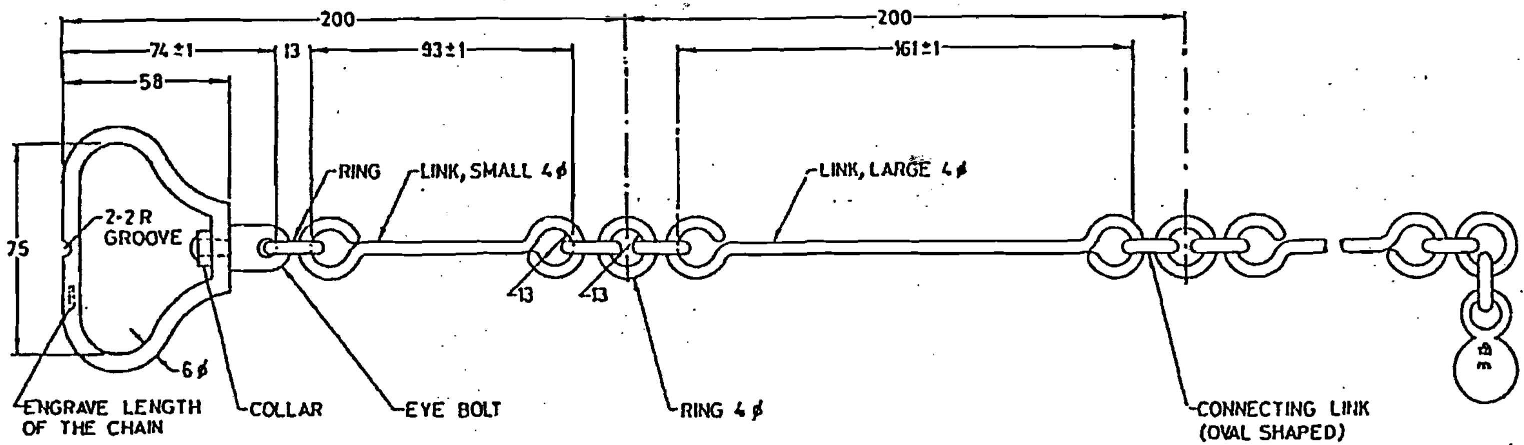


Fig. 3.1 Nomenclature and dimensions of different parts of chain (all dimensions in mm).

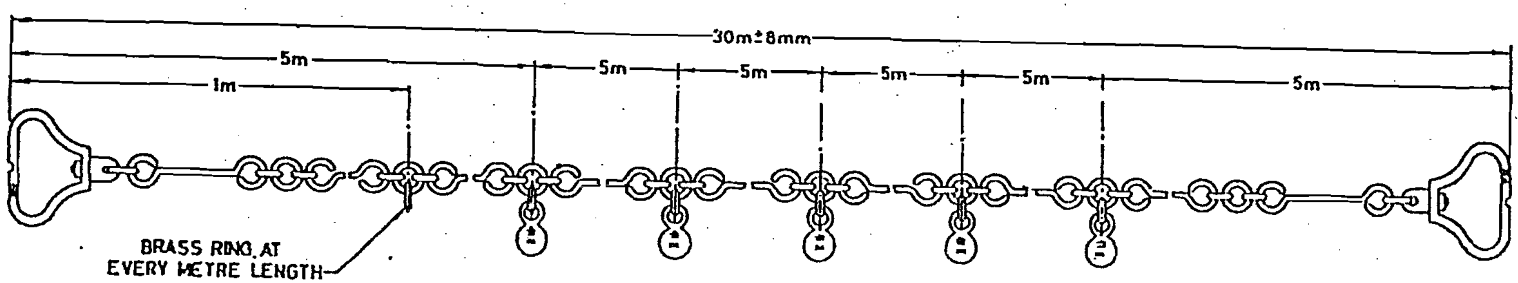


Fig. 3.2 30 Meter chain.

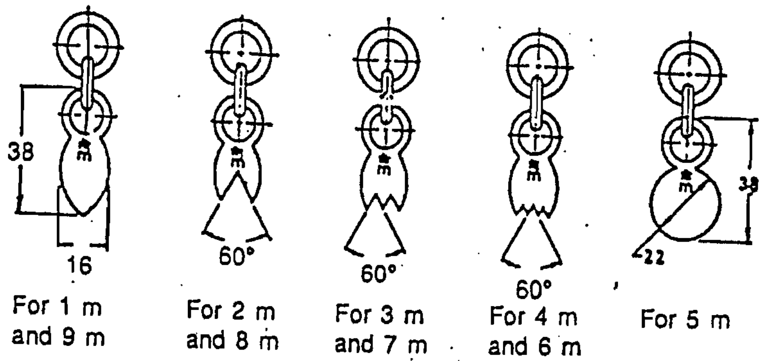


Fig. 3.3 Shapes of tallies for chains (5 m and 10 m).

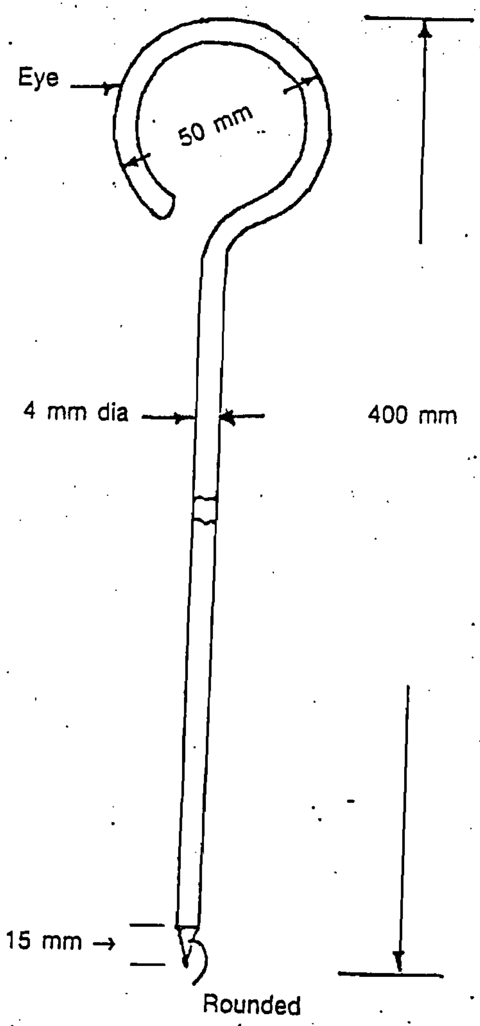


Fig. 3.4 Details of arrow or chain pin.

points of a survey line. The typical dimensions are 25 mm × 25 mm in cross section and 150 mm long with a nail at the top.

Ranging poles and rods are used to make measurements along a straight line. Details are shown in Fig. 3.5.

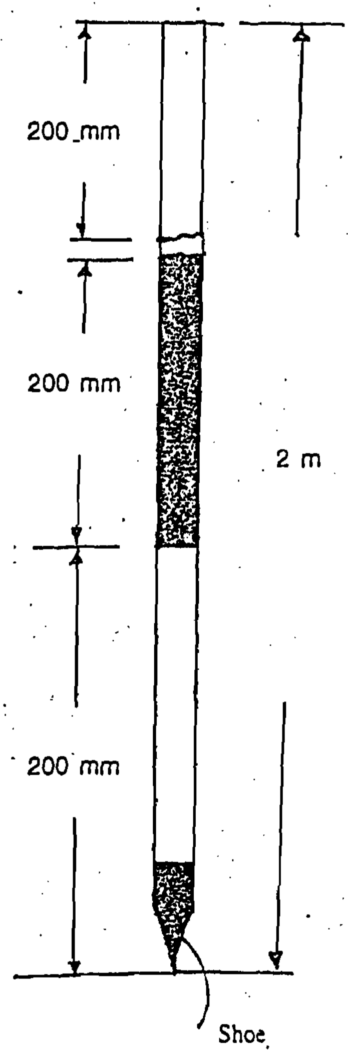


Fig. 3.5 Details of ranging rod.

Plumb bobs are used to project a point on the ground upto the tape or to project a point on the tape down to the ground. Details are shown in Fig. 3.6.

3.4 MEASUREMENT BY CHAIN

There are basically two types of measurements—(i) On level ground, (ii) On uneven ground.

In level ground the line to be measured is marked at both ends and at

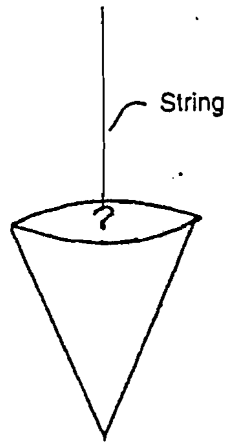


Fig. 3.6 Plumb bob.

intermediate point where necessary so that a clear sight is obtained. Sometimes a theodolite is used for ranging. The follower holds the rear end of the chain at the station point and by movements of his arms directs the arrow or ranging rod held by the leader for the purpose into true alignment. The leader then pulls the chain taut and inserts an arrow in the ground to mark the end. After relevant work in this chain line is over, the leader again pulls on the chain leaving an arrow to mark the position of the end of the first length. The follower holds the rear end of the chain against this and directs the leader into alignment as before. After the chain has been pulled taut and the further end marked by the second arrow, the follower picks up the first and carries it with him. The number of arrows in the hand of the follower at any time will indicate the number of complete chain lengths measured. After 10 chains have been laid down the follower hands over the ten arrows to the leader and the same procedure is carried out for the next ten lengths.

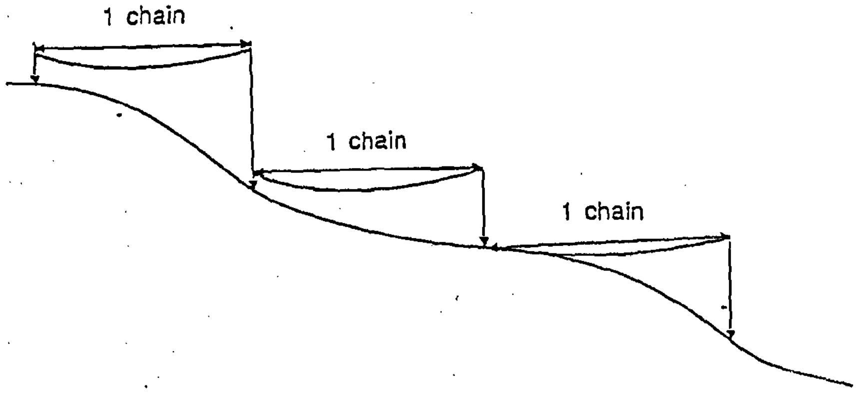
In uneven or sloping ground the distance may be directly measured in small horizontal stretches or steps as shown in Fig. 3.7a or indirectly by measuring the sloping distance along the slope and then getting the horizontal distance analytically by measuring the slope by means of a clinometer or measure the difference in elevation between the points (Fig. 3.7b).

For accurate measurements and in all important surveys, the lengths are now measured with a tape and not with a chain. For higher precision a taping tripod or taping buck must be used. The taping buck usually (i) is rigid in use, (ii) is easily aligned, (iii) is portable, (iv) permits easy transfer of chaining point to or from the ground (v) can easily act as a back sight.

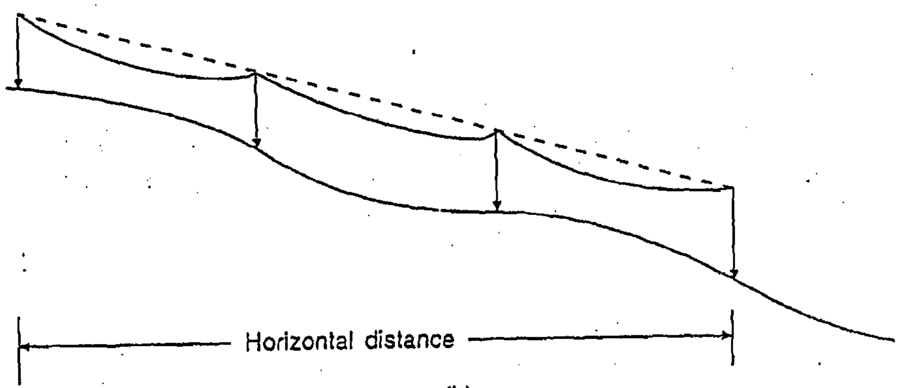
Since taping is usually done on the slope when tripods are used, the elevations of the tops of tripod must be ascertained simultaneously as the taping proceeds to determine the horizontal distance.

3.5 REDUCTIONS TO MEASUREMENT IN SLOPE

There are two ways in which this reduction can be made. When the slope angle α is known the horizontal distance is



(a)



(b)

Fig. 3.7 Measurement on slope: (a) chain held horizontally, (b) chain held on slope.

$$H = S \cos \alpha \tag{3.1}$$

where S is the inclined length.

To determine the accuracy with which the vertical angle must be measured in order to meet a given relative accuracy in the resulting horizontal distance, we differentiate the above equation with respect to α and get

$$dH = -S \sin \alpha \, d\alpha \tag{3.2}$$

Relative accuracy is then

$$\frac{dH}{H} = -\frac{S \sin \alpha \, d\alpha}{S \cos \alpha} = -\tan \alpha \, d\alpha \tag{3.3}$$

When the slope is expressed in terms of difference in elevation as in Fig. 3.8, we have

$$H = \sqrt{S^2 - h^2} \tag{3.4}$$

The expression on binomial expansion becomes

$$H = S - \frac{h^2}{2S} - \frac{h^4}{8S^3} \tag{3.5}$$

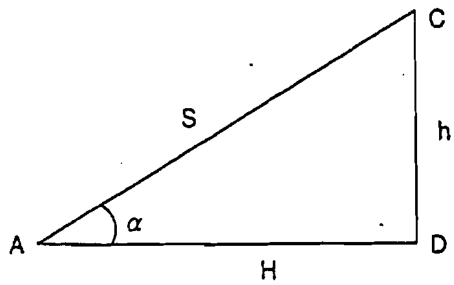


Fig. 3.8 Reduction in slope.

If we neglect the 3rd term

$$C = S - H = \frac{h^2}{2S} \tag{3.6}$$

and

$$dC = \frac{hdh}{S} \tag{3.7}$$

If both sides are divided by S the relative accuracy becomes

$$\frac{dC}{S} = \frac{hdh}{S^2} \tag{3.8}$$

Slightly different expressions can be derived as follows:

$$\text{Correction} = \text{hypotenusal allowance}$$

$$= AC - AD = AD \sec \alpha - AD$$

$$= AD\left(1 + \frac{\alpha^2}{2} + \frac{5\alpha^4}{24} + \ldots\right) - AD$$

$$\approx AD \frac{\alpha^2}{2} \tag{3.9}$$

If the slope is expressed as 1 to n $\alpha = \frac{1}{n}$.

Example 3.1 In chaining a line what is the maximum slope (a) in degrees and (b) as 1 in n which can be ignored if the error from this source is not to exceed 1 in 1500?

Solution

(i) Let α be expressed in degrees.

$$\alpha^\circ = \frac{\alpha \cdot \pi}{180} \text{ radian}$$

Error is 1 in 1500 $= \frac{AD}{1500}$

$$\frac{AD}{1500} = \frac{AD}{2}\left(\frac{\alpha\pi}{180}\right)^2$$

$$\frac{\alpha\pi}{180} = \sqrt{\frac{2}{1500}}$$

$$\alpha = \frac{180}{\pi}\sqrt{\frac{2}{1500}}$$

$$\alpha = 2.092^\circ$$

(ii)

$$\frac{AD \cdot \alpha^2}{2} = \frac{AD}{2n^2} = \frac{AD}{1500}$$

$$n = \sqrt{\frac{1500}{2}} = 27.386.$$

Therefore slope is 1 in 27.386.

Example 3.2 With what accuracy must a difference in elevation between two ends of a 30 m tape be known if the difference in elevation is 2.28 m and the accuracy ratio is to be at least 1 in 25,000.

Solution

We have

$$\frac{dC}{S} = \frac{h\,dh}{S^2} = \frac{1}{25{,}000}$$

$$dh = \frac{S^2(1) \cdot (1)}{25{,}000 \times h}$$

$$= \frac{30^2(1)(1)}{(25{,}000)(2.28)} = .0157894 \text{ m}$$

3.6 SYSTEMATIC ERRORS IN LINEAR MEASUREMENT BY CHAIN OR TAPE

The principal systematic errors in linear measurement made with a chain or tape are (i) Incorrect length, (ii) Tape or chain not horizontal, (iii) Fluctuations in temperature, (iv) Incorrect tension or pull, (v) Sag, (vi) Incorrect alignment, and (vii) Chain or tape not straight.

3.6.1 INCORRECT LENGTH

Incorrect length of a tape or chain is one of most important errors. It is systematic. A tape or chain is of nominal or designated length at the time of manufacture but with use it will seldom remain at its original length. It should be frequently compared with a standard length to find out the discrepancy. The correction to be applied is known as *absolute correction* C_a and is given by:

$$C_a = \text{True length} - \text{nominal length} \tag{3.10}$$

If the true length is shorter than the nominal length the correction should be subtracted while if the true length is greater than the nominal length, the correction is to be added.

3.6.2 CHAIN OR TAPE NOT HORIZONTAL

When the chain or tape is inclined but assumed to be horizontal an error in measurement is introduced. The horizontal distance is always less than the inclined length, hence the correction is always subtractive. The correction is given by Eqs. (3.6) and (3.9).

3.6.3 FLUCTUATIONS IN TEMPERATURE

A chain or tape is of standard length at a particular temperature. If the ambient temperature changes, the length of the tape also changes, the change being 1.15×10^{-5}/°C. The temperature correction C_t is, therefore

$$C_t = L\alpha\,(T - T_s) \tag{3.11}$$

where L is the length, α is the coefficient of thermal expansion, T is the temperature at which the measurement is made and T_s is the standardization temperature. Temperature effect is less pronounced on a cloudy day or early in the morning or late in the afternoon. Since the coefficient of invar is very small (3.6×10^{-7}/°C), invar tapes will give better result than steel and should be used in surveys of high order.

3.6.4 INCORRECT TENSION OR PULL

A tape or chain is of standard length under a particular pull. In field operation the pull applied may be more or less which will introduce an error. Steel being elastic there will be extension or contraction given by the expression

$$\frac{(P - P_s)L}{A \cdot E} \tag{3.12}$$

which should be applied as pull correction C_P. Here P is the actual pull applied P_s is the standard pull, L is the length of the chain or tape, A is the cross sectional area and E is the modulus of elasticity of steel. E for steel is 2.1×10^7 N/cm^2. For important work a spring balance should be used to determine the exact pull. Otherwise, sometimes more pull or sometimes less pull will be applied though the tendency is to apply less pull than the standard.

3.6.5 SAG

A tape or chain supported at the two ends will always sag, i.e. the mid point will be at a lower level compared to the two ends. As a result the chord or horizontal length will be less than the curved length. Assuming the curve to take an approximate shape of a parabola, the difference between sagged length and chord length is given as

$$L_s - d = \frac{8v^2}{3d} \tag{3.13}$$

where

L_s = unsupported length of tape
d = chord length
v = sag in the middle.

Also by taking moment about one of the supports for half the tape length we get,

$$P_1 v = \frac{wd^2}{8} \tag{3.14}$$

where w is the weight of the tape/unit length and P_1 is the pull.

Combining the two equations, sag correction becomes

$$L_s - d = \frac{8v^2}{3d} = \frac{8}{3d}\left(\frac{wd^2}{8P_1}\right)^2 = \frac{w^2 d^3}{24P_1^2}$$

Substituting L in place of d to simplify the result, we get

$$\text{Sag correction } C_s = \frac{w^2 L^3}{24P_1^2} \tag{3.15}$$

Sag correction is always negative as the correct length is always less than the measured length.

Normal tension when applied to a tape or chain will increase the length in such a way that sag correction will be compensated so that no sag correction will be necessary. This pull P_n is given by the expression:

$$P_n = \frac{0.204\, W \sqrt{AE}}{\sqrt{P_n - P_s}} \tag{3.16}$$

which is to be solved by trial and error.

Free tension is that tension which when applied will eliminate the need for corrections required due to tape standardization, temperature, sag and tension. This means

$$C_a + C_t - C_s + C_p = 0 \tag{3.17}$$

Like Eq. (3.16), Eq. (3.17) can also be solved by trial and error to find the free tension P_f.

Equation (3.16) can be derived as follows:

$$\text{Sag correction} = \frac{w^2 L^3}{24P_n^2} = \frac{W^2 L}{24P_n^2}$$

$$\text{Pull correction} = \frac{(P_n - P_s)L}{AE}, \text{ where } P_s \text{ is standard pull}$$

Equating

$$\frac{W^2 L}{24P_n^2} = \frac{(P_n - P_s)L}{AE}$$

$$P_n = \frac{\sqrt{AE}}{\sqrt{P_n - P_s}} \cdot W\left(\frac{1}{24}\right)^{1/2}$$

$$= \frac{\sqrt{AE}\ W(0.204)}{\sqrt{P_n - P_s}} = \frac{0.204\ W\sqrt{AE}}{\sqrt{P_n - P_s}}$$

3.6.6 INCORRECT ALIGNMENT

In taking a number of chain or tape measurements along a line the tape or chain may be off line and thus introduce systematic error. Equation (3.1) can be used to determine correct horizontal length with α being the horizontal off line angle instead of the slope angle. The correction is always subtractive.

3.6.7 CHAIN OR TAPE NOT STRAIGHT

When a chain or tape is not straight but gets bent due to bending of the links of the chain or bending of part of tape, the reading will always be more than the actual distance and the correction will be always subtractive. However, the magnitude is difficult to obtain unless compared with a standard chain or tape.

3.7 RANDOM ERRORS

The difference between systematic and random errors has already been explained. The systematic errors in chain or tape survey may become random when there is uncertainty about their magnitude and sign. Some of the random errors in chaining or taping are: (i) Incorrect determination of temperature, (ii) Incorrect application of pull, (iii) Deflection of plumb bob due to wind, (iv) Incorrect fixation of taping pin, (v) Incorrect reading.

Table 3.1 summarizes different characteristic of the types of errors discussed. Errors can be instrumental (*I*), natural (*N*) or personal (*P*):

Table 3.1 Classification of Errors

Type of errors	Classification	Systematic (S) or random (R)
Tape length	*I*	S
Temperature	*N*	S or R
Pull	*P*	S or R
Sag	*N, P*	S
Alignment	*P*	S
Tape not level	*P*	S
Plumbing	*P*	R
Marking	*P*	R
Interpolation	*P*	R

Example 3.3 A 30 m tape weighs 12 g/m and has a cross sectional area of 0.020 cm^2. It measures correctly when supported throughout under a tension of 85 newton and at a temperature of 20°C. When used in the field, the tape is only

supported at its ends, under a tension of 85 newton. The temperature is 13.5°C. What is the distance of zero and 30 mark under these conditions?

Solution There is variation from standardized condition as regards (i) Temperature, (ii) Support at the ends instead of throughout.

(i) Correction for temperature

$$= L \cdot \alpha (T - T_s)$$
$$= 30 \times 1.15 \times 10^{-5}(20 - 13.5)$$
$$= .00224 \text{ m.}$$

This correction is additive as the length of the tape is more than the standard because of rise in temperature.

(ii) Sag correction $= \dfrac{w^2 L^3}{24P^2}$

Weight of tape = 12 g/m = .012 kg/m = 0.12 N/m.

$$C_s = \frac{(0.12)^2(30^3)}{(24)(85^2)}$$
$$= .00224 \text{ m.}$$

The sag correction is negative as the correct length is always less than the measured length.

Total correction = 0.00224 − .00224 = 0

Distance between 0 and 30 mark = 30 m.

Example 3.4 A 30 m steel tape measured 30.0150 m when standardized fully supported under a 70 newton pull at a temperature of 20°C. The tape weighed 0.90 kg (9N) and had a cross sectional area of 0.028 cm^2. What is the true length of the recorded distance *AB* for the following condition? (Assume all full tape lengths except in the last one.)

Recorded distance *AB*	Average temperature	Means of support	Tension	Elevation difference per 100 m.
114.095 m	12°	Suspended	100 N	2.5 m

Solution

(i) Correction for absolute length $= + \dfrac{(30.0150 - 30.00)}{30.00} \times 114.095$

$$= + 0.0570 \text{ m}$$

(ii) Temperature correction $= L\alpha (T - T_s)$

$$= 114.095 \times 1.15 \times 10^{-5} \times (12° - 20°)$$
$$= -0.01049674 \text{ m}$$

(iii) Pull correction $= \dfrac{(P - P_s) L}{AE}$

$$= \frac{(100 - 70)(114.095)}{(0.028)(2.1 \times 10^7)} = +.0058211 \text{ m}$$

(iv) Sag correction $= \dfrac{W^2 L}{24P^2}$ (negative)

$$= \left(\frac{9^2 \times 30}{24 \times 100^2}\right)(3) + \frac{\left(\frac{9}{30} \times 24.095\right)^2 (24.09)}{24 \times 100^2}$$

$$= .030375 + .005245 = .0356208 \text{ (negative)}$$

(v) Correction for slope $= -\dfrac{d^2}{2L}$

$$= -\frac{2.5 \times 2.5}{2 \times 100} = -.03125 \text{ m/100 m}$$

For 114.095 m, slope correction

$$= -\frac{.03125}{100} \times 114.095 = -.0356546 \text{ m}$$

Hence total correction

$$= +.0570 - .01049 + .0058 - .0356 - .03565$$

$$= -.01894.$$

Corrected length = 114.076 m.

Example 3.5 A steel tape of length 30 m standardized on the flat under a pull of 49 N has a width of 12.70 mm and a thickness of 0.25 mm. It is to be used on the site to measure lengths of 30 m to an accuracy of ± 1/10,000. Assuming that the ends of the tape are held at the same level and that the standardizing temperature for the tape obtains on the site, determine the increases in tension to be applied to realize that accuracy. Take the density of steel as 7750 kg/m^3, Young's Modulus as 20700 MN/m^2 and the acceleration due to gravity as 9.806 m/s^2. [Salford]

Solution

Permissible error on 30 m $= \pm \dfrac{30 \times 1000}{10{,}000} = \pm 3$ mm

Weight of unit length of tape $= .0127 \times .00025 \times 1 \times 7750$

$= .024606$ kg/m $= .24129$ N/m

Let P be the pull applied.

$$\text{Pull correction} = \frac{(P - 49)(30)}{(.0127 \times .00025)(20700)(10^6)}$$

$$\text{Sag correction} = \frac{(.24129)^2 \times 30^3}{24P^2}$$

When the error is $\pm \frac{1}{10{,}000}$,

$$\pm 3 \text{ mm} = \frac{(P-49)(30)(10^3)}{(.0127 \times .00025)(20700)(10^6)} - \frac{(.24129)^2 \times 30^3}{24P^2} \times 10^3$$

$$= (P-49)(.045646) - \frac{65498.472}{P^2}$$

Solving by trial and error, when

$P = 50$	RHS = .045646 − 26.199 = − 26.153
$P = 100$	RHS = 2.3279 − 6.5498 = − 4.22
$P = 150$	RHS = 4.610 − 2.911 = 1.698
$P = 200$	RHS = 6.892 − 1.637 = 5.255

− 3 will lie between 100 and 150 App. value = 107.5 N
+ 3 will lie between 150 and 200 App. value = 167 N.

Example 3.6 A tape which was standardized on the flat under a tension P_s was used in catenary to measure the length of a base line. Show that the nominal corrections for pull and sag must be modified by factors of $\pm\, \delta P/(P - P_s)$ and $\mp\, 2\delta P/P$ respectively if an error of $\pm\, \delta P$ occurred in the applied field tension P.

The length of the line was deduced as 659.870 m, the apparent field tension being 178 N. Determine (i) The nominal corrections for pull and sag which would have been evaluated for each 30 m tape length, and (ii) The corrected length of the line if the actual field tension was 185 N.

The tape which had a mass of 0.026 kg/m and a cross-sectional area of 3.25 mm² was standardized on the flat under a pull of 89 N. Take Young's modulus as 155,000 MN/m² and the acceleration due to gravity as 9.806 m/s².

Solution
Theoretical Part

(i) Pull correction $C_P = \dfrac{(P - P_s)\, L}{AE}$

$$\delta C_P = \frac{\delta P \cdot L}{AE}$$

$$\pm \frac{\delta C_P}{C_P} = \pm \frac{\delta PL}{AE}\,\frac{AE}{L}\,\frac{1}{(P-P_s)}$$

$$= \pm \frac{\delta P}{(P-P_s)}$$

(ii) Sag correction $C_s = \dfrac{w^2 L^3}{24P^2}$

$$\delta C_s = -\frac{w^2L^3}{24}\cdot\frac{\delta P}{P^3}\cdot 2$$

$$\pm\frac{\delta C_s}{C_s} = \mp\frac{w^2L^3}{24}\cdot\frac{\delta P}{P^3}\cdot\frac{24P^2}{w^2L^3}\cdot 2 = \mp\frac{2\delta P}{P}.$$

Mathematical Part

$$\text{Pull correction for 178 N} = \frac{(178-89)(30)\times 1000}{3.2\times 155000}$$

$$= +\,5.383 \text{ mm}$$

$$\text{Sag correction for 178 N} = -\frac{w^2L^3}{24P^2}$$

$$= -\frac{(.026\times 9.806)^2\times 30^3}{24\times 178^2}\times 1000 = -\,2.3 \text{ mm}$$

$$\text{Modification factor for pull} = \pm\frac{\delta P}{P-P_s}$$

$$= +\frac{(185-178)}{(178-89)} = +\,.07865$$

$$\text{Modification factor for sag} = -\frac{2\delta P}{P} = -\frac{2\times 7}{178} = -\,.07865$$

Change in correction for 30 m tape = (5.38)(.07865) – (2.3)(.07865)

= .423137 – .180895 = .242242

$$\text{No. of 30 m tapes} = \frac{659.87}{30} \approx 22$$

Correction = .242242 × 22 = 5.329324 mm

Correct length = 659.870 + .005 = 659.875 m

Example 3.7 A steel tape, 30 m long was standardized on the flat, under a pull of 89 N. If the tape had a cross-sectional area of 3 mm² and a mass of 0.024 kg/m, determine the field tension to be applied in order that the correction in tension was equal in magnitude to the correction for sag. What error was induced in the sag correction by an error of + 6 N in that tension? Young's modulus = 155,000 MN/m².

Solution

$$\text{Pull correction} = \frac{(P-P_s)\,L}{AE} = \frac{(P-89)(30)(1000)}{(3)(155{,}000)}$$

$$\text{Sag correction} = \frac{w^2L^3}{24P^2} = \frac{(.024\times 9.806)^2\,(30^3)}{24P^2}\,(1000)\text{ mm}$$

By trial P = 140 N	Pull correction = 3.29 Sag correction = 3.16	}
P = 138 N	Pull correction = 3.16 Sag correction = 3.27	}
P = 139 N	Pull correction = 3.22 Sag correction = 3.22	}

$$\text{Change in sag correction} = -3.22 \times \frac{2\delta P}{P}$$

$$= -\frac{3.22 \times 2 \times 6}{139} = -0.27798 \text{ mm.}$$

Example 3.8 A copper transmission line 12.7 mm diameter is stretched between two points 300 m apart at the same level, with a tension of 5 kN when the temperature is 35°C. It is necessary to define its limiting positions when the temperature varies. Making use of the corrections for sag, temperature, and elasticity normally applied to base line measurements in catenary, find the tension at a temperature of −15°C and the sag in the two cases. Young's modulus for copper is 68,950 MN/m^2, its density 8890 kg/m^3 and its coefficient of linear expansion 15×10^{-6}/°C. [London University]

Solution Weight of transmission line/m

$$= \frac{\pi}{4}(12.7^2)\left(\frac{1}{10^6}\right)(1)(8890)(9.806) = 11.043125 \text{ N/m}$$

Initial length of line

$$= 300 + \frac{(11.043125)^2(300)^3}{(24)(5^2)(10^6)} = 305.48777 \text{ m}$$

With this length of line a better approximation for sag

$$= \frac{(11.043125)^2(305.48777)^3}{(24)(5)^2(10^6)} = 5.79403 \text{ m}$$

Hence correct length of line = 305.79403 m

$$\text{Amount of sag} = \frac{wL^2}{8T} = \frac{(11.043125)(305.794)^2}{(8)(5)(1000)} = 25.81 \text{ m}$$

When the temperature falls to −15°C, let T_1 be the tension.

Total present length of transmission line

$$300 + \frac{w^2L^3}{24T_1^2} = 300 + \frac{(11.043125)^2(305.794)^3}{(24)(10^6)(T_1)^2} = 300 + \frac{145.29}{T_1^2}$$

Contraction of wire $L\alpha t$

$$= (305.794)(15)(10^{-6})(35 - (-15)) = 0.2293455 \text{ m}$$

$$\text{Extension due to increase in tension} = \frac{(T_1 - 5)(305.794)(10)^3}{(\pi/4)(12.57)^2(68950)}$$

Equating,

$$300 + \frac{145.29}{T_1^2} = 305.79403 - 0.2293455 + 0.03573(T_1 - 5)$$

By trial and error $T_1 = 5.11$ kN

$$\text{New sag} = \frac{(11.043125)(305.564)^2}{(8)(5.11)(1000)} = 25.222 \text{ m}$$

Example 3.9 A tape of nominal length 30 m is standardized in catenary at 40 N tension and found to be 29.8850 m. If the mass of the tape is 0.015 kg/m, calculate the horizontal length of a span recorded as 16 m.

Solution Standardized chord length = 29.8850 m

$$\text{Sag correction } C_s = \frac{(.015 \times 9.806/40)^2 \times 30^3}{(24)}$$

$$= + .0152 \text{ m}$$

Standardized arc length = 29.9002

Standardization error per 30 m = – .0998 m

Recorded arc length = 16.000 m

$$\text{Standardization error} = \frac{(16.000)(-.0998)}{30} = -.0532 \text{ m}$$

Standardized arc length = 15.9468 m

$$\text{Sag correction} = C_s \times \left(\frac{16.00}{30}\right)^3 = -.0023 \text{ m}$$

Standardized chord length = 15.9468 – .0023 = 15.9445 m

(i) *Tape used vertically for measurement*

When a measurement is taken keeping the tape vertical say, in mining operations, the tape will extend under its own weight which can be obtained as follows (Fig. 3.9).

Let m = mass of tape/unit length

g = acceleration due to gravity

A = cross sectional area of the tape

E = Young's modulus

Force acting on element $dx = mg \cdot x$

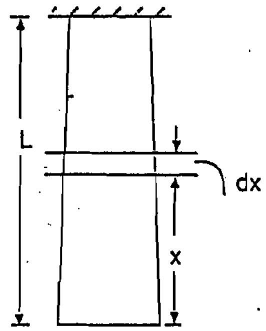


Fig. 3.9 Tape measured vertically.

$$\text{Extension } \delta e = \frac{mg \cdot x \cdot \delta x}{AE}$$

$$\text{Integrating } e = \frac{mg \cdot x^2}{2AE} + C$$

When x varies from 0 to L.

$$e = \frac{mg \cdot L^2}{2AE}$$

(ii) *Sag correction with the supports not at the same level*

When the supports are not at the same level, sag correction

$$C_s' = C_s \cos^2 \theta \left(1 + \frac{wL}{P} \cdot \sin \theta\right) \tag{3.18}$$

when tension P is applied at the higher end and is equal to

$$C_s \cos^2 \theta \left(1 - \frac{wL}{P} \cdot \sin \theta\right) \tag{3.19}$$

when tension P is applied at the lower end. Here θ is the angle of inclination with the horizontal and when θ is small C_s' becomes $C_s \cos^2 \theta$. The above formulae (3.18) and (3.19) include the effect of slope and as such separate slope correction is not necessary.

Example 3.10 Calculate the elongation of a 30 m tape suspended under its own weight at (a) 30 m from top; (b) 10 m from top. Given that $E = 20.7(10^{10})\text{N/m}^2$, mass of the tape is 0.0744 kg/m and the cross sectional area is $9.6(10^{-6})\text{m}^2$.

Solution

(i) $e = \dfrac{mg \cdot L^2}{2AE} = \dfrac{(.0744)(9.806)(30)^2}{(2)(9.6)(10^{-6})(20.7)(10^{10})} = 0.0001652$ m.

(ii) At 10 m from top $x = 20$ m.

$$e = \frac{(.0744)(9.806)(20)^2}{(2)(9.6)(10^{-6})(20.7)(10^{10})} = .00007342 \text{ m.}$$

Example 3.11 A nominal distance of "30 m" was set out with a 30 m tape from a mark on the top of one peg to a mark on the top of another. the tape being in catenary under a pull of 90 N. The top of one peg was 0.370 m below the other. Calculate the horizontal distance between the marks on the two pegs. Assume density of steel $7.75(10^3)$ kg/m^3, section of tape 3.13 mm by 1.20 mm, Young's modulus $2(10)^5$ N/mm^2.

Solution

Weight of tape/unit length = $(3.13)(1.2)(10)^{-6}(7.75)(10^3) = .029109$ kg/m.

$$\text{Correction for catenary} = \frac{w^2L^3}{24P^2}$$

$$= \frac{(2.9109 \times 9.806)^2(30)^3(10)^{-4}}{(24)(90)^2} = .0113 \text{ m}$$

$$\theta = \tan^{-1}\frac{37}{30} = 0.7066°$$

$$\sin\theta = 0.01233, \quad \cos\theta = 0.9999$$

$$\cos^2\theta = 0.9998$$

When tension is applied at the top end

$$C_s' = -(.0113)(.9998)\left(1 + \frac{(.029109)(9.806)(.01233)}{90}\right)$$

$$= -(.0113)(.9998)(1 + .0000391) = -.0112981 \text{ m}$$

Horizontal distance = 30 − .0112981 = 29.9887019 m.

When tension is applied at the lower end

$$C_s' = -.0112972 \text{ m.}$$

Horizontal distance = 30 − .0112972 = 29.9887028 m.

3.8 CHAIN AND TAPE SURVEY OF A FIELD

A field may be completely surveyed by a chain and tape or by tape only. In fact this was the only method available before instruments for measuring angles were developed. Now EDM equipments have brought this method to use again.

The method consists of dividing a field into a number of triangles and measuring the sides of each triangle. The field may be covered by a chain of triangles as in Fig. 3.10 or by a number of triangles with a central station as in Fig. 3.11. The triangles should be such that lengths of the sides do not differ widely when they become well conditioned triangles. If they differ widely the triangle is "ill conditioned". This type of survey is suitable for surveys of small extent on open ground with simple details. However, the following basic principles should be followed:

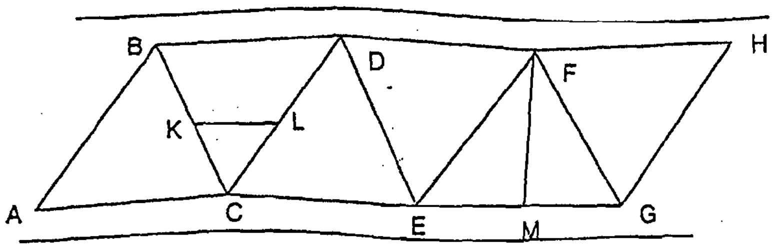


Fig. 3.10 Chain of triangles.

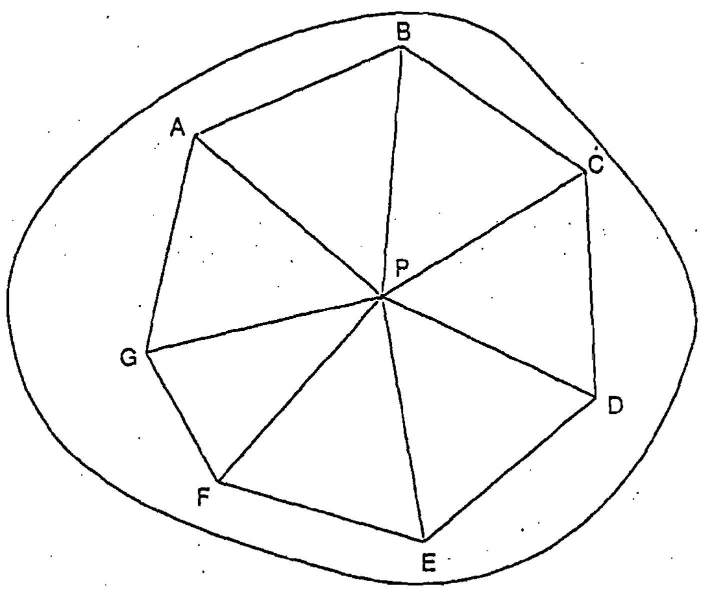


Fig. 3.11 Polygon with central station.

1. Always work from the whole to the part. The area should always be covered with as big triangles as possible. The tie line can then be plotted to fix details.
2. Always make provisions for adequate checks. Hence we have check lines. In Fig. 3.10. *A*, *B*, *C*, *D*, ... are station points, *AB*, *BC*, *AC*, *BD*, ... are chain lines, *FM* is a check line and *KL* is a tie line.

The interior details are usually plotted with respect to the chain line by taking measurements perpendicular to them when we have perpendicular offsets and sometimes taken at an angle to the chain line when we have oblique offsets.

In Fig. 3.12 *AB* is the chain line, *PQ* and *RS* are perpendicular offsets. In Fig. 3.12 *P* can be plotted if *AQ* and *PQ* are known where *PQ* is the perpendicular offset and *AQ* is the chain length. Similarly in Fig. 3.13 *P* can be plotted if *AP*, *PQ* and *AQ* are known where *AP*, *PQ* are oblique offsets and *AQ* is the chain length. To avoid error offsets should be as small as possible.

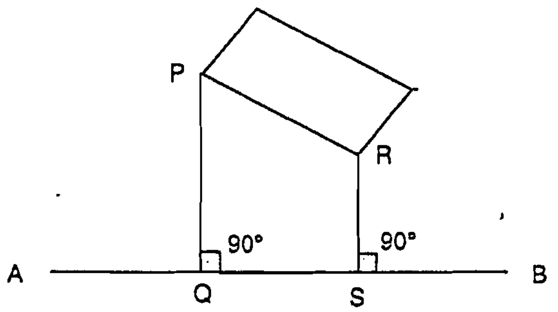


Fig. 3.12 Perpendicular offsets.

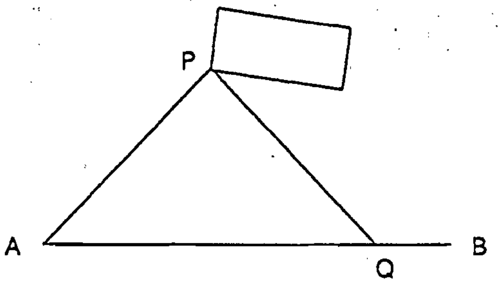


Fig. 3.13 Oblique offsets.

3.9 ERROR IN OFFSET

The offset may not be set exactly at right angle to the chain line but deviate from right angle through a small angle α. The horizontal displacement PP_2 in plotting is then $l \sin \alpha / S$ cm, where l = length of offset in meters and S = scale (1 cm = S meter) as shown in Fig. 3.14.

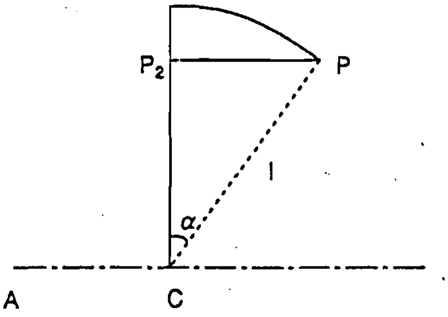


Fig. 3.14 Error in offset angle.

When there is error both in length and direction the total error is PP_2 as shown in Fig. 3.15. Taking PP_1P_2 as a right angled triangle

$$PP_2 = \sqrt{PP_1^2 + P_1P_2^2}$$

$$PP_1 = \delta l \quad \text{and} \quad P_1P_2 = l \sin \alpha$$

giving

$$PP_2 = \sqrt{\delta l^2 + (l \sin \alpha)^2}$$

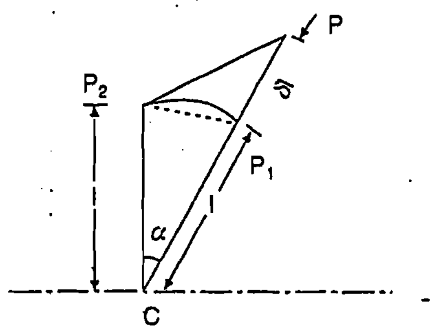


Fig. 3.15 Error in length and angle of offset.

Example 3.12 · Find the maximum permissible error in laying off the direction of offset so that the maximum displacement may not exceed 0.25 mm on the paper, given that length of the offset is 10 meters, the scale is 20 m to 1 cm and the maximum error in the length of the offset is 0.3 m.

Solution

$$PP_2 = \sqrt{\delta l^2 + (l \sin \alpha)^2}$$

Here $$PP_2 = 0.25 \text{ mm}$$

$$\delta l = \frac{0.3}{20} \times 10 \text{ mm} = 0.15 \text{ mm}$$

$$l = \frac{10}{20} \times 10 \text{ mm} = 5 \text{ mm}$$

$$0.25 = \sqrt{(0.15)^2 + (5 \sin \alpha)^2}$$

or $$(5 \sin \alpha)^2 = 0.25^2 - 0.15^2$$

or $$\sin \alpha = \frac{\sqrt{0.25^2 - 0.15^2}}{5}$$

or $$\alpha = 2.292° = 2°17'31''$$

3.10 INSTRUMENTS FOR SETTING OUT RIGHT ANGLES

The instruments used to set offsets at right angles to the chain line are (i) Cross staff, (ii) Optical square, and (iii) Prism square.

3.10.1 CROSS STAFF

The cross staff is the simplest instrument for setting out right angles. Two types of cross staff are shown in Fig. 3.16(a) and (b). Figure 3.16(a) shows the open cross staff with two pairs of vertical slits giving two lines of sights at right angles to each other. Figure 3.16(b) shows a French cross staff. It is essentially an octagonal brass box with slits cut in each face so that opposite pairs form sight

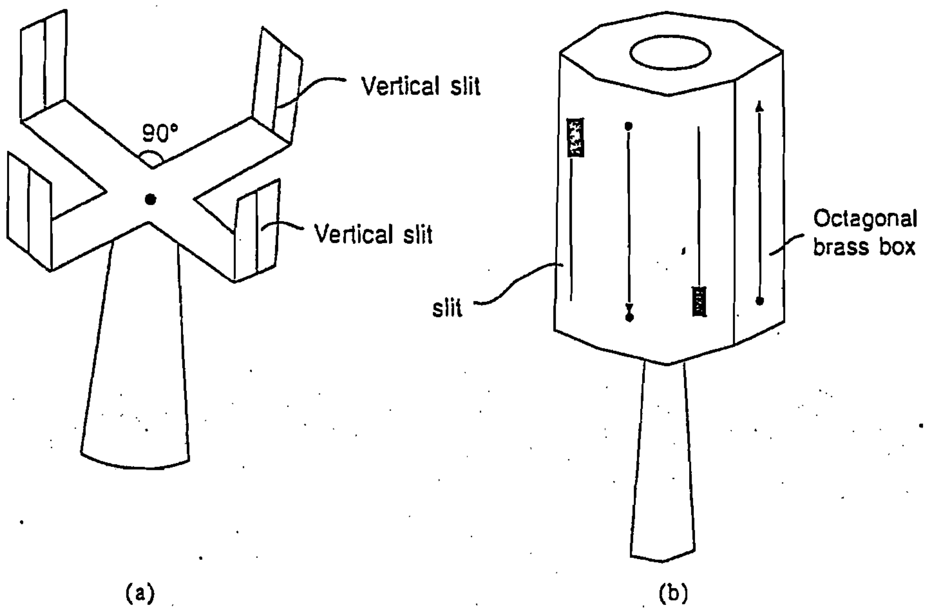
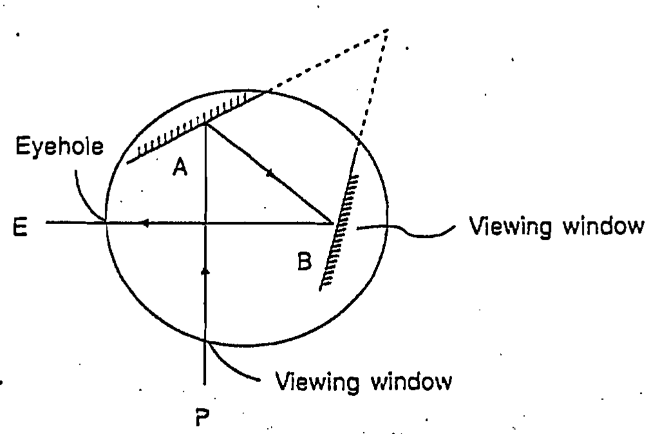


Fig. 3.16 (a) Schematic diagram of open cross staff. (b) French cross staff.

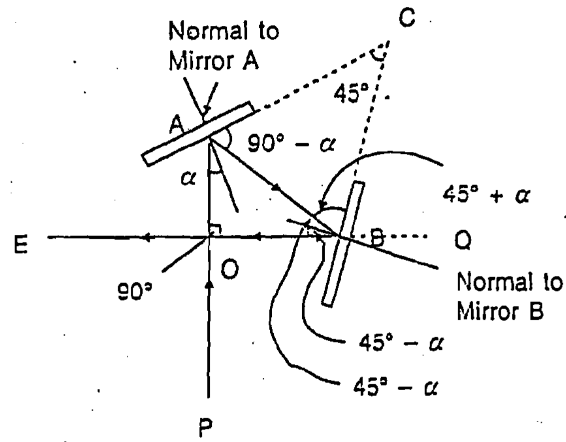
lines. The instrument is mounted over a short ranging rod and two sights, are observed through slits at right angles to each other. The other two pairs enable angles of 45° and 135° to be set out.

3.10.2 OPTICAL SQUARE

This is a handy instrument with three openings and is based on optical principle as shown in Fig. 3.17(a). If two mirrors *A* and *B* are fixed at an angle of 45°, rays from a point *P* will get reflected at first mirror *A* and then again get reflected at *B* to meet the eye *E*. The lines *PA* and *BE* are at right angles which can be seen from Fig. 3.17(b).



(a) Optical square.



(b) Path of rays in optical square.

Fig. 3.17 Optical Square.

If the incident ray *PA* makes an angle α with the normal, the reflected ray *AB* will also make the same angle. Hence

$$\angle CAB = 90° - \alpha$$

with $$\angle C = 45°$$

$$\angle ABC = 45° + \alpha$$

and therefore $$\angle ABE = 90° - 2\alpha$$

Hence $$\angle AOB = 90°.$$

In the optical square the mirror *B* is half silvered. To see whether the two lines are at right angles, observer at *E* sees a pole at *Q* through the unsilvered portion and the image of the pole at *P* through silvered portion of the mirror *B*. When the two poles appear coincident the two lines *PO* and *EQ* are at right angle and *O* is the foot of the perpendicular from *P* on *EQ* at *O*.

3.10.3 PRISM SQUARE

The same principle as described in optical square is followed in the working of prism square as shown in Fig. 3.18. The advantage of prism square is that the angle 45° is always fixed and needs no adjustment unlike in the optical square.

3.11 MISCELLANEOUS PROBLEMS IN CHAINING

In practical surveying many types of obstacles are encountered which can be classified as (i) Obstacles to ranging but not chaining. (ii) Obstacles to chaining but not ranging, and (iii) Obstacles to both chaining and ranging.

Case (i) can be further subdivided into two groups:

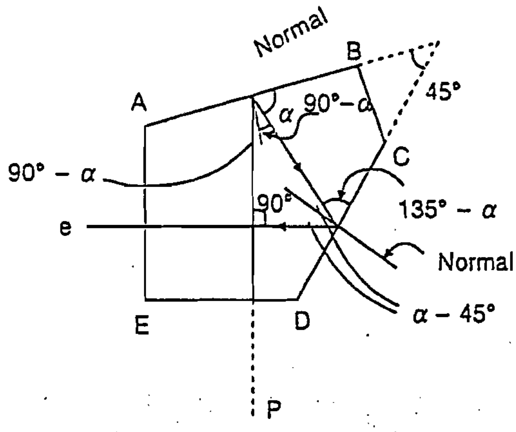


Fig. 3.18 Path of rays in prism square.

(a) When both ends of the line may be visible from intermediate points on the line.

(b) When both ends are not visible from intermediate points.

In case (a) recourse is to be taken to *reciprocal ranging*. As shown in Fig. 3.19 two intermediate points M_1 and N_1 are selected such that from M_1 and N_1 and B are visible. Similarly from N_1 both M_1 and A are visible. First a range man at M_1 will ask the range man at N_1 to move to N_2 such that M_1N_2B are in one line. Similarly range man at N_2 will ask range man M_1 to move to M_2 such that AM_2N_2 are in one line. The process will be repeated till A, M, N, B are in one line as shown in Fig. 3.19.

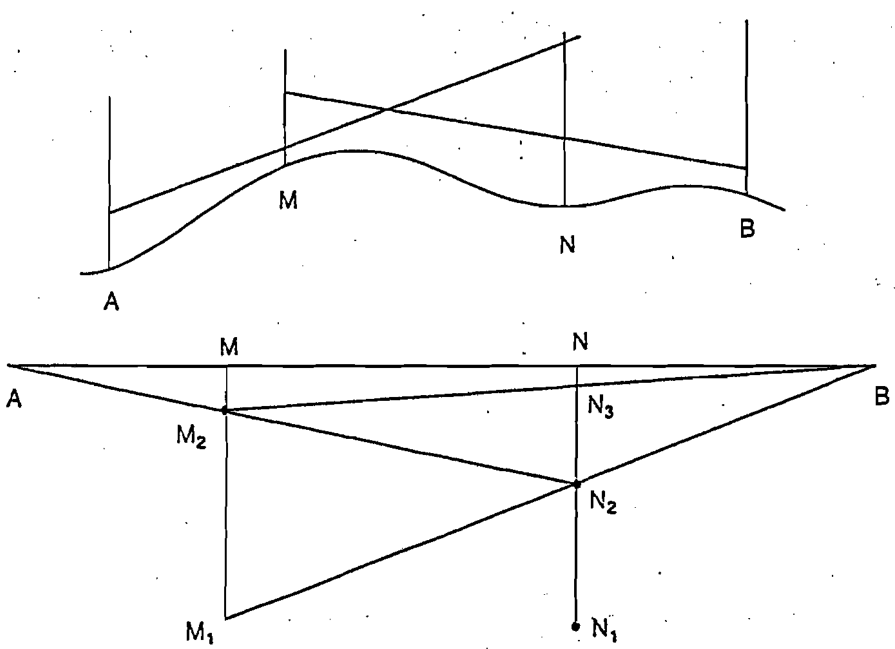


Fig. 3.19 Reciprocal ranging (two ends visible).

In case (b) as shown in Fig. 3.20, a random line AB_1 should be chosen such that B_1 is visible from B and BB_1 is perpendicular to the random line. Then computing C_1C and D_1D from consideration of proportionate triangle C and D can be plotted. Finally CD is joined and prolonged.

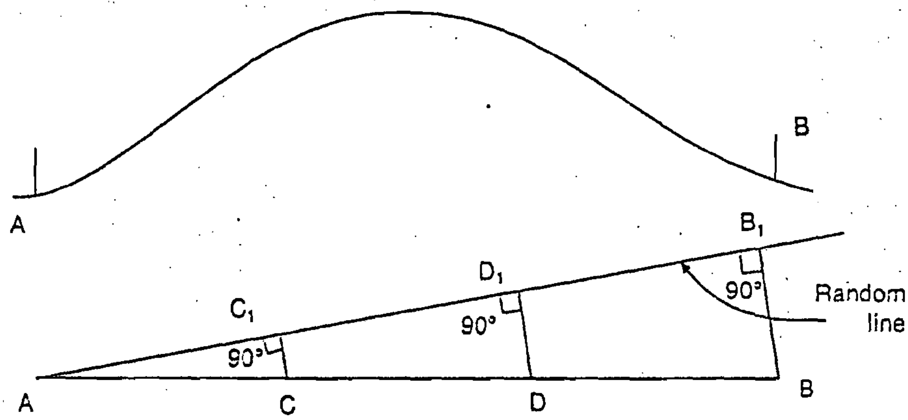


Fig. 3.20 Reciprocal ranging (two ends not visible).

Obstacles to chaining but not ranging are encountered while crossing rivers. Obstacles to both chaining and ranging occur while chaining across a building. These are exemplified by solving a few typical problems.

Example 3.13 A survey line ABC crossing a river at right angles cuts its banks at B and C (Fig. 3.21). To determine the width BC of the river, the following operation was carried out.

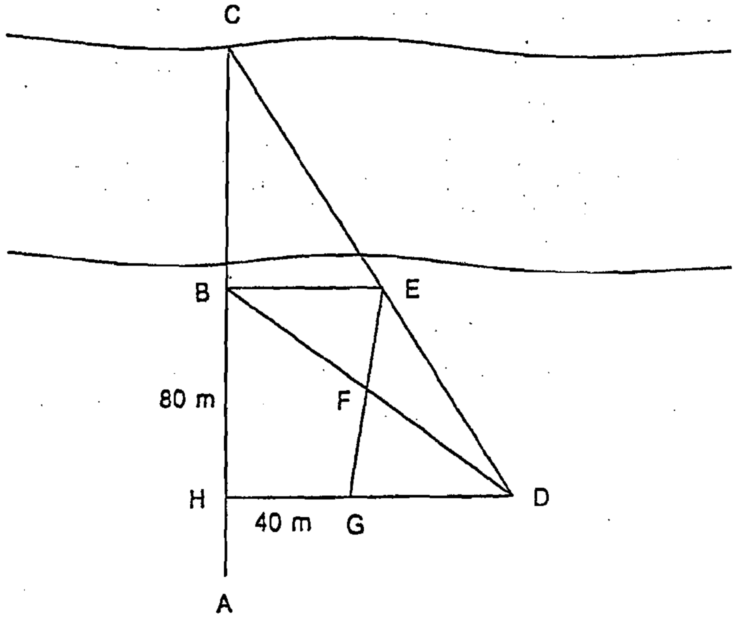


Fig. 3.21 Example 3.13.

A line *BE* 60 m long was set out roughly parallel to the river. Line *CE* was extended to *D* and mid-point *F* and *DB* was established. Then *EF* was extended to *G* such that *FG* = *EF*. And *DG* was extended to cut the survey line *ABC* at *H*. *GH* and *HB* were measured and found to be 40 m and 80 m respectively. Find the width of the river.

[AMIE, Summer 1981]

Solution As *BF* = *FD* and *EF* = *FG*, *BE* and *GD* are parallel and equal. Hence *GD* = 60 m

From ratio and proportion

$$\frac{BE}{HD} = \frac{CB}{CH} = \frac{60}{100}$$

or

$$\frac{CB}{CH - CB} = \frac{60}{100 - 60} = \frac{60}{40}$$

or

$$CB = \frac{80 \times 60}{40} = 120 \text{ m}.$$

Example 3.14 A river is flowing from west to east. For determining the width of the river two points *A* and *B* are selected on southern bank such that distance *AB* = 75 cm (Fig. 3.22). Point *A* is westwards. The bearings of a tree *C* on the northern bank are observed to be 38° and 338° respectively from *A* and *B*. Calculate the width of the river.

[AMIE, Summer 1982]

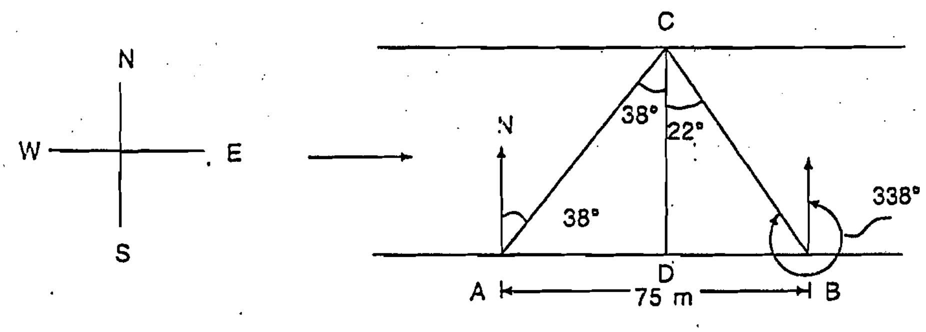


Fig. 3.22 Example 3.14.

Solution Let *CD* be the width of the river.

$$\frac{AD}{CD} = \tan 38°$$

$$\frac{DB}{CD} = \tan 22°$$

Hence $AD = CD \tan 38°$

$DB = CD \tan 22°$

Adding $AD + DB = 75 = CD(\tan 38° + \tan 22°)$

or $$CD = \frac{75}{\tan 38° + \tan 22°} = 63.27 \text{ m}$$

Example 3.15 *AB* is a chain line crossing a lake. *A* and *B* are on the opposite sides of the lake. A line *AC*, 800 m long is ranged to the right of *AB* clear of the lake. Similarly another line *AD*, 1000 m long is ranged to the left of *AB* such that the points *C*, *B*, and *D* are collinear (Fig. 3.23). The lengths *BC* and *BD* are 400 m and 600 m respectively. If the chainage at *A* is 1262.44 m, calculate the chainage of *B*. [AMIE, Winter 1985]

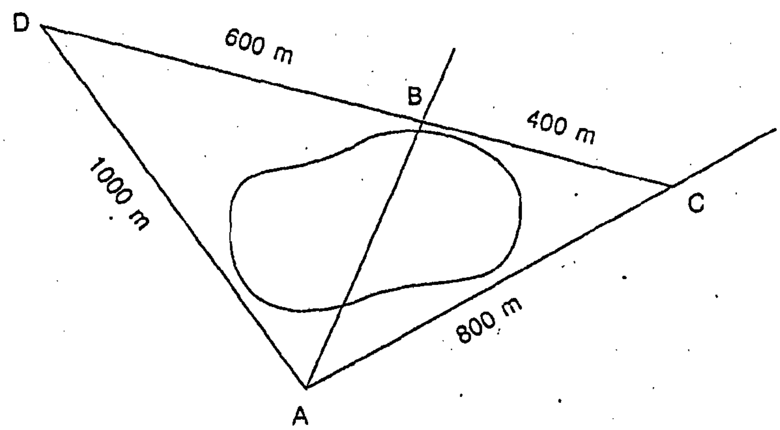


Fig. 3.23 Example 3.15.

Solution From triangle *ACD*,

$$\cos D = \frac{CD^2 + AD^2 - AC^2}{2AD \cdot AC}$$

$$= \frac{1000^2 + 1000^2 - 800^2}{(2)(1000)(800)} = 0.68$$

From ΔADB, $$\cos D = \frac{BD^2 + AD^2 - AB^2}{2DB \cdot AD}$$

or $$AB^2 = BD^2 + AD^2 - 2DB \cdot AD \cdot \cos D$$

$$= 600^2 + 1000^2 - (2)(600)(1000)(0.68) = 544000$$

$$AB = 737.56356 \text{ m}$$

$$\text{Chainage at } B = 1262.44 + 737.56$$

$$= 2000.00 \text{ m}$$

Example 3.16 A survey line *AB* is obstructed by a high building. To prolong the line beyond the building, a perpendicular *BC* 121.92 m long is set at *B*. From *C* two lines *CD* and *CE* are set out at angles of 30° and 40° with *CB* respectively (Fig. 3.24). Determine the lengths *CD* and *CE* so that *D* and *E* may be on the prolongation of *AB*. If the chainage of *B* is 95.10 m find the chainage of *D*. Draw a sketch showing all the points.

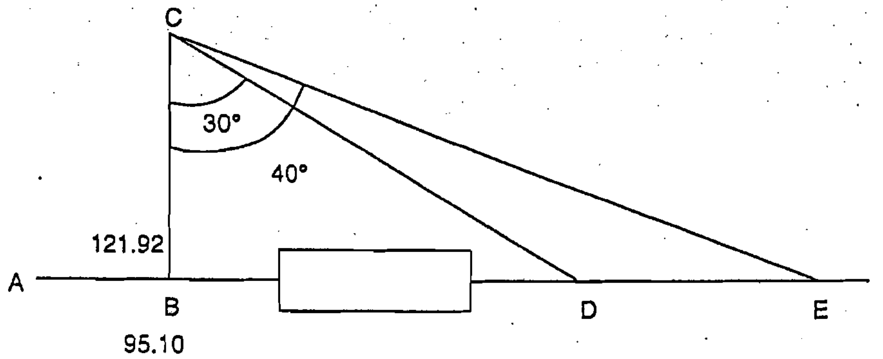


Fig. 3.24 Example 3.16.

Solution $CD \cos 30° = 121.92$ m

$$CD = \frac{121.92}{\cos 30°} = 140.78 \text{ m}$$

$$CE = \frac{121.92}{\cos 40°} = 159.16 \text{ m}$$

$$BD = 121.92 \tan 30° = 70.39 \text{ m}$$

$$\text{Chainage of } D = \text{Chainage of } B + BD$$

$$= 95.10 + 70.39 = 165.49 \text{ m}$$

3.12 FIELD WORK FOR CHAIN SURVEYING

In chain surveying only linear measurements can be taken with the help of chain or tape. No angular measurement is possible. Hence the principle of chain survey or *chain triangulation* as it is sometimes called, is to provide a skeleton or framework of a number of connected triangles as triangle is the only simple figure that can be plotted from the lengths of sides measured in the field. The intersection points of the sides are called stations and these are established by placing ranging rods at station points after reconnaissance survey of the site. The following points should be considered while selecting survey stations or survey lines.

1. Survey stations should be mutually visible.
2. Number of survey lines should be as small as possible.
3. There should be atleast one long back bone line in the survey upon which the surveyor forms the triangles.
4. The lines should preferably run through level ground.
5. The triangles formed should be well conditioned.
6. There should be sufficient checklines.
7. The offsets should be as short as possible. Hence the survey lines should pass close to the objects.

3.12.1 BOOKING THE SURVEY

The data obtained in the field are recorded systematically in an oblong book of size about 200 mm by 120 mm which is known as field book. It opens lengthwise and usually has two lines spaced about 15 to 20 mm apart ruled down the middle of each page. This is double line field book and the distance along the chain line is entered within the double lines. The important steps before starting a survey are:

1. Make a rough sketch in the field book showing the locations of chosen stations and chain lines.
2. The bearing from true or magnetic north of atleast one of the lines should be shown.
3. The stations should be located from three or more points and enough information should be plotted so that they can be relocated if necessary.

The following are the guide lines in recording a field book.

1. Begin each line at the bottom of a page.
2. Sufficient space should be kept in the field book between different chainages. Plotting in the field book need not be to scale.
3. Small details should be plotted in an exaggerated scale.
4. Clear sketches of all details should be shown in the field book. Nothing should be left to memory.
5. Bookings should be done systematically starting with the side having more details.

Figures 3.25, 3.26 and 3.27 show the rough sketch of a plot, the station points and the double line entry in the field book.

3.12.2 CONVENTIONAL SYMBOLS

Different features in survey are represented by different symbols and colours. Figure 3.28 shows some conventional symbols commonly used.

3.12.3 DEGREE OF ACCURACY OF CHAINING

The degree of accuracy which can be attained depends on (i) fineness of graduation of the chain, (ii) nature of the ground, (iii) time and money available, (iv) field

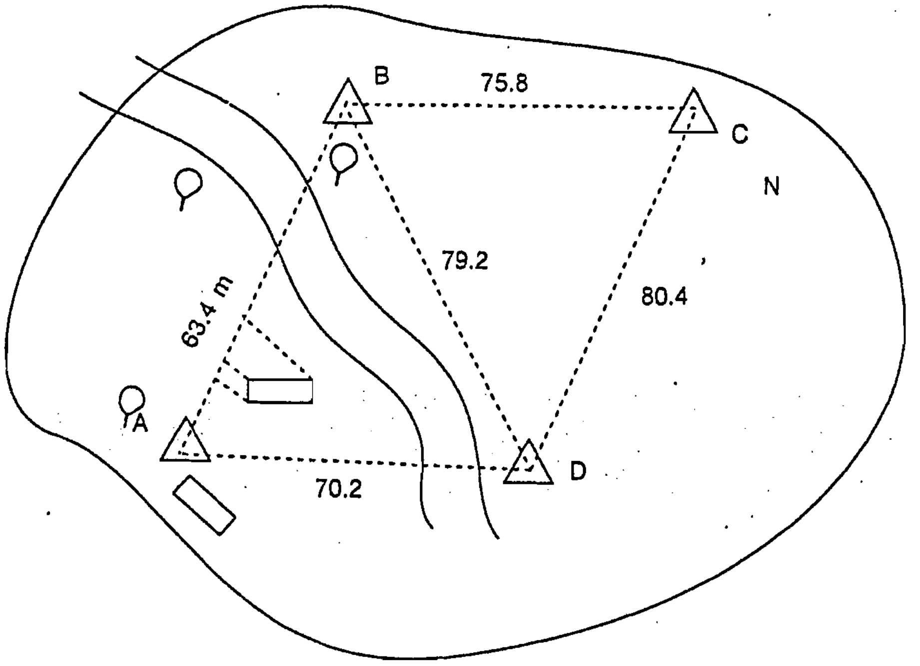


Fig. 3.25 Rough sketch of the plot, stations and chain lines.

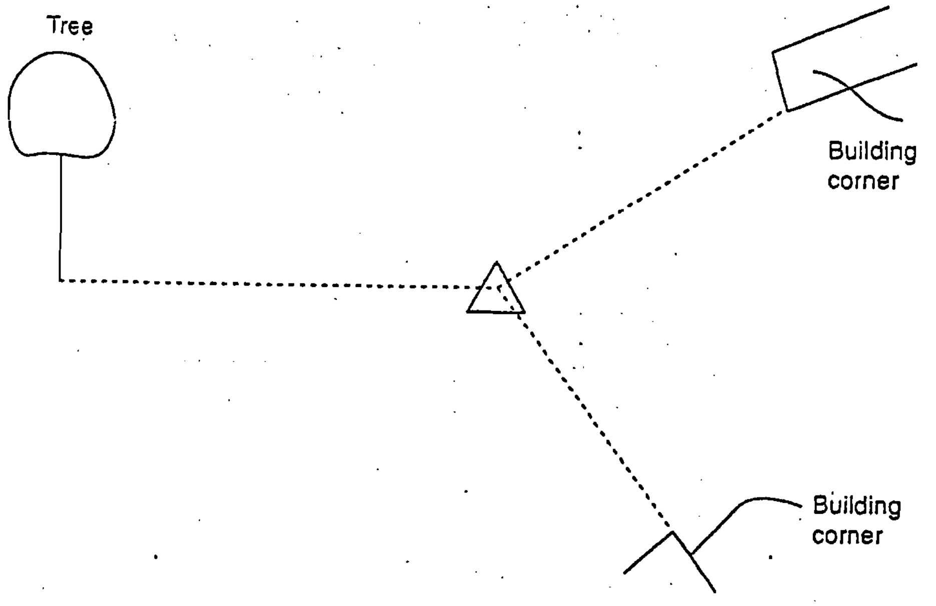


Fig. 3.26 Fixing of a station.

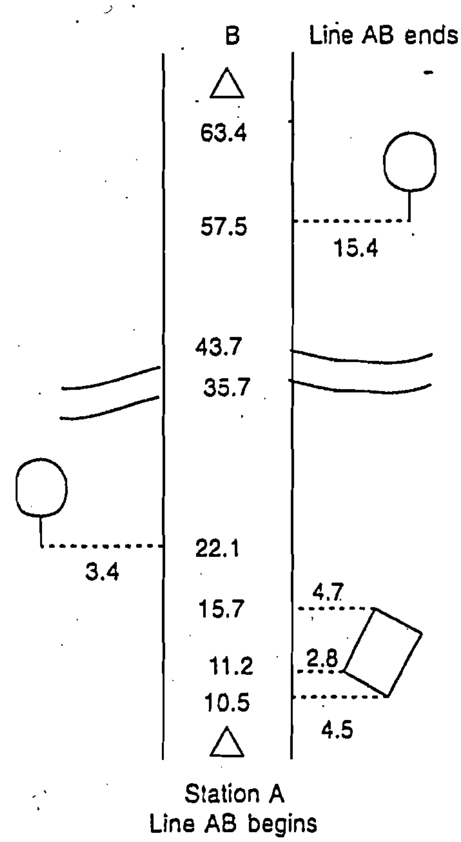


Fig. 3.27 A typical page of double entry field book.

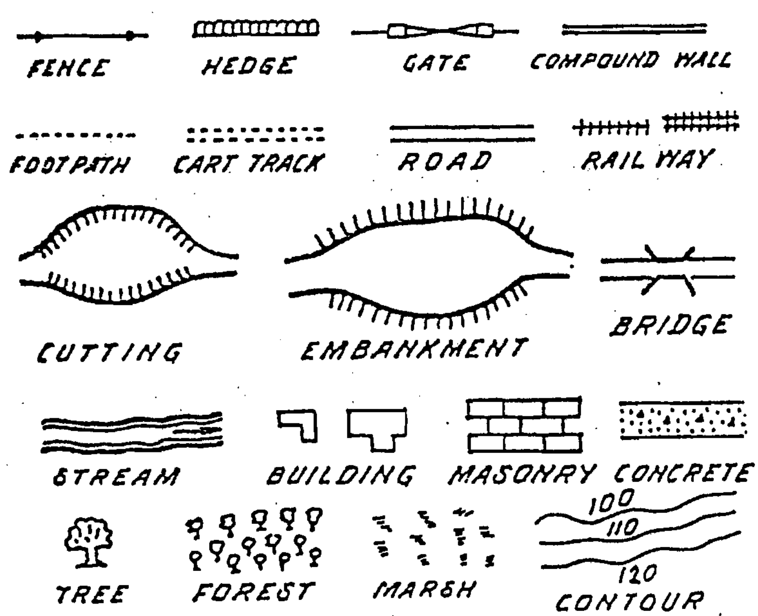


Fig. 3.28 Conventional signs or symbols.

conditions, (v) technical competence of the staff, etc. The following accuracy of chaining can usually be obtained under most suitable conditions.

1. For measurements with chain on rough or hilly ground 1 in 250.
2. For measurement with tested chain, plumb bob 1 in 1,000.
3. For measurement with steel tape 1 in 2,000 to 1 in 20,000.
4. For measurement with invar tape 1 in 20,000 to 1 in 1,000,000.

The accuracy varies from 1 in 100 to 1 in 200 when measurement is done through pacing or pedometer.

PROBLEMS

3.1 Give different methods of measuring horizontal distances. Give an advantage and a disadvantage of each.

3.2 Explain the principle of chain surveying.

3.3 What is a well conditioned triangle? Why it is necessary to use well conditioned triangle?

3.4 State the principles involved in choosing stations for a chain and tape survey.

3.5 What is an offset? What are different types of offsets? Why offsets should be as small as possible?

3.6 Explain the various points which you will keep in mind while recording entries in a field book. Give a neat sketch of a page of the field book.

3.7 A 30 m chain was found to be 15 cm long after chaining 1524 m. The same chain was found to 30.5 cm too long after chaining a total distance of 3048 m. Find the correct length of the total distance chained assuming the chain was correct at the commencement of chaining. [AMIE, May 1966]

3.8 *B* and *C* are two points on the opposite banks of a river along a chain line *ABC* which crosses the river at right angles to the bank. From a point *P* which is 45.720 m from *B* along the bank, the bearing of *A* is 215°30′ and the bearing of *C* is 305°30′. If the length *AB* is 60.960 m find the width of the river. [AMIE, May 1966]

3.9 A tape 100 m long was of standard length under a pull of 4 kg at 12°C. It was then used in catenary in three equal spans of 100/3 m each to measure a level line which was found to measure 3400 m. Calculate the true length of the line from the following data:

$$\text{Pull on tape} = 10 \text{ kg}$$

$$\text{Cross section of tape} = 5 \text{ mm} \times \frac{1}{2} \text{ mm}$$

$$\text{Weight of tape per cubic cm of steel} = 7.7 \text{ gm}$$

$$\text{Mean field temperature} = 20°\text{C}$$

$$\text{Coefficient of expansion} = 0.0000113$$

$$E = 21 \times 10^5 \text{ kg/cm}^2$$

[AMIE, Advanced Surveying, Winter 1978]

3.10 (a) Explain the principle, construction and use of an optical square.
(b) When is it necessary to adopt method of reciprocal ranging? Describe the procedure in detail.
(c) Explain briefly the method of chaining on sloping ground.
[AMIE, Surveying, Winter 1982]

3.11 (a) A survey line *AB* is running along different slopes as detailed below: There is a downward slope of 1 in 10 from station *A* to chainage 238 m. The ground has an angle of elevation of 8°15′ from chainage 238 m to chainage 465 m. There is a rise of 25 m from chainage 465 m to station *B* having chainage of 665 m. All the measurements of chainages have actually been taken along the ground. It was also found that the 20 m chain used for chaining was 5 cm too long throughout the work. Calculate the correct horizontal distance from station *A* to station *B*.
(b) State clearly the degree of accuracy required to be achieved in measuring horizontal distances under different conditions.
(c) Draw explanatory sketches to show (i) Well conditioned triangle, (ii) Tie line, (iii) Check line. [AMIE, Surveying, Summer 1983]

3.12 (a) What factors should be considered in selecting stations of a chain survey?
(b) What are offsets? Discuss the relative merits of different types of offsets? Why is it desirable that offsets should be as short as possible?
(c) *A* and *B* are two points 150 m apart on the near bank of a river which flows from east to west. The bearings of a tree on the far bank as observed from *A* and *B* are N 50°E and N 43°W respectively. Determine the width of the river.
[AMIE, Surveying, Winter 1984]

3.13 (a) Explain with sketches how to chain past (i) a river, (ii) a building.
(b) Two ranging rods, one of 2.50 m and the other of 1.00 m length were used in order to find the height of an inaccessible tower. In the first setting, the rods were so placed that their tops were in line with the top of the tower. The distance between the rods was 15 m. In the second setting the rods were ranged on the same line as before. This time the distance between the rods was 30 m. If the distance between the two longer rods was 90 m, find the height of the tower.
[AMIE, Surveying, Winter 1984]

3.14 (a) Describe the method of ranging a line across a ridge when the terminal stations are not intervisible
(b) *AB* is a chain line crossing a lake *A* and *B* are on the opposite sides of the lake. A line *AC* 800 m long is ranged to the right of *AB* clear of the lake. Similarly another line *AD* 1000 m long is ranged to the left of *AB* such that the points *C*, *B* and *D* are collinear. The lengths *BC* and *BD* are 400 m and 600 m respectively. If the chainage of *A* is 1262.44 m, calculate the chainage of *B*.
[AMIE, Winter 1985]

3.15 (a) Draw the conventional signs, as used on topography map for the following objects and indicate their colour:

(i) Pucca building, (ii) Temple, (iii) Surveyed tree, (iv) Embankment, (v) Road culvert on a drain, (vi) Railway bridge over river.

(b) Describe any three of the following operations in a chain surveying:

(i) Measurement of lengths on sloping ground,
(ii) Criteria for selection of chain survey stations, ,
(iii) Crossing a wide river as an obstacle to chaining,
(iv) Booking chainage and offset measurement entries in a field book.

4

Electronic Distance Measurements

4.1 INTRODUCTION

Electronic distance measuring instruments, as the name implies utilizes electromagnetic energy for measuring distances between two points. To facilitate understanding, basic electronic concepts are first discussed.

4.2 BASIC CONCEPTS

Electromagnetic waves can be represented in the form of periodic sinusoidal waves as shown in Fig. 4.1.

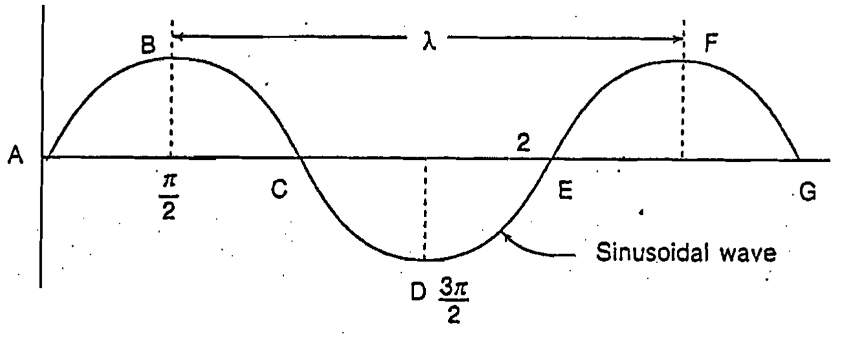


Fig. 4.1 Wavelength.

The time taken for an alternating current to go through one complete cycle of values is called *period* of the wave. One cycle of the wave motion is completed when one period has been completed and the number of cycles per unit of time is called *frequency*. The unit of frequency is hertz (Hz) which is one cycle per second. The linear length λ of a wave is the wavelength which can be determined as a function of the frequency f and the velocity of electromagnetic radiation C as $\lambda = C/f$.

Oscillators convert *DC* drawn from an energy source (battery) into an *AC* of continuous sinewaves. A continuous wave does not include any information or intelligence but it can be effectively used as a carrying agent.

The fundamental information transmitted by the carrierwave in electronic surveying is a sinusoidal wave form to be used for measurements. The process of superimposing the desired sinewave or other periodic signal on to the carrierwave is called *modulation*.

Three main types of modulation are frequently used in electronic distance measurements. They are: (i) Amplitude modulation, (ii) Frequency modulation, and (iii) Phase modulation.

In amplitude modulation, the frequency and the phase of the carrierwave do not change but the strength and amplitude V_c of the carrierwave, $V = V_c \sin \omega_c \cdot t$ alternates sinusoidally with an amount $V_c' = \alpha \cdot V_c$. The coefficient α indicates the depth or the *degree* of modulation; it is defined as V_c'/V_c where V_c' is the amplitude of modulation wave. During modulation, the amplitude of the carrierwave thus alternates between the limits $V_c + V_c'$ and $V_c - V_c'$ as shown in Fig. 4.2.

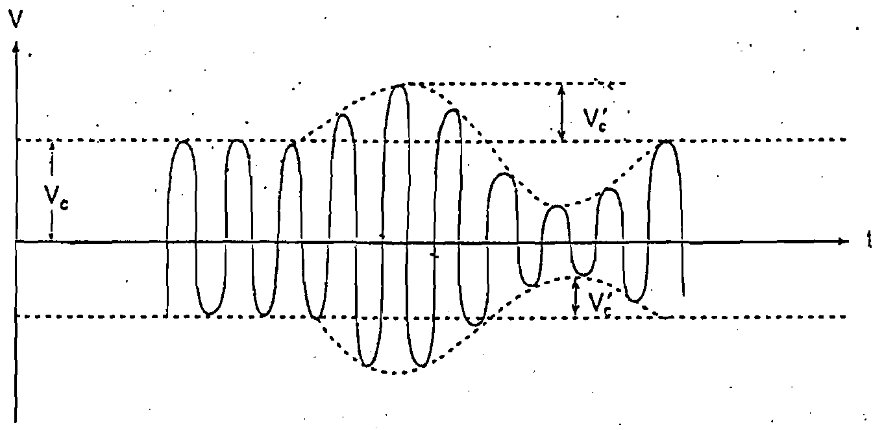


Fig. 4.2 Amplitude modulation.

In *frequency modulation* (Fig. 4.3) the amplitude of the carrierwave is kept constant but the frequency varies according to the amplitude and polarity of the modulation signal. The carrier frequency is increased during one half cycle of the modulation signal and decreased during the other half cycle; thus the frequency is least positive and highest when the modulation is most positive.

In *phase modulation* (Fig. 4.3) as in frequency modulation the amplitude of the carrierwave remains constant but the phase of the carrierwave is varied according to the phase of the modulation wave.

4.3 CLASSIFICATION OF ELECTROMAGNETIC RADIATION

The total spectrum of electromagnetic radiation used in electronic and electro-optical distance measurements encompasses wavelengths from the visible light of about $5(10)^{-7}$ to about $3(10^4)$ m at the radio frequency region. Frequencies are large numbers and always positive powers of the basic unit, the hertz; periods are small numbers and always negative powers of the basic unit. Table 4.1 lists

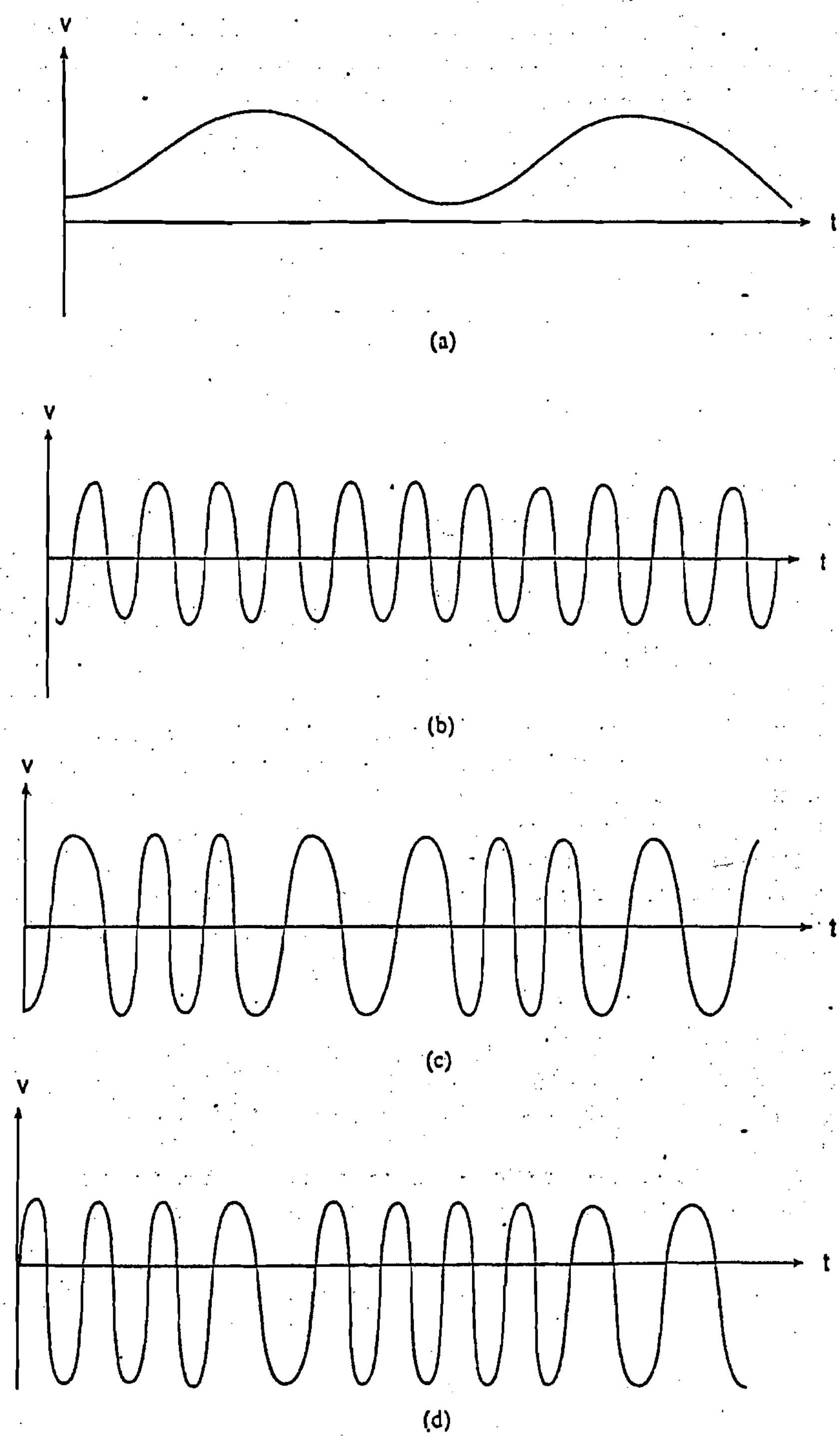


Fig. 4.3 Frequency and phase modulation: (a) Modulation wave. (b) Carrierwave, (c) Frequency modulated carrierwave (frequency changes). (d) Phase modulated carrierwave (phase changes). (Ref. 2)

Table 4.1 Relationship of Frequency, Period and Wavelengths (Ref. 2)

Frequency	Period $T = 1/f$	Wavelength $\lambda = cT$
Hertz (Hz)	1 sec	$3\ (10)^8$ m
kHz = 10^3 Hz	1 m sec = 10^{-3} sec	$3\ (10)^5$ m
MHz = 10^6 Hz	1 μ sec = 10^{-6} sec	$3\ (10)^{12}$ m
GHz = 10^9 Hz	1 n sec = 10^{-9} sec	$3\ (10)^{-1}$ m
THz = 10^{12} Hz	1 p sec = 10^{-12} sec	$3\ (10)^{-4}$ m

relationships of frequency, period and wavelengths when the propagation velocity c is given the approximate value of $c = 3(10^5)$, km/sec. Table 4.2 gives a commonly used classification of different frequency and wavelength.

Table 4.2 Commonly Used Frequencies and Wavelengths (Ref. 2)

Classification	Symbol	Frequency	Wavelength
Very low frequency	VLF	10–30 kHz	30,000–10,000
Low frequency	LF	30–300 kHz	10,000–1,000
Medium frequency	MF	300–3000 kHz	1,000–100
High frequency	HF	3–30 MHz	100–10
Very high frequency	VHF	30–300 MHz	10–1.0
Ultra high frequency	UHF	300–3000 MHz	1.0–0.1
Super high frequency	SHF	3–30 GHz	0.1–0.01
Extremely high frequency	EHF	30– GHz	0.01–

4.4 BASIC PRINCIPLE OF ELECTRONIC DISTANCE MEASUREMENT

In measuring the distance between two points electronically, an alternating signal travels from one point to the other, it is reflected or returned in some manner and then is compared with the phase of the original signal to determine the travel time for the round trip. If the distance is to be measured by direct time comparison with an accuracy corresponding to the nearest cm, a time of approximately 67 (10^{-12}) sec must be distinguishable. This interval of time is difficult to measure directly but it can be resolved by a phase measurement of a signal with a period of 67 (10^{-9}) sec corresponding to 15 mc per sec. This signal frequency is too low for direct transmission, so a much higher carrier frequency is employed and the signal appears as a modulation frequency. Depending on the type of carrierwave employed, EDM instruments can be classified as:

1. Microwave instruments.
2. Visible light instruments.
3. Infrared instruments.

Light frequencies permit the use of optical corner reflectors at the section stations but requires an optically clear path between the two stations. Microwave systems can operate through fog and clouds although an optically clear path is required if the vertical angle between two stations must be determined to convert into a horizontal distance. The presence of fog or clouds may cause a loss in accuracy and prevent a reliable estimate.

4.5 COMPUTING THE DISTANCE FROM THE PHASE DIFFERENCES

One complete cycle at the modulation frequency corresponds to the time required for the signal to travel one half wavelength in both direction. The distance being measured corresponds to many half wavelengths at the modulation frequency plus some fraction of a half wavelength. All the instruments use data from several frequencies to overcome this ambiguity.

Four modulation frequencies are used in determining the total length being measured. Several measurements are made at one of these frequencies in order to reduce inherent electronic errors in the system. The four frequencies are f_A, f_B, f_C and f_D. The corresponding half lengths are $\lambda_A/2$, $\lambda_B/2$, $\lambda_C/2$ and $\lambda_D/2$. The wavelength λ is related to c, the velocity of propagation and the frequency f by

$$\lambda = \frac{c}{f} \tag{4.1}$$

Representative nominal values for these quantities are given in Table 4.3.

Table 4.3 Nominal Frequency and Half Wavelengths

Frequency in cycle/sec	Half wavelength in m
$f_A = 10.00\ (10^6)$	$\frac{\lambda_A}{2} \cong \frac{3(10)^8}{2(10)^7} = 15$
$f_B = 9.99\ (10^6)$	$\frac{\lambda_B}{2} = \frac{f_A}{f_B} \cdot \frac{\lambda_A}{2} \cong \frac{15}{0.999}$
$f_C = 9.90\ (10^6)$	$\frac{\lambda_C}{2} \cong \frac{15}{0.99}$
$f_D = 9.0\ (10^6)$	$\frac{\lambda_D}{2} \cong \frac{15}{0.9}$

With the exception of λ_A, the modulation wavelengths are not useful directly but the wavelengths associated with the frequency differences allow the ambiguity introduced by the long path to be resolved. These relations are given in Table 4.4.

Phase differences needed in the determination of distance d can be derived as follows:

$$A - B = \frac{2d}{\lambda_A} - \frac{2d}{\lambda_B} = \frac{2d}{\lambda_A}\left(1 - \frac{\lambda_A}{\lambda_B}\right) = \frac{2d}{\lambda_A}\left(1 - \frac{f_B}{f_A}\right) \tag{4.2}$$

Table 4.4 Frequency Differences and Equivalent Half Wavelengths

Frequency differences in cycles/sec	Equivalent half wavelength in meters
$f_A - f_B = 10^4$	$\frac{\lambda_{A-B}}{2} \equiv \frac{3 \times 10^8}{2 \times 10^4} \equiv 15,000$
$f_A - f_C = 10^5$	$\frac{\lambda_{A-C}}{2} \equiv 1{,}500$
$f_A - f_D = 10^6$	$\frac{\lambda_{A-D}}{2} \equiv 150$
$f_A - f_O = 10^7$	$\lambda_A \equiv 15.$

Equation (4.2) can be rearranged to obtain an expression for the distance d.

$$A - B = 2d\left(\frac{f_A}{c} - \frac{f_B}{c}\right) = 2d \cdot \frac{f_A - f_B}{c} = \frac{d}{\lambda_{A-B}/2}$$

or

$$d = (A - B)\,\frac{\lambda_{A-B}}{2}$$

in which half wavelengths are obtained from Table 4.4 and the phase differences from the instrument readings. A hypothetical example of how distances are derived and total distance obtained is shown in Table 4.5. It can be seen from the table that only the first figure to the right of the decimal point is used in calculating the coarse contributions to d since other significant figures are included in later terms of the summation. All significant figures beyond the decimal point in the $A - O$ term are used. The final length is obtained by summing all the part lengths.

Table 4.5 Summation of Distances Contribution from Phase Difference

Hypothetical phase difference in cycles	Equivalent $\lambda/2$ in meters	Distance contribution in meters
$A - B = 0.92$	15,000	$\frac{0.9(\lambda_{A-B})}{2} = 13{,}500.00$
$A - C = 9.21$	1,500	$0.2\,\frac{(\lambda_{A-C})}{2} = 300.00$
$A - D = 92.11$	150	$\frac{0.1(\lambda_{A-D})}{2} = 15.00$
$A - O = 921.124$	15	$0.124\,\frac{(\lambda_{A-O})}{2} = 1.86$
		13,816.86 m

Modern EDMs use the decade modulation technique. When the modulation frequency is 15 MHz, the half wavelength is 10 m. The phase meter reading then gives distance between 0 and 9.999 m. When the modulation frequency is brought down to 1.5 MHz, the half wavelength is 100 m and the phase meter gives tens of meters. When it is still brought down to 0.15 MHz, the half wavelength is 1000 m and we get hundreds of meters. Finally a 15 kHz frequency will give the number of thousand meters. The distance 13,816.86 m will then be obtained as summation of 6.86, 10.00, 800 and 13,000.

4.6 BRIEF DESCRIPTION OF DIFFERENT TYPES OF INSTRUMENTS

Geodimeters. All geodimeters employ visible light as the carrier. The measuring set consists of an active transmitter and receiver at one end of the line to be measured and a passive retro directive prism reflector at the other end. Continuous light emission in the transmitter is intensity (amplitude) modulated, using a precision radio frequency generator and an electro-optical shutter to form sinusoidal light intensity waves. The distance is obtained by comparing the phases of outgoing modulation waves with those received by the receiving component after reflection from the distant reflector. All reflectors in the modern instruments are based on the retro directivity principle. Each unit in the reflector is a retro directive prism made of three mutually perpendicular reflecting surfaces.

The prism is often called a *corner reflector* because, for stability, it is made by cutting a corner from a solid glass cube. Light entering the prism reflects from each of the three surfaces and afterwards this triple reflection returns to the instrument parallel to the incident beam. As regards illumination power, the alignment of the reflector with respect to the geodimeter instrument is not critical and alignment errors of the order of 10° may be tolerated.

Tellurimeter. The tellurimeter uses microwaves at about 3, 10 or 35 GHz as the carrier. The measuring set consists of two active units with a transmitter or receiver; one is called the master and the other the remote unit. The carrier frequencies of the two units differ slightly making it possible to utilize intermediate frequency (IF) amplification.. Because the carriers are microwaves, the beam widths are narrow—between 2 and 20°. Measuring can be carried on either at night or day time, through haze or light rain, although heavy rainfall may reduce the working range. The bare outlines of the measuring principle consist of a frequency modulated carrierwave from the master station being sent to the remote station where it is received and retransmitted to the master station. There the phase difference between the transmitted and received modulation or pattern waves is compared. Knowing the phase difference or by decade modulation technique distance can be determined.

Hewlett-Packard 3800. This is a modern EDM instrument. Block diagram of the instrument is shown in Fig. 4.4. The transmitter uses a GaAs diode which emits amplitude modulated (AM) infrared light. Frequency of modulation is precisely controlled by a crystal oscillator. The intensity variation or amplitude variation is properly represented by sine waves. Environmental correction factor can be directly dialed into the transmitter to slightly vary the frequency so that a constant wavelength is maintained despite atmospheric variations. Hence no adjustment of distance is

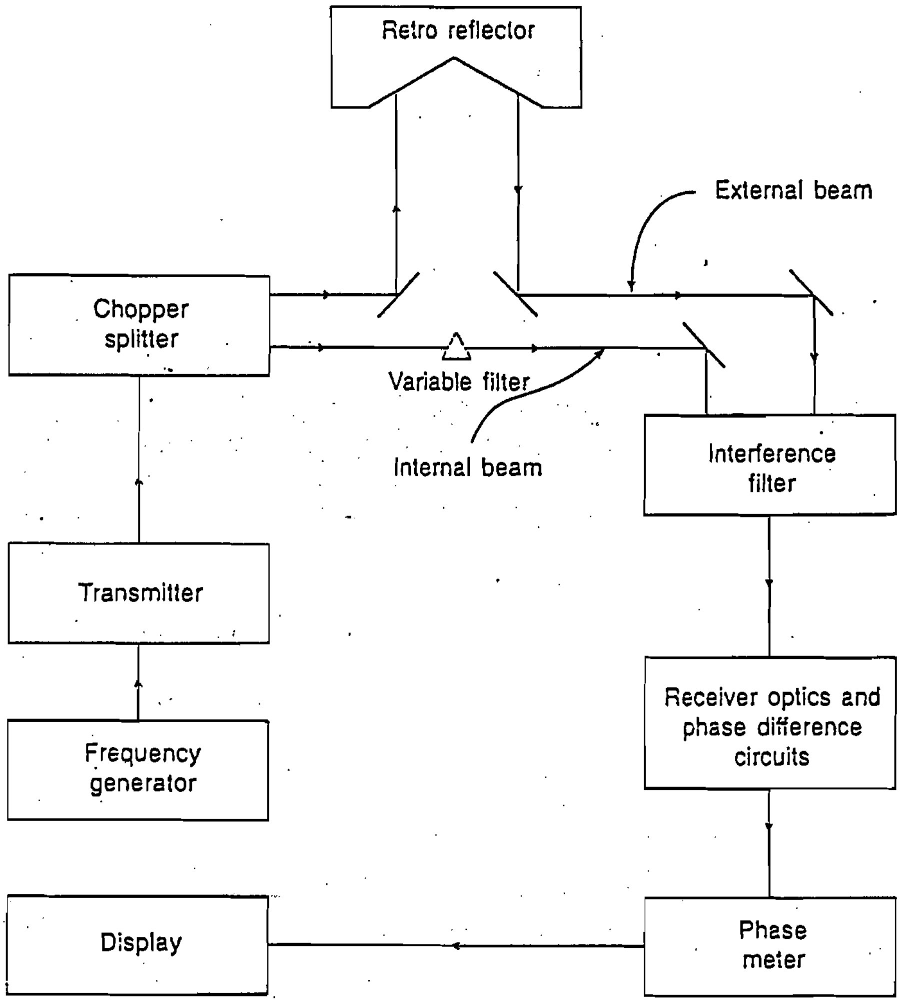


Fig. 4.4 General flow diagram of Hewlett-Packard 3800.

necessary at a later stage. Humidity has little effect on the propagation of infrared light and hence is not measured.

With the refractive index n = 1.0002783 and c_0 = 299,792.5 km/sec the basic modulation frequency representing 10 m distance is f_1 = 14.985 MHz. The other modulation frequencies are:

$$f_2 = 1.4985 \text{ MHz}$$

$$f_3 = 149.85 \text{ kHz}$$

$$f_4 = 14.985 \text{ kHz}$$

The time sharing between the transmitted signals and the reference signals is made in the chopper beam splitter which divides the modulated light coming from the Ga–As transmitted diode into two separate beams alternately. The light signal to be transmitted is then focussed into a beam. This beam is sent to the distant reflector. Both external and internal light signals then pass through an interference filter located just in front of the receiver diode. This filter helps to reject signals of other wavelengths (e.g. visible sunlight) without eliminating the modulation signal from the carrier. None of the beam splitting, chopping and filtering processes will cause phase shifts between the transmitted and reference light signals. The internal and external beams are then converted to electrical energy. A phase meter converts the phase difference into direct current having a magnitude proportional to the differential phase. This current is connected to a null meter which automatically adjusts itself to null the current. The fractional wavelength is converted to distance during the nulling process and displayed on instrument dials.

Modern version of HP 3800 is fully automatic and a built-in computer averages the distance measurements without being affected by interruptions of the beam. The digital light emitting diode display gives the operator the distance and classifies its measuring quality in three ways. A steady numerical display indicates a good, solid measurement within the instruments specified accuracy; a flashing display means that conditions are such that the measurements are marginal, and a flashing "O" display warns the operator that under present conditions a valid measurement cannot be made.

4.7 TOTAL STATION INSTRUMENTS

Modern surveying system typically consists of an electronic total station, electronic field book and softwares used in the office for processing data.

The total station's function is to measure horizontal and vertical angles and slope distances in a single integrated unit. It is usually connected to an electronic field book. The field system (total station and field book) is usually controlled via the field book. The principal reason for this is that the field book keyboard does not transfer the force used to press the keys to the total station.

In operation the total station is set up over the required point and its height over the survey station measured. Then the operator points at a prism/target and initiates a reading. Usually this is done by pressing a key on the field book. In some systems it may be a key on the electronic total station.

While the basic data sent by the electronic total station consist of slope distance, horizontal angle and vertical angle, other data may be included in the data stream. This may include units settings, parts per million (ppm) value (for the electronic distance meter EDM), prism constant being used, etc. Additionally, calculated values such as coordinates, azimuths, and horizontal distances may be transmitted. Electronic total stations can also have a variety of functions to improve efficiency and accuracy. Some of these may be corrections for collimations, curvature and refraction and horizontal and vertical angles to compensate for the tilt of the vertical axis.

The electronic field book's basic function is to store the raw data gathered in the field, including horizontal and vertical angles, slope distances, heights of instruments and targets, temperature and pressure, point numbers and descriptive codes.

One of the most crucial aspects of the electronic data collection concept is data flow. Traditional surveying techniques force one to view surveying as a data gathering activity. This view does not recognize the fact that once a design is completed based on the survey data there usually is a need to transfer this design on to the topography. In modern surveying, therefore, setting out is given equal importance as data gathering.

4.8 EFFECT OF ATMOSPHERIC CONDITIONS ON WAVE VELOCITY

The velocity of electromagnetic radiation is constant in vacuum (at the velocity of light) but when affected by the atmosphere it is retarded in direct proportion to the density of air. Because of refraction the direction and speed change. The refractive index usually symbolized as n is related to the dielectric constant μ of the air in the following way:

$$n = \sqrt{\mu}$$

The instantaneous velocity c of the radiation at any point within the atmosphere is a function of the speed of light c_0 and the refractive index n and is given as

$$c = c_0/n$$

where c_0 is a constant and is taken as 299,792.5 km/s.

Refractive index is a function of temperature, pressure and humidity. Humidity is given as the partial pressure of the water vapour in air. In field observations it is almost always obtained by the simultaneous observations of wet and dry bulb readings of a psychrometer.

In electro-optical instruments the light forms groups of waves of slightly different lengths even if it is monochromatically filtered. Since the velocity of such a group wave differs a little from the velocity of equivalent or effective wave, a special group index of refraction must be determined. International Association of Geodesy General Assembly (1963) has recommended that the Barrell and Sears (1939) formula may be used to calculate n_g or group index of refraction when λ is the equivalent or effective wavelength of radiation in micrometers.

$$(n_g - 1)\, 10^7 = 2876.04 + \frac{3(16.288)}{\lambda^2} + \frac{5(0.136)}{\lambda^4}$$

For the kinds of light used in EDM's the values of λ are:

(a) Mercury vapour = 0.5500
(b) Incandescent = 0.5650
(c) Red laser = 0.6328
(d) Infrared = 0.900–0.930

To compute the ambient refractive index for lightwaves, the Barrell and Sears, (1939) formula is usually applied as follows:

$$N_a = \left[\frac{n_g - 1}{1 + \alpha t}\frac{P}{760} - \frac{5.5(10)^{-8}}{1 + \alpha t}e\right]10^6$$

where $N_a = (n_a - 1)\ 10^6$

P = Total pressure

e = Partial pressure of water vapour in millimeters of mercury

α = Heat expansion coefficient of air 0.00367

t = Temperature in degree Centigrade.

The effect of water vapour is small on propagation of lightwaves but is of great significance when microwaves are used. For microwaves Essen-Froome formula can be used which is as follows:

$$N_m = \frac{103.49}{T}(P - e) + \frac{86.26}{T}\left(1 + \frac{5748}{T}\right)e$$

where T is in Kelvin units, P and e are in mm of Hg.

To investigate the sensitivity of the atmospheric parameters T, P and e or their partial effect on N, partial differentiation of the formulae is to be done. If error is to be restricted to one part per million (ppm) the allowable standard errors in observing T, P and e are:

For lightwave

$$m_T = \pm\ 1.0°\text{C}$$

$$m_P = \pm\ 3.6 \text{ mb (millibar)}$$

$$m_e = \pm\ 25.6 \text{ mb (millibar)}$$

For microwave

$$m_T = \pm\ 0.8°\text{C}$$

$$m_P = \pm\ 3.7 \text{ mb}$$

$$m_e = \pm\ 0.23 \text{ mb}$$

From above, it is obvious that the allowable uncertainty in observing temperature and pressure is of the same magnitude in each case. Humidity is the most critical parameter to be observed in connection with measurements made by using radiowaves. The required accuracy by which humidity must be known is about 100 times greater in radiowave propagation than it is when light emission is utilized. In high precision geodetic or geophysical applications, therefore, light is the carrying agent of preference.

4.9 INSTRUMENTAL ERRORS IN EDM

Apart from error due to atmospheric refraction, error in EDM may occur for not having the effective centre of the reflector plumbed over the far end of the line as shown in Fig. 4.5.

The distance through which light travels in the glass tube during retro reflection is $a + b + c$ which in turn is equal to $2t$. The distance t is measured from the face

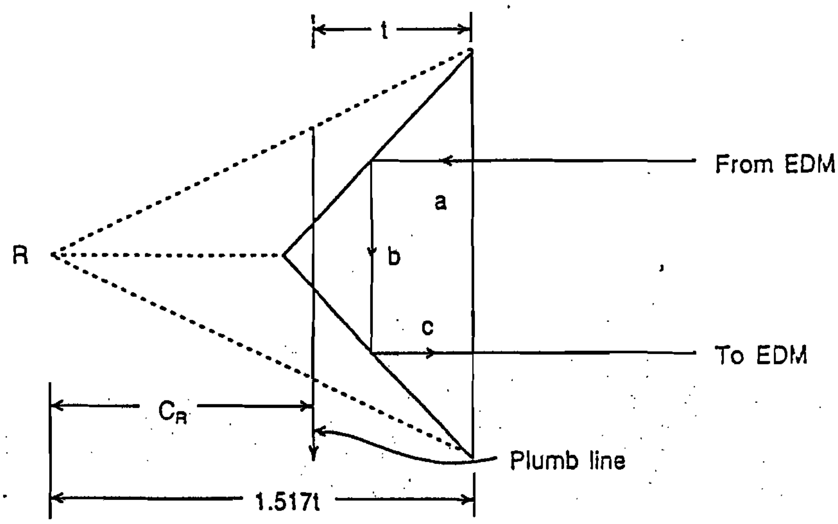


Fig. 4.5 Reflector correction.

of the reflectors to the corner of the glass tube. The equivalent air distance through which the light travels is 1.517 (2*t*) on account of the refractive index of glass.

The effective corner of the cube is at *R* and represents the end of the line and hence C_R is the correction to be applied to the measured line. As different combination of reflectors are used at different times of measurement the "reflector constant" is not the same for all setting of the instrument. This has to be determined for each reflector-instrument combination known as C_1. This can be obtained by measuring a distance electronically and also very accurately by means of an invar tape.

Another way of determining C_1 is to take measurements *AB*, *BC* and *AC* all in a straight line. Then

$$(\text{Measured } AB + C_1) + (\text{Measured } BC + C_1)$$

$$= \text{Measured } AC + C_1$$

or $$C_1 = \text{Measured } AC - (\text{Measured } AB + \text{Measured } BC)$$

Microwave instruments may suffer from a phenomenon known as *Ground Swing*. This is due to multiple reflection of microwaves from ground or water surface. Errors from this source can be reduced by elevating the master and remote units as high above ground as possible and averaging a number of measurements taken from both ends.

4.10 REDUCTION OF SLOPE MEASUREMENTS IN EDM

EDM measures slope distance between stations. Many instruments automatically reduce the horizontal distance. In some cases, it is to be done manually. Reduction of slope distance to horizontal can be based on difference in elevation or in vertical angle. The method is explained with the help of examples.

Example 4.1 (i) If electromagnetic energy travels 299,792.5 km/sec under given conditions what unit of distance corresponds to each millimicro second of time?

(ii) The speed of electromagnetic energy through the atmosphere at a standard barometric pressure of 760 mm of mercury is accepted as 299,792.5 km/sec for measurements with an EDM instrument. What time lag in the equipment will produce an error of 15 m in the distance to a target 80 km away?

(iii) (a) If an EDM has a purported accuracy capability of ± (5 mm + 5 ppm) what error can be expected in a measured distance of 800 m? (b) If a certain EDM instrument has an accuracy capability of ± (7 mm + 7 ppm) what is the precision of measurements in terms of $1/x$ for line length of 3000 m?

Solution

(i) 1 millimicro second $= 10^{-9}$ sec

Distance travelled/sec $= 299{,}792.5$ km

Distance travelled in 1 millimicro second

$$= 299{,}792.5\,(10^{-9})\text{ km}$$
$$= 2.997925\,(10^{-4})\text{ km}$$
$$= 2.997925\,(10^{-1})\text{ m}$$
$$= 29.97\text{ cm}$$

(ii) 15 m $= .015$ km

Velocity of light $= 299{,}792.5$ km/sec

Hence, time lag $= \dfrac{.015}{299{,}792.5}\text{ sec} = 5.00345 \times 10^{-8}\text{ sec}$

(iii) (a) Accuracy $= \pm\,(5\text{ mm} + 5\text{ ppm})$

$$= \pm\left(5 + \frac{5(800)(10^3)}{10^6}\right) = \pm\,9\text{ mm}$$

(b) Accuracy $= \pm\,(7\text{ mm} + 7\text{ ppm})$

$$= \pm\left(7\text{ mm} + \frac{7(3000)(10^3)}{10^6}\right)$$
$$= \pm\,28\text{ mm}$$

In terms of $1/x$ $= \dfrac{28}{(3000)(10^3)} = \dfrac{1}{107{,}142}$

$$\approx \frac{1}{107{,}000}$$

Example 4.2 What is the refractive index of red laser light at a temperature of 20°C and barometric pressure of 710 torr? Neglect the effect of vapour pressure. What is the velocity through this air? What is the modulated wavelength if the modulating frequency is 24 MHz?

Solution For red laser light $\lambda = 0.6328$. Hence refractive index of standard air for the laser carrier is given by

$$(n_g - 1)\ 10^7 = 2876.04 + \frac{(3)(16.288)}{\lambda^2} + \frac{5(0.136)}{\lambda^4}$$

$$= 2876.04 + \frac{3(16.288)}{(0.6328)^2} + \frac{5(0.136)}{(0.6328)^4}$$

$$= 3002.3078$$

$$n_g = 1.0003002.$$

Refractive index in air under given atmospheric condition neglecting effect c vapour pressure.

$$N_a = \left[\frac{n_g - 1}{1 + \alpha t} \cdot \frac{P}{760}\right] 10^6$$

$$= \left(\frac{.0003002}{1 + (0.00367)(20)}\right)\left(\frac{710}{760}\right)(10^6)$$

$$= (.000261272)\ (10^6)$$

$$(n_a - 1)(10^6) = (.000261272)\ (10^6)$$

$$n_a = 1.000261272.$$

The velocity of light through this atmosphere

$$V_a = \frac{c_0}{n_a}$$

where c_0 is the velocity of light in vacuum taken as 299,792.5 km/s

$$= \frac{299{,}792.5}{1.000261272} = 299{,}714.21 \text{ km/s}$$

Modulated wavelength

$$= \frac{299{,}714.21}{(24)(10^6)} = (12488.092)(10^{-6})\text{km}$$

$$= 12.488092 \text{ m}$$

Example 4.3 Microwaves are modulated at a frequency of 70 MHz. They are propagated through an atmosphere at a temperature of 12°C, atmospheric pressure of 712 torr, and a vapour pressure of 7.6 torr. What is the modulated wavelength of these waves?

Solution For microwaves,

$$N_m = \frac{103.49}{T}(P - e) + \frac{86.26}{T}\left(1 + \frac{5748}{T}\right)e$$

$$= \frac{103.49}{285}(712 - 7.6) + \frac{86.26}{285}\left(1 + \frac{5748}{285}\right)7.6$$

$$= 255.78371 + 48.693013$$

$$= 304.47672$$

$$n_m = 1 + .0003044$$

$$= 1.0003044$$

$$V_m = \frac{299,792.5}{1.0003044} = 299,702.47 \text{ km/sec}$$

Modulated wavelength

$$\lambda = \frac{299,702.47}{(70)(10^6)} = 4.2814638 \text{ m.}$$

Example 4.4 In a straight line *ABC*, *AB* measures 354.384 m, *BC* measures 282.092 m and *AC* measures 636.318 m using a particular EDM reflector combination. A line measures 533.452 m with this instrument-reflector combination. What is the correct length of the line?

Solution

$$C_l = \text{Measured } AC - (\text{Measured } AB + \text{Measured } BC)$$
$$= 636.318 - (354.384 + 282.092)$$
$$= -0.158 \text{ m.}$$

Correct length of the line $= 533.452 - 0.158$

$$= 533.294 \text{ m.}$$

Example 4.5 The height of an EDM set up at *M* is 1.495 m. The height of the reflector set up at *P* is 1.295 m. The height of the theodolite at *M* used to measure the vertical angle is 1.615 m. The height of the target at *P* on which the vertical sight is taken is 1.385 m. The slope distance after meterological corrections is 1650.452 m. The measured vertical angle is + 3°02′32″. What is the horizontal distance between *M* and *P*?

Solution

In Fig. 4.6
E = Position of EDM
T = Position of theodolite
R = Position of reflector
S = Position of target

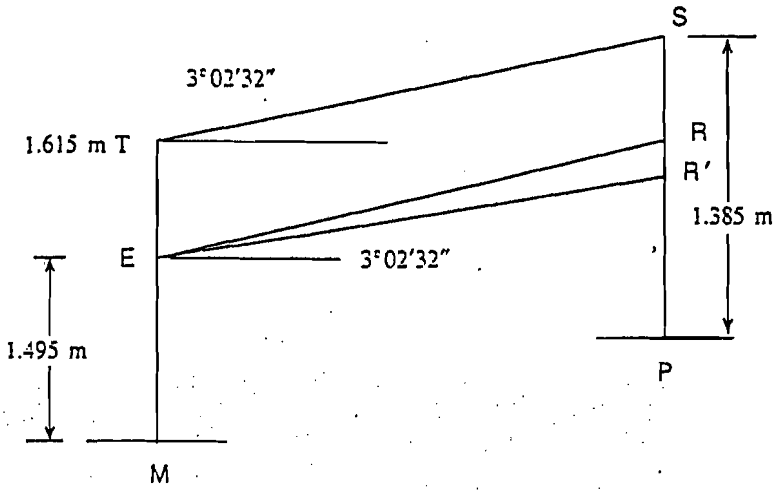
From the data given

$ME = 1.495$ m $\quad MT = 1.615$ m

Therefore, $TE = MT - ME = 1.615 - 1.495 = 0.12$ m

$PR = 1.295 \quad PS = 1.385$

Therefore, $SR = 1.385 - 1.295 = 0.09$ m



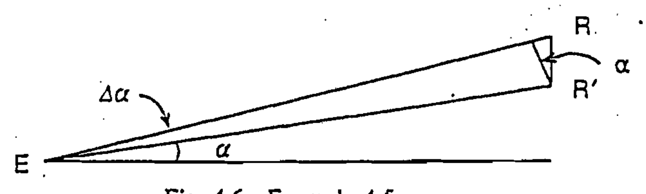


Fig. 4.6 Example 4.5.

From *E*, *ER'* is drawn parallel to *TS* giving *RR'* = 0.12 – 0.09 = .03 m

$$\Delta\alpha \text{ in sec} = \frac{RR' \cos \alpha}{ER'} = \frac{(0.03)(\cos 3°02'32'')}{1650.452}\ (206{,}265) \text{ sec}$$

$$= 3.74396''$$

$$\alpha + \Delta\alpha = 3°\ 02'35.74''$$

Horizontal distance = *ER* cos ($\alpha + \Delta\alpha$)

$$= 1650.452 \cos 3°02'35.74''$$

$$= 1648.1244 \text{ m}$$

Example 4.6 In Fig. 4.7 a vertical angle of – 8°06'20" was recorded. The EDM instrument was standard mounted and offset a distance of 0.20 m above the theodolite axis. If the theodolite and reflector heights are equal, what is the corrected horizontal distance for a recorded slope distance of 75.65 m?

Solution

Measured angle α_m (by Theodolite) = – 8°06'20"

$$\Delta\alpha''_v = \frac{0.20 \cos 8°06'20''}{75.65}\ (206{,}265)''$$

$$= 539.87''$$

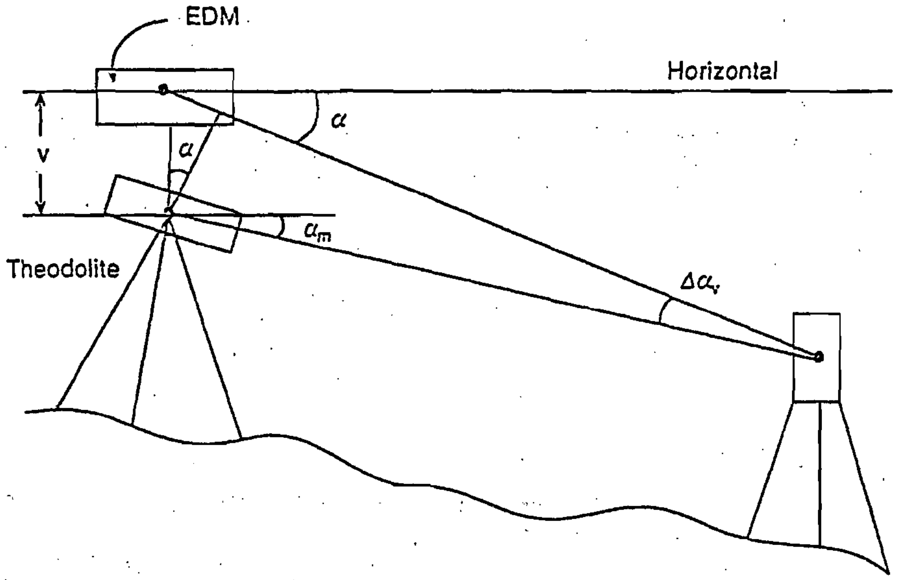


Fig. 4.7 Example 4.6.

$$\alpha = \alpha_m + \Delta\alpha_v = 8°06'20'' + 08'59.87''$$

$$= 8°15'19.87''$$

$$\text{Horizontal length} = 75.65 \cos 8°15'19.87''$$

$$= 74.866 \text{ m}$$

Example 4.7 A slope distance of 940.07 m (corrected for meteorological conditions) was measured from A to B whose elevations were 643.41 m and 568.39 m above datum respectively (Fig. 4.8). Find the horizontal length AB if heights of the EDMI and reflector were 1.205 m and 1.804 m above their respective stations.

Solution

Here $CD = L \quad h_e = 1.205$ m

$$h_r = 1.804 \text{ m}$$

$$d = 643.410 + h_e - (\text{Elev. of } B + h_r)$$

$$= 643.410 + 1.205 - (568.39 + 1.804)$$

$$= 73.911 \text{ m}$$

$$H = \sqrt{L^2 - d^2} = \sqrt{940.07^2 - 73.911^2}$$

$$= 937.1599 \text{ m}$$

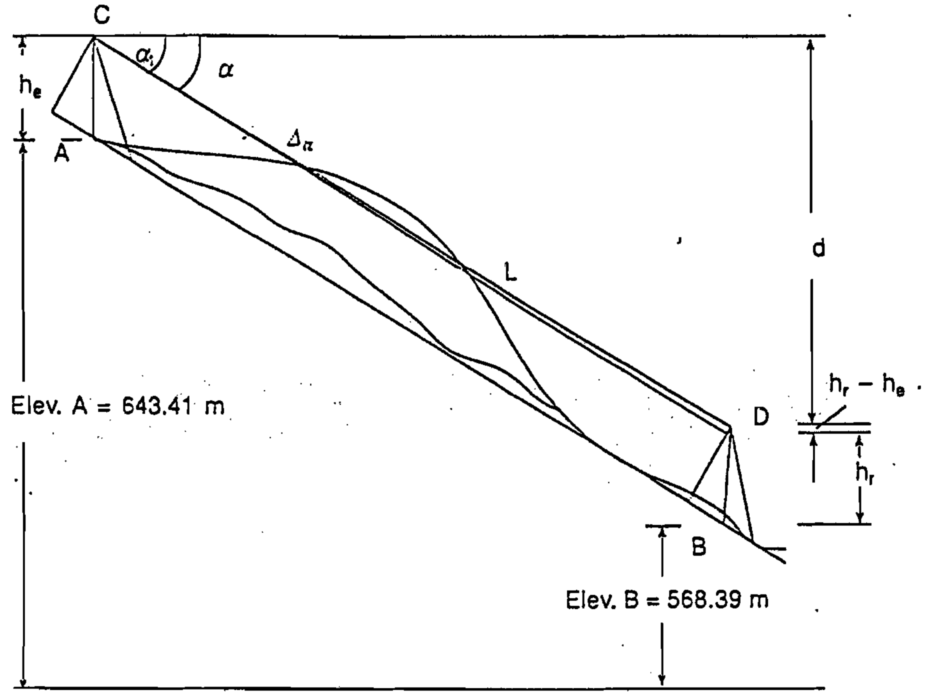


Fig. 4.8 Example 4.7.

Example 4.8 The formula given in a manufacturer's instruction manual for computing the atmospheric correction (C_m) to measure electro-optical distance measurement is

$$C_m = \frac{1.00028195}{\left[\frac{0.000294335}{1 + 0.00366086\,t} \times \frac{P}{1013} + 1\right]}$$

where t = ambient atmospheric temperature (°C)

P = ambient atmospheric pressure (mb)

Corrected slope distance = measured slope distance × C_m

The modulated wavelength of the instrument (λ_s) is 20.00000 m corresponding to a frequency of 14.985400 MHz at specified meteorological reference data of 12°C (t) and 1013 mb (P) and carrier wavelength (λ) of 0.860 μm.

A survey line forming part of a precise test network was measured with the instrument and a mean value of 2999.097 m recorded. The mean ambient temperature t and pressure P were 13.4°C and 978.00 mb respectively.

Compute the atmospheric correction using the formula given in the instruction manual and from first principles, and compare the results. Assume the velocity of electromagnetic radiation in free space to be 299,792.5 km/s.

It was later discovered that the field barometer was in error by + 24 mb. Compute the correction in the distance due to this error. What conclusions can be drawn from these calculations?

Aide memoire:

$$n_a = 1 + \frac{n_g - 1}{\alpha T} \times \frac{P}{1013.25}$$

$$n_g = 1 + \left[28760.4 + \frac{3 \times 162.88}{\lambda^2} + 5 \times \frac{1.36}{\lambda^4}\right] \times 10^{-8}$$

$$\lambda_s = \frac{C_0}{f n_s}$$

where

n_a = group refractive index of atmosphere,
n_g = group refractive index of white light (1.000294),
n_s = group refractive index for standard conditions,
α = 3.661×10^{-3} K^{-1},
T = ambient temperature (K),
P = ambient pressure (mb),
λ_s = modulated wavelength (20.000000 m),
λ = carrier wavelength (0.860 μm)
C_0 = velocity of electromagnetic radiation in free space (299792.5 km/s)
f = modulation frequency (14.98540 MHz) [Eng. Council]

Solution From manufacturer's formula:

$$C_m = \frac{1.00028195}{\left[\frac{0.000294335}{1 + 0.00366086t} \times \frac{P}{1013} + 1\right]}$$

$$= \frac{1.00028195}{\left[\frac{0.000294335}{1 + 0.00366086 \times 13.4} \times \frac{978.00}{1013} + 1\right]}$$

$$= 1.000011.$$

From 1st principles:

$$n_g = 1 + \left[28760.4 + \frac{3 \times 162.88}{\lambda^2} + \frac{5 \times 136}{\lambda^4}\right] \times 10^{-8}$$

when $\lambda = 0.860$

$$n_g = 1 + \left[28760.4 + \frac{3 \times 162.88}{.860^2} + \frac{5 \times 136}{0.860^4}\right] \times 10^{-8}$$

$$= 1.00029433513.$$

With temperature of 12°C and P = 1013 mb.

$$n_a = 1 + \frac{0.0002943}{3.661 \times 10^{-3} \times 285} \times \frac{1013}{1013.25}$$

$$= 1.000281993$$

$$\lambda_s = \frac{299792500}{1.000281993 \times 14985400} = 20.00.$$

At temperature of 13.4°C and 978.00 mb of pressure,

$$n_a = 1 + \frac{0.0002943}{3.661 \times 10^{-3} \times 286.4} \times \frac{978}{1013.25}$$

$$= 1.000270919$$

$$\text{ratio} = \frac{1.000281993}{1.000270919} = 1.000011$$

$$n_a = 1 + \frac{n_g - 1}{\alpha T} \times \frac{P}{1013.25}$$

$$\delta n_a = \frac{n_g - 1}{\alpha T} \times \frac{\delta P}{1013.25}$$

$$= \frac{.00029433513}{3.661 \times 10^{-3} \times 286.4} \times \frac{\delta P}{1013.25}$$

$$= 2.77046 \times 10^{-7} \quad \delta P$$

$$\delta n_a \times 10^6 = .277 \ \delta P$$

with $\delta P = +24$

$$\delta n_a \times 10^6 = .277 \times 24 = 6.648$$

With length 2999.097 m

Correction in distance $= 2999.097 \times 6.648 \times 10^{-6}$

$= .02$ m

Error due to incorrect reading of pressure is small.

REFERENCES

1. Harrison, A.E., "Electronic Surveying: Electronic Distance Measurements", *Journal of the Surveying and Mapping Division*, Proceedings of the American Society of Civil Engineers, Vol. 89, No. 503, October 1963, pp. 97–116.
2. Laurila Simo, H., *Electronic Surveying and Navigation*, New York: John Wiley & Sons, 1976.

PROBLEMS

4.1 Explain the principles of electronic distance measurement.

4.2 How does electro-optical instrument differ from EDM instrument?

4.3 If an EDM instrument has a purported accuracy capability of ± (5 mm + 5 ppm) what error can be expected in a measured distance of (a) 600 m (b) 3 km?

4.4 If a certain EDM instrument has an accuracy capability of ± (7 mm + 7 ppm) what is the precision of measurements, in terms of $1/x$, for line lengths of (a) 30 m (b) 150 m (c) 2000 m?

4.5 To callibrate an EDM instrument, distances *AC*, *AB* and *BC* along a straight line were measured as 2436.24 m, 1205.45 m and 1230.65 m respectively. What is the instrument constant for this instrument? Compute the length of each segment corrected for the instrument constant?

4.6 Discuss the errors in electronic distance measurements.

4.7 Which causes a greater error in a line measured with an EDMI?
   (a) A 2°C variation of temperature from the standard.
   (b) A neglected atmospheric pressure difference from standard of 2 m of mercury.

4.8 Calculate the horizontal length between *A* and *B* if in Example 4.7, h_e, h_r, $elev_A$, $elev_B$ and the measured slope length *L* are 1.7 m, 1.45 m, 275.25 m, 329.12 m and 428.09 m respectively.

4.9 Calculate the horizontal length in Example 4.6 if the vertical angle is + 10°45′30″. EDM instrument is standard mounted and offset a distance of 0.25 m vertically above the theodolite axis and the recorded slope distance is 59.83 m.

4.10 What is the velocity of mercury vapour light at a temperature of 10°C and barometric pressure of 710 torr?

4.11 Microwaves are propagated through an atmosphere of 75°F, atmospheric pressure of 715 torr and a vapour pressure of 12.5 torr. If the modulating frequency is 30 MHz, what is the modulated wavelength?

4.12 Determine the velocity of red laser light through an atmosphere at 30°C and elevation 1700 m.

5

Levelling I

5.1 INTRODUCTION

Levelling involves measurements in vertical direction. With the help of levelling difference in elevation between two points or level of one point with respect to another point of known elevation can be determined. Levelling helps in (i) knowing the topography of an area, (ii) in the design of highways, railways, canals, sewers, etc, (iii) locating the gradient lines for drainage characteristics of an area, (iv) laying out construction projects, and (v) calculating volume of earth work, reservoir, etc.

5.2 BASIC DEFINITIONS

Figure 5.1 illustrates some of the basic terms defined below as used in levelling.

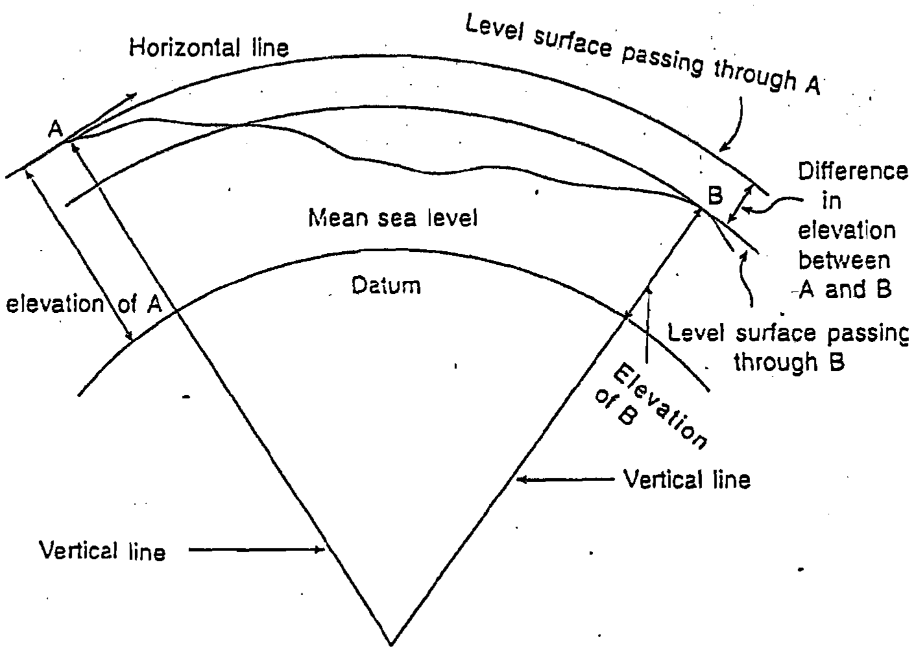


Fig. 5.1 Basic terms in levelling.

Vertical Line. It follows the direction of gravity at any point on the earth's surface and is indicated by a plumb at that point.

Horizontal Line. A line at any point which is perpendicular to the vertical line at that point.

Level Surface. It is a continuous surface that is perpendicular to the plumb line. A large body of still water unaffected by tidal waves is the best example of level surface. For small areas level surface is taken to be a plane surface.

Mean Sea Level. The average height of the sea's surface for all stages of the tide over a very long period (usually 19 years).

Datum. Any level surface to which elevations are referred (for example, mean sea level).

Bench Mark (B.M.). It is a point of known elevation above or below a datum. It is usually a permanent object, e.g. top of a metal disc set in concrete, top of a culvert, etc.

5.3 CURVATURE AND REFRACTION

From Fig. 5.1 it is apparent that difference in level between A and B is measured by passing level lines through the points A and B. However, levelling instruments provide horizontal line of sight and as a result curvature error occurs. In addition due to refraction in the earth's atmosphere the ray gets bent towards the earth introducing refraction error. Figure 5.2 illustrates these errors.

Neglecting small instrument height SA, OA can be taken as the radius of the earth.

From geometry of circle

$$AB(2R + AB) = d^2$$

As AB is very small compared to diameter of the earth

$$AB \cdot 2R = d^2$$

or

$$AB = \frac{d^2}{2R} \qquad (5.1)$$

The diameter of the earth is taken as

$$12.734 \text{ km}$$

Hence curvature correction

$$AB = \frac{d^2}{12.734} \text{ km}$$

$$= 0.078\, d^2 \text{ m} \qquad (5.2)$$

when d is expressed in km.

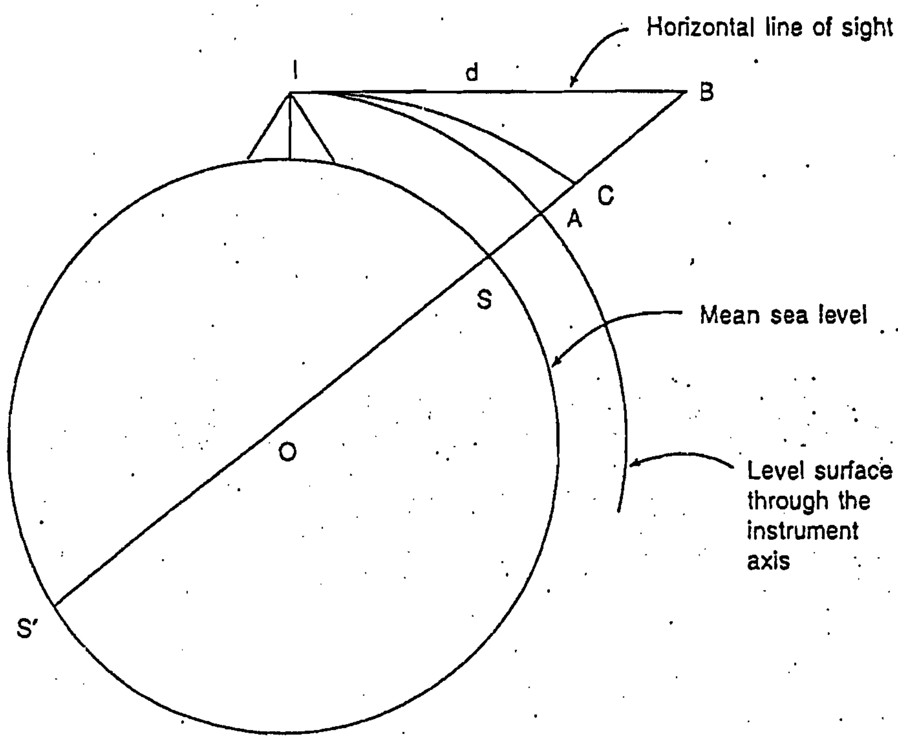


Fig. 5.2 Curvature and refraction correction: I = instrument station; S = staff station; AB = curvature error; BC = error due to refraction; AC = combined error due to curvature and refraction; SB = staff normal to earth's surface; $IB = d$, distance of the staff from the instrument.

The radius of the ray IC bent due to refraction is taken as seven times the radius of the earth. Consequently the refraction correction is taken as 1/7th of the curvature correction. From Fig. 5.1, it can be seen that refraction correction reduces the curvature correction and hence the combined correction is 6/7th of $.078d^2$ m, i.e. $0.067d^2$ m when d is expressed in km.

The correction is subtractive from the staff reading.

Example 5.1 Determine the distance for which the combined correction is 5 mm.

Solution Correction in $m = 0.067d^2$, where d is in km

$$d^2 = \frac{.005}{.067}$$

or

$$d = \sqrt{\frac{.005}{.067}} = 0.273 \text{ km}$$

$$= 273 \text{ m}$$

Example 5.2 What will be the effect of curvature and refraction at a distance of (i) 100 m (ii) 1 km (iii) 50 km (iv) 100 km?

Solution

(i) $E_a = 0.067(.01)^2 = 6.7(10^{-6})$ m
(ii) $E_b = .067(1)^2 = .067$ m
(iii) $E_c = .067(50)^2 = 167.5$ m
(iv) $E_d = .067(100)^2 = 670$ m

From the above result, it is seen that curvature and refraction correction may be neglected for small lengths of sights but should invariably be taken for long sights.

Example 5.3 A sailor standing on the deck of a ship just sees the top of a lighthouse. The top of the light house is 30 m above sea level and the height of the sailor's eye is 5 m above sea level. Find the distance of the sailor from the light house. [AMIE, Summer 1979]

Solution

$$h = 0.067\ d^2 \text{ m}$$

where h is in m, d is in km

$$h_1 = 30 = .067 D_1^2$$

or $$D_1 = 21.16 \text{ km}$$

Similarly $$D_2 = \sqrt{\frac{5}{.067}} = 8.64 \text{ km}$$

Hence total distance $D = 21.16 + 8.64 = 29.80$ km

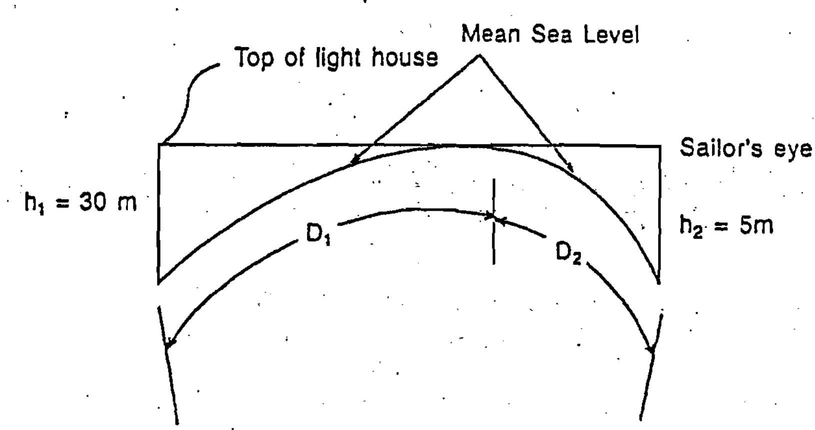


Fig. 5.3 Example 5.3.

5.4 LEVELLING INSTRUMENTS

In levelling, distant objects are to be viewed and measurements taken. A measuring telescope and *not* a viewing telescope forms the main part of a levelling instrument. Telescopes are broadly of two types. Figure 5.4(a) shows a Kepler's or astronomical telescope. Rays from the object AB after refraction from the objective O. are brought to focus before they enter the eyepiece *E* and in consequence a real inverted image is formed in front of the eyepiece. If the lens is so placed that *ba* is situated within the focal length, the rays after refraction at *E* appear to the eye to proceed from *b'a'*, a virtual image conjugate to *ba*. The object *AB* thus appears magnified, inverted and placed at *b'a'*. In Galileo's telescope (Fig. 5.4(b)) the rays refracted by the objective *O* are intercepted by a concave eyepiece *E* before a real image is formed. On entering the eye, they therefore appear to diverge from the vertical image *ab* which is magnified and erect.

For viewing purpose Galileo's telescope is more suitable than Kepler's telescope as an erect image is obtained in the former. However, for measuring purposes Kepler's telescope is more suitable as a real inverted image is formed in front of the eyepiece. In surveying telescope there is a diaphragm carrying crosshairs placed in front of the eyepiece. The line joining the intersection of the

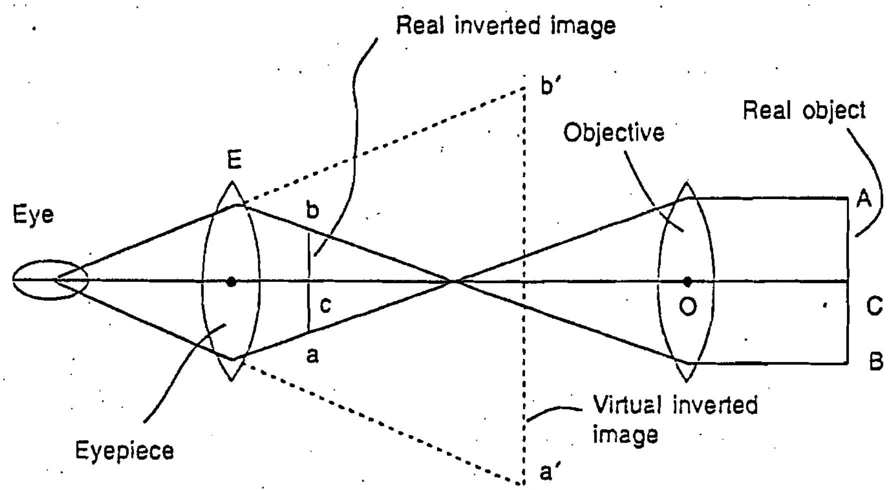


Fig. 5.4(a) Kepler's telescope.

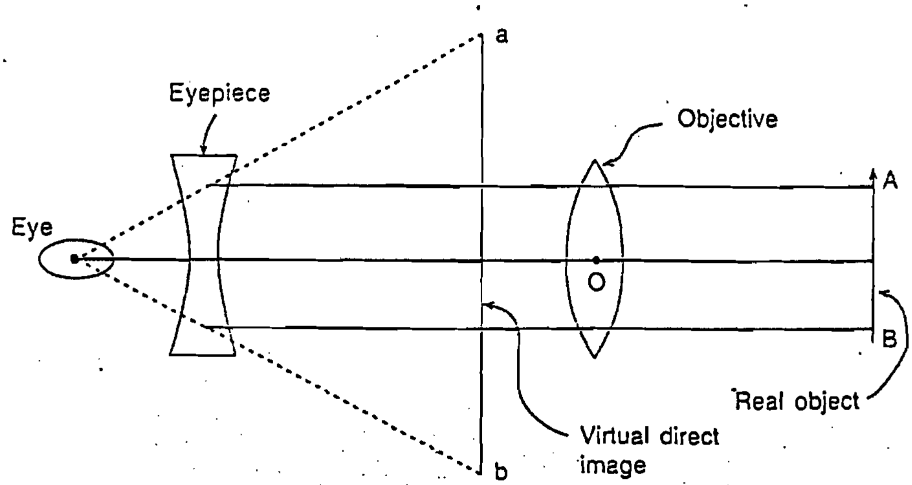


Fig. 5.4(b) Galileo's telescope.

crosshairs with the centre of the objective provides a definite line of sight known as *line of collimation.* In surveying telescope, the real image is formed in the plane of the crosshairs. The eyepiece magnifies both the image and the crosshairs simultaneously and distortion or other defects produced in the passage of the rays through the eyepiece affects both to the same degree.

5.5 CLASSIFICATION OF SURVEYING TELESCOPE

The surveying telescope is broadly of two types—(i) External focussing; (ii) Internal focussing.

By focussing is meant bringing the image of the object in the plane of the crosshairs. If it is done by changing the position of the objective relative to the crosshair, it is *external focussing.* If it is done by moving an additional concave lens between the object and crosshairs it is *internal focussing.* Figure 5.5(a) shows a section through an external focussing telescope while Fig. 5.5(b) shows a section through an internal focussing telescope. In external focussing telescope the objective which is mounted on the inner tube can be moved with respect to the diaphragm which is fixed inside the outer tube. The movement is done by a rack and pinion

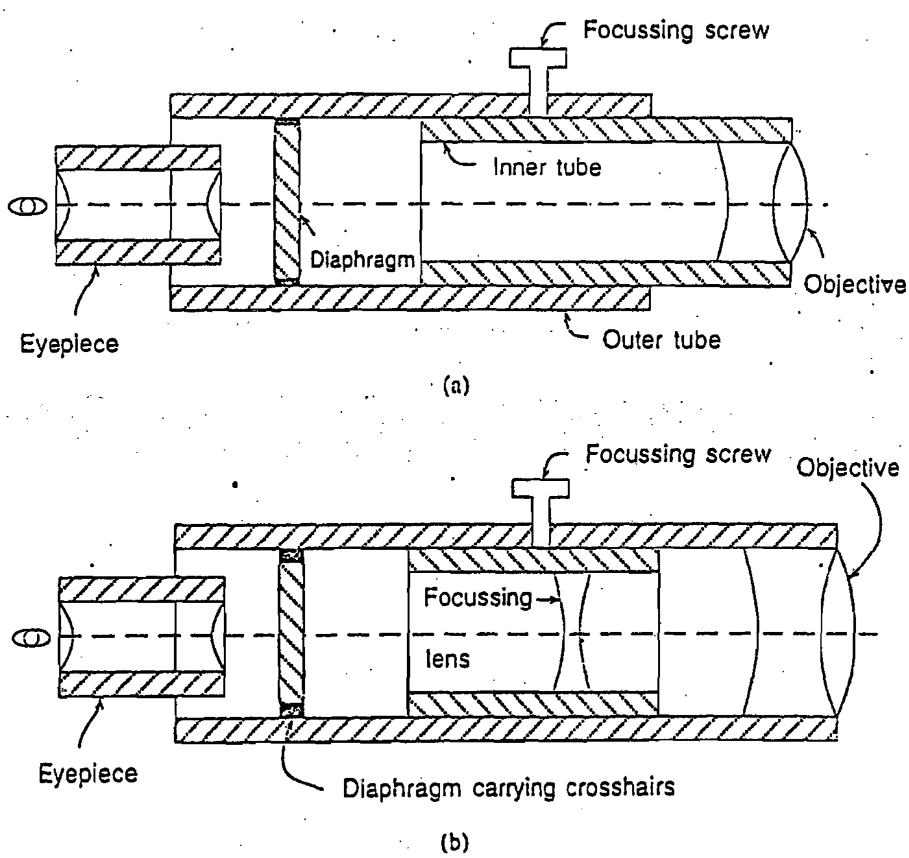


Fig. 5.5 (a) External focussing telescope. (b) Internal focussing telescope.

arrangement operated by focussing screw. As can be seen from Fig. 5.5(b) in the internal focussing telescope, both the objective and the diaphragm carrying crosshairs are mounted inside the outer tube and the distance between them is fixed. An additional double concave lens is mounted on a short tube which can move to and fro between the diaphragm and the objective by means of focussing screw.

5.6 LENS FORMULA

The lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ can be applied to find v when u and f are known. The conventions to be followed in this book are:

Distances are measured inwards towards the lens, and

u = object distance and is positive in a direction opposite to the direction of rays coming from the object
v = image distance and is positive in the direction of rays
f = focal length, positive for convex lens and negative for concave lens

Figure 5.6(a) gives the ray diagram for external focussing telescope. Figure 5.6(b) gives that for an internal focussing telescope. The advantages of internal focussing are:

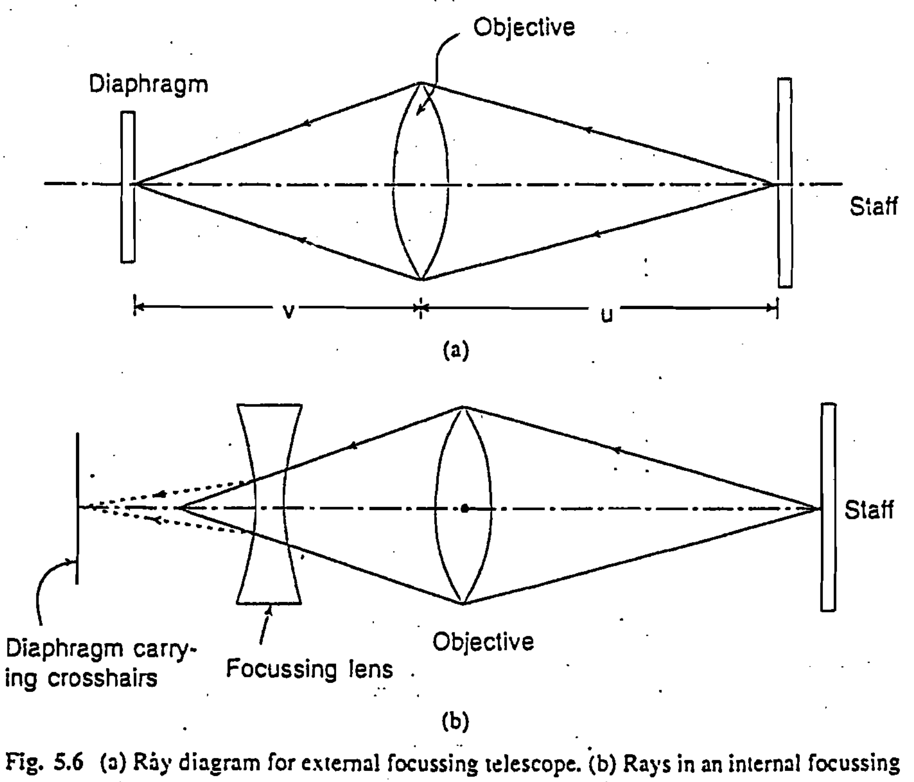


Fig. 5.6 (a) Ray diagram for external focussing telescope. (b) Rays in an internal focussing telescope.

(1) Less overall length of the tube of the telescope.

(2) Less imbalance.

(3) Less wear of rack and pinion.

(4) Better optical properties.

(5) Line of sight is not affected much during focussing operation.

(6) When used as a tacheometer, additive constant is very small.

Its disadvantages are:

(i) Less brightness of the image. (ii) Interior of the telescope cannot be cleared and repaired in the field.

Example 5.4 An internal focussing lens has an object glass of 200 mm focal length. The distance between the object glass and the diaphragm is 250 mm. When the telescope is at infinity focus the internal focussing lens is exactly midway between the objective and diaphragm. Determine the focal length of the focussing lens.

At infinity focus the optical centre of the focussing lens lies on the line joining the optical centre of the objective and the crosshairs but deviates laterally 0.025 mm from it when the telescope is focussed at 7.5 m. Calculate the angular error in seconds due to this cause. [L.U.]

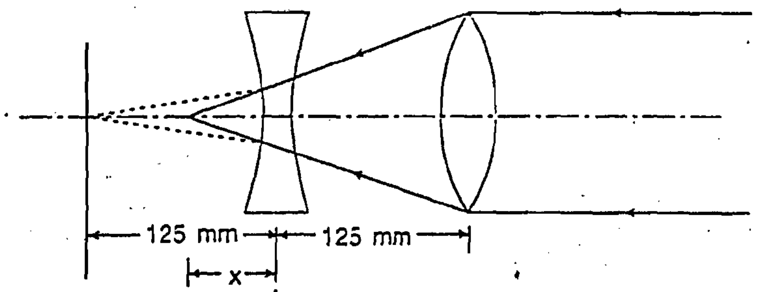


Fig. 5.7(a) Example 5.4.

Solution Using the lens formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Without the focussing lens

$$u = \infty \text{ and is positive}$$

$$f = 200$$

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{200}$$

giving $$v = 200 \text{ mm}$$

$$x = 75 \text{ mm}$$

For the focussing lens, object distance is negative as they are measured towards the lens in the direction of light but image distance is positive,

$$-\frac{1}{75}+\left(\frac{1}{+125}\right)=\frac{1}{f}$$

or $$\frac{1}{f}=-\frac{1}{75}+\frac{1}{125}=-\frac{2}{375}$$

or $$f=-187.5 \text{ mm (concave)}$$

when the telescope is focussed at 7.5 m without the focussing lens

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{+7500}=\frac{1}{200}$$

or $$\frac{1}{v}=\frac{1}{200}-\frac{1}{7500}$$

or $$v=205.479 \text{ mm}$$

with the shifting of the focussing lens. Let

$$u=x$$

then $$v=x+250-205.479$$

$$=x+44.521$$

Using the lens formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{x+44.521}-\frac{1}{x}=-\frac{1}{187.5}$$

which gives $$x=71.77 \text{ mm}$$

The distance between the lens and objective then becomes 205.479 – 71.77 = 133.709 mm.

When the focus is at 7.5 m with lateral displacement of concave lens .025 mm the position of the lenses are as shown in Fig. 5.7(b). xx_1 is the displacement at the level of 1st image because of lateral displacement of 0.025 mm of the concave lens.

From Fig. 5.7(b)

$$\frac{xx_1}{44.521}=\frac{.025}{44.521+71.77}$$

or $$xx_1=.00957 \text{ mm}$$

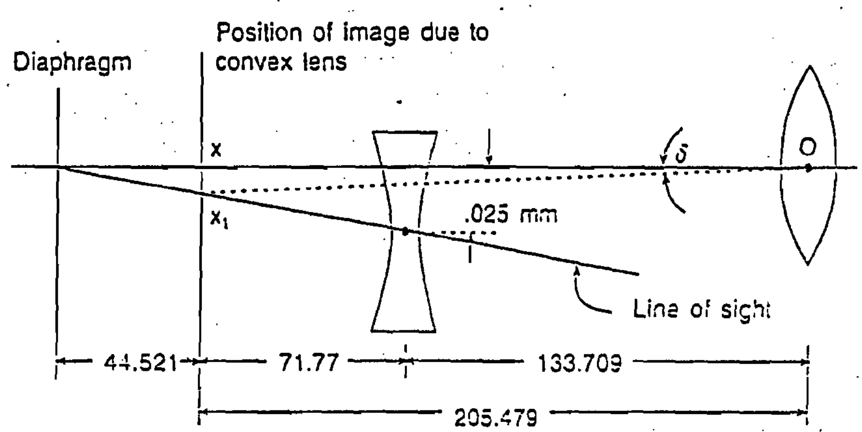


Fig. 5.7(b) Example 5.4.

Angular error $\delta = \frac{xx_1}{ox} = \frac{.00957}{205.479} (206265'')$

$= 9.62''$

5.7 ENGINEER'S LEVELS

There are three general types of engineer's levels. These are: (i) Dumpy level (ii) Tilting level and (iii) Automatic or self levelling level.

Though the design of the three types differs, the operating principle is the same. The major parts of a dumpy level are:

1. A telescope
2. A bubble tube
3. A vertical spindle
4. A levelling head
5. A tripod

The schematic diagram of a dumpy level is shown in Fig. 5.8.

The telescope tube and the vertical spindle are cast together as one piece. The spindle revolves in the socket of the levelling head. The levelling head consists of two parallel plates held apart by three levelling screws. The upper parallel plate is called the *tribrach*. The lower plate, known as *trivet stage*, is screwed on top of a tripod when the instrument is to be used. The telescope can be rotated in the horizontal plane about its vertical axis.

Telescope of a dumpy level is normally internal focussing. It is a metal tube containing four main parts:

(a) Objective lens
(b) Negative lens
(c) Diaphragm or reticule, and
(d) Eyepiece

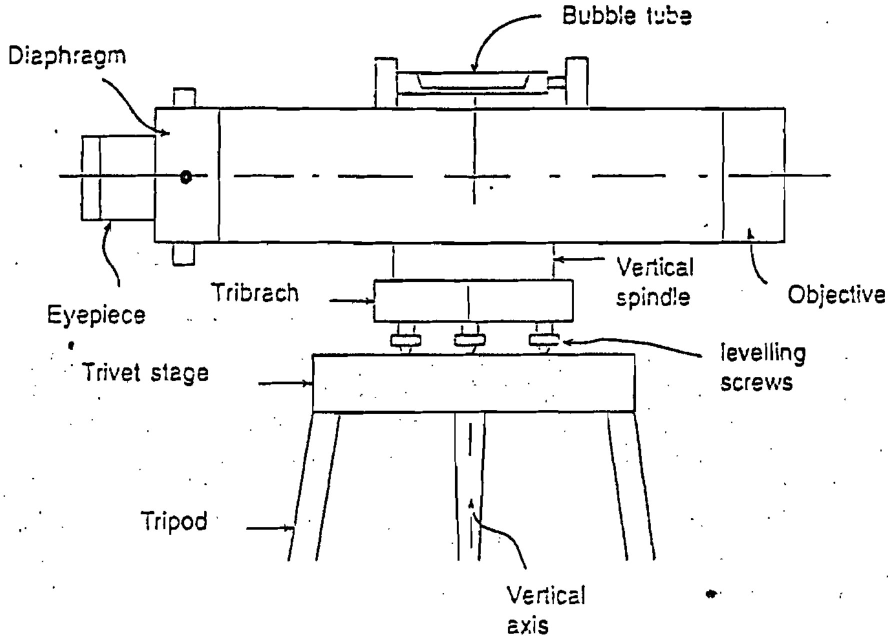


Fig. 5.8 Dumpy level.

Objective Lens. A single lens has many defects like (i) chromatic aberration, (ii) spherical aberration, (iii) coma, (iv) astigmatism, (v) curvature of field, and (vi) distortion. To avoid the first two defects as much as possible the objective lens is composed of both crown and flint glass as shown in Fig. 5.9.

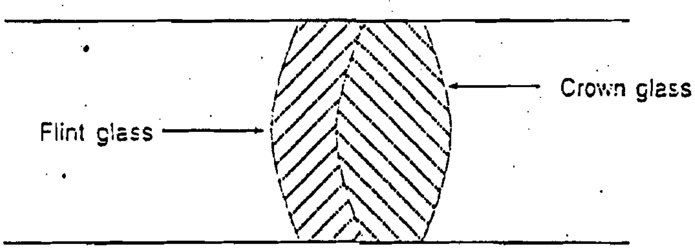


Fig. 5.9 Objective lens.

As a result, chromatic aberration and spherical aberration are nearly eliminated. To minimize loss due to reflection, the lenses are given a thin uniform coating which has an index of refraction smaller than that for glass.

Negative Lens. The negative lens should be mounted on a sliding tube co-axially inside the tube carrying the objective lens. The optical axis of both the lenses should be the same and movement of the negative lens during focussing operations should not introduce deviation of either of the lens axes.

Diaphragm. The diaphragm carries the reticle containing a horizontal and vertical hair. The diaphragm is fitted inside the main tube by means of four capstan headed screws with the help of which the position of the crosshairs inside the tube can be adjusted slightly, both horizontally and vertically and a slight rotational movement is also possible. Previously the crosshairs were made of spider web or filaments of platinum or glass stretched across an annular ring. In many modern instruments, a thin glass plate with lines ruled or etched and filaments of dark metal deposited in them, serves as reticle. Sometimes, two additional horizontal lines are added, one above and another below the usual horizontal hair. The additional hairs are known as stadia hairs and are used in computing distances by stadia tacheometry. Figure 5.10 shows the diaphragm and reticle. Figure 5.11 shows different arrangements of crosshairs.

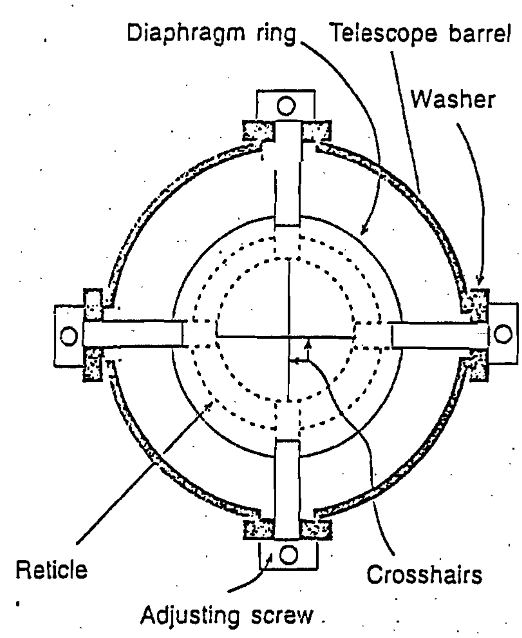


Fig. 5.10 Diaphragm carrying crosshairs.

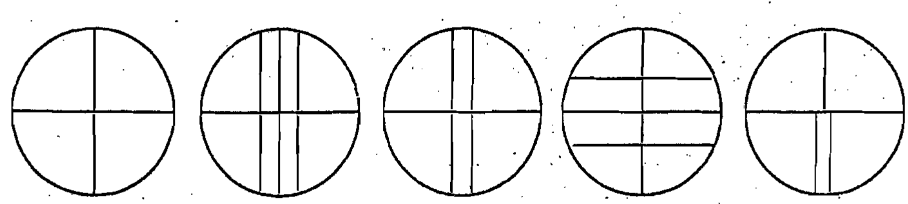


Fig. 5.11 Different types of crosshairs.

Eyepiece. The eyepiece lenses magnify the image together with the crosshairs in order to give the surveyor ability to sight and read accurately the levelling rod graduations. The image formed by the objective and the focussing lens is inverted. Some eyepiece erect the image to give a normal view when it is known as *erecting* eyepiece. Normally, however, the image is seen magnified and inverted through the eyepiece.

Ideally, the eyepieces should reduce chromatic and spherical aberration. Lenses of the same material are achromatic if their distance apart is equal to the average of their focal lengths, i.e.

$$d = \frac{1}{2}(f_1 + f_2)$$

If their distance apart is equal to the differences between their focal lengths, spherical aberration is reduced. i.e. $d = f_1 - f_2$.

For surveying purposes the diaphragm must be between the eyepiece and the objective. The most suitable is Ramsden's eyepiece (Fig. 5.12).

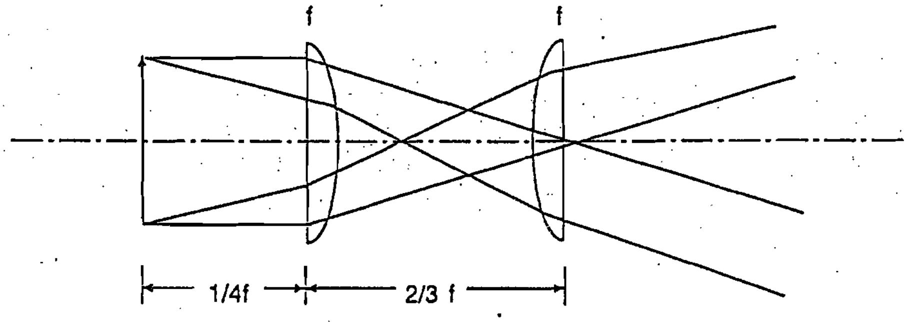


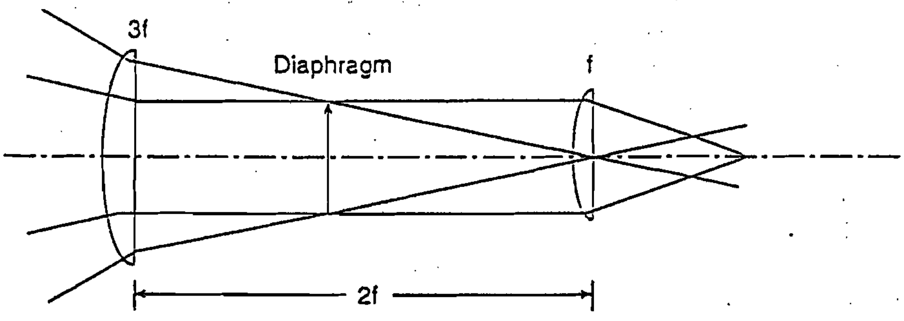
Fig. 5.12 Ramsden's eyepiece.

It can be seen that this eyepiece satisfies the condition for elimination of neither chromatic aberration nor spherical aberration. Here,

$$d = \frac{2}{3}f \quad \text{instead of } \frac{1}{2}(f + f) = f$$

$$= \frac{2}{3}f \quad \text{instead of } f - f = 0$$

Huygen's eyepiece (Fig. 5.13) satisfies the conditions but the focal plane lies between the lenses. This eyepiece, however, is not generally used with the telescopes for measuring instruments because it does not correct the image of the diaphragm which is put between the two lenses and is thus only viewed through one of them with the consequence that its image is distorted. This introduces error in measurement. However it is used in Galileo telescope.



Chromatic condition $\frac{1}{2}(3f + f) = 2f = d$

Spherical condition $3f - f = 2f = d$

Fig. 5.13 Huygen's eyepiece.

The third type of eyepiece is erecting eyepiece (Fig. 5.14). As seen in the figure, it consists of four plano-convex lenses. It gives erect image of the object. Its disadvantages are: (i) Loss of brilliancy of the image due to two additional lenses, (ii) Uncertain definition, and (iii) Larger length of the eyepiece.

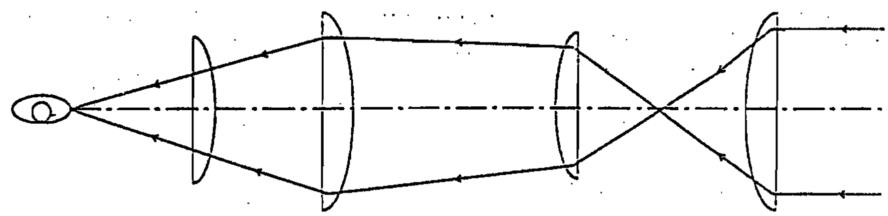


Fig. 5.14 Erecting eyepiece.

Level tube works on the principle that the surface of a still liquid, being at every point normal to the direction of gravity is a level surface. It is of glass tube sealed at both ends that contains a sensitive liquid and small air bubble. The liquid must be non-freezing, quick acting and relatively stable in length for normal temperature variation. Earlier, alcohol, chloroform or sulphuric acid or petroleum ether was used as a liquid. Nowadays, purified synthetic alcohol is used. The upper surface of the tube—and sometimes also the lower surface—is ground to form a longitudinal circular curve. The sectional elevation and the plan of a level tube are shown in Fig. 5.15. The capstan headed screws at the ends help in adjustment of the level tube.

The sensitivity of the level tube depends on the radius of curvature (R) and is usually expressed as an angle (θ) per unit division (d) of the bubble scale. This angle may vary from 1″ to 2″ in the case of a precise level, upto 10″ to 30″ on an engineer's level. The radius to which the tube of an engineer's level is ground is usually between 25 to 50 m. This can be determined in the field by observing the staff readings at a known distance from the level by changing the bubble position by means of a foot screw or tilting screw as shown in Fig. 5.16. From Fig. 5.16

$$\tan n\theta = \frac{S}{l}$$

Since θ is very small, $\tan n\theta \approx n\theta$

or

$$n\theta_{rad} = S/l$$

$$\theta_{rad} = \frac{S}{nl}$$

$$\theta_{sec} = \frac{206265S}{nl}$$

where S = difference in staff readings a and b

n = number of divisions the bubble is displaced between readings

l = distance of staff from instrument

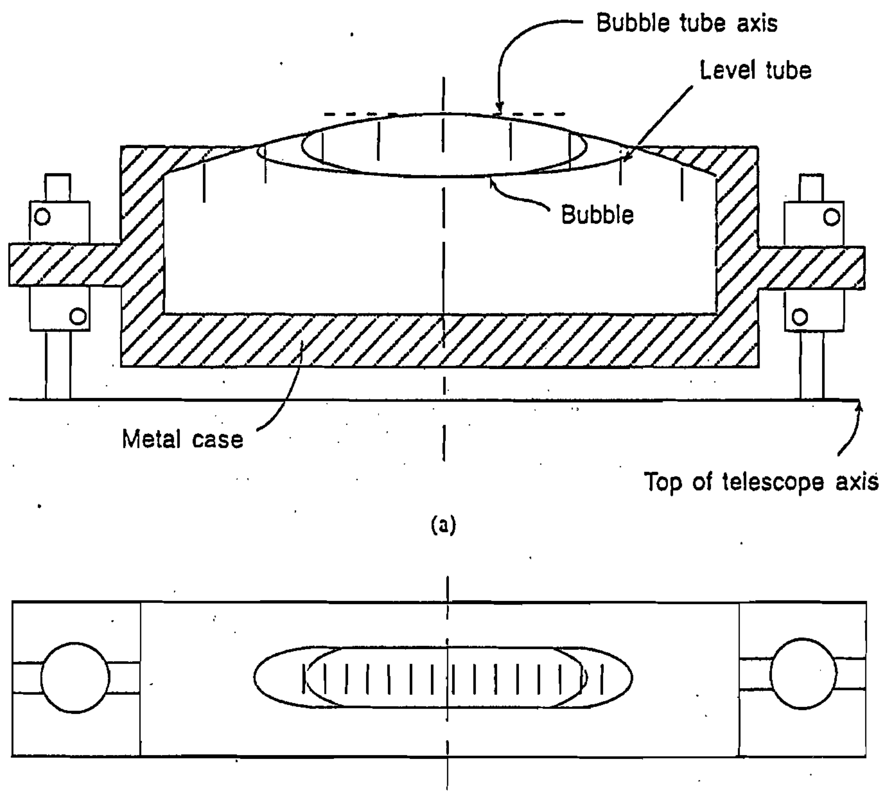


Fig. 5.15 A bubble tube: (a) Cross section. (b) plan.

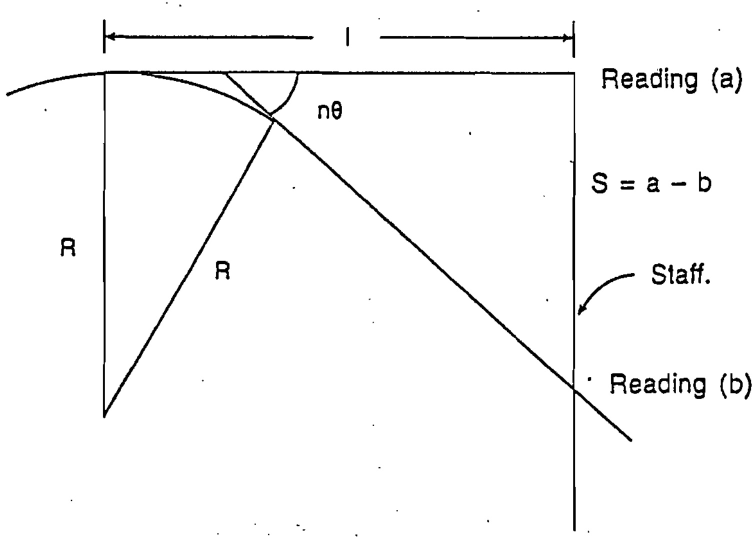


Fig. 5.16 Sensitivity of bubble tube.

If $\quad d =$ length of one division of the bubble tube then

$$d = R\theta_{rad}$$

or

$$R = d/\theta$$

$$= \frac{ndl}{S}$$

A tube is said to be more sensitive if the bubble moves by more divisions for a given change in the angle. The sensitiveness of a bubble tube can be increased by:

(a) Increasing internal radius of the tube.
(b) Increasing diameter of the tube.
(c) Increasing length of the bubble.
(d) Decreasing roughness of the wall
(e) Decreasing viscosity of the liquid.

The sensitivity of the bubble tube should tally with the accuracy achievable with other parts of the equipment. If the bubble is graduated from the centre then an accurate reading is possible by taking readings at the objective and eyepiece ends (Fig. 5.17).

From Fig. 5.17

$$L = \text{Length of bubble}$$

$$= O_1 + E_1 = O_2 + E_2$$

$$XX = \frac{O_1 + E_1}{2} - E_1$$

$$YY = \frac{O_2 + E_2}{2} - O_2$$

Total movement $n = \dfrac{O_1 - E_1}{2} + \dfrac{E_2 - O_2}{2}$

Figures 5.18 and 5.19 show the details of an adjustable leg of a tripod stand and a fixed leg tripod as per I. S. An adjustable tripod is advantageous for set ups in rough terrain but the type with fixed leg may be slightly more rigid. A sturdy tripod in good condition is necessary to obtain the best results for a fine instrument.

Example 5.5 A three screw dumpy level, setup with the telescope parallel to two foot screws is sighted on a staff 100 m away. The line of sight is depressed by manipulating the foot screws until the bubble on the telescope reads 4.1 at the object glass end and 14.4 at the eye piece end, these readings representing divisions from a zero at the centre of the bubble tube. The reading on the staff was 0.930 m. By similarly elevating the sight the bubble readings were—*O* 12.6, *E* 5.7 and staff reading 1.025 m.

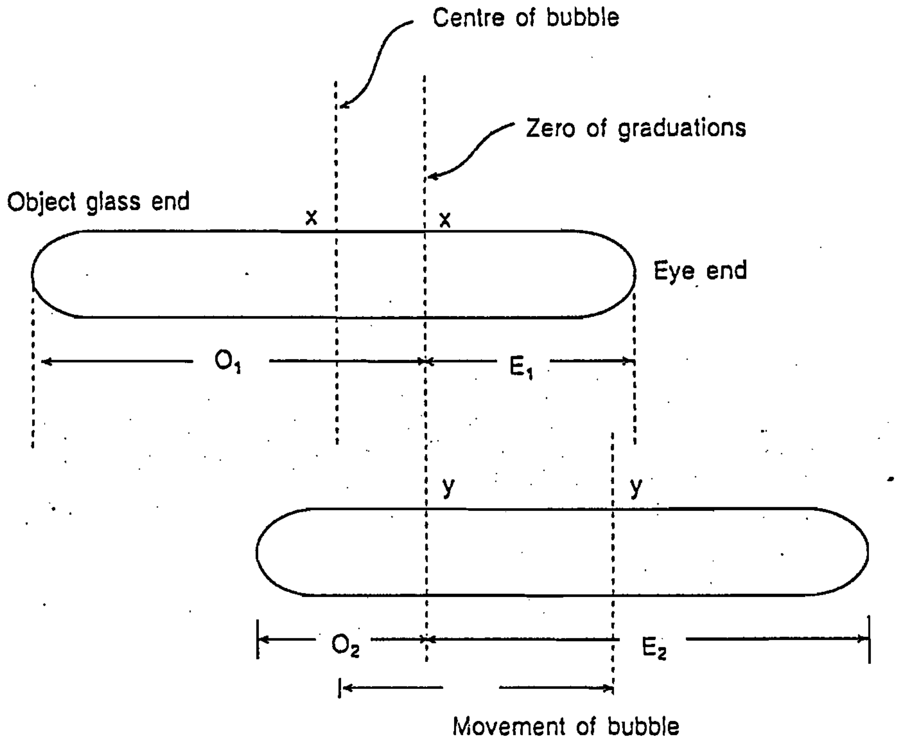


Fig. 5.17

Determine the sensitivity of the bubble and the radius of curvature of the bubble tube if the length of one division is 2.50 mm. [L.U.]

Solution

$$n = \frac{O_1 - E_1}{2} + \frac{E_2 - O_2}{2}$$

$$= \frac{4.1 - 14.4}{2} + \frac{5.7 - 12.6}{2}$$

$$= -\frac{10.3 + 6.9}{2} = -\frac{17.2}{2} = -8.6 \text{ divisions}$$

(Negative because the line of sight is depressed and the bubble moves to the eyepiece end initially.)

$$\text{Sensitivity of bubble } \theta_{sec} = \frac{206265S}{nl}$$

$$= \frac{206265(1.025 - 0.930)}{8.6(100)}$$

$$= 22.78''$$

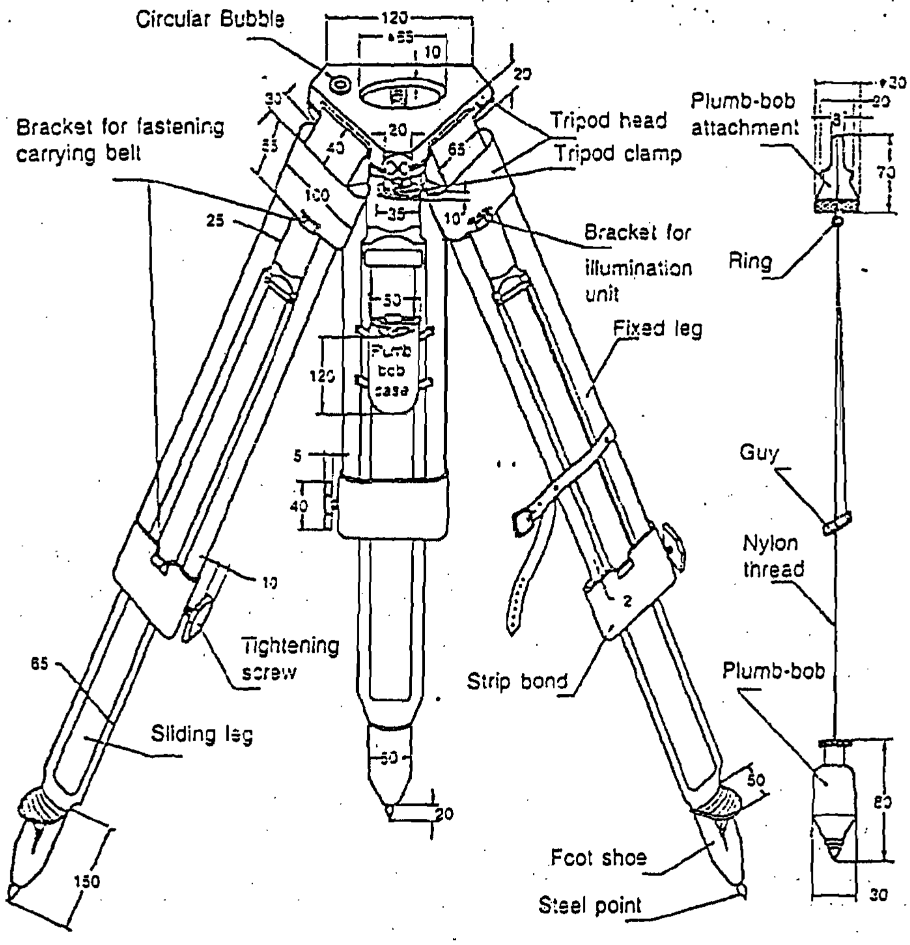


Fig. 5.18 Dimensions and nomenclature of tripod for surveying instruments (adjustable leg).

$$R = \frac{n \cdot d \cdot l}{S} = \frac{(8.6)(2.5)(100)}{.095}$$

$$= 22631.5 \text{ mm}$$

$$= 22.631 \text{ m}$$

5.8 TILTING LEVEL

In dumpy level, if the level is in adjustment and if the line of sight is made horizontal by bringing the bubble to the centre of its run, the vertical axis automatically becomes truly vertical. In tilting level, the line of sight can be made horizontal by a tilting screw even though the vertical axis is not exactly vertical. It was initially developed for precise levelling work but nowadays is used for general purpose. A bull's eye (circular) spirit level is available for quick approximate levelling or a ball and socket arrangement (with no levelling screws) permits the

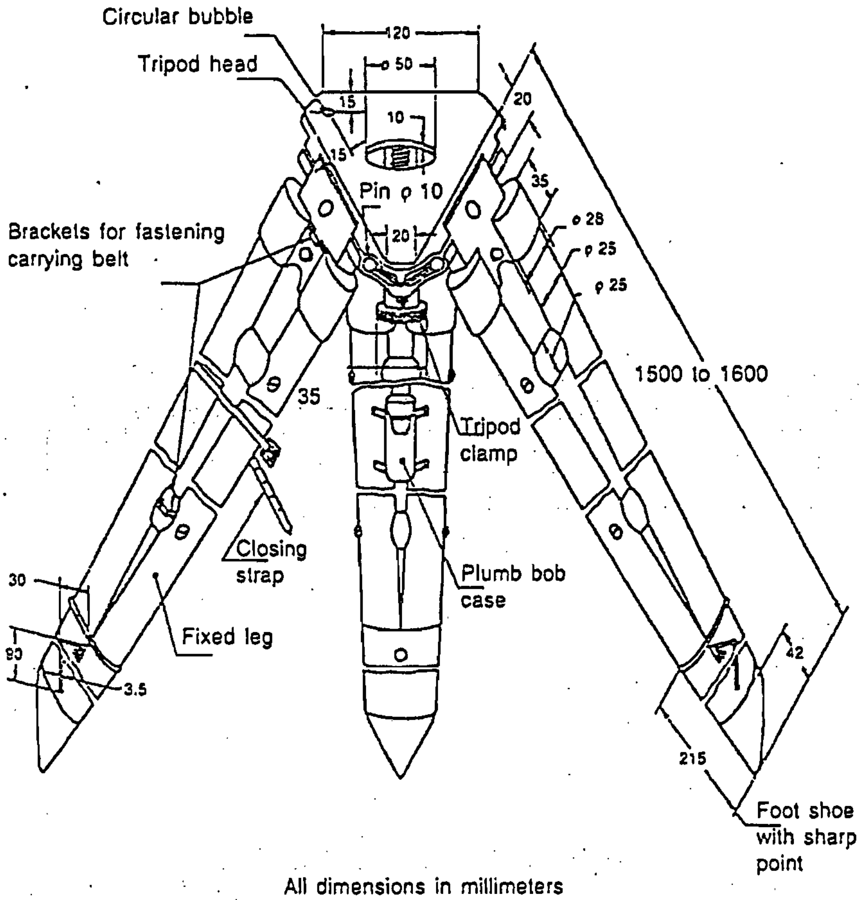


Fig. 5.19 Dimensions and nomenclature for fixed leg tripod for surveying instruments.

head to be tilted and locked nearly level. The exact level is obtained by tilting or rotating the telescope slightly in a vertical plane about a fulcrum at the vertical axis of the instrument without changing its height. A micrometer screw under the eyepiece controls this movement. When the level is not horizontal, the observer sees the main level tube as two half images of opposite ends of the bubble. These half images are brought into superposition and made visible by a prismatic arrangement directly over the bubble. The observer then tilts the telescope until the two half images are made to coincide in which position the bubble is centred. Figure 5.20 shows a split bubble before and after coincidence.

Advantages of tilting level are accuracy and quickness. The level can be made horizontal just before the observation. Figure 5.21 shows schematically a tilting level.

5.9 AUTOMATIC OR SELF-LEVELLING LEVEL

This is the most popular variety of levels. The ease and rapidity with which the instrument makes error free readings has made it popular.

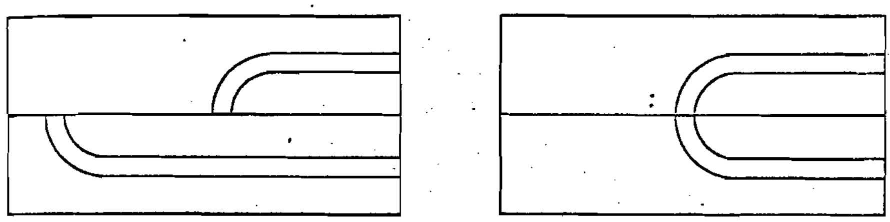


Fig. 5.20 Coincidence bubble.

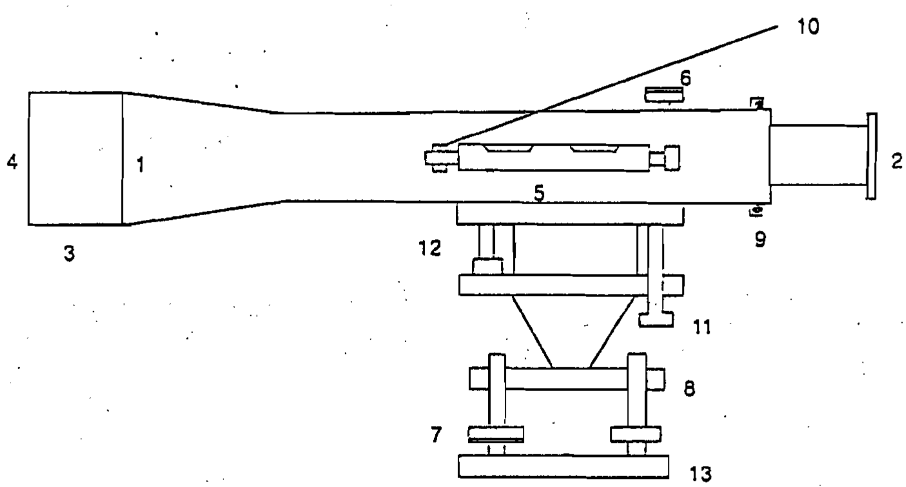


Fig. 5.21 Tilting level: 1. Telescope. 2. Eyepiece. 3. Ray shade. 4. Objective end. 5. Level tube. 6. Focussing screw. 7. Foot screw. 8. Tribrach. 9. Diaphragm adjusting screws. 10. Bubble tube fixing screws. 11. Tilting screws. 12. Spring loaded plunger. 13. Trivet stage.

All levelling operations depend on the establishment of a line of collimation perpendicular to the direction of gravity. In making a conventional level ready for operation the skill and time of the operator is needed for accurate centring of its sensitive telescope bubble. But in making an auto level ready for operation only approximate levelling of its circular bubble is needed and then its in-built compensator takes over and makes the level ready for operation automatically in no time.

As apparent from Fig. 5.22, the compensator (which freely hangs in correct vertical position) takes the horizontal ray from the staff to the centre of diaphragm for correct readings inspite of any possible residual tilt. Technical data of an Indian automatic level is as follows:

Telescope

Image	– Erect
Magnification	– 24 times
Multiplication constant	– 100
Shortest reading distance	– 1.5 m
Longest reading distance	– 200 m
Clear objective aperture	– 35 mm

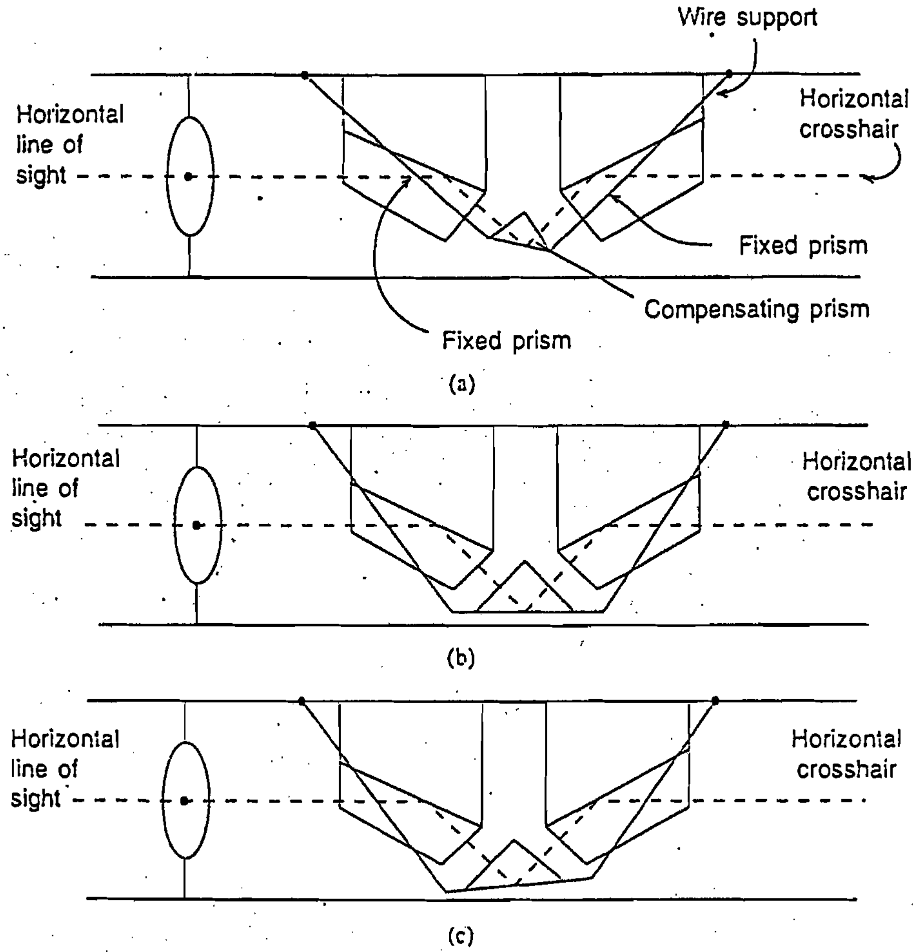


Fig. 5.22 Compensator of a self-levelling level: (a) When telescope tilts up, compensator swings backward. (b) Telescope horizontal. (c) When telescope tilts down compensator swings forward.

Circular level

Sensitivity – 10 mm/2 mm

Automatic levelling

Setting accuracy = ± 0.8 sec

Compensator range = ± 15 min

Horizontal circle

Diameter – 110 mm

Graduation interval – 1°

5.10 SOME IMPORTANT OPTICAL TERMS

Resolving Power. It is the ability of lens for distinguishing details. Its value is usually stated as the maximum number of lines per millimeter that can be seen as separate lines in the image. The resolving power depends on the diameter of the objective lens actually used (effective aperture). It is given by the empirical formula

$$R = \frac{140 \text{ sec}}{D}$$

where D is the diameter of the lens aperture in mm. If D is 30 mm R comes to 4.67. To distinguish details human eye requires a minimum resolving power of 60. This can be obtained with an instrument with $R = 4.67$ if it is magnified 13 times.

Magnification. It is the ratio between the angle subtended at the eye by the virtual image and that subtended by the object. Magnifying power of telescope is measured as the ratio of the focal length of the objective to that of the eye piece. Large magnification causes (i) Reduction of brilliancy of image, (ii) Waste of time in focussing, and (iii) Reduction of the field of view. Usually, therefore, magnification is restricted to 2 to 3 times $60/R$.

Definition. The quality of definition in a telescope is its capacity to produce a sharp image. It depends on eliminating optical defects like chromatic aberration and spherical aberration from the eyepiece and objective.

5.11 SOME IMPORTANT OPTICAL DEFECTS

Chromatic Aberration. When white light is refracted through a glass prism it is split into its component colours, the red end of the spectrum being refracted less than the violet end. This phenomenon, known as *dispersion*, makes accurate focussing difficult, the image being surrounded by a rainbow like boundary. This is chromatic aberration is shown in Fig. 5.23(a). To remedy this defect two lenses, one concave of flint glass and the other a convex lens of crown glass are cemented together with balsam as already explained in Section 5.10.

Spherical Aberration. It arises due to the spherical surface of the lens and prevents accurate focussing due to the rays incident on the lens being refracted more than the rays incident at the centre (Fig. 5.23(b)). This can be remedied by using only the central portion of the lens which also cuts down the amount of light entering the eye. Usually, therefore a combination of lenses is used so that aberration of one eliminates that of the other. For example, in objective a convex and a

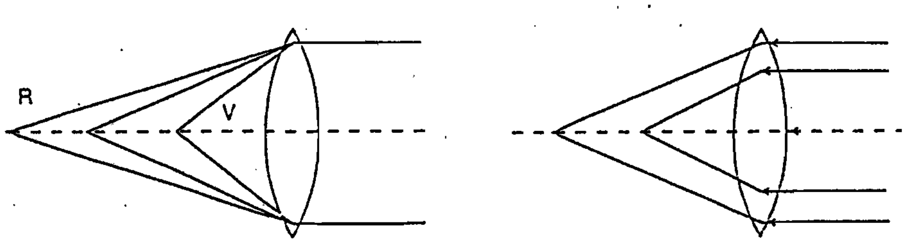


Fig. 5.23 (a) Chromatic aberration. (b) Spherical aberration.

concave lens are cemented together. In Ramsden's eyepiece two plano convex lenses are kept at a fixed distance apart.

5.12 THE LEVELLING STAFF

A level staff is a graduated rod of rectangular section. It is usually made of teakwood. It may also be of fibre glass or metal. Two main classes of rod are:

1. Self-reading which can be read by the instrument operator which sighting through the telescope and noting the apparent intersection of the cross wires on the rod. This is the most common type.

2. Target rods having a movable target that is set by a rod person at the position indicated by signals from the instrument—man.

A levelling staff can be of
(a) Solid, i.e. of one piece—Fig. 5.24(a).
(b) Folding when it can be folded to smaller length—Fig. 5.24(b).
(c) Telescopic when the staff can be shortened by putting one piece inside another—Fig. 5.24(c).

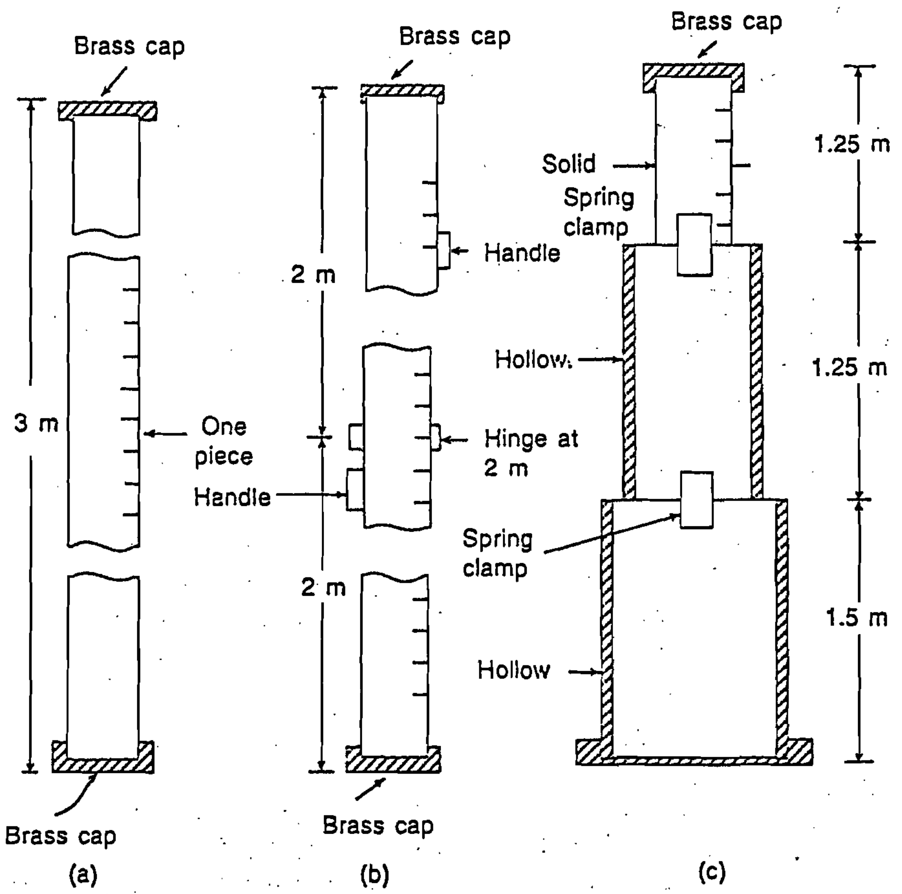
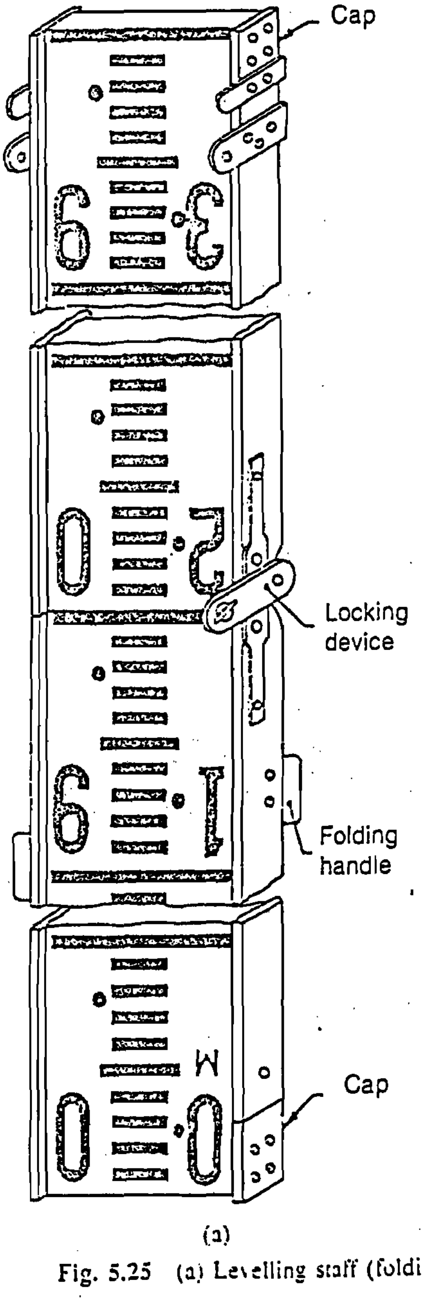


Fig. 5.24 Different types of levelling staff.

Solid staff being of one piece gives more accurate reading. Folding staff is light and convenient to handle. As per IS-1779–1961, the width and thickness of staff are 75 mm and 18 mm respectively. The staff can be folded to 2 m length. To ensure verticality the staff has a circular bubble of 25 minute sensitivity. Each meter is subdivided into 200 subdivisions, the thickness of the graduation being 5 mm. Details of a levelling staff (Folding type) are shown in Fig. 5.25(a) and (b). In telescopic staff shown in Fig. 5.24(c), the topmost part is solid and the other two parts are hollow.



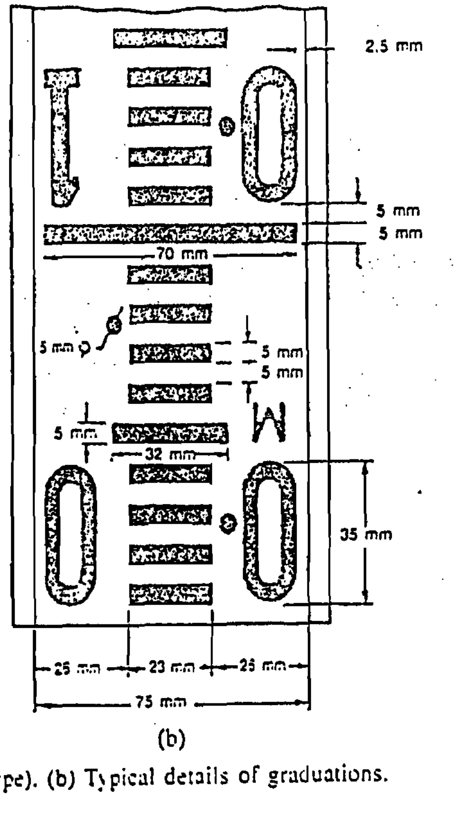


Fig. 5.25 (a) Levelling staff (folding type). (b) Typical details of graduations.

The two top pieces when pulled up are kept in position by brass flat spring clamps at the back of each piece fixed at its lower end. While using the telescope staff care should be taken to ensure that the three parts are fully extended. The telescopic staff is not as accurate as a folding staff because of possible slippage between the parts.

Target staff has sliding target equipped with vernier. It is used for long distance sighting when it becomes difficult to take staff reading directly. The target is a small metal piece of circular or oval shape about 125 mm diameter. It is painted red and white in alternate quadrants. For taking reading the level man directs the staffman to raise or lower the target till it is bisected by the line of sight. The staff holder then clamps the target and take the reading. Apparent advantage of target staff is accurate reading but it takes more time. On the other hand self-reading staff is quicker. Moreover, for self-reading staff only one trained staff, that is, instrument man is required but for target staff reading both instrument man and target man should be adequately trained. Fig. 5.26 gives details of a target staff.

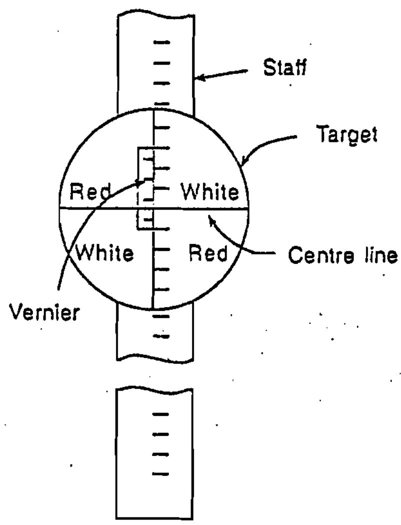


Fig. 5.26 Target staff.

5.13 PARALLEL PLATE MICROMETER

As shown in Fig. 5.27a parallel glass plate is usually fitted in front of the objective of a precise or geodetic level. It enables the interval between the horizontal crosshair and the nearest staff division to be read directly to 0.1 mm. The parallel glass plate can be tilted forward or backward by means of a micrometer head at the eye end of the telescope. Due to refraction a ray of light parallel to the telescope axis is displaced upwards and downwards by an amount proportional to the amount of tilt. When the plate is vertical no displacement occurs. The plate is tilted till a full reading of the staff coincides with the crosshair. The displacement *d* gives the fractional reading which is obtained directly from the micrometer drum. The theory can be derived as follows:

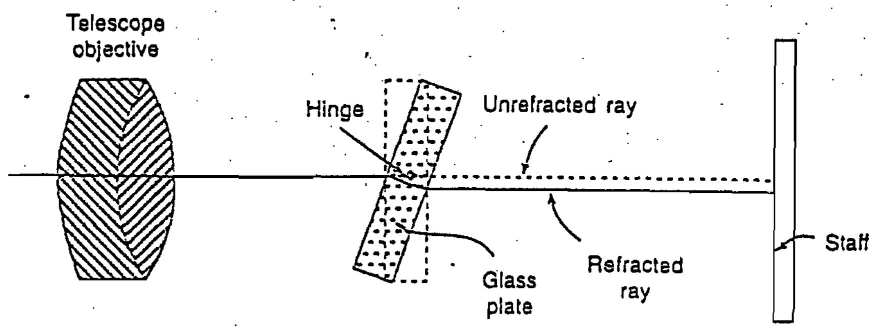


Fig. 5.27(a) Parallel plate micrometer.

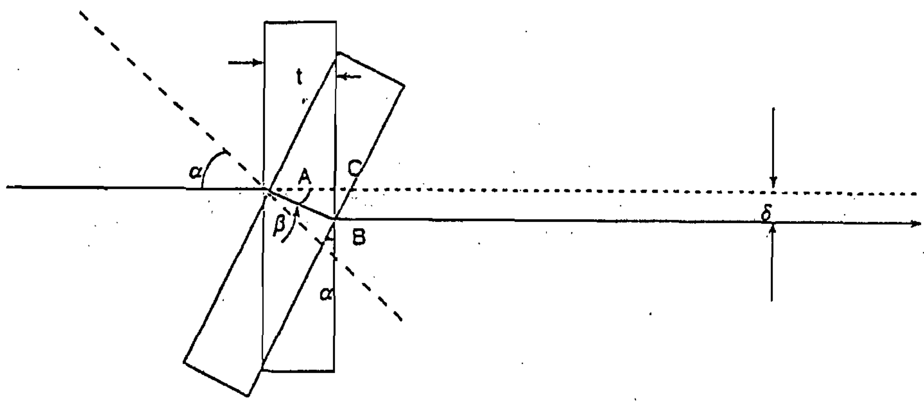


Fig. 5.27(b) Derivation of equation.

Let μ be the refractive index of the glass used in the plate (Fig. 5.27b). From ΔABC,

$AB \cos \beta = AC = t$, thickness of the glass plate

$$AB = \frac{t}{\cos \beta}$$

$$BC = \delta = AB \sin(\alpha - \beta)$$

$$= \frac{t}{\cos \beta} \cdot \sin(\alpha - \beta)$$

$$= \frac{t}{\cos \beta} \cdot (\sin \alpha \cos \beta - \cos \alpha \sin \beta)$$

$$= t\left(\sin \alpha - \cos \alpha \frac{\sin \beta}{\cos \beta}\right)$$

$$= t \sin \alpha \left(1 - \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \beta}{\cos \beta}\right)$$

From laws of refraction

$$\frac{\sin \alpha}{\sin \beta} = \mu \quad \text{or} \quad \sin \beta = \frac{\sin \alpha}{\mu} \quad \text{or} \quad \cos \beta = \sqrt{1 - \left(\frac{\sin \alpha}{\mu}\right)^2}$$

which gives

$$\delta = t \sin \alpha \left(1 - \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \alpha/\mu}{\sqrt{1 - (\sin \alpha/\mu)^2}}\right)$$

$$= t \sin \alpha \left(1 - \frac{\sqrt{1 - \sin^2 \alpha}}{\sqrt{\mu^2 - \sin^2 \alpha}}\right)$$

If α is small $\sin \alpha \to \alpha$ and $\sin^2 \alpha \to 0$, and

$$\delta = t\alpha \left(1 - \frac{1}{\mu}\right)$$

$$= t\alpha \cdot \frac{\mu - 1}{\mu}$$

$$= k\alpha \quad \text{where} \quad k = t \cdot \frac{\mu - 1}{\mu}$$

which shows that the displacement is directly proportional to the angle of rotation α of the plate provided. The angle α is small.

Example 5.6 If the index of refraction from air to glass is 1.6 and the parallel plate prism is 16 mm thick, calculate the angular rotation of the prism to give a vertical displacement of the image of 1 mm.

Solution

$$\delta = t \cdot \alpha \cdot \frac{\mu - 1}{\mu}$$

$$1 = 16 \cdot \alpha \cdot \frac{1.6 - 1}{1.6}$$

$$\alpha = \frac{1.6}{16(0.6)} = 0.1667 \text{ rad}$$

$$= 9°33'$$

5.14 TEMPORARY ADJUSTMENTS OF A DUMPY LEVEL

Temporary adjustments are done at every setting of the instrument in the field. They are:

1. *Setting Up*: Initially the tripod is set up at a convenient height and the instrument is approximately levelled. Some instruments are provided with a small circular bubble on the tribrach to check approximate levelling. At this stage the levelling screw should be at the middle of its run.

2. *Levelling Up*: The instrument is then accurately levelled with the help of levelling screws or foot screws. For instruments with three foot screws the following steps are to be followed:

(a) Turn the telescope so that the level tube is parallel to the line joining any two levelling screws as shown in Fig. 5.28(a).

(b) Bring the bubble to the centre of its run by turning the two levelling screws either both inwards or outwards.

(c) Turn the telescope through 90° so that the level tube is over the third screw or on the line perpendicular to the line joining screws 1 and 2. Bring the bubble to the centre of its run by the third foot screw only rotating either clockwise or anticlockwise Fig. 5.28(b).

(d) Repeat the process till the bubble is accurately centred in both these conditions.

(e) Now turn the telescope through 180° so that it is again parallel to levelling screws 1 and 2 (Fig. 5.28(a)). If the bubble still remains central, the adjustment is allright. If not, the level should be checked for permanent adjustments.

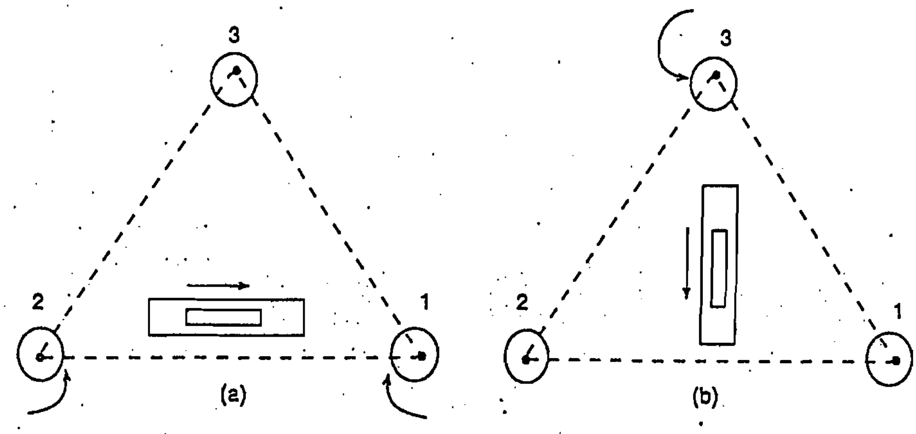


Fig. 5.28 Turning foot screws to level bubble tube.

3. *Focussing*: This is done in two steps. First step is focussing the eyepiece. This is done by turning the eyepiece either in or out until the crosshairs are sharp and distinct. This will vary from person to person as it depends on the vision of the observer. The next step is focussing the objective. This is done by means of the focussing screw where by the image of the staff is brought to the plane of the crosshairs. This is checked by moving the eye up or down when reading the crosshair does not change with the movement of the eye as the image and the crosshair both move together.

5.15 TERMS USED IN LEVELLING

The following terms are frequently used in levelling

1. *Station:* This is a point where a levelling staff is held for taking observations with a level.

2. *Height of the Instrument* (HI): This has two meanings. It may mean height of the instrument above the ground at the station where the instrument is placed. However, usually it means elevation of the line of sight or line of collimation with respect to the datum. Line of collimation is an imaginary line joining the optical centre of the objective with the intersection of crosshairs and its continuation.

3. *Back Sight:* It is the first reading taken at a station of known elevation after setting up of the instrument. This reading gives the height of the instrument (elevation of the line of collimation) as.

Elevation of line of collimation = Known elevation + Back sight

4. *Intermediate Sight* (IS): As the name suggests these are readings taken between the 1st and last reading before shifting the instrument to a new station.

5. *Fore Sight* (FS): This is the last reading taken before shifting an instrument to a new station.

6. *Turning Point or Change Point:* For levelling over a long distance, the instrument has to be shifted a number of times. Turning point or change point connects one set of instrument readings with the next set of readings with the changed position of the instrument. A staff is held on the turning point and a foresight is taken before shifting the instrument. From the next position of the instrument another reading is taken at the turning point keeping the staff undisturbed which is known as back sight.

7. *Reduced Level* (RL): Reduced level of a point is its height relative to the datum. The level is calculated or reduced with respect to the datum.

5.16 DIFFERENT METHODS OF LEVELLING

In levelling it is desired to find out the difference in level between two points. Then if the elevation of one point is known, the elevation of other point can be easily found out. In Fig. 5.29, the instrument is placed at *C* roughly midway between two points *A* and *B*. The staff readings are shown in the figure.

From the figure the reduced level of *B* can be derived as 100.50 + 1.51 – 0.57 = 101.44 mm. From the readings it can also be observed that if the second reading is smaller than the 1st reading, it means that the second point is at a higher level than the first. This is also known as *direct levelling*.

In trigonometrical levelling the difference in elevations is determined indirectly from the horizontal distance and the vertical angle. Since trigonometric relations are utilized in finding the difference in elevation it is known as trigonometrical levelling. It is used mainly to determine elevations of inaccessible points such as mountain peaks, top of towers, etc. as shown in Fig. 5.30.

In barometric levelling, the principle that pressure decreases with rise in

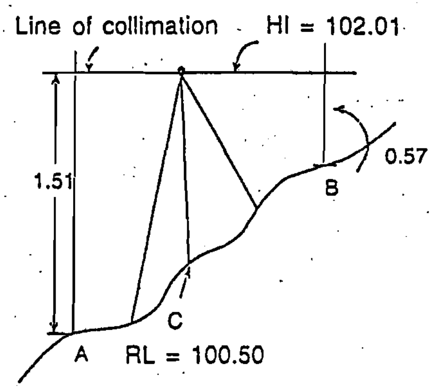


Fig. 5.29 Direct levelling.

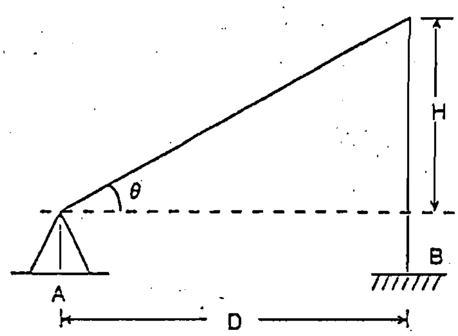


Fig. 5.30 Trigonometrical levelling.

elevation is used. Hence it is possible to determine the difference in elevation between two points by measuring the pressure difference between the points by either mercury barometer or aneroid barometer. As the aneroid barometer is strong and sturdy it is preferred to the mercury barometer which is fragile and cumbersome. However, aneroid barometer is less accurate compared to the mercury barometer.

PROBLEMS

5.1 Explain how surveyors and engineers can often ignore the error caused by curvature and refraction in levelling work.

5.2 What errors may be introduced in using telescope's focussing screws?

5.3 Define optical axis of a lens.

5.4 List in tabular form, for comparison, the advantages and disadvantages of a tilting level versus an automatic level.

5.5 Describe the method of operation of a parallel plate micrometer in precise levelling. If the index of refraction from air to glass is 1.6 and the parallel plate prism is 15 mm thick, calculate the angular rotation of the prism to give a vertical displacement of the image of 0.0001 m.

5.6 Describe with the aid of a sketch the function of an internal focussing lens in a surveyor's telescope and state the advantages and disadvantages of internal focussing as compared with external focussing.
In a telescope, the object glass of focal length 178 mm is located 230 mm from the diaphragm. The focussing lens is midway between these when a staff 20 m away is focussed. Determine the focal length of the focussing lens. (L.U., BSc.)

5.7 Define the following terms.
(a) level surface, (b) horizontal plane, (c) vertical plane, (d) vertical line, (e) elevation of a point, (f) line of collimation, (g) benchmark, (h) change point, (i) datum, (j) back sight, (k) foresight.

5.8 Give the definition of sensitiveness of bubble tube in levelling and its effect in accuracy of levelling. [AMIE, November 1964]

5.9 Sketch a modern tilting level, name its parts and describe step by step how it is used. What is the main advantage of this type of level over the Dumpy level?

5.10 Work out the true difference in level between points A and B if curvature and refraction effects are taken into account in the following case:

Level set up over point A	
Staff held over point B	
R.L of point A	= 100.000 m
Height of instrument at point A	= 1.000 m
Reading at staff on point B	= 2.000 m
Distance AB	= 300 m
Assume diameter of earth	= 12,742 km

[AMIE, May 1969]

5.11 (a) Explain the effects of curvature and refraction in levelling.
(b) An observer standing on the deck of a ship just sees a lighthouse. The top of the lighthouse is 64 m above sea level and the height of the observer's eye is 9 m above sea level. Find the distance of the observer from the lighthouse.

5.12 If the bubble tube of a level has a sensitiveness of 35″ per 2 mm division, find the error in staff reading on a vertical staff at a distance of 100 m caused by the bubble bending $1^1/_2$ divisions out of centre.

6

Levelling II

6.1 INTRODUCTION

Direct levelling can be broadly classified as:

(i) Differential levelling, (ii) Check levelling, (iii) Fly levelling, (iv) Profile levelling, (v) Cross sectional levelling, and (vi) Reciprocal levelling.

6.2 DIFFERENTIAL LEVELLING

Figure 6.1 shows the plan, and sectional elevation of a road way along which a line of level is being taken. The figure also explains the different terms used in connection with differential levelling.

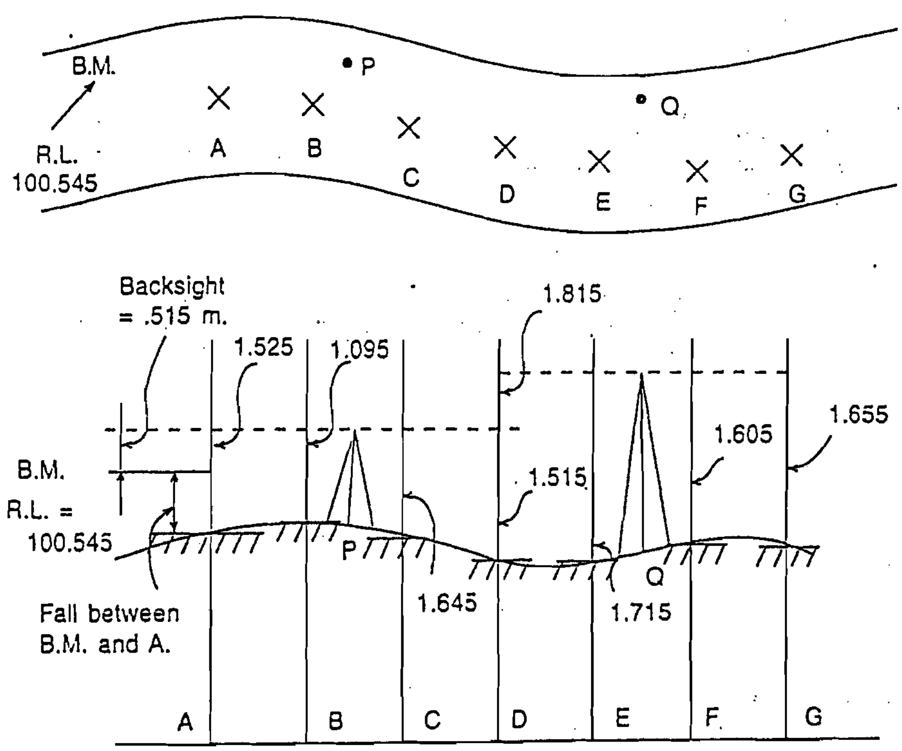


Fig. 6.1 Differential levelling.

The instrument is set up at a convenient position *P* and the level staff is placed over the B.M. of reduced level 100.545 m. This is the first reading taken after setting up the instrument and is known as *back sight*. Let its value be 0.515 m. The staff is now held at points *A*, *B* and *C* in turn and readings known as *intermediate sights* are taken. The last reading taken with this set up of instrument is at *D*. It is found that no further readings are possible after *D* due to (i) poor visibility and (ii) change in level of the ground surface or some obstacle in the line of sight. The last reading on *D* is known as *fore sight*. After this, the instrument is shifted to point *Q*. The point *D* is called *change point* or *turn point* because it is the staff position during which the position of the level is being changed.

The instrument is then setup at *Q*, levelled and a staff reading is again taken on point *D*. This is back sight for the second set up of the instrument. Intermediate sights are taken on *E* and *F* and the last reading, a fore sight is taken at *G* before the instrument is shifted again.

6.3 LEVEL BOOK

Instead of writing the readings in a sketch and giving suitable descriptions, the whole process of levelling is systematically shown in a level book and reduced levels of different points found out. There are two methods of reducing levels. (i) Rise and fall method, and (ii) Height of collimation method. Complete bookings and reductions in the two methods are given in Table 6.1.

Table 6.1 Rise and Fall Method

Back-sight	Inter-sight	Fore-sight	Rise	Fall	Reduced level	Distance in m	Remarks
0.515					100.545		Bench mark
	1.525			1.010	99.535	0	Staff Stn. *A*
	1.095		0.430		99.965	30	Staff Stn. *B*
	1.645			0.550	99.415	60	Staff Stn. *C*
1.815		1.515	0.130		99.545	90	Staff Stn. *D* (Change point)
	1.715		0.100		99.645	120	Staff Stn. *E*
	1.605		0.110		99.755	150	Staff Stn. *F*
		1.655		0.050	99.705	180	Staff Stn. *G*
Σ = 2.330		Σ 3.170	Σ 0.770	Σ 1.610			

Check: Σ Back-sight – Σ Fore-sight
= (2.330) – (3.170) = – 0.840
Σ Rise – Σ Fall
= (0.770) – (1.610) = – 0.840
Last R.L. – 1st R.L.
= (99.705) – (100.545) = – 0.840

6.3.1 RISE AND FALL METHOD

Each reading is entered on a different line in the applicable column, except at change points where a fore-sight and a back-sight occupy the same line. This is to connect the line of sight of one set up of the instrument with the line of sight of the second set up of the instrument. From Fig. 6.1, it can be seen that they are not at the same level. R.L. of change point *D* is obtained from the first line of sight by comparing intermediate sight 1.645 with foresight 1.515, i.e. a rise of 0.130 m. For the R.L. of next point *E*, back sight 1.815 is compared with intermediate sight 1.715, i.e. a rise of 0.100 m (Table 6.2). At the end of the table arithmetic checks are shown.

The checks are:

$$\Sigma \text{ Backsights} - \Sigma \text{ Foresights} = \Sigma \text{ (Rises)} - \Sigma \text{ (Falls)}$$
$$= \text{Last R.L.} - \text{First R.L.}$$

Table 6.2 Height of Collimation Method

Back-sight	Inter-sight	Fore-sight	Ht. of collimation	Reduced level	Distance	Remarks
0.515			101.060	100.545		B.M.
	1.525			99.535		
	1.095			99.965		
	1.645			99.415		
1.815		1.515	101.360	99.545		
	1.715			99.645		
	1.605			99.755		
		1.655		99.705		
Σ 2.330	Σ 7.585	Σ 3.170				

Check: Σ Backsights $-$ Σ Foresights = 2.330 − 3.170
= − 0.840

Last R.L. − First R.L. = 99.705 − 100.545
= − 0.840

6.3.2 HEIGHT OF COLLIMATION METHOD

The height of collimation is obtained by adding the staff reading which must be backsight to the known R.L. of the point on which the staff stands. R.Ls. of all the other points are obtained by subtracting the staff reading from the height of collimation. When the instrument is changed a new height of collimation is obtained by again adding new backsight with R.L. of the last point obtained from previous set up of the instrument. The arithmetic checks to be applied are:

$$\Sigma \text{ B.S.} - \Sigma \text{ F.S.} = \text{Last R.L.} - \text{First R.L.}$$

In this case intermediate R.Ls. remain unchecked compared with rise and fall method where errors in all R.Ls. are detected. However, reduction is easier with height of collimation method when the intermediate sights are large in number. When a reading ends on an intermediate sight, for checking purposes it should be taken as a foresight. For checking for subsequent reading this may be considered as backsight. To check intermediate R.L.'s the following formula may be used:

Σ Reduced levels less the first + Σ I.S. + Σ F.S.

= Height of each collimation × No. of applications.

For the example given:

L.H.S. = (99.535 + 99.965 + 99.415 + 99.545 + 99.645 + 99.755 + 99.705) + 7.585 + 3.170

= 708.32

R.H.S. = (101.06)(4) + (101.36)(3) = 708.32

No. of applications can be further explained as equal to: No. of I.S. and F.S deduced from it.

In the first set up of instrument we have 3 I.S. and 1 F.S. making total 4. In the second set up of the instrument we have 2 I.S. and 1 F.S. making total 3.

Example 6.1 Complete the levelling table given below. If an even gradient of 1 vertical in every 7 horizontal starts 1 m above peg 0, what is the height of the gradient above or its depth below peg 7? [I.C.E. Lond]

Table 6.3 Example 6.1

Station	Dist.	Back-sight	Inter-sight	Fore-sight	Rise	Fall	R.L.
B.M		3.10					193.62
0	0		2.56				
1	20		1.07				
2	40	1.92		3.96			
3	60	1.20		0.67			
4	80		4.24				
5	100	0.22		1.87			
6	120		3.03				
7	140			1.41			

Solution Complete levelling table is

Table 6.4 Example 6.1

Station	Distance	Back-sight	Inter-sight	Fore-sight	Rise	Fall	R.L.	R.L. of gradient
B.M.		3.10					193.62	
0	0		2.56		0.54		194.16	195.16
1	20		1.07		1.49		195.65	
2	40	1.92		3.96		2.89	192.76	
3	60	1.20		0.67	1.25		194.01	
4	80		4.24			3.04	190.97	
5	100	0.22		1.87	2.37		193.34	
6	120		3.03			2.81	190.53	
7	140			1.41	1.62		192.15	193.16
		Σ 6.44		Σ 7.91	Σ 7.27	Σ 8.74		

Check: 6.44 – 7.91 = –1.47 m

7.27 – 8.74 = – 1.47 m

Last R.L. – 1st R.L = 192.15 – 193.62 = –1.47 m

Height of gradient above peg 7 = 193.16 – 192.15

= 1.01 m above

Example 6.2 In order to find the rail levels of an existing railway, a point *A* was marked on the rail, then points at distances in multiples of 20 m from *A* and the following readings were taken:

Backsight 3.39 m or O.B.M. 23.10

Intermediate sights on *A*, *A* + 20 and *A* + 40, 2.81, 2.51 and 2.22 respectively. *A* + 60: change point: foresight 1.88, backsight 2.61. Intermediate sights on *A* + 80 and *A* + 100, 2.32 and 1.92 respectively; and finally a foresight of 1.54 on *A* + 120, all being in meters. Tabulate the above readings on the collimation system and then assuming the levels at *A* and *A* + 120 were correct, calculate the amounts by which the rails would have to be lifted at the intermediate points to give a uniform gradient throughout. Repeat the tabulation on the rise and fall system and apply what checks are possible in each case.

Solution (i) *Height of Collimation Method*

Table 6.5 Example 6.2

Back-sight	Inter-sight	Fore-sight	Height of Collimation	R.L.	Distance	R.L. on gradient	Diff.	Remarks
3.39			26.49	23.10				O.B.M 23.10
	2.81			23.68	0	23.68	0.00	Point *A*
	2.51			23.98	20	24.01	0.03	
	2.22			24.27	40	24.34	0.07	
2.61		1.88	27.22	24.61	60	24.68	0.07	Change Point
	2.32			24.90	80	25.01	0.11	
	1.92			25.30	100	25.34	0.04	
		1.54		25.68	120	25.68	0.00	
Σ 6.00	Σ 11.78	Σ 3.42						

Check: Σ (B.S.) – Σ (F.S.) = 6.00 – 3.42 = 2.58 m

Last R.L. – 1st. R.L. = 25.68 – 23.10

= 2.58 m

Σ Reduced levels less the 1st + Σ I.S. + Σ F.S.

= (23.68 + 23.98 + 24.27 + 24.61 + 24.90 + 25.30 + 25.68) + 11.78 + 3.42

= 187.62 m

Σ Each instrument height × (No. of I.S. and F.S. deduced from it)

= 26.49 (4) + 27.22 (3)

= 187.62 m

(ii) *Rise and Fall Method*

Table 6.6 Example 6.2

Backsight	Intersight	Foresight	Rise	Fall	R.L.	Distance	Remarks
3.39					23.10		
	2.81		0.58		23.68	0.00	
	2.51		0.30		23.98	20.00	
	2.22		0.29		24.27	40.00	
2.61		1.88	0.34		24.61	60.00	
	2.32		0.29		24.90	80.00	
	1.92		0.40		25.30	100.00	
		1.54	0.38		25.68	120.00	
Σ 6.00		Σ 3.42	Σ 2.58	Σ 0.00			

Check: Σ B.S. – Σ F.S. = 6.00 – 3.42 = 2.58

Σ Rise – Σ Fall = 2.58 – 0.00 = 2.58

Last R.L. – 1st. R.L. = 25.68 – 23.10 = 2.58

Example 6.3 The under noted readings in meters on a levelling staff were taken along a roadway *AB* with a dumpy level, the staff being held in the 1st case at a starting point *A* and then at 20 m intervals: 0.765, 1.064, [0.616], 1.835, 1.524. The level was then moved forward to another position and further readings were taken. These were as follows; the last reading being at *B*: 2.356, 1.378, [2.063], 0.677, 2.027. The level of *A* is 41.819 m. Set out the readings and complete the bookings. Calculate the gradient from *A* to *B*. (Figures in brackets denote inverted staff readings) [R.I.C.S.]

Solution The readings are setout in the table below. Inverted staff readings are taken as negative.

Table 6.7 Example 6.3

Backsight	Intersight	Foresight	Ht of collimation	R.L.	Distance	Remark
0.765			42.584	41.819	0	A
	1.064			41.520	20	
	[0.616]			43.200	40	
	1.835			40.749	60	
2.356		1.524	43.416	41.060	80	
	1.378			42.038	100	
	[2.063]			45.479	120	
	0.677			42.739	140	
		2.027		41.389	160	B
Σ 3.121	Σ 2.275	Σ 3.551				

Check: $\sum$ B.S. – $\sum$ F.S. = 3.121 – 3.551
= – 0.430

Last R.L. – 1st R.L. = 41.389 – 41.819
= – 0.430

$\sum$ Reduced level less the 1st + $\sum$ I.S. + $\sum$ F.S.
= 344.00

$\sum$ Each instrument height × (No. of I.S. and F.S. deduced from it)
= 42.584 × 4 + 43.416 × 4
= 344.00

$$\text{Gradient from } A \text{ to } B = \frac{0.430}{160} = 1 \text{ in } 372.09$$

Example 6.4 The following figures are staff readings taken in order on a particular scheme, the backsights being underlined.

<u>0.813</u>, 2.170, 2.908, 2.630, 3.133, <u>3.752</u>, 3.277, 1.899, 2.390, <u>2.810</u>, 1.542, 1.274, 0.643.

The first reading was taken on a benchmark 39.563. Enter the readings in level book form, check the entries, and find the reduced level of the last point. Comment on your completed reduction.

Solution

Table 6.8 Example 6.4

Backsight	Intersight	Foresight	Ht. of collimation	R.L.	Remarks
0.813			40.376	39.563	B.M.
	2.170			38.206	
	2.908			37.468	
	2.630			37.746	
3.752		3.133		37.243	
	3.277			37.718	
	1.899			39.096	
2.810		2.390		38.605	
	1.542			39.873	
$\sum$ 7.375	$\sum$ 14.426	$\sum$ 5.523			

Difference of $\sum$ Backsight and $\sum$ Foresight
(Last intermediate sight should be considered as foresight)

= 7.375 – 5.523 – 1.542

= 0.310

Difference of last R.L. and first R.L.

= 39.873 − 39.563

= 0.310

Σ R.L. less the 1st + Σ I.S. + Σ F.S. = 305.955 + 14.426 + 5.523

= 325.904

Σ Each instrument height × (Nos. of I.S. and F.S. deduced from it).

= 40.376 × 4 + 40.995 × 3 + 41.415 × 3

= 325.904

Example 6.5 A page of an old level book had been damaged by white ants and the readings marked × are missing. Find the missing readings with the help of available readings and apply arithmetical check. [AMIE, Summer 1979]

Table 6.9 Example 6.5

Distance in m	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
	×			×	209.510	B.M.
0		1.675			×	
30		×			210.425	
60		3.355			209.080	
×	0.840		×	209.520	×	C.P
120		×			208.275	
150		×			210.635	Underside of bridge girder
×	×		2.630	×	×	×
210		×			206.040	
240		1.920			205.895	
270			×		205.690	

Solution Missing readings can be obtained as follows:

(i) Difference in R.L. between 240 and 270

= 205.895 − 205.690

= 0.205

Hence F.S. reading corresponding to 270 m chainage

= 1.920 + 0.205 = 2.125 m

(ii) R.L. at 210 = 206.040

R.L. at 240 = 205.895

Difference in R.L. = 0.145

Hence intermediate sight corresponding to 210

= 1.920 + 0.145

= 2.065 m

(iii) With R.L. = 205.895

and I.S. = 1.920

Height of instrument becomes 207.815.

(iv) After 150 m distance will be 150 + 30 = 180 m

With F.S. = 2.630 and H.I. = 209.520

R.L. at 180 = 206.890 m

Corresponding backsight = 207.815 – 206.890

= 0.925 m

(v) At 150, R.L. = 210.635

Ht. of instrument = 209.520 m

Inverted staff reading = 1.115 m

(vi) At 120 m I.S. = 209.520 – 208.275 = 1.245 m

(vii) After 60 m distance will be 90 m

with backsight 0.840 and H.I. = 209.520

R.L. = 208.680

(viii) Difference in R.L. between 60 and 90

= 209.08 – 208.68

= 0.400 m

Hence F.S at 90 m = 3.355 + 0.400

= 3.755

H.I. = 209.080 + 3.355

= 212.435 m

(ix) At 30 m, R.L. = 210.425

H.I. = 212.435

Hence I.S. = 2.010

(x) With I.S. = 1.675

R.L = 212.435 − 1.675

= 210.760 m

Writing the missing readings, we have the following table:

Table 6.10 Example 6.5

Distance in m	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
—	2.925			212.435	209.510	
0		1.675			210.760	
30		2.010			210.425	
60		3.355			209.080	
90	0.840		3.755	209.520	208.680	C.P.
120		1.245			208.275	
150		1.115			210.635	Under side of bridge girder staff inverted
180	0.925		2.630	207.815	206.890	C.P.
210		1.175			206.040	
240		1.920			205.895	
270			2.125		205.690	
	Σ 4.690		Σ 8.510			

Difference of Σ F.S. − Σ B.S. = 8.510 − 4.690

= 3.820

Difference of 1st R.L and Last R.L. = 209.510 − 205.690

= 3.820

Example 6.6 The following consecutive staff readings were taken on pegs at 15 m interval on a continuously sloping ground: 0.895, 1.305, 2.800, 1.960, 2.690, 3.255, 2.120, 2.825, 3.450, 3.895, 1.685, 2.050 (Stn. A) R.L. of station *A* where the reading 2.050 was taken is known to be 50.250.

From the last position of the instrument two stations *B* and *C* with R.L. 50.800 and 51.000 respectively are to be established without disturbing the instrument. Workout the staff reading at *B* and *C* and complete all the work in level book form. [AMIE, Winter 1982]

Solution Since it is a continuously sloping ground with same set up of instrument there will be continuous increase of reading. When there is a sudden change, it indicates change of instrument point. The readings are tabulated as follows:

Table 6.11 Example 6.6

Distance	B.S.	I.S.	F.S.	Rise	Fall	Ht. of instrument	R.L.	Remarks
0	0.895					56.485	55.590	
15		1.305					55.180	
30	1.960		2.800			55.645	53.685	C.P.
45		2.690					52.955	
60	2.120		3.255			54.510	52.390	C.P.
75		2.825					51.685	
90		3.450					51.060	
105	1.685		3.895			52.300	50.625	C.P.
120		2.050					50.250	Station *A*
		1.500					50.800	Station *B*
			1.300				51.000	Station *C*
Σ	6.660	13.820	11.250					

Check: Σ B.S. – Σ F.S. = 6.660 – 11.250 = – 4.590

1st R.L. – Last R.L. = 55.590 – 51.00 = – 4.590

Σ R.L. less the 1st + Σ I.S. + Σ F.S. = 519.62 + 13.82 + 11.25

= 544.69

Σ Each Instrument Height × (No. of I.S. and F.S. deduced from it).

= 56.485 × 2 + 55.645 × 2 + 54.51 × 3 + 52.3 × 3 = 544.69

6.4 CHECKING OF LEVELS

The arithmetic checks carried out after each example above indicate only correctness of arithmetical computations. They do not indicate that levels of the points are also correct. There will always be errors in field work and it is always necessary to get an idea of the magnitude of error. This can be obtained by taking the level back to the original benchmark or to another point of known elevation or benchmark. It is advisable to make the length of foresight and backsight equal to eliminate common instrumental errors. Figures 6.2(a) and 6.2(b) show the two types of check.

In Fig. 6.2(a) to check the level difference between *A* and *B*, the line of level

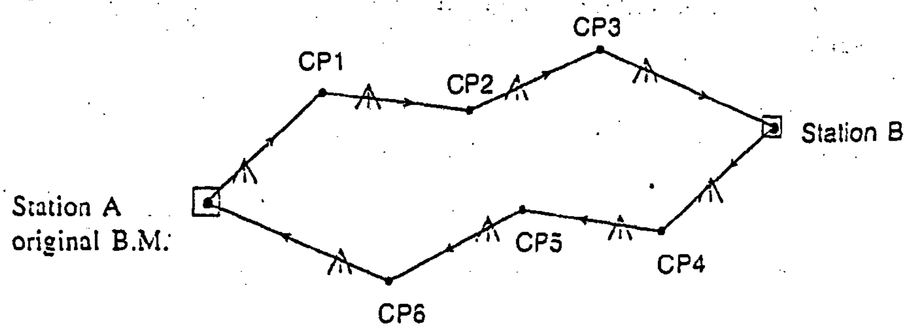


Fig. 6.2(a) Checking of levels (closed circuit).

is brought back to the original station A. In such a case station B has to be made a change point, that is, after taking foresight at B instrument should be changed to a new position and backsight taken.

In Fig. 6.2(b) line of level is taken to another point P of known R.L. In both cases we can compute the error in levelling. In the first case the level difference should be zero. In the second case, it should be known R.L. The discrepancy represents the error of closure of the circuit and should be very small. If a large difference occurs there must be some mistake in either (i) computation or, (ii) in reading of the rod or (iii) in entering the field notes.

6.5 ERRORS IN LEVELLING

As explained earlier, in levelling it is possible to make blunders, systematic errors and accidental errors. Proper note keeping and systematic field work will eliminate the first two while multiple readings can reduce the third to a minimum. Blunders in levelling may occur due to, (i) using a wrong point for a benchmark, (ii) reading rod incorrectly, (iii) reading on the stadia cross hair instead of the middle crosshair, and (iv) reading wrong numbers. Systematic errors occur when the instrument is out of adjustment; for example, when the line of sight is not horizontal when the bubble is at the centre of its run. When a survey starts from a point and loops back to the same point, the accidental errors in reading, sighting and atmospheric conditions are proportional to the number of setups and/or distances between benchmarks.

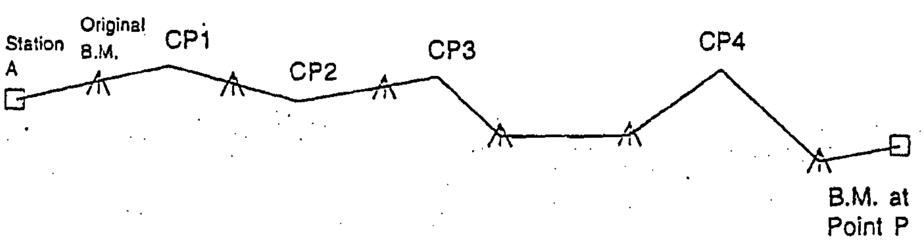


Fig. 6.2(b) Checking of levels with known R.L.

Errors can also be classified as (i) Instrumental errors, (ii) Personal errors, (iii) Natural errors.

6.5.1 INSTRUMENTAL ERRORS

1. *Level out of adjustment* Normally when the bubble is in the centre of its run line of sight is horizontal. But with imperfect adjustment, with bubble at the centre, line of sight remains inclined. This can be removed by checking the permanent adjustment of the level frequently. It can also be eliminated by keeping the backsight and foresight equal. Figure 6.3 shows the error caused by inclined line of sight. This is known as *collimation error*.

If $DA = DB$, $e_1 = e_2$ and the difference of reading is the true difference of level between A and B as the errors get cancelled.

As $DC > DA$, $e_3 > e_1$ and the difference of reading does not give the true difference.

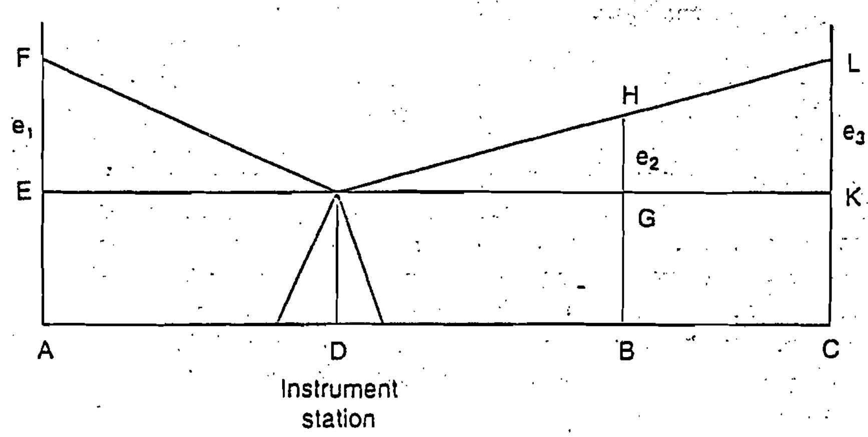


Fig. 6.3 Collimation error.

(2) Other instrumental errors are: (i) Sluggish bubble, (ii) Defective staff, (iii) Defective tripod, (iv) Faulty focussing tube.

Sluggish bubble and faulty focussing tube will lead to inclined line of sight and hence erroneous reading. Defective staff will give wrong reading and so also defective tripod.

6.5.2 PERSONAL ERRORS

1. *Bubble not properly centred* This is a very serious error because of line of sight will be horizontal only when bubble is central. Hence a habit should be formed for checking at the beginning and at the end of each reading.

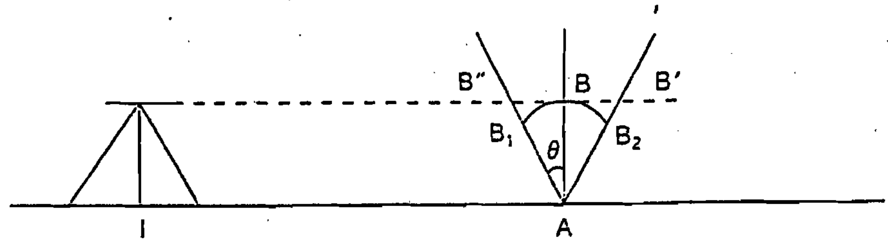
2. *Parallax* Parallax, that is, when the image and the crosshair do not exactly coincide, leads to error in rod reading. This can be avoided by proper focussing and checked by moving the eye up and down.

3. *Non verticality of the staff* The rod should be plumb when the reading is made. Some levelling rods are fitted with circular levels at the back so that verticality can be ensured by keeping the bubble central. This can also be done by moving the rod slowly backward and forward and taking the minimum reading as shown in Fig. 6.4. From the figure it can be seen that

$$AB'' \cos\theta = AB$$

or

$$AB'' = AB \sec\theta$$



I—Instrument Station; AB—Vertical Staff; AB′, AB″—Inclined Staff

Fig. 6.4 Error due to non-verticality.

The error due to non verticality is given by

$$e = AB'' - AB = AB(\sec \theta - 1)$$

It is obvious that this error increases as AB increases or as θ increases. Hence it is advisable to use small height of the staff.

4. *Telescopic staff not fully extended* In India telescopic staff is more frequently used. When working with telescopic staff, it should be fully extended, all the parts of the staff should remain truly vertical and graduation should be continuous from one piece of staff to another.

5. *Sighting error* Error in sighting occurs in poor weather conditions and in long sights. It is also dependent on the coarseness of the crosshair and the type of rod. The error is accidental.

6.5.3 NATURAL ERRORS

1. *Curvature and refraction* This error has already been explained is Sec. 5.3. It is of negligible quantity for ordinary levelling. It can be practically eliminated by keeping the backsight and foresight distances equal. In precise levelling when the backsight and foresight are not equal, a correction should be applied as already explained in Sec. 5.3. Moreover, levelling may be discontinued for a few hours during midday or shorter sights may be taken.

2. *Wind vibration* High wind shakes the instrument and thus disturbs the bubble and the rod. Precise levelling work should never be done under high wind.

3. *Temperature variation* Temperature may cause unequal expansion of the various parts of the instrument. One end of the bubble tube may be heated more than the other, the bubble then moves to the warmer end causing error. The level should, preferably, be protected from the direct rays of the sun. The rod may also expand due to temperature. For precise work invar rods may be used.

4. *Settlement of tripod or turning point* If the tripod settles between taking the backsight and foresight readings, the observed foresight will be too small and the elevation of the turning point will be too large. Similarly if the change point settles between taking a foresight and the following backsight, the next observed backsight will be too great and H.I. calculated will also be too great. Thus settlement of tripod or C.P. leads to systematic error as the resulting elevation will always be too high.

6.6 REDUCING ERRORS AND ELIMINATING MISTAKES IN LEVELLING

Errors in levelling can be reduced but never fully eliminated by systematic adjustment and manipulation of both the level and staff. The following points should be kept in mind for an accurate levelling: (i) bubble should be checked before and after each reading, (ii) rod with circular bubble should be used; (iii) length of foresight and backsight should be made equal; (iv) usual field check should be done; (v) usual field book checks should be observed; (vi) telescope should be shaded from sun; (vii) line of sight should be at least 0.3 m above intervening terrain. This will reduce errors and detect mistakes.

6.7 COLLIMATION CORRECTION

It is not always possible in practice to make backsight equal to foresight. It is also not possible to always ensure horizontal line of sight. Hence collimation error invariably occurs and a collimation correction should be applied. This is also known as *C-factor correction* in which *C* represents the inclination of the line of sight when the level bubble is centred. In Fig. 6.5(a) the line of collimation is inclined upwards even if the bubble is central. The error is $D \tan \alpha \approx D\alpha$ as α is small and the correction is $-D\alpha$. This is often expressed as CD where C is the

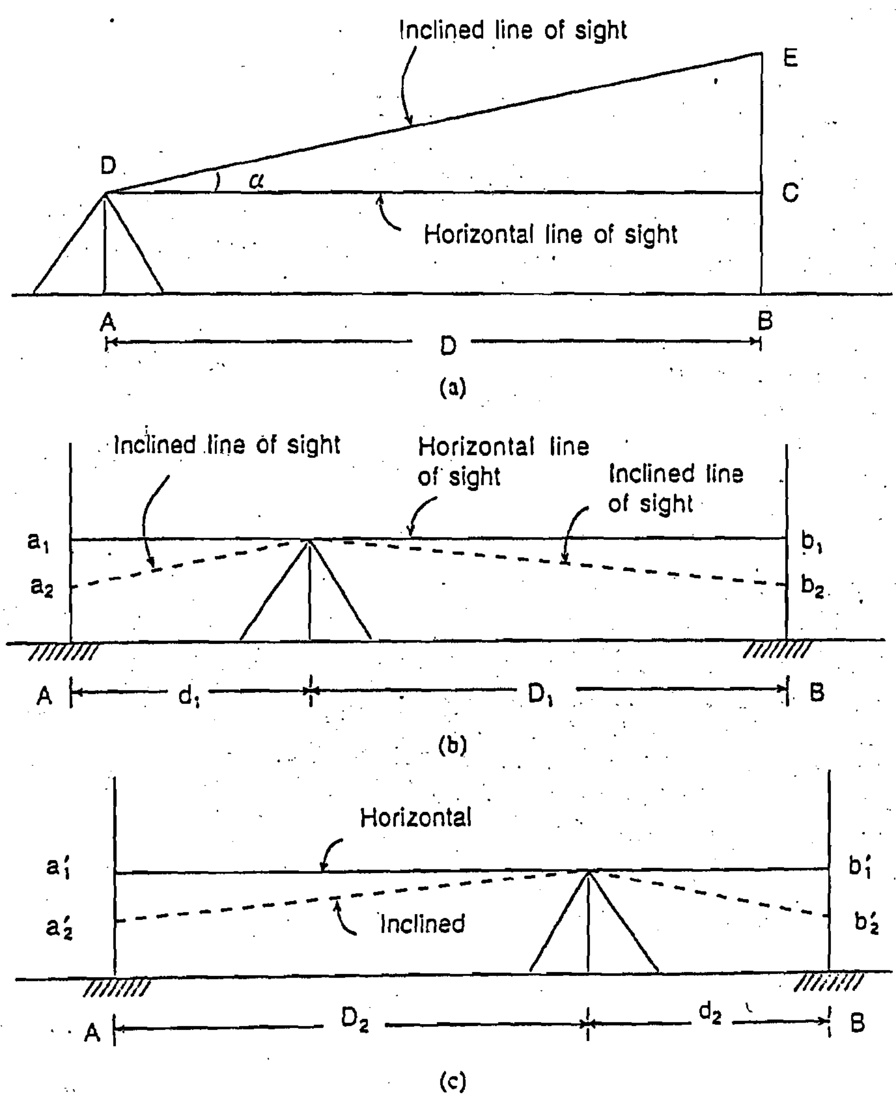


Fig. 6.5 (a) Line of collimation inclined upwards. (b) Set up 1. (c) Set up 2.

correction factor. In this case $C = -\alpha$. Hence if C is positive the line of sight is inclined downward. To determine C, set up the instrument between A and B as shown in Figs. 6.5(b) and (c). The line of collimation is assumed to be downward, i.e. C positive. From the figures,

$$a_1a_2 = Cd_1 \qquad b_1b_2 = CD_1$$

$$a_1'a_2' = CD_2 \qquad b_1'b_2' = Cd_2$$

For Set up 1
Correct difference between A and B

$$= Aa_1 - Bb_1$$

$$= Aa_2 + Cd_1 - (Bb_2 + CD_1) \tag{6.1}$$

For Set up 2
Correct difference between A and B

$$= Aa_1' - Bb_1'$$

$$= Aa_2' + CD_2 - (Bb_2' + Cd_2) \tag{6.2}$$

Equating (6.1) and (6.2), we get

$$Aa_2 + Cd_1 - (Bb_2 + CD_1) = Aa_2' + CD_2 - (Bb_2' + Cd_2)$$

$$\text{Transposing } C = \frac{(Aa_2 + Bb_2') - (Bb_2 + Aa_2')}{(D_1 + D_2) - (d_1 + d_2)}$$

$$= \frac{\text{Sum of short distance reading} - \text{Sum of long distance reading}}{\text{Sum of long distances} - \text{Sum of short distances}}$$

Once the C factor is known, this can be applied for necessary correction for unequal backsight and foresight as shown in Fig. 6.5(d).

The correction to be applied as shown in the Fig. 6.5(d) is C factor times (Σ F.S. intervals $-$ Σ B.S. intervals).

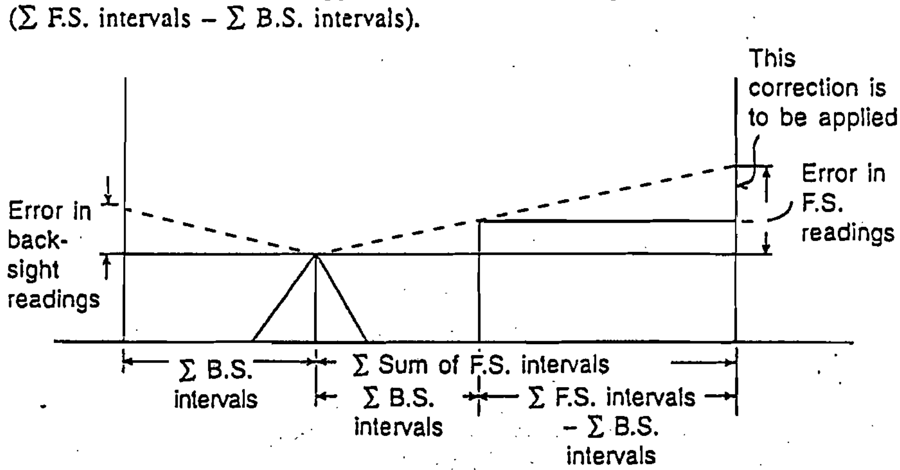


Fig. 6.5(d) Collimation correction (backsights and foresights not equal).

Example 6.7 A level set up in a position 30 m from peg *A* and 60 m from peg *B* reads 1.914 m on a staff held at *A* and 2.237 m on a staff held at *B*, the bubble having been carefully brought to the centre of its run before each reading. It is known that the reduced levels of the top of the pegs *A* and *B* are 87.575 m and 87.279 O.D respectively (Fig. 6.6). Find (a) Collimation error; (b) The readings that would have been obtained had there been no collimation error. [L.U.]

Solution

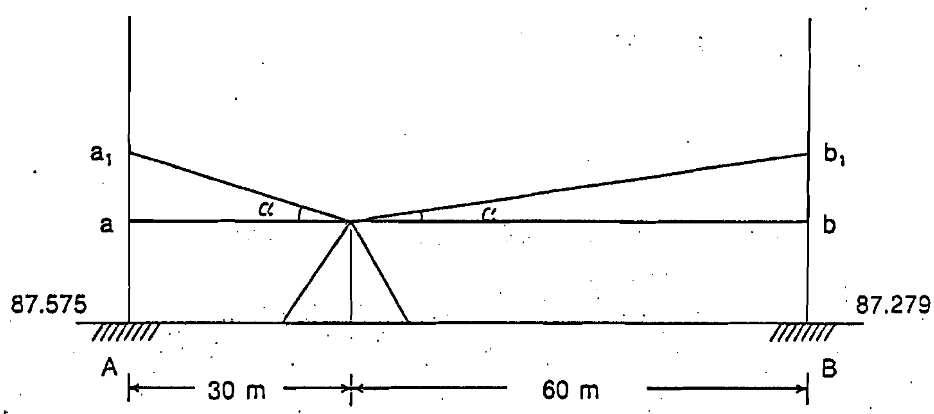


Fig. 6.6 Example 6.7.

Let us assume that the error is positive, i.e. the line of collimation is upward. True difference of level between *A* and *B*

$$= 87.575 - 87.279$$

$$= 0.296$$

and *A* is at a higher level than *B*.

This is equal to $(Bb_1 - b_1b) - (Aa_1 - a_1a)$

$$= (2.237 - 60\alpha) - (1.914 - 30\alpha)$$

$$= -30\alpha + 0.323$$

Therefore $0.296 = -30\alpha + 0.323$

or $30\alpha = 0.027$ m

$\alpha = 0.027$ per 30 m upward

Reading at $A = Aa_1 - a_1a$

$$= 1.914 - 30\alpha$$

$$= 1.914 - 0.027$$

$$= 1.887 \text{ m}$$

Reading at $B = 2.237 - 60\alpha$

$$= 2.237 - 0.054$$

$$= 2.183 \text{ m}$$

Example 6.8 The following staff readings were obtained when running a line of levels between two benchmarks *A* and *B*.

1.085 (A), 2.036, 2.231, 3.014, change point, 0.613, 2.003, 2.335, C.P., 1.622, 1.283, 0.543, C.P., 1.426, 1.795, 0.911.

Enter and reduce the readings in an accepted form of field book. The reduced levels of the bench marks at *A* and *B* were known to be 43.650 m and 41.672 m respectively.

It is found after readings have been taken with the staff supposedly vertical as indicated by a level on the staff that the level is 5° in error in the plane of the staff and instrument. Is the collimation error of the instrument elevated or depressed? What is its value in seconds if the backsights and foresights averaged 30 m and 60 m respectively? (L.U)

Solution The data are tabulated in level book form and the R.L. of the different points calculated.

Table 6.12(a) Example 6.8

Distance	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
	1.085			44.735	43.650	A 43.650
		2.036			42.699	
		2.231			42.504	
	0.613		3.014		41.721	
		2.003			40.331	
	1.622		2.335		39.999	
		1.283			40.338	
	1.426		0.543		41.078	
		1.795			40.709	
			0.911		41.593	B 41.672
	Σ 4.746		Σ 6.803			

Difference between Σ F.S. – Σ B.S.

$$= 6.803 - 4.746 = 2.057$$

Difference between 1st R.L. and last R.L.

$$= 43.650 - 41.593$$

$$= 2.057 \text{ m}$$

But the staff was held 5° off the vertical hence,

Corrected staff reading = (observed staff reading) cos 5°

$$\text{Correction} = -(1 - \cos 5°) \times \text{staff reading}$$

$$= -0.0038 \times \text{staff reading}$$

A second table is drawn with the corrected staff readings for backsight and foresight only.

Table 6.12(b) Example 6.8

B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1.081			44.731	43.650	*A* 43.650
0.611		3.003	42.339	41.728	
1.616		2.326	41.629	40.013	
1.421		0.541	42.509	41.088	
		0.908		41.601	*B* 41.672
Σ 4.729		Σ 6.778			
Diff = 6.778 − 4.729 = 2.049			43.650 − 41.601 = 2.049		Actual difference = 1.978

The observed difference in level is too great as the actual difference is 1.978 and as the foresights exceeded the backsights in length the collimation is upward.

$$\text{Collimation error} = \frac{2.049 - 1.978}{240 - 120} \times 206265 \text{ seconds}$$

$$= 122 \text{ seconds}$$

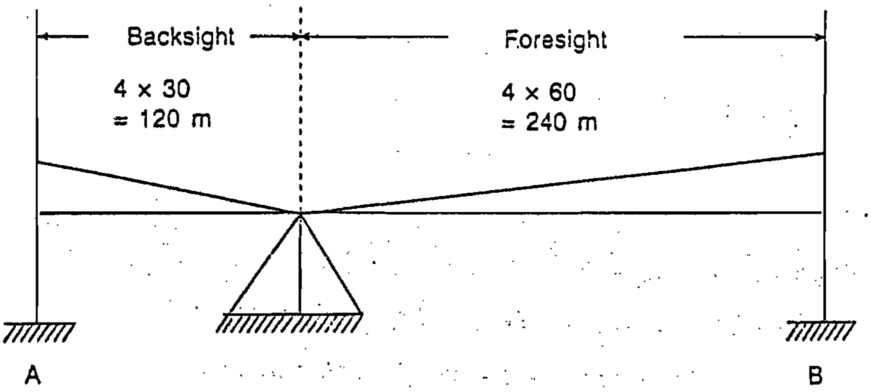


Fig. 6.7 Example 6.8.

6.8 CHECK LEVELLING

It is used for checking of elevations at the end of day's work.

6.9 FLY LEVELLING

It is a quick but approximate method of levelling. Long distances are taken as sights. It is used for reconnaissance of an area or for approximate checking of levels.

6.10 PROFILE LEVELLING

As the name suggests, it shows a profile, that is, a line depicting ground elevations at a vertical section along a survey line. This is necessary before a rail road, highway, transmission line, side walk or sewer line can be designed. Usually a line of level is run along the centre line of the proposed work as shown in Fig. 6.1. Level is taken every 15 m or 30 m interval, at critical points where there is a sudden change of levels, at the beginning or end of curve. The basic objective is to plot accurately the elevation of the points along the line of levels. The procedure is exactly the same as in differential levelling as explained in Sec. 6.2. It is necessary to take staff readings along the centre line, book them properly in the level book, compute the R.L.'s of different points and apply suitable arithmetic checks. It is also necessary to start from a B.M. of known R.L. also close with a known R.L. so that suitable field checks are applied. It is however, not essential to put the instrument along the centre line. It can be placed any where if necessary, off the centre line, so that large number of readings can be taken and foresights and backsights are made approximately equal. It is now necessary to plot the profile or longitudinal section. To show the distortions of the ground the elevations are plotted on a much larger scale after taking a suitable datum than the longitudinal distances. Based on the example given in Sec. 6.2 a typical longitudinal section is shown in Fig. 6.8.

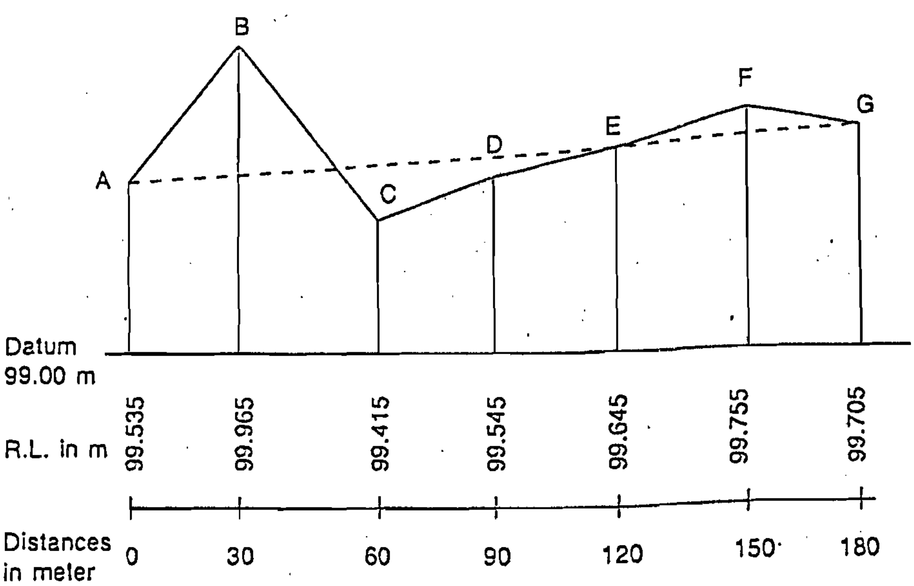


Fig. 6.8 Longitudinal section. Scale: horizontal 1 cm = 15 m, vertical 1 cm = 20 cm.

After the longitudinal or profile section is drawn, it is necessary to have a smooth surface: This is known as grade line which is selected on various considerations like: (i) minimum amount of cutting and filling of earth work; (ii) balancing the cut and fill; and (iii) keeping the slope within allowable limit.

If points *A* and *G* are joined by a straight line the slope of the line becomes (99.705 – 99.535)/180 or 1/1059 which is very small and is within allowable limit. This may not, however, ensure equal volumes of cut and fill and suitable adjustments of grade line may be necessary to ensure this condition.

Example 6.9 The levelling shown in the field sheet given below was undertaken during the laying out of a sewer line. Determine the height of the ground at each observed point along the sewer line and calculate the depth of the trench at points *X* and *Y* if the sewer is to have a gradient of 1 in 200 downwards from *A* to *B* and is to be 1.280 m below the surface at *A* [R.I.C.S]

Solution The tables show the problem and solution.

R.L. of sewerline at *A* = 99.645 – 1.280 = 98.365 m

At *X*, 40 m from *A* R.L. $= 98.365 - \frac{40}{200}$

$= 98.165$ m

At *Y*, 120 m from *A* R.L. $= 98.365 - \frac{120}{200}$

$= 97.765$

Hence depth of trench at *X* = 99.919 – 98.165

= 1.754

Depth of trench at *Y* = 100.204 – 97.765

= 2.439

Table 6.13 Example 6.9

B.S.	I.S.	F.S.	Distance (m)	Remarks
3.417				B.M. 98.002 m
1.390		1.774	0	
	1.152		20	
3.551		1.116	40	Point *X*
0.732		1.088	60	
2.384		3.295	80	
	1.801		100	
	1.999		120	Point *Y*
1.936		2.637	140	Point *B*
		1.161		B.M. 100.324

Details of Field sheet.

Table 6.14 Example 6.9

B.S.	I.S.	F.S.	Rise	Fall	R.L.	R.L. of sewer	Distance	Remark
3.417					98.002			
1.390		1.774	1.643		99.645	98.365	00	*A*
	1.152		0.238		99.883		20	
3.551		1.116	0.036		99.919	98.165	40	*X*
0.732		1.088	2.463		102.382		60	
2.384		3.295		2.563	99.819		80	
	1.801		0.583		100.402		100	
	1.999			0.198	100.204	97.765	120	*Y*
1.936		2.637		0.638	99.566		140	Point *B*
		1.161	0.775		100.341			B.M. 100.324
Σ 13.410		Σ 11.071	Σ 5.738	Σ 3.399				

Σ B.S. – Σ F.S. = 13.410 – 11.071 = 2.339

Σ Rise – Σ Fall = 5.738 – 3.399 = 2.339

Last R.L. – First R.L. = 100.341 – 98.002 = 2.339

Example 6.10 In running fly levels from a benchmark of R.L. 183.185, the following readings were obtained:

Backsight: 2.085, 1.025, 1.890, 0.625
Foresight: 1.925, 2.820, 0.890

From the last position of the instrument five pegs at 25 meters interval are to be set out on an uniformly falling gradient of line 100 with the 1st peg to have a R.L. of 182.350. Determine the staff readings required for setting the tops of the five pegs on the given gradient [AMIE, Summer 1986]

Solution The data and the required readings are given in a tabular form.

Table 6.15 Example 6.10

S. No.	Dist.	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1		2.085			185.270	183.185	B.M.
2		1.025		1.925	184.370	183.345	
3		1.890		2.820	183.440	181.550	
4		0.625		0.890	183.175	182.550	
5	0		0.825			182.350	Peg 1
6	25		1.075			182.100	Peg 2
7	50		1.325			181.850	Peg 3
8	75		1.575			181.600	Peg 4
9	100			1.825		181.350	Peg 5
		Σ5.625	Σ4.800	Σ7.460			

Check: Σ B.S. – Σ F.S. = 5.625 – 7.460 = – 1.835

Last R.L. – 1st R.L. = 181.350 – 183.185 = – 1.835

Σ R.L. less the 1st + Σ I.S. + Σ F.S = 1456.695 + 4.800 + 7.460

= 1468.955

Σ (Each instrument height × No. of I.S. and F.S. from it)

= (185.27)(1) + (184.37)(1) + (183.175)(5)

= 1468.955

6.11 CROSS SECTIONAL LEVELLING

For laying a pipeline or sewerline only longitudinal section is adequate because the width of the line is small. In the case of roads and railways apart from longitudinal section, cross sections at right angles to the centre line of the alignment are required at some regular intervals. This is necessary to know the topography of the area which will be required for the roads and railways and also to compute the volume of cut and fill for the construction work. Figure 6.9(a) shows the plan. Figure 6.9(b) shows the cross section and the table shows the entry in the level book. Cross section is usually plotted in the same horizontal and vertical scale.

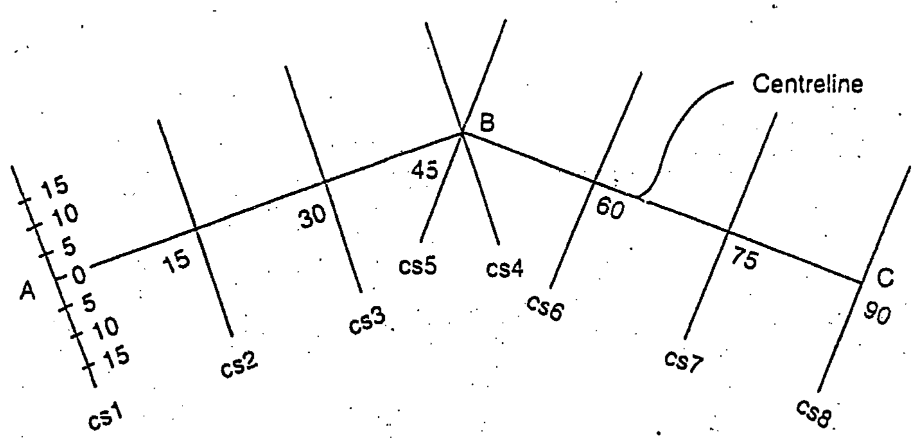


Fig. 6.9(a) Plan

Station	Distance (m)			B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
	L	C	R						
B.M				1.415			106.820	105.405	
A		0			1.875			104.945	
	5				1.795			105.025	
	10				1.625			105.195	
	15				1.540			105.280	
			5		1.535			105.285	
			10		1.685			105.135	
			15		1.805			105.015	

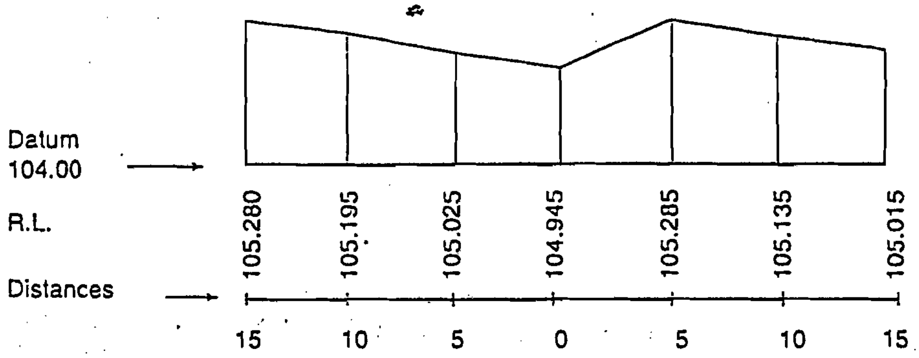


Fig. 6.9(b) Cross section Cs1. Scale: Horizontal 1 cm = 2.5 m, Vertical 1cm = 0.5m.

6.11.1 SIGHT RAILS AND BONING RODS

Sight rails and boning rods are used for excavation purposes associated with the grading of drains and sewers. The sight rails are established at fixed points along the excavation line at a height above the formation level equal to the length of the boning rod. The formation level compared with the surface level gives the depth of excavation. When the boning rod is in line with sight rails the excavation is correct depth (Fig. 6.10).

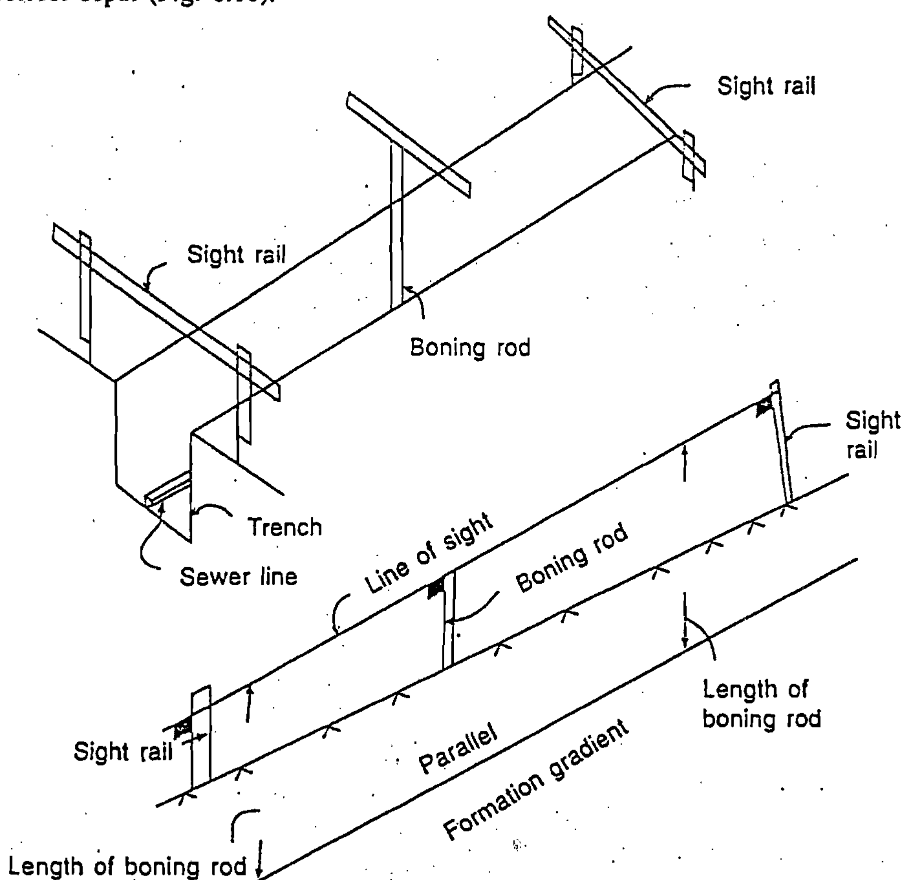


Fig. 6.10 Sight rails and boning rods.

Example 6.11 *A, B, C, D, E* and *F* are the sites of manholes 100 m apart on a straight sewer. The natural ground can be considered as a plane surface rising uniformly from *A* to *F* at a gradient of 1 vertically in 500 horizontally, the ground level at *A* being 31.394 m. The level of the sewer invert is to be 28.956 m at *A*, the invert then rising uniformly at 1 in 200 to *F*. Sight rails are to be set up at *A, B, C, D, E* and *F* so that a 3 m boning rod or traveller can be used. The backsights and foresights were made approximately equal and a peg at ground level at *A* was used as datum. Draw a level book showing the readings. [L.U.]

Solution

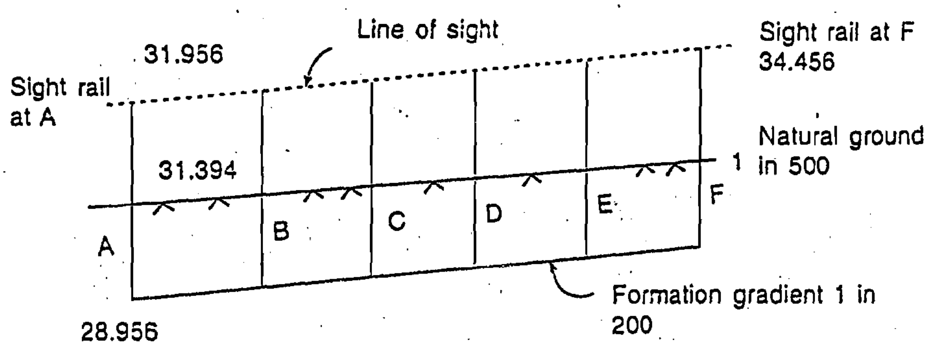


Fig. 6.11 Example 6.10.

(i)

Ground level at $A = 31.394$
Invert level at $A = 28.956$

Difference in level = 2.438

(ii) Ground level at $B = 31.394 + \frac{100}{500} = 31.594$

Invert level at $B = 28.956 + \frac{100}{200} = 29.456$

Difference in level = 2.138

(iii)

Ground level at $C = 31.794$
Invert level at $C = 29.956$

Difference = 1.838

(iv)

Difference at D = 1.538
Difference at E = 1.238
Difference at F = 0.938

The level book is shown in Table 6.16.

Level book

Table 6.16 Example 6.11

B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1.815 (assumed)			33.209	31.394	Ground level at *A*
	4.253			28.956	Invert Level at *A*
	1.253			31.956	Sight rail at *A*
	1.615			31.594	Ground level at *B*
	3.753			29.456	Invert level at *B*
1.129		0.753	33.585	32.456	Sight rail at *B*
	1.791			31.794	Ground level at *C*
	3.629			29.956	Invert level at *C*
0.994		0.629	33.950	32.956	Sight rail at *C*
	1.966			31.994	Ground level at *D*
	3.494			30.456	Invert level at *D*
1.059		0.494	34.515	33.456	Sight rail at *D*
	2.321			32.194	Ground level at *E*
	3.559			30.956	Invert level at *E*.
1.149		0.559	35.105	33.956	Sight rail at *E*
	2.711			32.394	Ground level at *F*
	3.649			31.456	Invert level at *F*
—		0.649		34.456	Sight rail at *F*
Σ 6.146 − 3.084 Diff. = 3.062		Σ 3.084	Diff. 34.456 − 31.394	= 3.062	

6.12 RECIPROCAL LEVELLING

While crossing a river or ravine it is not possible to put the level midway so that the backsight and foresight are equal. Sight distance, however, is long and errors due to (i) collimation, i.e. inclined line of sight, (ii) curvature and refraction are likely to occur. To avoid these errors two observations are made. As shown in Fig. 6.12 instrument is placed near station *A* and observations are made on staffs at *A* and *B*. Similarly instrument is placed near *B* and staff readings are taken on *B* and *A*.

From first set of readings:

$$\text{difference in level} = d = BB_1 = a_1 + c + e - r - b_1$$

$$= (a_1 - b_1) + (c - r) + e$$

From second set of readings:

$$\text{difference in level} = d = AA_1 = -(b_2 + c + e - r - a_2)$$

$$= (a_2 - b_2) - (c - r) - e$$

(− sign as difference is measured at *A* instead of at *B*)

By adding $$2d = (a_1 - b_1) + (a_2 - b_2)$$

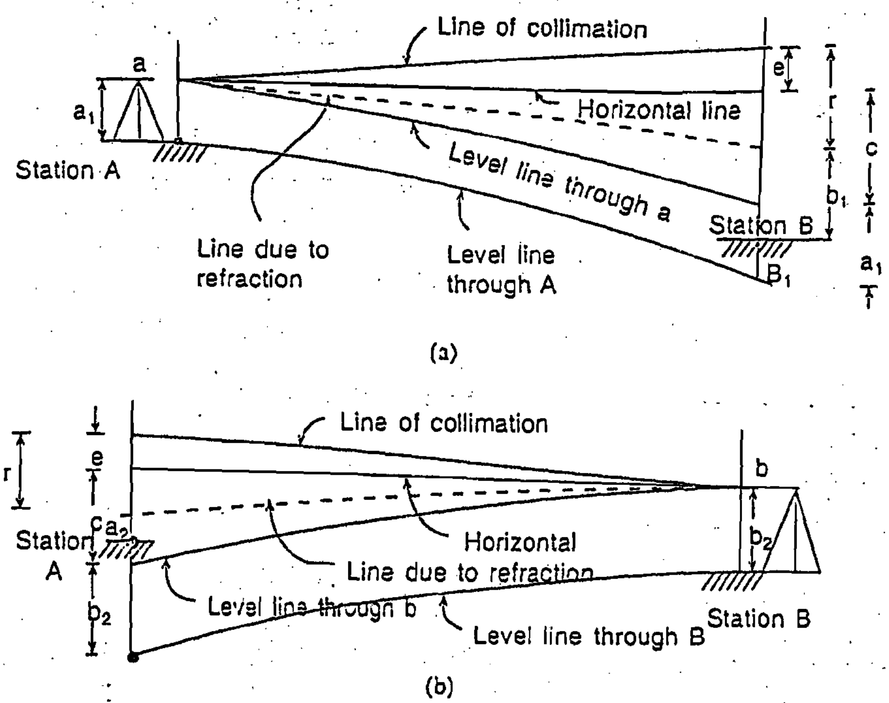


Fig. 6.12 Reciprocal levelling.

or $$d = \frac{1}{2}\left[(a_1 - b_1) + (a_2 - b_2)\right]$$

Subtracting $$2(c - r + e) = \left[(a_2 - b_2) - (a_1 - b_1)\right]$$

or $$c - r + e = \frac{1}{2}\left[(a_2 - b_2) - (a_1 - b_1)\right]$$

Here c = curvature error

r = refraction error

e = error due to collimation

If the combined error due to curvature and refraction are known error due to collimation can be found out.

Example 6.12 The results of reciprocal levelling between stations *A* and *B* 250 m apart on opposite sides of a wide river were as follows.

Level at	Height of eyepiece (m)	Staff reading
A	1.399	2.518 on *B*
B	1.332	0.524 on *A*

Find (a) The true difference of level between the stations.

(b) The error due to imperfect adjustment of the instrument assuming the mean radius of the earth 6365 km. (L.U.)

Solution Since the staff is very close to A and B in 1st and 2nd setup respectively, the height of the eyepiece is taken as the staff reading.

$$\text{True difference of level} = \frac{1}{2}\,[(a_1 - b_1) + (a_2 - b_2)]$$

$$= \frac{1}{2}\,[(1.399 - 2.518) + (0.524 - 1.332)]$$

$$= -\,0.964 \text{ m}$$

indicating that A is at a lower level than B.

$$\text{Total error} = \frac{1}{2}\,[(a_2 - b_2) - (a_1 - b_1)]$$

$$= \frac{1}{2}\,[(-\,0.808) - (-\,1.119)]$$

$$= +\,0.156 \text{ m}$$

Error due to curvature and refraction

$$= \frac{L^2}{2R}\,[1 - 2\text{ m}]$$

$$= \frac{250^2}{2(6365)(1000)}\,[1 - 2(.07)]$$

$$= .00422 \text{ m}$$

$$\text{Error due to collimation} = +\,0.156 - .004$$

$$= +\,0.152 \text{ in } 250 \text{ m}$$

$$\text{Hence error/100 m} = +\,0.06 \text{ m}$$

6.13 TWO PEG TEST

The two peg test is a familiar test to find the error of line of collimation of a level. Figure. 6.13 shows the fundamentals of the test. Initially the length AB is measured and the level is placed at the middle of two pegs A and B and the staff readings taken (Fig. 6.13(a)). The difference of readings gives the true difference in level between points A and B. The level is then shifted along line AB either towards A or B through a known distance and the readings taken as shown in (Figs. 6.13(b) and (c)). From the readings and the known distances it is possible to calculate the collimation error.

Example 6.13 A modern dumpy level was set up at a position equidistant from two pegs A and B. The bubble was adjusted to its central position for each reading as it did not remain quite central when the telescope was moved from A to B. The readings on A and B were 1.481 m and 1.591 m respectively. The instrument was then moved to D so that the distance DB was about five times the distance DA and the readings with the bubble central were 1.560 m and 1.655 m respectively. Was the instrument in adjustment?

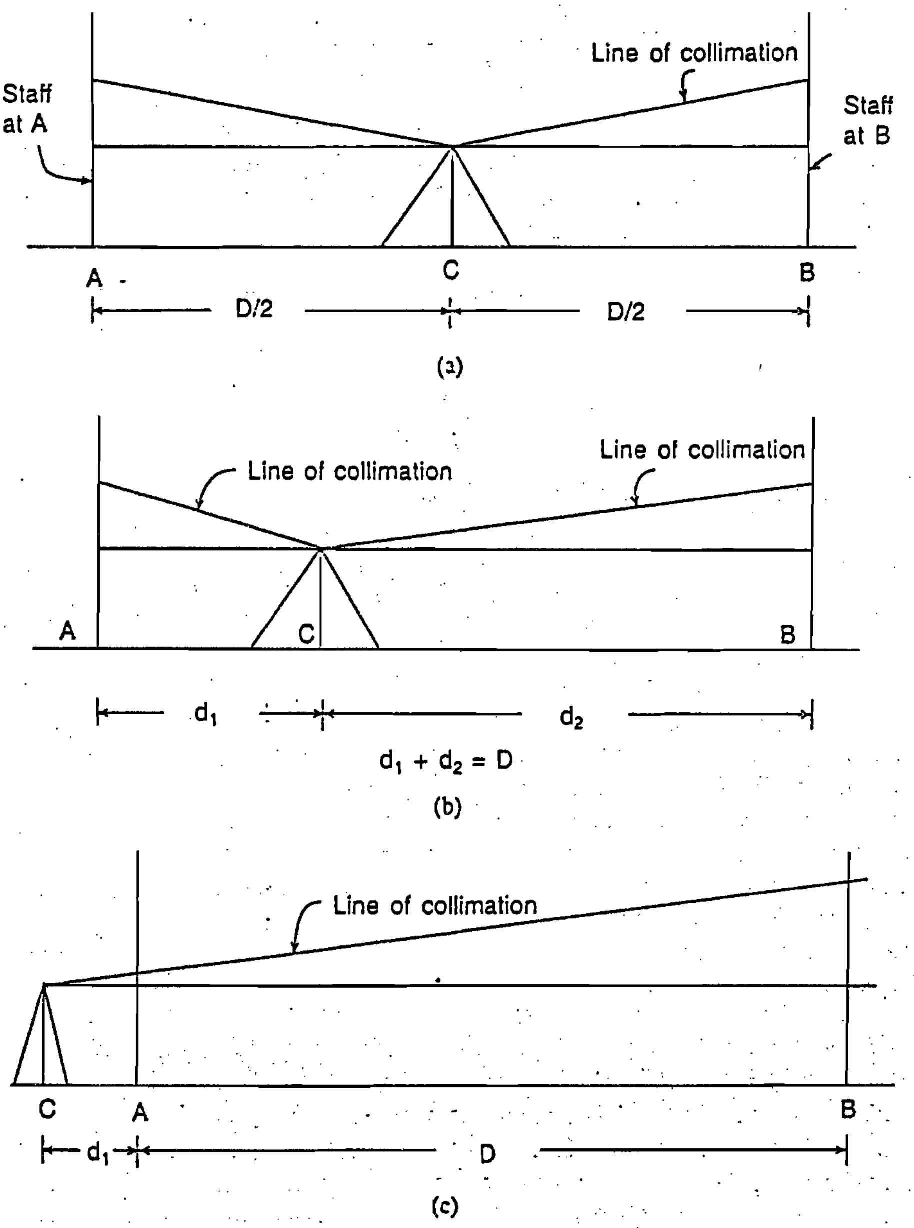


Fig. 6.13 Two peg test.

Solution Figure 6.14 shows the two positions of the instrument with corresponding staff readings.

True difference of level = 1.591 – 1.481

= 0.110 m

A is at a higher level than *B*.

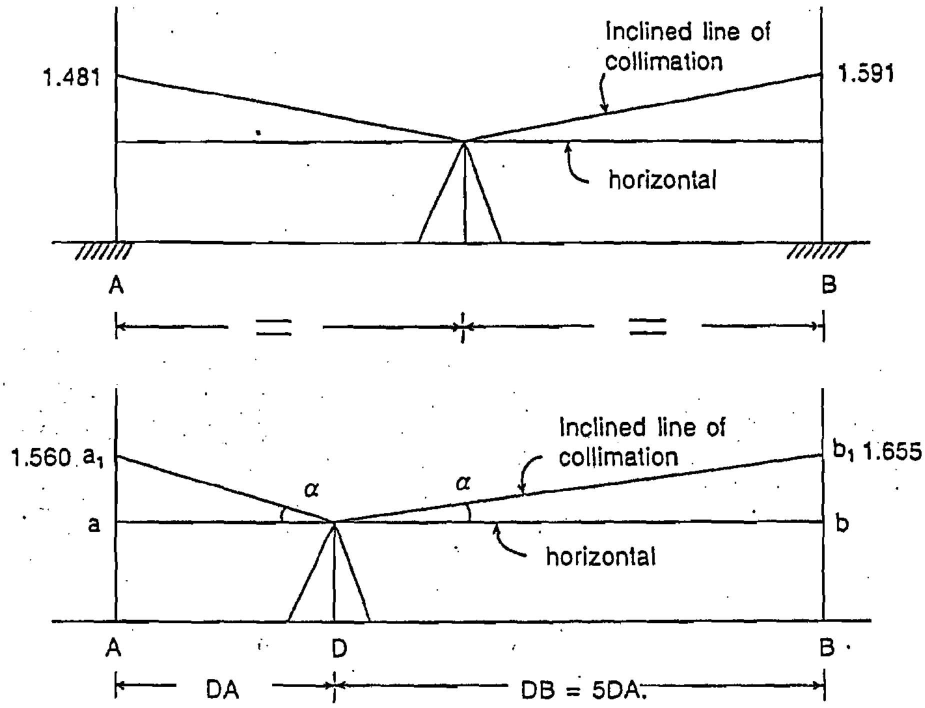


Fig. 6.14 Example 6.13.

In the second set of readings as angle α of the line of collimation is constant, if error on A is e, on B it is $5e$. Hence

$$Bb + 5e - (Aa + e) = 1.655 - 1.560$$

or
$$(Bb - Aa) + 4e = 0.095 \text{ m}$$

But $Bb - Aa$ = true difference in level

$$= 0.110 \text{ m}$$

Hence,
$$4e = 0.095 - 0.110$$
$$= -0.015$$

or
$$e = -.004 \text{ m}$$

indicating line of collimation was downward.

6.14 THREE WIRE LEVELLING

In ordinary levelling staff is read against only the middle horizontal crosshair whereas in three wire levelling staff is read against all the three horizontal crosshairs and recorded. These three readings are averaged to get a better value. Since the wires are equally spaced the difference of reading between the upper and middle wires should be equal to that between middle and lower wire. If they do not agree within the accuracy of the instrument, the observation should be repeated. The difference between the upper and lower reading provides the staff intercept necessary to calculate the sight distance and to check whether backsight and foresight are equal. A typical page of the level book used for three wire levelling in given below.

Staff station	B.S.	F.S.	H.I.	R.L.	Distance reading		Remarks
					B.S.	F.S.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
BM1							
CP1							
CP2							
CP3							
BM2							
	Arithmetic check						

6.15 ERROR, ADJUSTMENT AND PRECISION OF LEVEL

In levelling error is likely to be more when the length of the line is more or the number of set up of the instrument is more. Since standard error or probable error is directly proportional to the square root of the length of a line and weight is inversely proportional to the square of probable error weight is inversely proportional to the length of a line in a level circuit. Similarly weight is inversely proportional to the number of instrument set ups. Weight also varies directly with the number of repetitions. Therefore adjustment or calculation of most probable value of R.L. of a point in a level line is based on the above principles. When misclosure of level is known, i.e. when levelling ends at a point of known elevation or at the starting point, discrepancy is adjusted in proportion to length from the starting point. In multi loop circuits a benchmark should be made common to both the circuits. Though theory of least square is the best method for adjusting such a circuit, approximate adjustments can be made. The outer loop should be adjusted first. The adjustment required for the common point is found out. From the adjustment necessary for the common point, the adjusted value of the starting point is found out as also misclosure in the inner loop. This misclosure is again adjusted. Finally, the outer loop is again adjusted based on new adjustment of the inner loop. Examples 6.17 and 6.18 show the procedure.

Example 6.14 The difference in level between two points A and B was found by three routes—(1) via C and D, (2) via E, F and G, (3) via H, distances being as follows:

Route 1 $AC = 180$ m $CD = 282$ m $DB = 228$ m
Route 2 $AE = 144$ m $EF = 156$ m $FG = 324$ m, $GB = 270$ m
Route 3 $AH = 264$ m $HB = 369$ m

The sections on Route 1 were levelled eight times, those on Route 2 twice and those on Route 3 four times and the differences in level were found to be 8.275 m, 8.292 m and 8.285 m respectively. If the probable error in any section for a single levelling is proportional to the square root of the length of that section, find the most probable value of the difference in level between *A* and *B*. (Salford)

Solution Most probable value is the weighted mean of the observed values. Weight is inversely proportional to the square of probable error and directly proportional to the number of repetitions.

$$w_1 : w_2 : w_3 = n_1/l_1 : n_2/l_2 : n_3/l_3.$$

where n_1, n_2 and n_3 = no. of repetitions

l_1, l_2 and l_3 = corresponding lengths

Here $n_1 = 8$ $n_2 = 2$ $n_3 = 4$

$l_1 = 180 + 282 + 228 = 690$ m

$l_2 = 144 + 156 + 324 + 270 = 894$ m

$l_3 = 264 + 369 = 633$ m

Most probable value = Weighted mean

$$= \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$$

$$= \frac{(8.275)\left(\frac{8}{690}\right) + 8.292\left(\frac{2}{894}\right) + 8.285\left(\frac{4}{633}\right)}{\frac{8}{690} + \frac{2}{894} + \frac{4}{633}}$$

$$= 8.279 \text{ m}$$

Example 6.15 In a topographical survey, the difference in level between two points *A* and *B* is found by three routes—via *C*, *D*, *E*, and *F*, via *G* and *H* and via *J*, the distances being

Route 1	AC = 120 m	CD = 162 m	DE = 300 m	EF = 258 m	FB = 132 m
Route 2	AG = 240 m	GH = 306 m	HB = 384 m		
Route 3	AJ = 294 m	JB = 462 m			

The sections on Route 1 are each levelled four times, those on Route 2 eight times, and on Route 3 twice; the established differences in level thus obtained being 30.81 m, 30.57 m and 31.08 m respectively. If the probable error in any section at each levelling is proportional to its length, and the usual laws for combination of readings hold, find the most probable value for the difference in level between *A* and *B*. (L.U., B.Sc.,)

Solution From the given data,

$l_1 = 120 + 162 + 300 + 258 + 132 = 972$ m

$l_2 = 240 + 306 + 384 = 930$ m

$l_3 = 294 + 462 = 756$ m

$$w_1 : w_2 : w_3 = 4/972 : 8/930 : 2/756$$

Most probable value = Weighted mean

$$= \frac{\frac{4}{972}(30.81) + \frac{8}{930}(30.57) + \frac{2}{756}(31.08)}{\frac{4}{972} + \frac{8}{930} + \frac{2}{756}}$$

$$= \frac{0.1268 + 0.2630 + 0.0822}{0.00411 + 0.0086 + 0.0026}$$

$$= 30.72 \text{ m}$$

Example 6.16 A line of levels is carried from B.M.A whose elevation is 146.522 m, to a new B.M.P requiring 10 set ups. The measured difference in elevation is – 3.436 m. A line is carried from B.M.B whose elevation is 146.851 m to B.M.P requiring six set ups. The measured difference in elevation is – 3.755 m. A line is carried from B.M.C whose elevation is 132.768 m to B.M.P requiring four set ups. The measured difference in elevation is + 10.312 m. Compute the weighted elevation of B.M.P and the standard error of this elevation. [Moffit]

Solution Standard error is proportional to the square root of the number of set ups required. Hence standard errors of lines 1, 2 and 3 are proportional to $\sqrt{10}, \sqrt{6}$ and $\sqrt{4}$ respectively. Weights are inversely proportional to square of standard errors, hence,

$$w_1 : w_2 : w_3 = \frac{1}{10} : \frac{1}{6} : \frac{1}{4}$$

Level of B.M.P., by 1st route = 146.522 – 3.436

= 143.086 m

by 2nd route = 146.851 – 3.755

= 143.096 m

by 3rd route = 132.768 + 10.312

= 143.080 m

Weighted elevation

$$= \frac{\frac{1}{10}(143.086) + \frac{1}{6}(143.096) + \frac{1}{4}(143.08)}{\frac{1}{10} + \frac{1}{6} + \frac{1}{4}}$$

$$= 143.08632 \text{ m}$$

Standard error of the weighted mean

$$v_1 = 143.086 - 143.08632 = -.00032$$

$$v_2 = 143.096 - 143.08632 = +.00968$$

$$v_3 = 143.080 - 143.08632 = -.00632$$

$$v_1^2 = 1.024 \times 10^{-7}, \ p_1 v_1^2 = 1.024(10^{-8})$$

$v_2^2 = 9.37 \times 10^{-5} \quad p_2 v_2^2 = 1.562(10^{-5})$

$v_3^2 = 3.99 \times 10^{-5} \quad p_3 v_3^2 = 9.990(10^{-6})$

$$\sum pv^2 = 2.562 \times 10^{-5}$$

Standard error of the weighted mean

$$= \sqrt{\frac{2.562 \times 10^{-5}}{0.5167 \times 2}}$$

$$= \pm .004979$$

Standard error of each of the measured elevation

$$= \sqrt{\frac{2.562 \times 10^{-5}}{2}}$$

$$= \pm .003579$$

$\sigma_1 = \pm (.003579)(10) = \pm .03579$

$\sigma_2 = \pm (.003579)(6) = \pm .02147$

$\sigma_3 = \pm (.003579)(4) = \pm .01432.$

Example 6.17 Figure 6.15 shows a closed circuit of levels with difference of levels between different points and the length between them. Compute the adjusted values of the levels of different points.

Solution

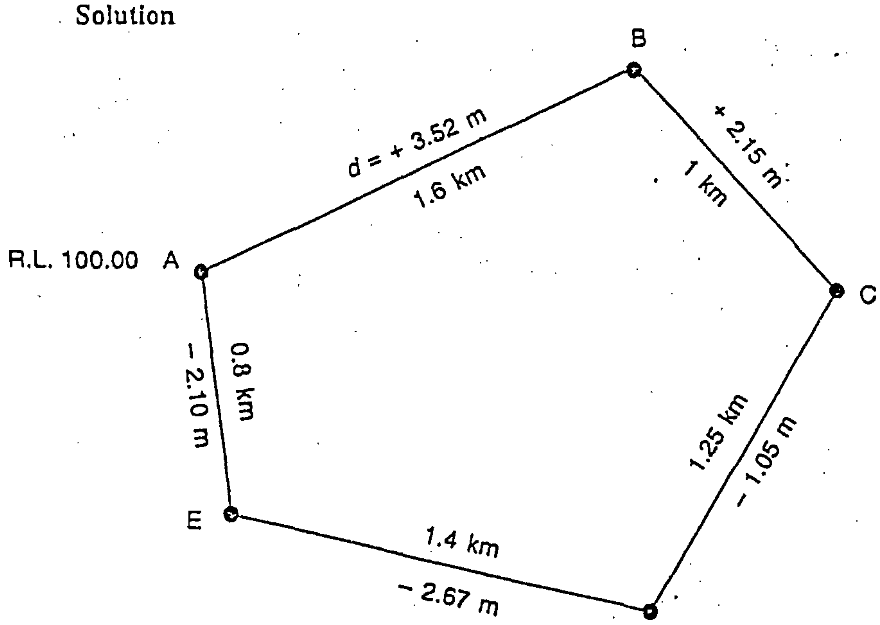


Fig. 6.15 Example 6.17.

Misclosure = + 3.52 + 2.15 – 1.05 – 2.67 – 2.10

= – 0.15 m over a length of 6.05 km

$$\text{Elevation adjustment at } B = 100.00 + 3.52 + \frac{0.15 \times 1.6}{6.05}$$

$$= 103.559 \text{ m}$$

$$\text{at } C = 100 + 3.52 + 2.15 + \frac{0.15(2.6)}{6.05}$$

$$= 105.734 \text{ m}$$

$$\text{at } D = 100 + 3.52 + 2.15 - 1.05 + \frac{0.15(3.85)}{6.05}$$

$$= 104.645 \text{ m}$$

$$\text{at } E = 100 + 3.52 + 2.15 - 1.05 - 2.67 + \frac{0.15(5.25)}{6.05}$$

$$= 102.08 \text{ m}$$

$$\text{at } A = 100 + 3.52 + 2.15 - 1.05 - 2.67 - 2.10 + \frac{0.15(6.05)}{6.05}$$

$$= 100 \text{ m}$$

Example 6.18 A level line is run from B.M.A and closes on the same point as shown in Fig. 6.16. B.M.B is included in both the loops. In the outer loop there are five change points while in the inner loop there are four change points. The elevations observed at different points are as follows:

	Elevation observed	1st correction	2nd correction
B.M.A	= 100.00		
C.P1	= 101.50		
B.MB(1)	= 101.85	101.85	
C.P.2	= 104.35	104.29	
C.P.3	= 103.75	103.63	
C.P.4	= 105.45	105.27	
C.P.5	= 103.45	103.21	
B.M.B(2)	= 102.15	102.85	101.93
C.P.6	= 101.30	101.00	
B.M.A	= 100.14.	99.84	100.00

Solution First the outer circuit, i.e. Loop 2 is adjusted. Starting with B.M.B(1) as 101.85 it closes on B.M.(2) as 102.15 giving a misclosure of + 0.30 m. In Loop 2 there are 5 instrument set ups and hence a correction of – 0.30/5 = – 0.06 m/set up. B.M(2) is to be reduced by 0.30 m. As C.P.6 and B.M.A is based on B.M.B as reference they are also reduced by the same amount

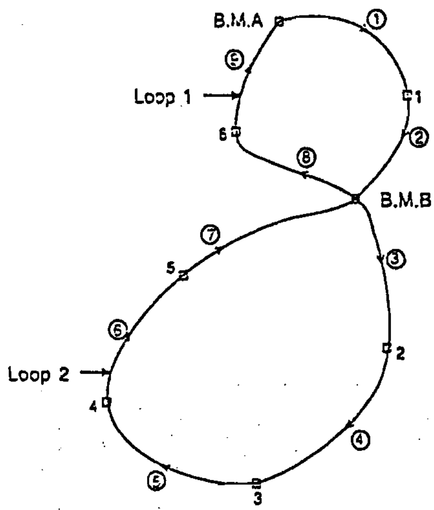


Fig. 6.16 Example 6.18.

giving R.L. of B.M.A as 99.84 m. This gives a misclosure in loop 1 as – 0.16 and is equal to 0.16/4 = 0.04 m per set up. The values in loop 2 are now adjusted. B.M.A is adjusted by four instrument set ups and B.M.B by 2 instrument set ups. Details are shown as 1st correction and 2nd correction in the example itself.

PROBLEMS

6.1 Draw a page of a typical levelling field book and explain how the readings are recorded.

6.2 Describe in detail the methods of reduction of levels and explain their merits and demerits.

6.3 Name the different sources of errors in levelling and explain how they can be eliminated or minimized.

6.4 (a) Derive an expression for the combined effect of earth's curvature and atmospheric refraction in levelling, given the diameter of earth as 12,740 km.

(b) The following notes refer to reciprocal levels:

Instrument near	Staff readings on P	Staff readings on Q	Remarks
P	1.850	2.850	PQ = 1055 m
Q	1.000	2.200	R.L of P = 126.100

Determine

(i) the true R.L. of Q (ii) the combined correction for curvature and refraction and (iii) the angular error, if any, in the collimation adjustment of instrument. [AMIE, Sec B, Winter 1984]

6.5 (a) Define with the help of neat sketches the following:
(i) Level surface, (ii) Horizontal surface, (iii) Backsight, (iv) Foresight, (v) Height of instrument, and (vi) Reduced level.

(b) The following figures were extracted from a "level field book", some of the entries being illegible. Insert the missing figures, check your results, and re-book all the figures using the "rise and fall" method.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	2.285						B.M No. 1
2	1.650						
3		2.105					
4	×		1.960				
5	2.050		1.925		0.300		
6		×		×		232.255	B.M No. 2
7	1.690		×	0.340			
8	2.865		2.100				
9			×	×		233.425	B.M No. 3

[AMIE, Winter 1986]

6.6 (a) Show that the reciprocal levelling eliminates effects of atmospheric refraction and earth's curvature as well as effect of in adjustment of the line of collimation.

(b) The following consecutive readings were taken with a level and 3 m levelling staff on continuously sloping ground at a common interval of 20 m.
0.605, 1.235, 1.860, 2.575, 0.240, 0.915, 1.935, 2.875, 1.825, 2.725
The reduced level of the 1st point was 192.120. Rule a page of a level field book and enter the above readings. Calculate the reduced levels of the points and also the gradient of the line joining the first and last points. [AMIE, Summer 1987]

6.7 (a) Discuss the effects of curvature and refraction in levelling. Find the correction due to each and the combined correction. Why are these effects ignored in ordinary levelling?

(b) In levelling between two points *A* and *B* on opposite sides of a river, the level was set up near *A* and the staff readings on *A* and *B* were 2.645 m and 3.230 m. respectively. The level was then moved and set up near *B*, the respective staff readings on *A* and *B* were 1.085 m and 1.665 m. Find the true difference of level between *A* and *B*.

[AMIE, Winter 1987]

6.8 (a) Describe the method of longitudinal levelling with the help of a suitable diagram and sketch a typical longitudinal section.

(b) The following is the page of a level field book from which several values are missing. Reconstruct the page and fill all the entries where × mark is present. Apply all necessary checks.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	1.385					100.00	B.M.
2		1.430			×	×	
3		×			0.395	×	
4			1.275	×		×	T.P.1
5	0.630		0.585	0.310		×	T.P.2
6		0.920			×	100.13	
7		×			0.210	×	
8			1.740		×	×	

[AMIE, Summer 1988]

6.9 (a) Derive expressions for curvature and refraction correction along with diagrams in levelling.

(b) Explain the operation of balancing backsight and foresight with the help of a diagram and describe its advantages.

(c) The following consecutive readings were taken with a level and 3 m levelling staff on a continuously sloping ground.
0.602, 1.234, 1.860, 2.574, 0.238, 0.914, 1.936, 2.872, 0.568, 1.824, 2.722
Determine the reduced levels of all the points of R.L. if first point was 192.122 m.

6.10 Reproduced below is the page in a level book. Fill in the missing data. Apply the usual checks.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	2.150					450.000	B.M. 1
2	1.645		?	0.500			
3		2.345			?		
4	?		1.965	?			
5	2.050		1.925		0.500		
6		?		?		451.730	B.M. 2
7	– 1.690		?	0.120			Staff held against ceiling
8	?		2.100		?		
9			?	?		449.000	B.M. 3
Sum	8.445						

[AMIE, Summer 1991]

6.11 (a) List out the permanent relationships that should exist between different principal lines of a perfect dumpy level.

(b) The distance between two bench marks A and B was 40 m. A dumpy level was placed at C on an extension of AB such that $AC = 60$ m. The following data was recorded:

Staff reading on B.M. A(R.L. 10.750) = 0.750
Staff reading on B.M. B(R.L. 11.750) = 1.750

(i) Was the line of collimation inclined upwards or downwards and by how much?

(ii) Calculate the readings that should be obtained on A and B to have a horizontal line of sight.

(iii) State in what direction and by how much the diaphragm has to be moved for adjustment. [AMIE, Winter 1980]

6.12 (a) Write in brief about the special points of 'autolevel'.

(b) The following consecutive staff readings were taken on pegs at 15 m interval on a continuously sloping ground.

0.895, 1.305, 2.800, 1.960, 2.690, 3.255, 2.120, 2.825, 3.450, 3.895, 1.685, 2.050 (station A)

R.L. of station A where the reading 2.050 was taken, is known to be 50.250.

From the last position of the instrument two stations B and C with R.L. 50.800 and 51.000 respectively are to be established without disturbing the instrument. Work out the staff readings at stations B and C and complete all the work in level book form.

[AMIE, Winter 1982]

6.13 (a) Draw of neat sketch of an interval focussing telescope of a level showing clearly all the important parts. State the function of each part.

(b) Calculate the error in staff reading on account of (i) Curvature of earth, (ii) Normal refraction in levelling operations if the staff is held at a distance of 800 m from the instrument.

(c) Show the above two errors clearly on a neatly drawn sketch and also the combined correction to be applied in this case.

(d) Following observations were taken for testing of a dumpy level:

(i) Instrument exactly at the mid-point of line AB
Staff reading at station A = 1.855
Staff reading at station B = 1.605

(ii) Instrument very near to station B
Staff reading at station A = 0.675
Staff reading at station B = 0.925

Find out from the above observations whether the line of collimation is in adjustment or not. If not in adjustment what is the nature and amount of error in distance AB? What will be the correct readings on staff at A and B from station B when the line of collimation is adjusted?

[AMIE, Summer 1983]

7

Permanent Adjustments of Levels

7.1 INTRODUCTION

Two types of adjustments are made on any surveying instrument—(i) Temporary; (ii) Permanent.

Temporary adjustments are those performed each time an instrument is used and should invariably be done each time the instrument is set up in the field. *Permanent* adjustments are usually done by the instrument makers. They should however, be checked periodically by the users and, if necessary, sent to the maker or may be done by the users themselves if experts are available.

7.2 PERMANENT ADJUSTMENTS OF A DUMPY LEVEL

The correct axes relationships for a properly adjusted level is shown in Fig. 7.1 They are as follows:

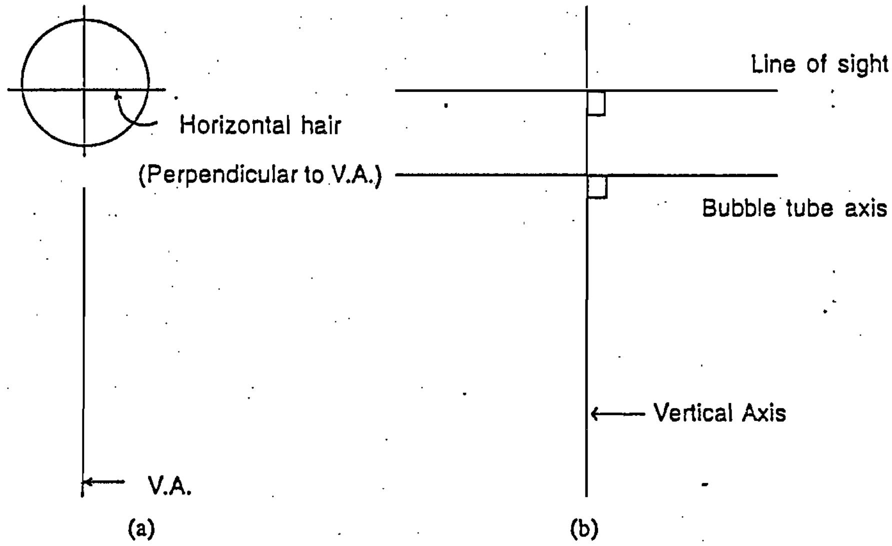


Fig. 7.1 Axes relationships of a level: (a) Side view. (b) Front view.

1. The axis of the bubble tube should be perpendicular to the vertical axis of the instrument.

2. The horizontal crosshair should lie in a plane perpendicular to the vertical axis.

3. The line of sight should be parallel to the axis of the bubble tube.

7.2.1 FIRST ADJUSTMENT

Purpose To make the axis of the level tube perpendicular to the vertical axis.

Test Set up the level, centre the bubble and revolve the telescope through 180°. If the bubble remains central, no adjustment is necessary. If not, the distance through which the bubble moves off the central position is double the error.

Correction

1. Bring the bubble half way back by raising or lowering one end of the level tube by means of capstan headed screws.

2. The other half is corrected by means of the two levelling screws parallel to the telescope.

3. Now rotate the telescope through 180° to see if the bubble still remains central. If not, the adjustment has to be repeated till the bubble remains central during one complete revolution of the telescope.

Explanation The adjustment is based on the principle of reversal which states that reversing the instrument position by rotation in a horizontal or vertical plane doubles any error present, enabling a surveyor to directly determine how much correction is needed. This can be easily shown with the help of a set square where the sides *AB* and *BC* are not exactly perpendicular but there is small angular error *e*. If the set square is now turned about *BC*, the error is doubled as shown in Fig. 7.2.

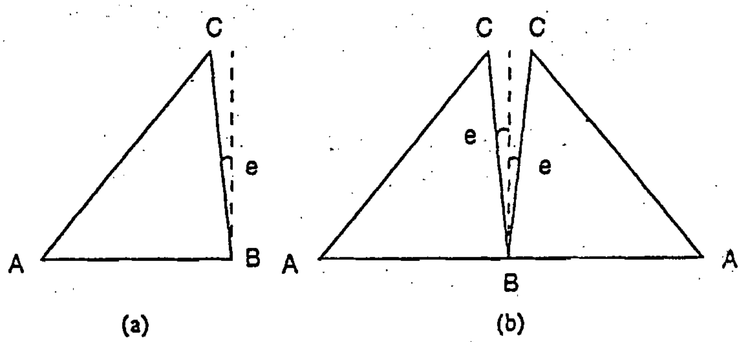


Fig. 7.2 Principle of reversal.

The same principle is applied in this adjustment. Figure 7.3(a) shows how level tube is brought to the centre of its run when the vertical axis is not truly vertical. This is initial condition. When the instrument is now rotated through 180°, *cd* remains as it is but the new position of *ab* becomes *a'b'* as the angle

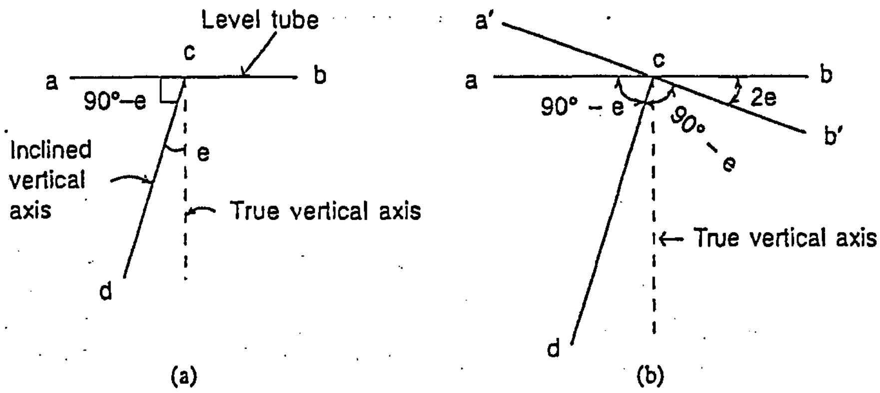


Fig. 7.3 Adjustment of bubble tube axis.

$90° - e$ remains fixed and *ac* becomes *cb'*. Hence the error from the horizontal is $2e$ which is double the error between the level tube axis and the vertical axis. Hence half the error is adjusted by the capstan headed screws as shown in Fig. 7.3(b).

7.2.2 SECOND ADJUSTMENT

Purpose To make the horizontal hair truly horizontal when the instrument is levelled.

Test Sight some well defined point *P* with one end of the horizontal hair. Rotate the telescope slowly on its vertical axis. If the horizontal crosshair moves over the point *P* throughout its length, the horizontal hair is truly horizontal. If not, the instrument is out of adjustment.

Adjustment Loosen the four capstan screws holding the reticle carrying the crosshairs. Rotate the reticle through the required small angle so that the horizontal hair becomes truly horizontal. The screws should be carefully tightened in its final position. Check again by sighting point *P* as before and repeat the process, if necessary.

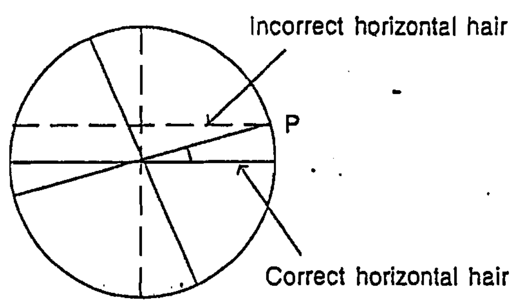


Fig. 7.4 Adjustment of horizontal crosshair.

7.2.3 THIRD ADJUSTMENT

Purpose To make the line of sight parallel to the axis of the bubble tube or to make the line of sight horizontal when the bubble is in the centre of its run.

Test This is done by means of Two-peg test. The instrument is first placed midway between two pegs *A* and *B* which are at least two chains apart as shown in Fig. 7.5.

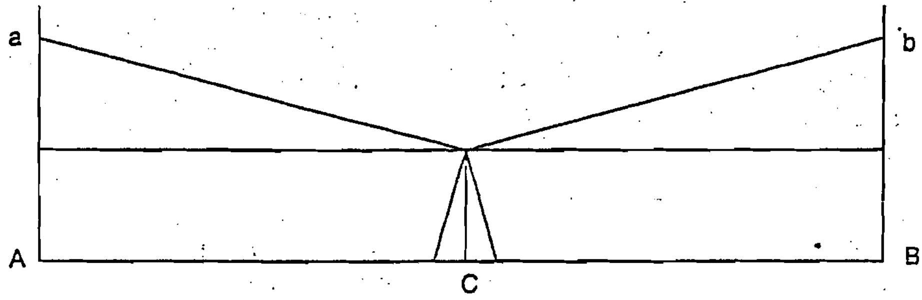


Fig. 7.5 Initial mid position of level.

Since the instrument is placed at the middle even if the line of sight is inclined, the difference of readings *Aa* and *Bb* will be the true difference of level between *A* and *B*.

Now the instrument is placed at one end *D* very close to the point *A*. Reading on *A*, *Aa′* is taken by sighting through the objective lens. The reading on *B*, *Bb′* is taken in the usual way (Fig. 7.6). If the difference in reading between *Aa′* and *Bb′* is the same as the previous reading *Aa* – *Bb*, the line of sight is horizontal and no adjustment is necessary.

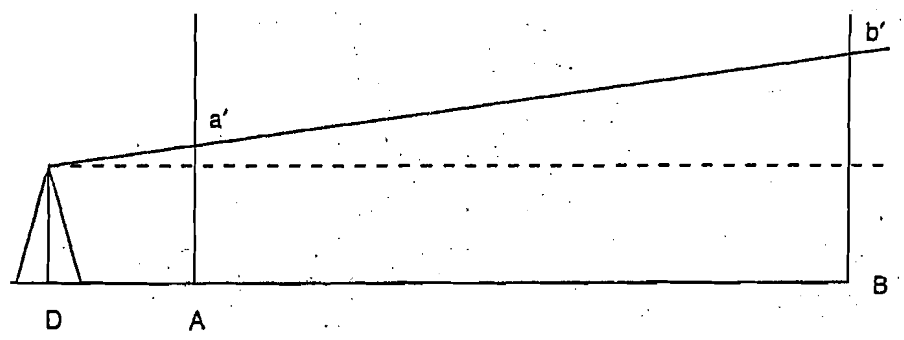


Fig. 7.6 Second position of level (two peg test).

Adjustment If difference in reading is not the same, compute the correct reading at *B* by adding or subtracting from the reading at *A*, the true difference of level between *A* and *B*. Loosen the top (or bottom) capstan screw holding the reticle and tighten the bottom (or top) screw to move the horizontal hair up or down to get the required reading at *B*. To get the correct value several trials will be necessary.

7.3 ADJUSTMENTS OF A TILTING LEVEL

The tilting level has three adjustments:

1. Horizontal crosshair should be truly horizontal when the bull's eye bubble is centred.

2. The circular bubble should stay in the centre as the telescope is rotated about the vertical axis.

3. Line of sight should be horizontal when the main sensitive bubble is centred.

These adjustments closely follow the adjustments of dumpy level with minor modifications.

7.3.1 FIRST ADJUSTMENT

The test is same as in dumpy level. A point P is seen at the end of the horizontal crosshair. The telescope is now rotated about its vertical axis to see if the point P remains in the horizontal hair. If not, adjustments are to be made by the capstan screws and rotating the crosshair reticle.

7.3.2 SECOND ADJUSTMENT

Purpose The purpose of this adjustment is to make the plane of the circular bubble perpendicular to the instrument's vertical axis.

Test The telescope is turned 180° in azimuth. If the bubble does not remain central, it is brought half way back to the centre in both directions by raising or lowering the bubble mount by means of capstan screws or spring screws as required. This adjustment is not essential as precise bubble tube is used to obtain a horizontal line of sight.

7.3.3 THIRD ADJUSTMENT

Purpose This adjustment makes the precise bubble tube axis parallel to the instrument's line of sight.

Test The two-peg test is applied to find out if the line of sight is horizontal when the bubble is centred.

Adjustment If not, the correct reading at B is computed and the crosshairs are brought to that reading by rotating the telescope's tilting drum. The precise bubble is then centred using the bubble tube adjusting nuts. If the bubble is the coincident type, raising or lowering one end of the bubble vial will bring them into coincidence.

7.4 ADJUSTMENTS OF AUTOMATIC LEVEL

Automatic levels have two principal adjustments: (i) Circular bubble; (ii) Line of sight.

Adjustments are the same as stated earlier. Before these adjustments are done on automatic levels, it should be checked that the compensator is functioning properly.

PROBLEMS

7.1 (a) What is the difference between the temporary adjustment and permanent adjustment of a dumpy level?

(b) Describe the permanent adjustments of a dumpy level. How can these be tested and done in the field? [AMIE, Summer 1978]

7.2 (a) Draw a neat sketch of a dumpy level and name parts thereon.

(b) List the 'temporary' and 'permanent' adjustments of a dumpy level.

(c) Describe the function of crosshairs and stadia hairs. [AMIE, Winter 1979]

7.3 List out the permanent relationships that should exist between the different principal lines of a perfect dumpy level. [AMIE, Winter 1980]

8

Angles and Directions

8.1 INTRODUCTION

Measurement of angles is basic to any survey operation. When an angle is measured in a horizontal plane it is horizontal angle, when measured in a vertical plane it is vertical angle. Angle measurements involve three steps: (i) Reference or starting line; (ii) Direction of turning; (iii) Angular value (Value of the angle). These are shown in Fig. 8.1.

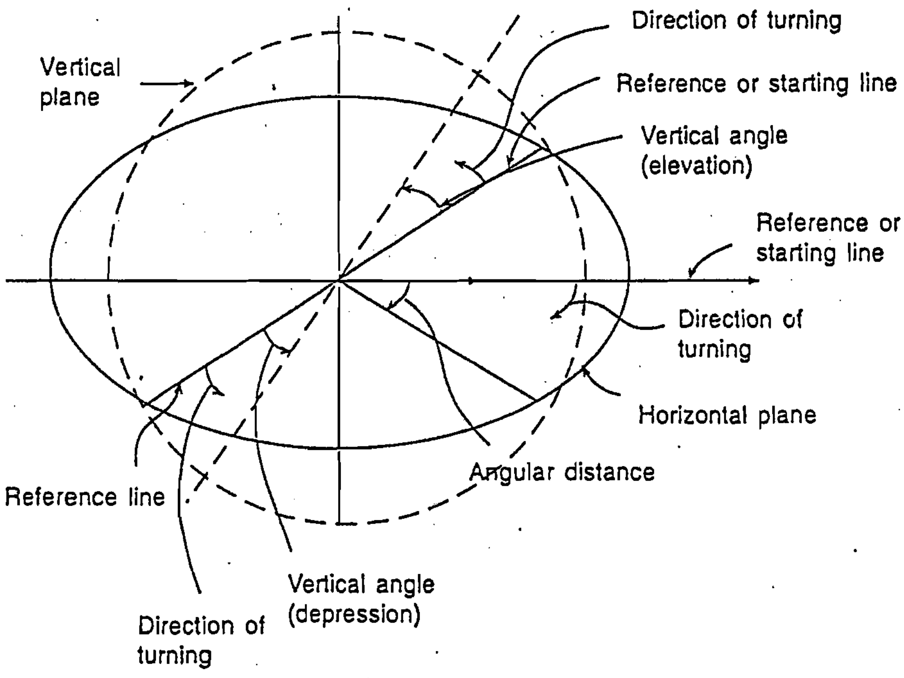


Fig. 8.1 Measurement of angle.

8.2 DIFFERENT TYPES OF HORIZONTAL ANGLES

As explained in Fig. 8.2 horizontal angles can be (i) Interior angles, or (ii) Deflection angles. Interior angles can be clockwise when the direction of turning is clockwise, or anticlockwise when the direction of turning is anticlockwise. Similarly deflection

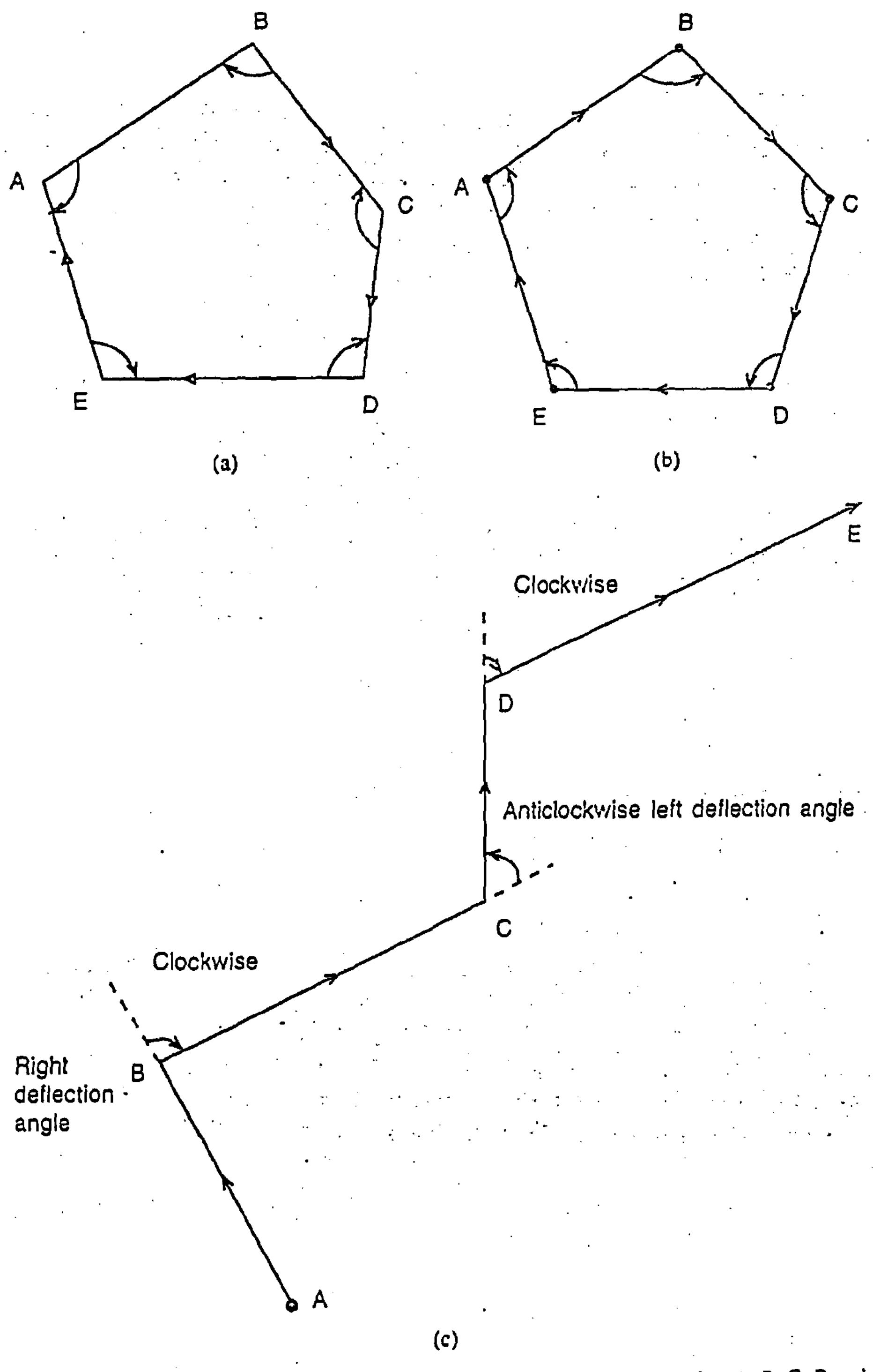


Fig. 8.2 Different types of angles: (a) Closed polygon—instrument station *A*, *B*, *C*, *D* and *E*, all angles measured clockwise. (b) Closed polygon—instrument stations *A*, *B*, *C*, *D* and *E*, all angles measured anticlockwise. (c) Deflection angles.

angles are measured clockwise or towards right and anticlockwise or towards left. Figure 8.2 explains the different measurements.

8.3 DIRECTION OF A LINE

Direction of a line is the horizontal angle from a reference line called the meridian. There are four basic types of meridians:

1. *Astronomic meridian* It is an imaginary line on the earth's surface passing through the north-south geographical poles.

2. *Magnetic meridian* It is the direction of the vertical plane shown by a freely suspended magnetic needle.

3. *Grid meridian* A line through a point parallel to the central meridian or y-axis of a rectangular coordinate system.

4. *Arbitrary meridian* An arbitrary chosen line with a directional value assigned by the observer. These are explained graphically in Fig. 8.3.

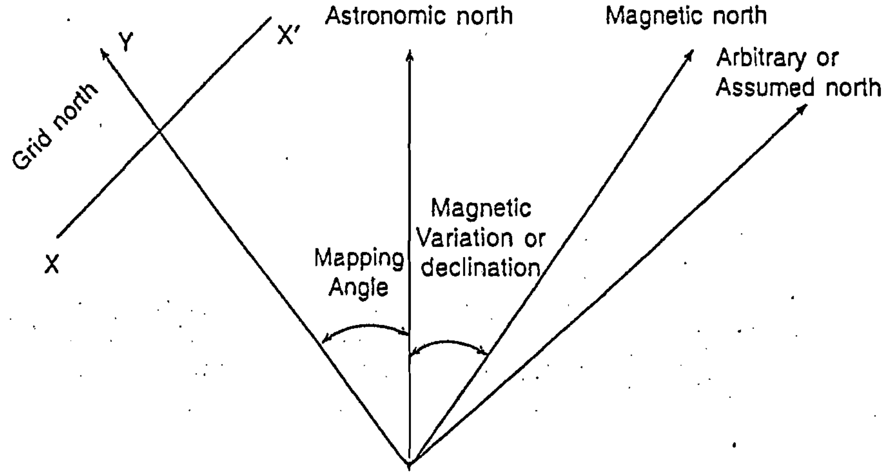


Fig. 8.3 Different north directions.

8.4 BEARINGS

Bearing of a line is measured from the north or south terminus of a reference meridian. It is always less than 90° and is designated by the quadrant in which it lies as shown in Fig. 8.4. From the figure it can be seen that

Bearing of OA = N 40°E
OB = S 25°E
OC = S 30°W
OD = N 45°W

Since bearing is with reference to N-S line angles are measured clockwise in the 1st (NE) and IIIrd (S.W) quadrants. It is measured anticlockwise in 2nd and 4th Quadrants (NW and SE). When bearings are measured with reference to astronomic or true meridian it is true bearing. If the bearing is from magnetic meridian, it is

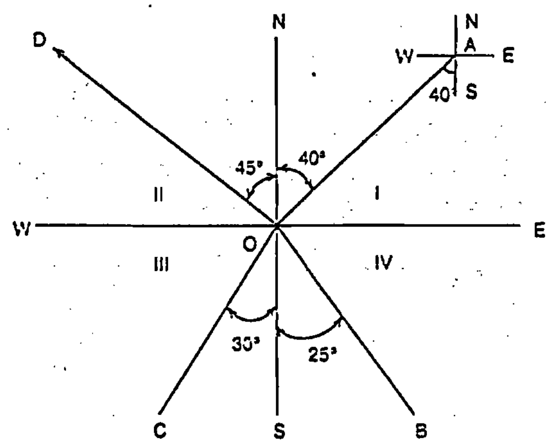


Fig. 8.4 Quadrantal bearing.

magnetic bearing and when from a grid it is grid bearing. If the instrument is set up at *O* and bearing of *OA* is taken it is forward bearing. But if the instrument is set up at *A* and bearing of *AO* is taken, it is forward bearing of *AO* but back bearing of *OA*. Hence back bearing of *AO* is S40°W. Back bearings thus have the same numerical value but opposite letters.

8.5 AZIMUTHS

Azimuths are angles measured clockwise from any reference meridian. They are measured from the north and vary from 0° to 360° and do not require letters to identify their quadrant. Figure 8.5 shows the azimuths of different lines whose bearings are given in Fig. 8.4.

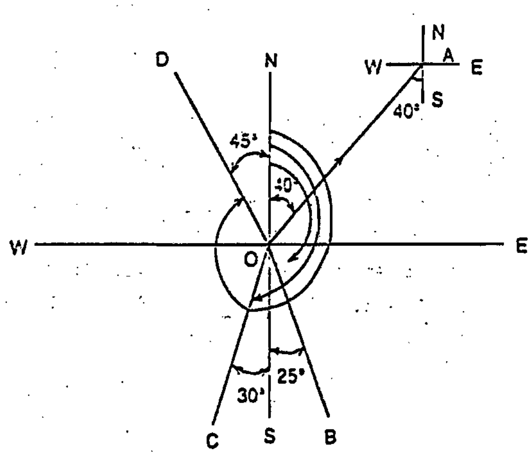


Fig. 8.5 Azimuth or whole circle bearing.

Azimuth of $OA = 40°$

$OB = 180° - 25° = 155°$

$OC = 180 + 30° = 210°$

$OD = 360° - 45° = 315°$

As stated before, forward azimuth of *OA* is 40°. The back azimuth of *OA* is forward azimuth of *AO* and from the figure it is clearly equal to 40° + 180° = 220°. Thus the forward azimuth and back azimuth differ by 180°. As before azimuths are true, magnetic, grid or assumed when they are measured with reference to true, magnetics grid or assumed meridian respectively. From a study of the bearings and azimuths of the lines *OA*, *OB*, *OC* and *OD*, the following observations are made:

(a) When a line is in the 1st quadrant the azimuth varies from 0 to 90° and azimuth and bearing of a line are the same (line *OA*).

(b) When a line is in the 2nd quadrant the azimuth lies between 90° and 180° and it can be obtained from bearing by subtracting it from 180° (line *OB*).

(c) When a line is in 3rd quadrant the azimuth lies between 180° and 270° and it can be obtained from bearing by adding 180° (line *OC*).

(d) Finally, when a line is in the 4th quadrant, the azimuth is obtained by subtracting the bearing from 360°. The azimuth will lie between 270° and 360° (line *OD*).

To convert azimuth to bearing the following rules should be followed:

(a) When the azimuth is between 0° to 90°, it lies in 1st quadrant and the bearing is the same as azimuth with the symbols NE.

(b) Between 90° to 180°, the line lies in the 2nd quadrant and the bearing is obtained by subtracting azimuth from 180° with the symbols SE.

(c) Between 180° and 270°, the line lies in the 3rd quadrant and bearing is obtained by subtracting 180° from azimuth with symbols SW.

(d) Finally, in the 4th quadrant, azimuth is subtracted from 360° to get bearing with the symbols NW.

Usually azimuth is known as whole circle bearing and bearing is designated as quadrantal bearing or reduced bearing. Table 8.1 gives a comparison between bearing and azimuth.

Table 8.1 Comparison between Bearing and Azimuth

Point	Bearing	Azimuth
1. Angle	Varies from 0° to 90°	Varies from 0° to 360°
2. Designation	It always lies between any two of the four letters N, S, E and W	No letter is necessary
3. Measurement	(i) Both clockwise and anticlockwise measurement is necessary	Measured always clockwise
	(ii) Measured from north and south	Measured only from north

Example 8.1 Convert the following azimuths to bearings

OA = 54°20′ *OB* = 154°25′ *OC* = 261°25′

OD = 312°38′

From Fig. 8.6 (i) bearing of *OA*, θ = N 54°20′E

Fig. 8.6 (ii) bearing of *OB*, θ = 180° − 154°25′
= S 25°35′E

Fig. 8.6 (iii) bearing of *OC* θ = 261°25′ − 180°
= S 81°25′W

Fig. 8.6 (iv) bearing of *OD* θ = 360° − 312°38′
= N 47°22′W.

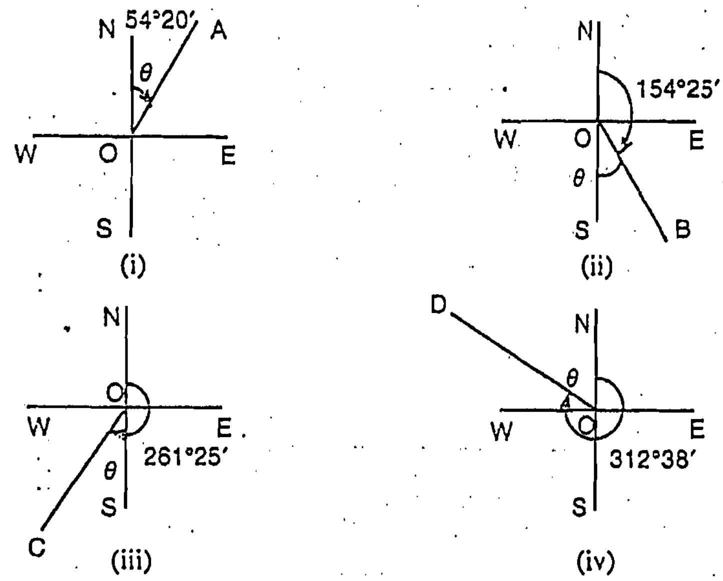


Fig. 8.6 Example 8.1.

Example 8.2 Convert the following quadrantal bearings to whole circle bearings.

OA = N 15°10′E *OB* = S 37°50′E

OC = S 49°40′W *OD* = N 80°25′W

From Fig. 8.7 (i) whole circle bearing of *OA* θ = 15°10′

Fig. 8.7 (ii) whole circle bearing of *OB* θ = 180° − 37°50′ = 142°10′

Fig. 8.7 (iii) whole circle bearing of *OC* θ = 180° + 49°40′ = 229°40′

Fig. 8.7 (iv) whole circle bearing of *OD* θ = 360° − 80°25′ = 279°35′

Example 8.3 The whole circle bearings of the sides of a traverse *ABCDEF* are given below. Compute the internal angles.

Bearing of *AB* = 290°45′
Bearing of *BC* = 250°48′
Bearing of *CD* = 196°12′
Bearing of *DE* = 175°24′

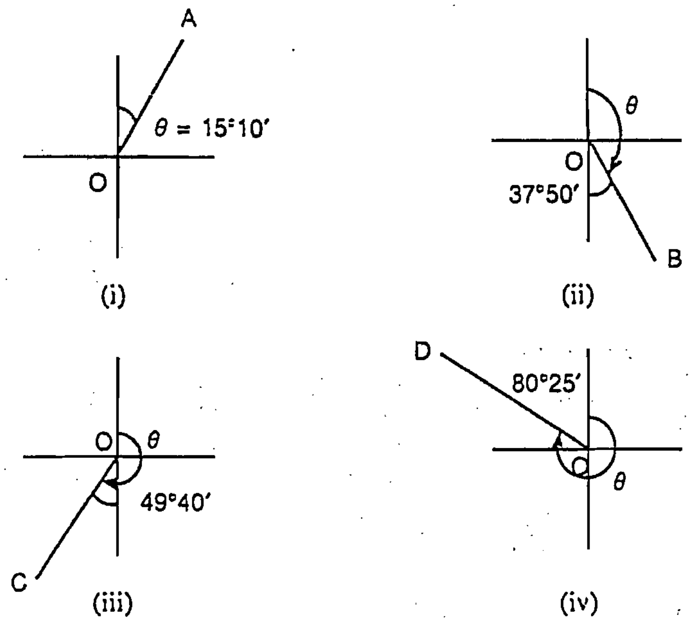


Fig. 8.7 Example 8.2.

Bearing of *EF* = 112°18′
Bearing of *FA* = 30°00′

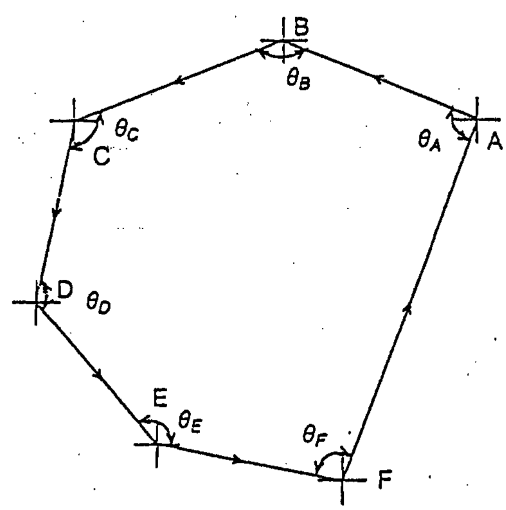


Fig. 8.8 Example 8.3.

Solution The backbearings and forebearings differ by 180°. Hence,

Bearing of *BA* = 110°45′
Bearing of *CB* = 70°48′
Bearing of *DC* = 16°12′
Bearing of *ED* = 355°24′

Bearing of FE = 292°18′
Bearing of AF = 210°00′

From the Fig. 8.8

θ_A = Bearing of AB – Bearing of AF
= 290°45′ – 210°00′
= 80°45′

θ_B = Bearing of BC – Bearing of BA
= 250°48′ – 110°45′
= 140°03′

θ_C = Bearing of CD – Bearing of CB
= 196°12′ – 70°48′
= 125°24′

θ_D = Bearing of DE – Bearing of DC
= 175°24′ – 16°12′
= 159°12′

From the Figure

θ_E = Bearing of EF + (360° – Bearing of ED)
= 112°18′ + (360° – 355°24′)
= 116°54′

θ_F = Bearing of FA + (360° – Bearing of FE)
= 30°00′ + (360° – 292°18′)
= 97°42′

Total internal angles of a closed traverse
= $(2n - 4)$ Right angles
= $(2 \times 6 - 4)$ Right angles
= 720°

$\theta_A + \theta_B + \theta_C + \theta_D + \theta_E + \theta_F$ = 80°45′ + 140°03′ + 125°24′ + 159°12′ + 116°54′ + 97°42′ = 720°00′

Calculations should always be based on a properly drawn sketch.

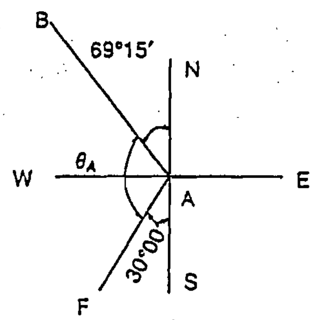
Example 8.4 The same problem when the bearings of the sides are expressed in quadrantal system.

Line	Bearing
AB	N 69°15′W
BC	S 70°48′W
CD	S 16°12′W
DE	S 04°36′E
EF	S 67°42′E
FA	N 30°00′E

Solution The backbearings of the lines are obtained by just changing N to S, E to W and vice versa.

Bearing	Backbearing	Value
AB	*BA*	S 69°15′E
BC	*CB*	N 70°48′E
CD	*DC*	N 16°12′E
DE	*ED*	N 04°36′W
EF	*FE*	N 67°42′W
FA	*AF*	S 30°00′W

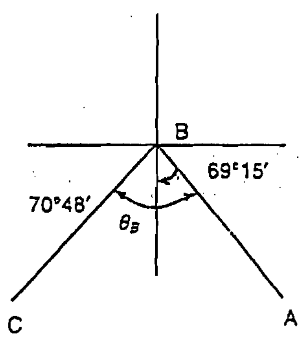
Drawing separately each node



(i)

From Fig. 8.9(i)

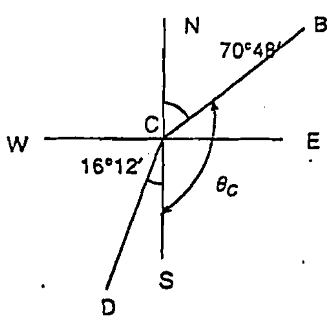
$$\theta_A = 180° - (69°15' + 30°00') = 80°45'$$



(ii)

From Fig. 8.9(ii)

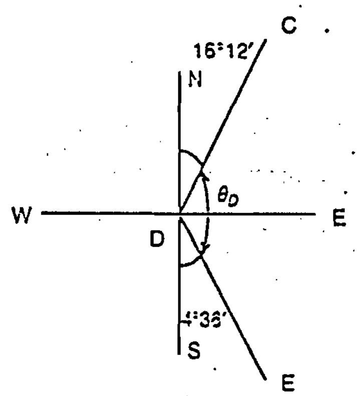
$$\theta_B = 70°48' + 69°15' = 140°03'$$



(iii)

From Fig. 8.9(iii)

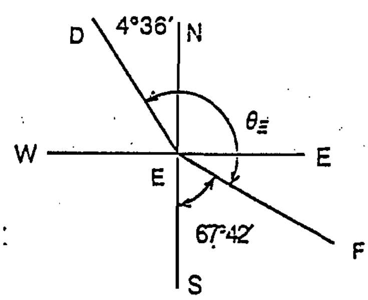
$$\theta_C = 180° - 70°48' + 16°12' = 125°24'$$



(iv)

From Fig. 8.9(iv)

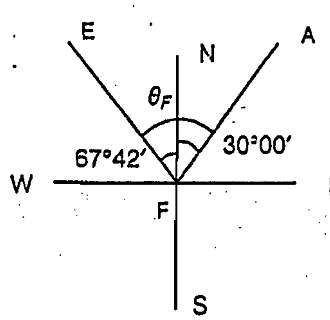
$$\theta_D = 180° - (16°12' + 4°36')$$
$$= 159°12'$$



(v)

From Fig. 8.9(v)

$$\theta_E = 180° - 67°42' + 4°36'$$
$$= 116°54'$$



(vi)

From Fig. 8.9(vi)

$$\theta_F = 67°42' + 30°00'$$
$$= 97°42'$$

Fig. 8.9 Example 8.4.

Example 8.5 Compute and tabulate the bearings of a regular hexagon given the starting bearing of side *AB* = S 50°10′E (Station *C* is easterly from *B*).

Solution The total internal angles of a regular hexagon are $(2n - 4)$ right angles, i.e. $2 \times 6 - 4 = 8$ right angles = 720°. Each angle, therefore, is 120°. The rough sketch of the hexagon is shown in Fig. 8.10.

Whole circle bearing of $AB = 180° - 50°10'$

$$= 129°50'$$

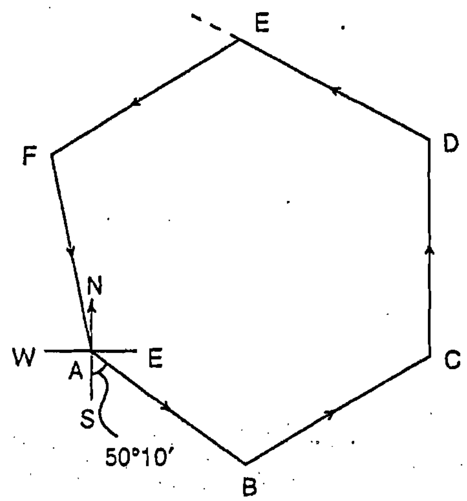


Fig. 8.10 Example 8.5.

The external angles are all 60° and the deflection at *B* from *AB* to *BC* is 60° towards left or anticlockwise. Since whole circle bearing is measured clockwise, this is subtracted from the previous bearing. Hence

Bearing of *BC* = 129°50′ – 60° = 69°50′

Bearing of *CD* = 69°50′ – 60° = 9°50′

Bearing of *DE* = 9°50′ – 60° = – 50°10′

Bearing negative means it has crossed the N-S line in the anticlockwise direction by 50°10′ (Fig. 8.11). It is anticlockwise because we are subtracting 60°, that is, moving in an anticlockwise direction by 60°.

True bearing then becomes 360° – 50°10′ = 309°50′

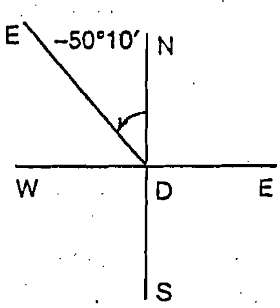


Fig. 8.11 Example 8.5.

Bearing of *EF* =	309°50′
	– 60°
	249°50′
Bearing of *FA*	– 60°
	189°50′
Bearing of *AB*	– 60°
	129°50′

as before.

9

Compass Survey

9.1 INTRODUCTION

The compass has been used by navigators and others for many centuries. The surveyor's compass is an instrument for determining difference in direction between any horizontal line and a magnetic needle, the needle pointing towards the magnetic north. Magnetic compasses, though of limited accuracy, have the advantage of giving reading directly in terms of directions or bearings referred to magnetic north. Prismatic compasses can either be used independently or in conjunction with other angle measuring instruments in orienting a map or plane table and making a survey or traverse.

9.2 PRINCIPLE OF COMPASS

The earth acts as a powerful magnet and like any magnet forms field of magnetic force which exerts a directive action on a magnetized bar of steel or iron. A freely suspended magnetic needle will align itself in a direction parallel to the lines of magnetic force of the earth at that point and indicate the magnetic north. The imaginary line on the surface of the earth joining a point and the true North and South geographical poles indicate the true north or Astronomical North. The horizontal angle between true north and magnetic north is known as *declination*. The earth's magnetic force not only aligns a freely suspended magnetic needle along magnetic north and south but also pulls or dips one end of it below the horizontal position. The angle of dip varies from 0° near the equator to 90° at the magnetic poles. To overcome this dip a small weight is placed on one side of the needle so that it can be adjusted until the needle is horizontal.

9.3 DECLINATION

Declination may be towards east or west. When the magnetic north is towards the west of true north, the declination is west or negative, when towards east, it is east declination or positive. Figure 9.1 explains this.

The declination at any location can be obtained (if there is no local attraction) by establishing a true meridian from astronomical observations and then reading the compass when sighting along the true meridian. A line on a map or chart connecting points that have the same declination is called isogonic lines. An agonic line consists of points having zero declination.

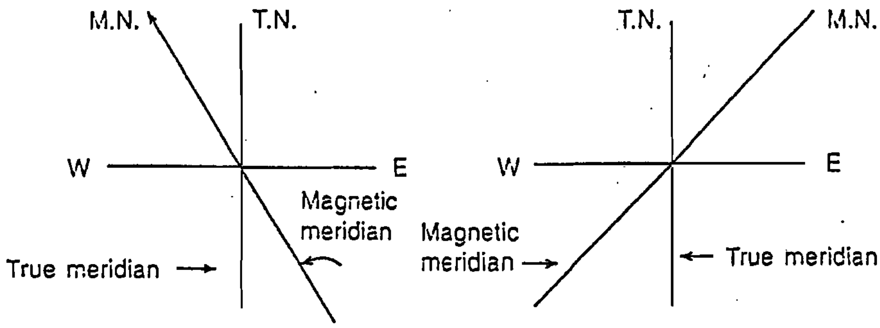


Fig. 9.1 (a) West or negative declination. (b) East or positive declination.

The declination at a place does not remain constant but changes with time. These changes are (i) Secular change, (ii) Annual change, (iii) Diurnal change, and (iv) Irregular change.

Secular variation of declination occurs over a long period of time approximately for 250 years. The magnitude also is very high. However, this variation does not follow any general law or mathematical law. It can be obtained only from detailed charts and tables derived from observations. For example, in London, the declination was 11°E in 1580 and 24°W in 1820. In Paris it was 11°E in 1680 to 22°W in 1820. Secular variation is very important in the work of the surveyor and unless mentioned otherwise variation in declination means secular variation.

Annual variation means variation over a year. It is roughly 1′ to 2′ in amplitude. It varies from place to place.

Diurnal variation means variation over a day. It depends on the following four factors:

1. Locality—It is greater near poles and less near equator.
2. Season—It is greater in summer than in winter.
3. Time—It is more during day and less during night. The rate of variation over 24 hr is quite irregular.
4. Year—The daily variation changes from year to year.

Irregular variation is caused by unpredictable magnetic disturbances and storms. The magnitude of variation is more than a degree.

9.4 PRISMATIC COMPASS

The names of different components of the prismatic compass are shown in Fig. 9.2. They are:

Sighting system The sighting system consists of sighting vanes hinged

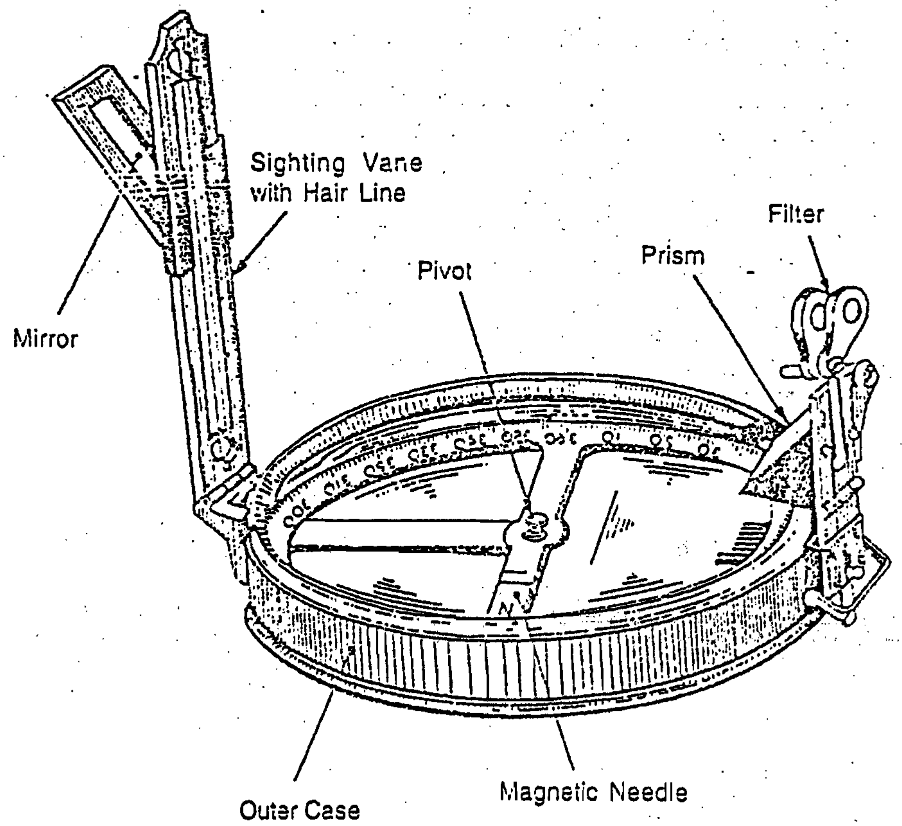


Fig. 9.2 Prismatic compass, non-liquid.

diametrically opposite to each other on the outer case. The eyeing vane has a vertical line slot in the metal housing of the magnifying prism and the front or sighting vane has a fine vertical hair line sight.

Magnetic assembly The magnetic assembly consists of a thin bar magnet fitted to a conical agate sapphire and pivoted on a sharp pin point.

Bearing indicating system The system consists of a graduated circular arc which forms an integral part of the needle.

Damping-cum-antiwear system The system consists of a device to lift the needle when the former is moved by one of the vanes normally kept folded when not in use.

Dip adjustment slide The dip adjusting slide consists of a small metal rider, which may slide along the needle to balance it in the horizontal plane.

Protective cover The protective cover consists of a disc of glass fitted on top of the brass case to protect the needle and the graduated circle.

Reading system The reading system consists of a magnifying prism attached to the outer case.

Coloured glasses Red and green glasses are provided near the eye vane which can be placed between the eye and prism to see objects against sun or other source of illumination.

With the compass held level and sighted along the local magnetic meridian, the following parts of the compass should lie in the same vertical plane. (a) Centre of the prism slide block; (b) Slot in the prism bracket; (c) South line in the outer box below the prism; (d) Tip of the pivot: (e) North line of the outer box; (f) Centre line marked on the hinge lug; and (g) sighting line on the vane.

The graduations on the aluminium ring increase clockwise from 0° to 360° with the zero of the graduations coinciding with the south end of the needle. The figures are engraved inverted on the aluminium ring. This arrangement directly gives the bearing of a line. When the needle points towards the north, the observer reads the south end of the needle through the triangular prism and reads 0° which is the bearing of true north. When the object vane is rotated, say clockwise, the reading increases. When it points exactly to the east the reading is 90°, when towards west, the reading is 270°. These are shown in Fig. 9.3.

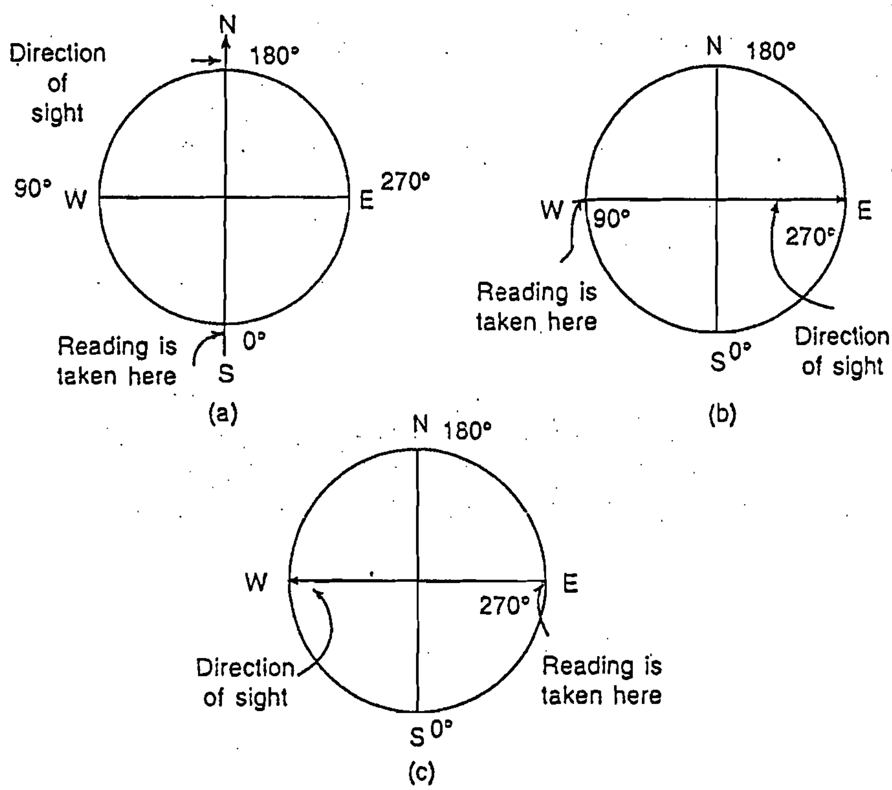


Fig. 9.3 Graduations of a prismatic compass.

9.5 SURVEYOR'S COMPASS

Figure 9.4 shows the essential parts of a surveyor's compass. As shown in the figure, the graduated ring is directly attached to the box and not with the needle. Hence the graduated card or ring is not oriented in the magnetic meridian as in the case of prismatic compass. For reading the graduated scale, the object is sighted first with the object and eye vane and reading is taken against the north

Case 2 When change in declination is given.

When there is an annual change of 'g' in magnetic declination for a period of n years, then the original and present declination can be related as:

$$\gamma'_{EW}\ \theta' = (\gamma_{EW}\ \theta + \delta_{EW} \cdot ng) \qquad (9.4)$$

where $\delta_{EW} = \begin{cases} 1 & \text{if the change in declination is eastward.} \\ -1 & \text{if the change in declination is westward.} \end{cases}$

If the right hand side of Eq. (9.4) is positive then $\gamma'_{EW} = 1$ and θ' = East. If it is negative then $\gamma'_{EW} = -1$ and θ' is west.

$$Z' = [\xi_{EW} \cdot \xi_{NS} \cdot \beta + 90\ (2 - \xi_{EW} - \xi_{EW}\xi_{NS})] - \delta_{EW}ng. \qquad (9.5)$$

It is interesting to note that Eq. (9.5) is a function of only the changes in magnetic declination, the original and present magnetic declinations are not needed. If Z' is greater than 360°, subtract 360° from it. if it is negative add 360° to it.

Example 9.2 Suppose the magnetic declination of a line is S 45°30'W in 1950. If the annual change is 2' eastward, what is the magnetic bearing in 1993?

Solution Given β = S45°30'W, hence

$$\xi_{NS} = -1,\ \xi_{EW} = -1$$

$$n = 43,\ (1993-1950)$$

$$g = 2'$$

$\delta_{EW} = 1$ as change is eastward.

$$\begin{aligned} Z' &= 45°30' + 90[(2 - (-1) - (-1)(-1)] - (1)(43)(2) \\ &= 45°30' + 90\ [2 + 1 - 1] - 1°26' \\ &= 45°30' + 180° - 1°26' \\ &= 224°04' \end{aligned}$$

Corresponding reduced bearing is S44°04'W

9.7.2 GRAPHICAL SOLUTION

Magnetic declination problem can be solved easily by drawing sketches of true north, magnetic north and also change of declination.

Example 9.3 At the time a survey was run, the magnetic declination was 6°50'E. The magnetic bearings of several lines observed at the time were as follows:

AB = N26°20'W; BC = S4°40'E.

CD = N2°15'E; DE = S58°00'E;

EF = N88°30'W.

These lines are to be retraced using a compass when the declination is 0°30'W. What bearings should be set off on the compass?

Solution (1) By graphical methods (Figs. 9.7(a) to 9.7(e)).
Line AB (Fig 9.7a)

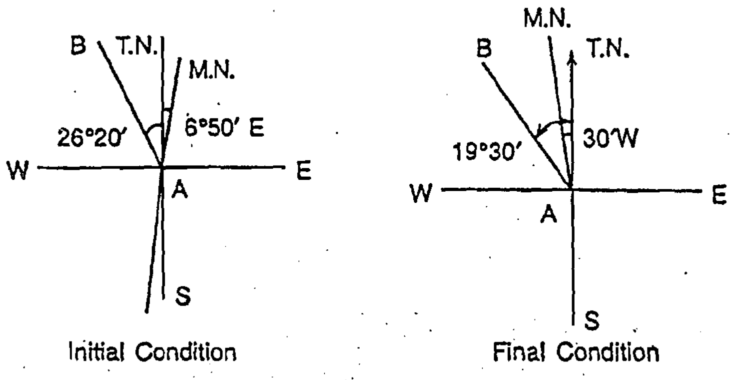


Fig. 9.7(a) Example 9.3.

Initial true bearing = 26°20′ – 6°50′ = N19°30′W
Final observed bearing = 19°30′ – 30′ = N19°00′W
Line BC (Fig. 9.7b)

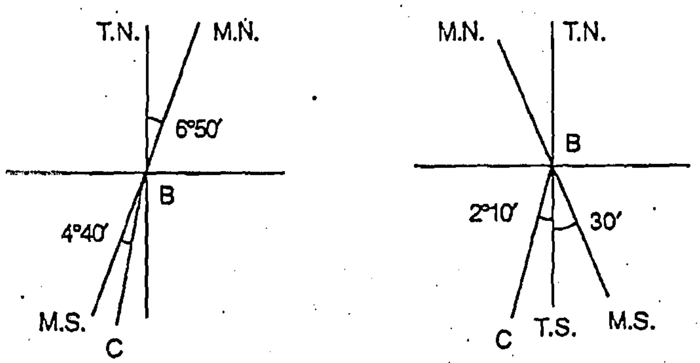


Fig. 9.7(b) Example 9.3.

Initial true bearing = 6°50′ – 4°40′ = S2°10′W
Final Observed bearing = 2°10′ + 0°30′ = S2°40′W
Line CD (Fig. 9.7c)

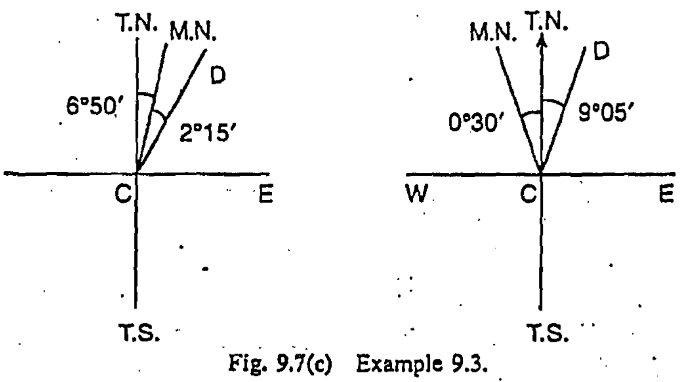


Fig. 9.7(c) Example 9.3.

Initial true bearing = 6°50′ + 2°15′ = N9°05′E

Final observed bearing = 9°05′ + 0°30′ = N9°35′E

Line DE (Fig. 9.7d)

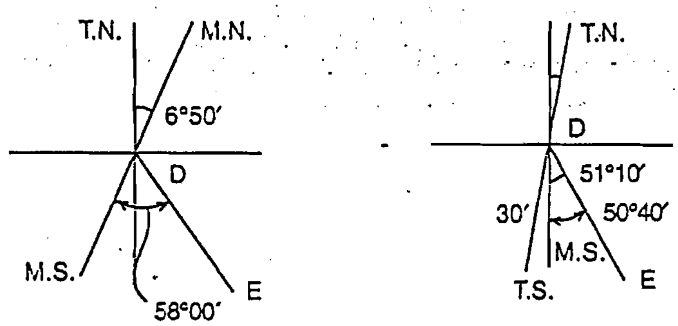


Fig. 9.7(d) Example 9.3.

Initial true bearing = 58°00′ – 6°50′ = S51°10′E

Final observed bearing = 51°10′ – 0°30′ = S50°40′E

Line EF (Fig.9.7e)

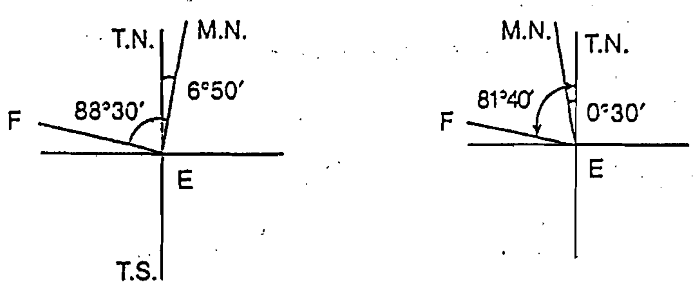


Fig. 9.7(e) Example 9.3.

Initial true bearing = 88°30′ – 6°50′ = N81°40′W.

Final observed bearing = 81°40′ – 0°30′ = N81°10′W.

(ii) By analytical methods

Present magnetic azimuth

$$Z' = [\xi_{EW}\,\xi_{NS}\beta + 90(2 - \xi_{EW} - \xi_{EW}\cdot\xi_{NS})] + \gamma_{EW}\cdot\theta - \gamma'_{EW}\cdot\theta'$$

Line AB. β = N26°20′W.

$\xi_{NS} = 1$ $\xi_{EW} = -1$

θ = 6°50′E $\gamma_{EW} = 1$

θ' = 0°30′W $\gamma_{EW} = -1$

$$Z' = (-1)(1)\cdot 26°20' + 90(2 - (-1) - (-1)(1)] + (1)(6°50') - (-1)(0°30')$$

$$= 341°$$

Quadrantal bearing N 19°W

Line BC. β = S 4°40′E

$\xi_{NS} = -1$

$\xi_{EW} = + 1$

$Z' = (+ 1)(- 1)\ 4°40' + 90(2 - (+ 1) - (+1)(-1)]$
$\quad + 1(6°50') - (- 1)(0°30')$
$= - 4°40' + 180° + 6°50' + 0°30'$
$= 182°40'$

Quadrantal bearing = S2°40′W.

Line CD β = N 2°15′E

$\xi_{NS} = 1$

$\xi_{EW} = 1$

$Z' = (1)(1)\ 2°15' + 90[2 - 1 - (1)(1)]$
$\quad + 1(6°50') - (- 1)(0°30')$
$= 2°15' + 6°50' + 0°30'$
$= 9°35'$

Quadrantal bearing = N9°35′E.

Line DE β = S58°00′E.

$\xi_{NS} = - 1$

$\xi_{EW} = + 1$

$Z' = (+ 1)(- 1)(58°00') + 90[(2 - (- 1) - (- 1)(1)]$
$\quad + 1\ (6°50') - (- 1)(0°30')$
$= - 58°00' + 360° + 6°50' + 0°30'$
$= 309°20'$

In quadrantal system S50°40′E

Line EF β = N88°30′W

$\xi_{NS} = + 1$

$\xi_{EW} = - 1$

$Z' = (-1)(1)(88°30') + 90\ [2 - (- 1) - (- 1)\ (1)]$
$\quad + 1(6°50') - (- 1)(0°30').$
$= - 88°30' + 360° + 6°50' + 0°30'$
$= 278°50'$

In quadrantal system = 360° − 278°50′ = N81°10′W

9.8 COMPASS TRAVERSE

This instrument is normally used for rapid and exploratory surveys. In such a case it is held in the hand and traverse sides are made long so that centring effect is reduced. But for accurate work, the compass is set over tripods and lines are measured with a chain or tape. The free needle method of surveying is used in which the needle is floated before each reading so that reading at each station is taken with respect to magnetic meridian. The usual steps in compass surveying are: (i) Reconnaissance, (ii) Marking and referencing stations, (iii) Running survey lines, (iv) Taking offset to the details, (v) Observing forebearings and backbearings at each station.

Figure 9.8 shows a typical compass traverse and how measurements and offsets are taken. Lengths *AB, BC, CD, DE, EF* and *FA* are measured. At each station two bearings are taken. For example, at *A*, bearing of *AB* and *AF* are measured. Then proceeding from *A* to *B*, the offsets on the right are taken.

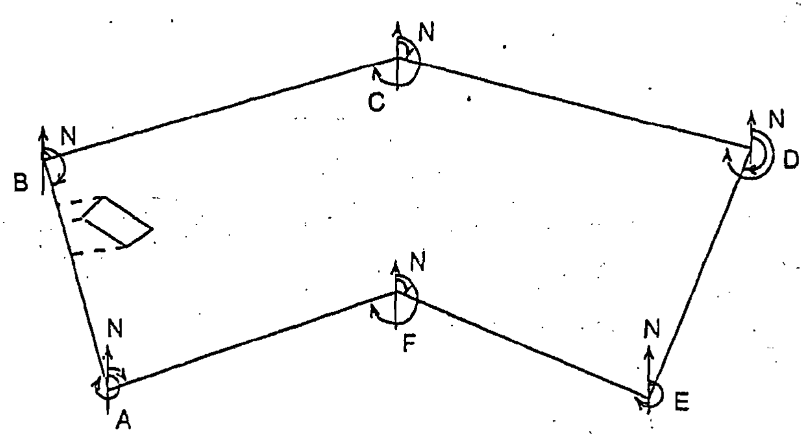


Fig. 9.8 Compass traverse.

9.8.1 ADJUSTMENT OF ANGLES OF A CLOSED TRAVERSE

From the bearings obtained from compass, the included angle at each station can be computed. They should add to $(2n - 4)$ right angles where n is the number of sides. There may be small discrepancy and this may be distributed equally among all the angles. If it is suspected that large error may occur at a particular station, the whole or major part of the error may be adjusted at that station. The error may also be adjusted in the bearings of the lines. If the total error of the bearing of the last line is e and number of lines are n, the bearing of the 1st line is adjusted by e/n, that of second line $2e/n$, and so on, so that adjustment of the last line is $ne/n = e$.

9.9 LOCAL ATTRACTION

Normally a magnetic needle points towards magnetic north and as such remains parallel to itself at all stations of the compass survey. However, if there are magnetite in the ground, wires carrying electric current, steel structures, iron pipes near a station, they deflect the needle and the needle no longer points to the true magnetic north. The difference between the true magnetic north and the north pointed by the magnetic needle at a particular station is known as Local attraction at a station and should be corrected before true bearings of the lines may be obtained. Normally the backbearing and forebearing of a line should differ by 180°. If they do not, it may be due to observational error or local attraction. If observational and instrumental errors are eliminated, the local attraction can be computed. Computation of local attraction and adjustment of compass traverse are shown through different examples.

Example 9.4 The forebearing and backbearing of line *PQ* were observed to be 205°30′ and 24°0′ respectively. It was known that station *Q* was free from local attraction. If the bearing of sun as observed from station *P* at local noon is 358°0′, find the true bearing of line *PQ* and also the declination at station *P*.

[AMIE Winter 1982]

Solution The forebearing and backbearing of a line should differ by 180°. Here the difference is 205°30′ – 24°0′ = 181°30′

Hence local attraction = 181°30′ – 180°

= 1°30′

As station *Q* is free from local attraction bearing at *P* (free from local attraction) is 180° + 24°0′ = 204°

True bearing of sun at local noon = 360°
Observed bearing = 358°
Hence error due to local attraction and declination combined = – 2°
(Correction to be added to observed bearing)
Error due to local attraction = + 1°30′
(Correction to be subtracted from observed reading)
Hence error due to declination = – 2° – 1°30′ = – 3°30′
Correction for declination = + 3°30′
When corrected for declination bearing becomes 204° + 3°30′ = 207°30′

Example 9.5 Following are the data regarding a closed compass traverse *PQRS* taken in clockwise direction.

(i) Forebearing and backbearing at station *P* = 55° and 135° respectively.
(ii) Forebearing and backbearing of line *RS* = 211° and 31° respectively.
(iii) Included angles ∠*Q* = 100°, ∠*R* = 105°.
(iv) Local attraction at station *R* = 2°W
(v) All the observations were free from all the errors except local attraction

From the above data calculate (i) local attractions at stations *P* and *S* (ii) corrected bearings of all the lines and tabulate the same. [AMIE Winter 1982]

Solution Since the forebearing and backbearing of *RS* differ by (211° – 31°) = 180°, local attractions at *R* and *S* are the same.

Local attraction at *R* = 2°W

Local attraction at *S* = 2°W

Hence Correct bearing of *RS* = 211° – 2° = 209°

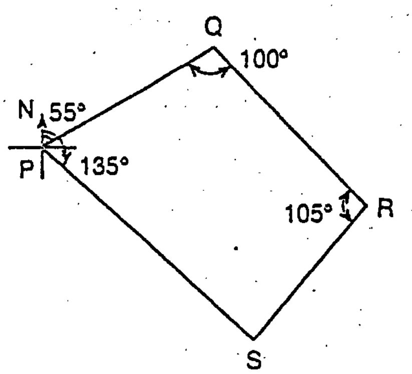


Fig. 9.9 Example 9.5.

Correct bearing of $SR = 31° - 2° = 29°$

Correct bearing of $RQ = 209° + 105° = 314°$

Correct bearing of $QR = 314° - 180° = 134°$

Correct bearing of $QP = 134° + 100° = 234°$

Correct bearing of $PQ = 54°$

Correct bearing of $PS = 134°$

Correct bearing of $SP = 314°$

Observed bearing of $PQ = 55°$

Hence Local attraction at P is $55° - 54° = 1°W$

Local attraction at $S = 2°W$

Example 9.6 The forebearings and backbearings of the lines of a closed compass traverse are as follows:

Line	Forebearing	Backbearing
AB	32°30′	214°30′
BC	124°30′	303°15′
CD	181°00′	1°00′
DA	289°30′	108°45′

Correct the bearings for local attraction and determine the true bearings of the lines if the magnetic declination at the place is 3°30′W.

Solution From the observations of the bearings of different lines, it is found that the forebearing and the backbearing of the line *CD* differ by 180°. Hence the two stations *C* and *D* are free from local attraction.

Observed bearing of *CD* = 181°00′

Declination W 3°30′

Hence	Correct bearing of *CD*	177°30′
	Correct bearing of *DC*	+ 180°00′
		357°30′
	Observed bearing of *DA*	= 289°30′
	Declination W	= – 3°30′
	Correct bearing of *DA*.	= 286°00′
	Correct bearing of *AD*, subtracting	180°00′
		106°00′

It differs from observed bearing of 108°45′ by (108°45′ – 106°00′) = 2°45′W.

Error due to magnetic declination = 3°30′W

Hence Local attraction at *A* = 3°30′ – 2°45′

= 0°45′E

Observed bearing of *AB* = 32°30′

Correct bearing of *AB* = 32°30′ – 2°45′

= 29°45′

Correct bearing of *BA* = 209°45′

Observed bearing of *BA* = 214°30′

Hence correction at *B* = 4°45′

Observed bearing of *BC* = 124°30′

Correct bearing of *BC* = 124°30′ – 4°45′

= 119°45′

Correct bearing of *CB* = 119°45′ + 180°

= 299°45′

Observed bearing of *CB* = 303°15′

Since *C* is free from local attraction, error is due only to declination of 3°30′W.

Hence Correct bearing at *C* = 303°15′ – 3°30′

= 299°45′

as obtained before

Example 9.7 The following forebearings and backbearings were observed in traversing with a compass.

Line	Forebearing	Backbearing
PQ	S 37°30′E	N 37°30′W
QR	S 43°15′W	N 44°15′E
RS	N 73°00′W	S 72°15′E
ST	N 12°45′E	S 13°15′W
TP	N 60°00′E	S 59°00′W

Calculate the interior angles and correct them for observational errors.

Solution

Interior angle at P = 37°30′ + 59°00′
= 96°30′

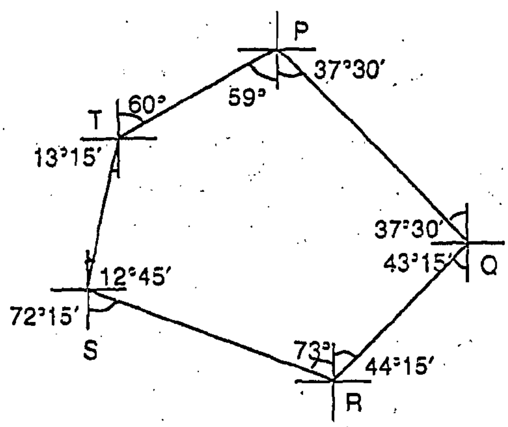


Fig. 9.10 Example 9.7.

Interior angle at Q = 180° – (37°30′ + 43°15′)
= 180° – 80°45′
= 99°15′

Interior angle at R = 44°15′ + 73°00′
= 117°15′

Interior angle at S = 180° – (12°45′ + 72°15′)
= 95°

Interior angle at T = 13°15′ + 90°00′ + (90° – 60°)
= 133°15′

Sum of interior angles = 541°15°

Theoretical sum = $(2n - 4)$ rt angles
= (2 × 5 – 4)(90)
= 540°

Hence Error = 541°15′ – 540° = 1°15′

Distributing the error equally in all angles, correction at each angle = $-(1°15')/5 = -15'$

The correct angles are

$$\angle P = 96°30' - 15' = 96°15'$$
$$\angle Q = 99°15' - 15' = 99°00'$$
$$\angle R = 117°15' - 15' = 117°00'$$
$$\angle S = 95°00' - 15' = 94°45'$$
$$\angle T = 133°15' - 15' = 133°00'$$
$$\overline{540°00'}$$

Example 9.8 (i) The magnetic bearing of sun at noon was 175°. Show with a sketch the true bearing of sun and the magnetic declination.

(ii) In an old map a survey line was drawn with a magnetic bearing of 202° when the declination was 2°W. Find the magnetic bearing when the declination is 2°E.

(iii) The true bearing of a tower T as observed from a station A was 358° and the magnetic bearing of the same was 4°. The back bearings of the lines AB, AC and AD when measured with a prismatic compass were found to be 296°, 346° and 36° respectively. Find the true forebearings of the lines AB, AC and AD.

[AMIE Summer 1991]

Solution From the Fig. 9.11(i) the declination is 5°E

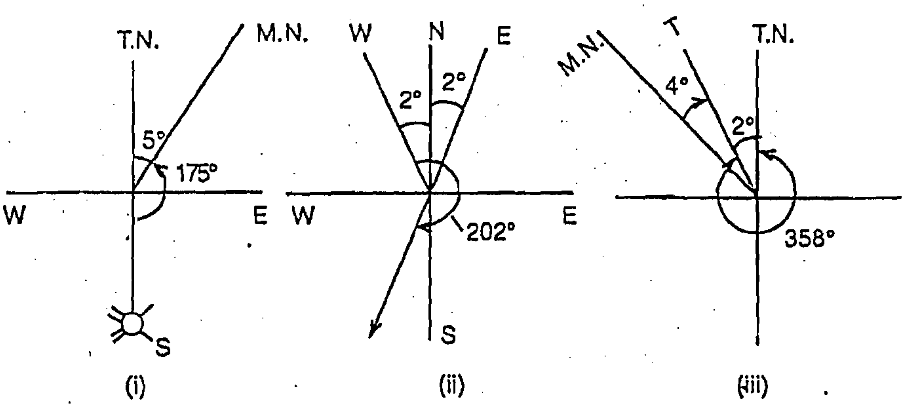


Fig. 9.11

(ii) From Fig. 9.11(ii)

True bearing = 202° − 2° = 200°

Magnetic bearing when declination is 2°E

= 200° − 2° = 198°

From Fig. 9.11(iii) Declination = 4° + 2° = 6°W
Backbearing of AB = 296°

Forebearig of AB = 116°
Declination = 6°
Correct forebearing = 110°
Backbearing of AC = 346°
Forebearing of AC = 166°
True bearing of AC = 160°
Backbearing of AD = 36°
Forebearing of AD = 216°
True bearing of AD = 210°

9.10 ADJUSTMENT OF A COMPASS TRAVERSE

A compass survey is usually plotted by drawing the length of a side with known bearing and then plotting other sides from the included angles. For accurate work, the coordinates of the traverse stations are computed from the length and bearings of lines and then plotted.

While plotting a closed traverse it is usually found that the last point does not fall exactly on the starting point. This introduces what is called a *closing error.* Usually Bowditch's rule is used for adjustment of the closing error. According to this rule, the closing error is adjusted by shifting each station by an amount which is proportional to its length from the starting station. Let $ABCDE$ be a closed traverse. As shown in Fig. 9.12, the plotted traverse is $ABCDE\,A_1$. Here AA_1 is the closing error. The movement of the stations should be parallel to the closing error and the amount should be, for

$$B = \frac{l_1}{l_1 + l_2 + l_3 + l_4 + l_5} \times AA_1$$

$$C = \frac{l_1 + l_2}{\text{Perimeter}} \times AA_1$$

$$D = \frac{l_1 + l_2 + l_3}{\text{Perimeter}} \times AA_1$$

$$E = \frac{l_1 + l_2 + l_3 + l_4}{\text{Perimeter}} \times AA_1$$

and finally A_1 should join A by moving $\frac{l_1 + l_2 + l_3 + l_4 + l_5}{\text{Perimeter}} \times AA_1$ i.e. AA_1 itself.

The adjustment can also be done graphically. The perimeter of the traverse is drawn on a suitable scale. Let it be aa' as shown in Fig. 9.12. At a' the closing error is plotted in original direction and in true magnitude. Let it be aa_1. Join aa' and draw parallels through b, c, d, e which cuts aa_1 in b_1, c_1, d_1, e_1. The plotted points B, C, D, E and A_1 should now be shifted by lengths bb_1, cc_1, dd_1, ee_1 and $a'a_1$ respectively. $AB_1C_1D_1E_1A$ is the adjusted traverse.

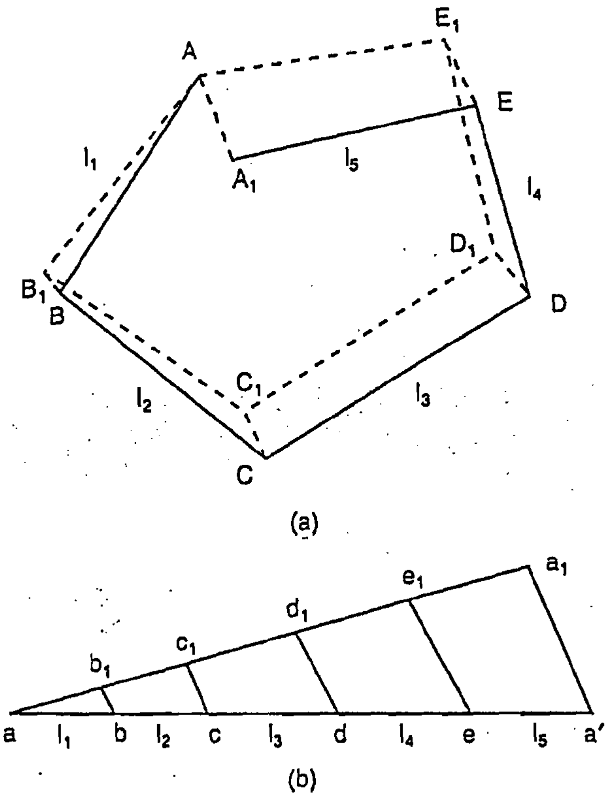


Fig. 9.12 Graphical adjustment of compass traverse.

9.11 ERRORS IN COMPASS SURVEYING

Errors in compass surveying may be due to the following causes:

(a) Instrument Errors
   (1) Compass out of level.
   (2) Needle not straight.
   (3) Movement of level sluggish.
   (4) Magnetism of needle weak.
   (5) Plane of sight not vertical
   (6) Line of sight not passing through centre of graduated ring.

(b) Personal Errors
   (1) Compass not properly levelled.
   (2) Compass not properly centred over the station.
   (3) Ranging rod or signal not accurately bisected.
   (4) Incorrect reading and recording of the graduated ring.

(c) Natural Errors
   (1) Variation in declination
   (2) Local attraction.
   (3) Magnetic changes in atmosphere due to clouds and storms.
   (4) Irregular variation in magnetic storms.

REFERENCE

1. Easa, M. Said, "Analytical Solution of Magnetic Declination Problem", *ASCE Journal of Surveying Engineering* Vol. 115, No. 3, August 1989, pp. 324–329.

PROBLEMS

9.1. (a) What are the advantages and disadvantages of compass survey? Describe the limits of precision of compass surveying. Where is compass survey normally used?

(b) What are the different forms of 'bearings' of a line? How would you convert one form of bearings to the others? [AMIE Summer 1978]

9.2 (a) What is local attraction? Briefly describe it.

(b) A compass was set on the station *A* and the bearing of *AB* was 309°15′. Then the same instrument was shifted to station *B* and the bearing of *BA* was found to be 129°15′. Is there any local attraction at station *A* or at the station *B*? Can you give a precise answer?

State your comment and support it with rational arguments.

(c) Describe in tabular form the relation between:

(i) Magnetic bearing and true bearing.

(ii) Forebearing and backbearing.

(iii) Whole Circle bearing and reduced bearing.

[AMIE Winter 1978]

9.3. While making a reconnaissance survey through woods, a surveyor with a hand compass, started from a point *A* and walked 1000 steps in the direction S 67°W and reached a point *B*. Then he changed his direction and walked 512 steps in the direction N 10°E and reached a point *C*. Then again he changed the direction and walked 1504 steps in the direction S 65°E and reached a point *D*. Now the surveyor wanted to return to his starting point *A*. In which direction must he move with the hand compass and how many steps must he walk to reach the point *A*?

[AMIE Summer 1979]

9.4. (a) Tabulate the differences between a prismatic and surveyor's compass.

(b) The following bearings were taken in running a compass traverse:

Line	F.B.	B.B.
AB	48°25′	230°00′
BC	177°45′	356°00′
CD	104°15′	284°55′
DE	165°15′	345°15′
EA	295°30′	79°00′

(i) State what stations are affected by local attraction and by how much.

(ii) Determine the corrected bearings.

(iii) Calculate the true bearings if the declination was 1°30′W.

[AMIE Summer 1980]

9.5. (a) In an old revenue map a survey line was drawn with a magnetic bearing of 196°00′ when the declination was 1°30′ E. Find the magnetic bearing of the line at a time when the declination was 2°30′W.

(b) The magnetic bearing of sun at noon was 170°. By means of a sketch show the true bearing of the line and show the true bearing of the line and the magnetic declination.

(c) The true bearing of a tower *T* as observed from a station *A* was 358°00′ and the magnetic bearing of the same was 8°00′. The backbearings of the lines *AB*, *AC* and *AD* when measured with a prismatic compass was found to be 290°00′, 340°00′ and 30°00′ respectively. Find the true forebearings of the lines *AB*, *AC* and *AD*. [AMIE Winter 1981]

9.6. (a) Differentiate clearly between a prismatic compass and a surveyor's compass

(b) Discuss the correctness of the following statements giving *reasons*:

(i) Local attraction affects included angles.

(ii) Zero is marked at south in case of a prismatic compass.

(iii) Dip of the magnetic needle is the angle between magnetic north and true north.

(iv) Coloured glasses are provided in a prismatic compass to avoid parallax.

(v) Bearing of the same line will not be constant over number of years.

(vi) Ball and socket arrangement is useful for permanent adjustment of a compass. [AMIE Summer 1982]

9.7. (a) The forebearing and backbearing of line *PQ* were observed to be 205°30′ and 24°00′ respectively. It was known that station *Q* was free from local attraction. If the bearing of sun as observed from station *P* at local noon is 358°00′, find the true bearing of the line *PQ* and also the declination at station *P*.

(b) Following is the data regarding a closed compass traverse *PQRS* taken in clockwise direction.

(i) Forebearing and backbearing at station *P* = 55° and 135° respectively.

(ii) Forebearing and backbearing of line *RS* = 211° and 31° respectively.

(iii) Included angles ∠*Q* = 100° and ∠*R* = 105°.

(iv) Local attraction at station *R* = 2°W.

(v) All the observations were free from all the errors except local attraction.

From the above data:

(i) Calculate the local attraction at stations *P* and *S*.

(ii) Calculate the corrected bearings of all the lines and tabulate the same.

[AMIE Winter 1982]

9.8. (a) State precisely the function of the following parts of a prismatic compass: (i) Rider, (ii) Lifting lever, (iii) Prism, (iv) Adjustable mirror, (v) Hinged glasses, (vi) Spring brake. Draw neat sketches to show these parts.

(b) In an anticlockwise closed traverse *ABCA* all the sides were equal. Magnetic forebearing of *BC* was observed to be 15°30′. The bearing of sun was observed to be 184°30′ at local noon with a prismatic compass. Calculate the magnetic bearings and true bearings of all the sides of the traverse. Tabulate the results and draw a neat sketch to show true bearings. [AMIE Summer 1983]

9.9. (a) What is meant by closing error in a closed traverse? How would you adjust it graphically.

(b) Write a brief note on variations in magnetic declination.

(c) The forebearings and backbearings of the lines of a closed compass traverse are as follows:

Line	Forebearing	Backbearing
AB	32°30′	214°30′
BC	124°30′	303°15′
CD	181°00′	1°00′
DA	289°30′	108°45′

Correct the bearings for local attraction and determine the true bearings of the lines, if the magnetic declination at the place is 3°30′W. [AMIE Summer 1984]

9.10. (a) What are the sources of error in compass survey? What precautions will you take to eliminate them?

(b) Convert the following whole circle bearings to quadrantal bearings: (i) 32°30′ (ii) 170°22′ (iii) 217°54′ (iv) 327°24′.

(c) Distinguish between the following terms:
 (i) True meridian and magnetic meridian
 (ii) Local attraction and declination.
 (iii) Trough compass and tubular compass. [AMIE Winter 1987]

9.11. (a) Differentiate between prismatic and surveyor's compass.

(b) Explain the following:
 (i) Magnetic meridian
 (ii) True bearing
 (iii) Declination
 (iv) Whole circle bearing
 (v) Isogonic lines
 (vi) Secular variation.

(c) A line was drawn to a magnetic bearing of S32°W, when the magnetic declination was 4°W. To what bearing should it be set now if the magnetic declination is 8°E?

(d) The following forebearings and backbearings were observed in traversing with a compass where local attraction was suspected:

Line	FB	BB
AB	65°30′	245°30′
CD	43°45′	226°30′
BC	104°15′	283°00′
DE	326°15′	144°45′

Determine the corrected FB, BB and true bearing of the lines assuming magnetic declination to be 5°20′W. [AMIE Winter 1993]

HINTS TO SELECTED QUESTIONS

9.1 (a) Advantages are: (i) Giving direct reading for direction (ii) Quick survey for rough work (iii) Valuable tool for geologists, foresters and others (iv) Bearing of oneline has no effect upon the observed direction of any other line (v) Obstacles such as trees can be passed readily by offsetting the instrument by a short measured distance from line.

Disadvantages are: (i) Not accurate due to various instrumental errors and undetected magnetic variations. Not suitable except for rough surveys (ii) Computational corrections for local attraction is necessary.

Limits of precision of compass survey: (i) Angular error should not exceed $15 \times \sqrt{n}$ minutes, where n is the number of sides of the traverse (ii) Linear error should be about 1 in 500.

Used for preliminary and rough work.

9.6 (b) (i) False, included angle is not affected as both the sides are equally affected by local attraction.
(ii) Correct, then whole circle bearing is directly obtained.
(iii) False, dip is vertical angle of depression of the magnetic needle.
(iv) False, they are provided to read against Sun or other source of illumination.
(v) Correct, bearing will change with change of declination.
(vi) Correct, permanent adjustment of surveyor's compass is done by ball and socket arrangement.

10

Theodolites

10.1 INTRODUCTION

The theodolite is a very useful instrument for engineers. It is used primarily for measuring horizontal and vertical angles. However, the instrument can be used for other purposes like (i) Prolonging a line, (ii) Measuring distances indirectly, and (iii) Levelling.

Theodolites these days are all transit theodolites. Here the line of sight can be rotated in a vertical plane through 180° about its horizontal axis. This is known as transitting and hence the name "transit". Theodolites can be broadly classified as (i) Vernier theodolites, (ii) Precise optical theodolites.

As the name suggests, in vernier theodolites verniers are used to measure accurately the horizontal and vertical angles. Generally 20″ vernier theodolites are used.

The precise optical theodolites uses an optical system to read both horizontal and vertical circles. The precision of angles can be as high as 1″.

10.2 MAIN PARTS OF A VERNIER THEODOLITE

Figure 10.1 shows the main parts of a vernier theodolite. The main components are described below.

Telescope

As already explained in detail in the chapter on levelling, the telescope is of measuring type, has an object glass, a diaphragm and an eyepiece and is internal focussing. When elevated or depressed, it rotates about its transverse horizontal axis (trunnion axis) which is placed at right angles to the line of collimation and the vertical circle which is connected to the telescope rotates with it. Typical data of a telescope used for a transit theodolite available in India are:

Objective Aperture = 38 mm
Magnification = 25 times
Tube length = 17 mm
Nearest focussing distance = 3 m
Field of view = 1° 30′

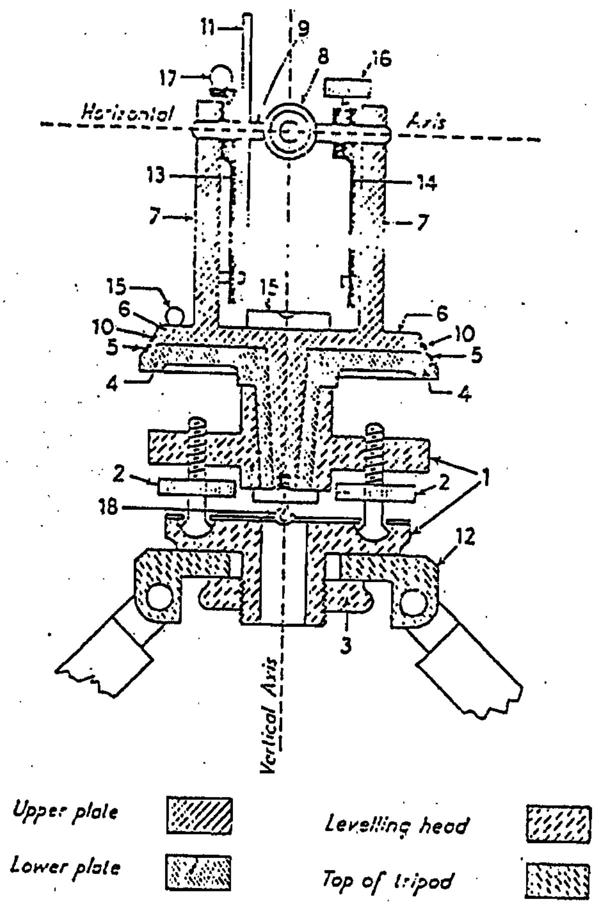


Fig. 10.1. Diagrammatical sectional drawing of a theodolite: (1) Tribrach and trivet (2) Footscrews (3) Clamping screw for centring (4) Lower plate (5) Graduated arc (6) Upper plate (7) Standards (8) Telescope (9) Horizontal axis (10) Verniers (11) Vertical circle (12) Tripod top (13) Vernier frame (14) Arm of vertical circle clamp (15) Plate levels (16) Vertical circle clamping screw (17) Level on vernier arm (18) Hook for plumb bob.

$$\text{Stadia ratio} = 100$$
$$\text{Addition constant} = 0.416$$

The trunnion axis is supported at its ends on the standards which are carried by the horizontal vernier plate. The function of the telescope is to provide the line of sight.

Vertical Circle

The vertical circle is rigidly connected to the transverse axis of the telescope and moves as the telescope is raised or depressed. The vertical circle is graduated in degrees with graduations at 20′. The graduations in each quadrant are numbered from 0° to 90° in opposite directions from the two zeros placed at the horizontal diameter of the circle. When the telescope is horizontal, the line joining the two zeros is also horizontal. The usual diameter of the circle is 127 mm, graduations on silver in degrees is 1/3° or 20′. Vernier reading with magnifier is 20″. Figure 10.2 shows the position of the vernier with respect to the axis of the telescope. The vertical circle is used to measure the vertical angle.

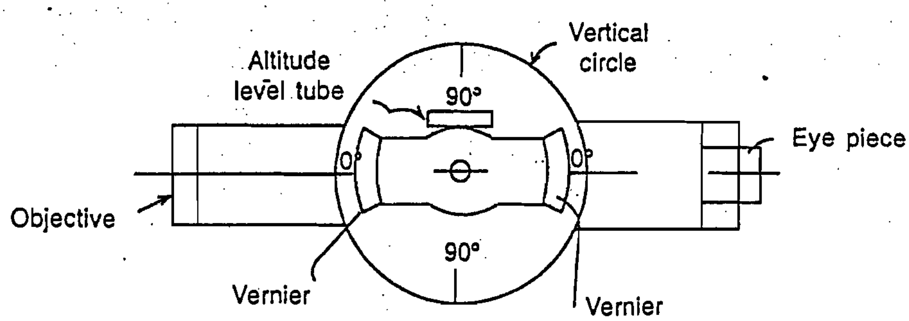


Fig. 10.2 Vertical circle, vernier and telescope.

Index frame (or T frame or vernier frame)

It consists of a vertical portion called dipping arm and a horizontal portion called an index arm. At the two extremities of the index arm are fitted two verniers to read the vertical circle. The index arm is fixed and is centred on the trunnion axis. Reading is obtained with reference to the fixed vernier when the vertical circle moves. A bubble tube often known as altitude bubble is fixed on the *T* frame. For adjustment purposes the index arm can be rotated slightly with the help of a clip screw fitted to the clipping arm at its lower end as shown in Fig. 10.3. The index arm helps in taking measurements of vertical angle with respect to horizontal plane.

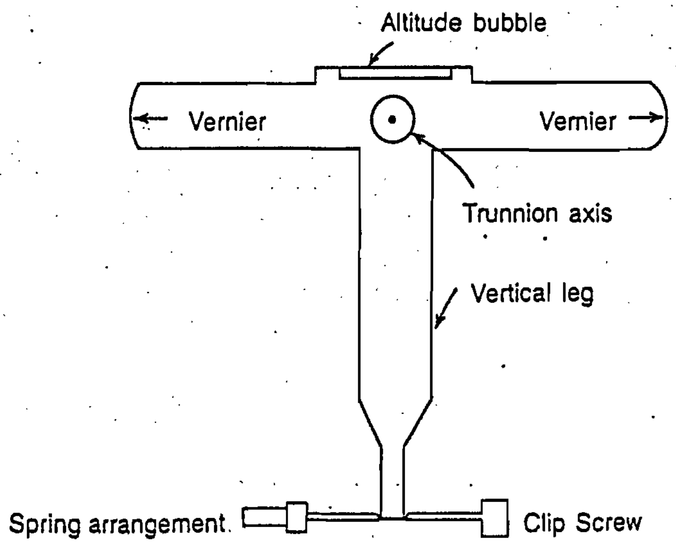


Fig. 10.3. Functioning of clip screw and vernier (vertical circle).

The standards or (A frame)

Two standards resembling letter *A* are fixed on the upper plate. The trunnion axis of the telescope is supported on these *A* frames. The *T* frame and also the arm of the vertical circle clamp are attached to the *A* frame.

The Upper Plate

Also called the vernier plate supports the standards at its upper surface. As shown in Fig. 10.4, the upper plate is attached to the inner spindle and carries two verniers with magnifiers at two extremities diametrically opposite. It carries an upper clamping screw and a corresponding tangent screw for purpose of accurately fixing it to the lower plate. When the upper plate is clamped to the lower plate by means of upper clamping screw, the two plates can move together. The upper

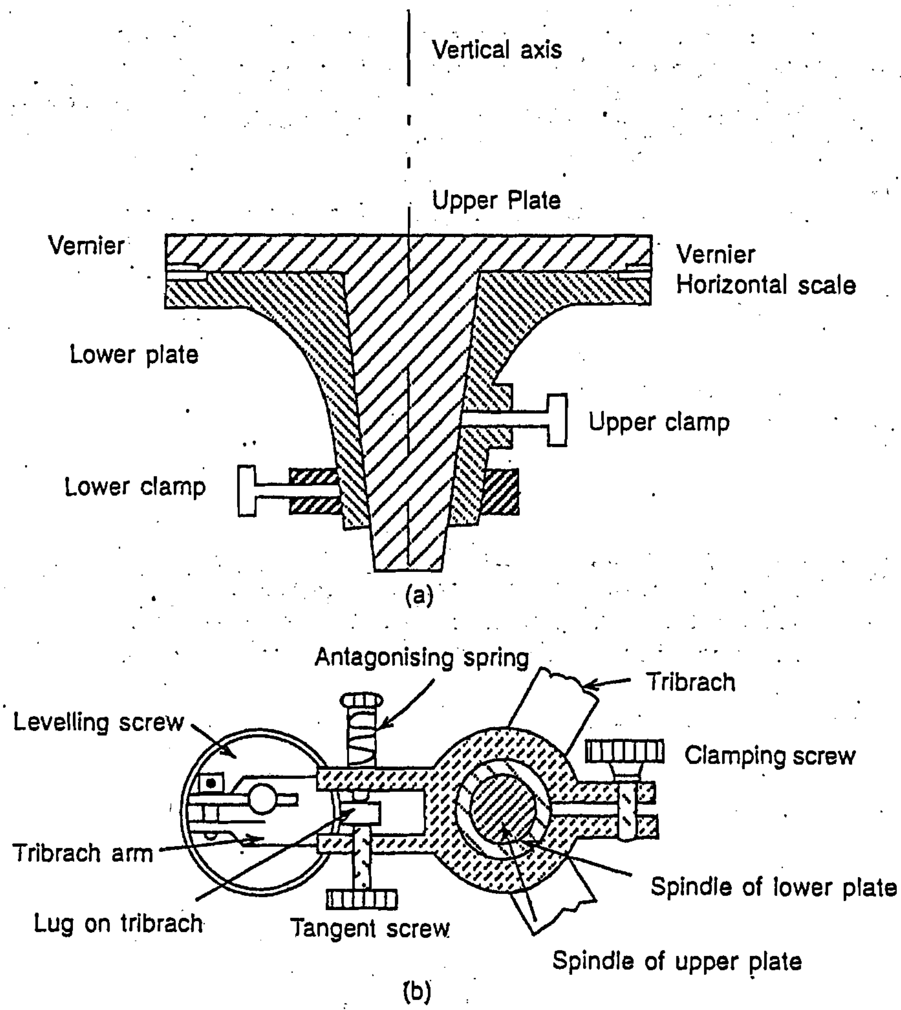


Fig. 10.4 (a) Longitudinal section through upper plate and lower plate. (b) Cross section of spindles.

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plate carries two plate levels placed at right angles to each other. One of the plate levels is kept parallel to the trunnion axis. The purpose of the plate levels is to make the vertical axis truly vertical.

The Lower Plate

Also called the scale plate, it carries the circular scale which is graduated from 0 to 360°. It is attached to the outer spindle which turns in a bearing within the tribrach of the levelling head. The lower plate is fixed to the tribrach with the help of the lower clamping screw. There is also the lower tangent screw to enable slow motion of the outer spindle. The diameter of the typical circular scale is 127 mm, graduation on silver in degrees is 1/3° or 20′ and vernier reading with magnifier is 20″. Lower plate is used to measure the horizontal angle.

The levelling head

It usually consists of two triangular parallel plates. The upper one is known as tribrach and carries three levelling screws at the three ends of the triangle. The lower plate also known as foot plate has three grooves to accommodate the three levelling screws. The lower ends of the grooves are enlarged into hemispherical shapes. There is a large central hole with thread in the trivet. The thread fits into the top of the tripod when mounting the theodolite. A plumb bob can be suspended from a hook at the lower end of the inner spindle to put the instrument exactly over the station. The levelling head is used to level the instrument horizontal.

The shifting head

It is a centring device which helps in centring the instrument over the station. It usually lies below the lower plate but above the tribrach. When the device is untightened, the instrument and the plumb bob can be moved independently of the levelling head when the foot plate has been screwed on to the tripod. Usually, therefore, the instrument is first approximately centred over the station by moving the tripod legs. Exact centring is then done by using the shifting head.

Other accessories

(a) Magnetic compass—theodolites of simpler types may be obtained fitted with a circular compass box in the centre of the upper plate.

(b) For rough pointing of the telescope towards the object, the telescope is generally fitted with a pair of external sights. They are provided on the top of the telescope for ease of initial sighting.

Tripod

The theodolite is fitted on a strong tripod when being used in the field. The tripod has already been explained in detail in connection with levelling (Fig. 10.5). The telescopic tripod is used for theodolites where accurate centring is required.

The following terms are frequently used in connection with theodolite and are given here for ready reference.

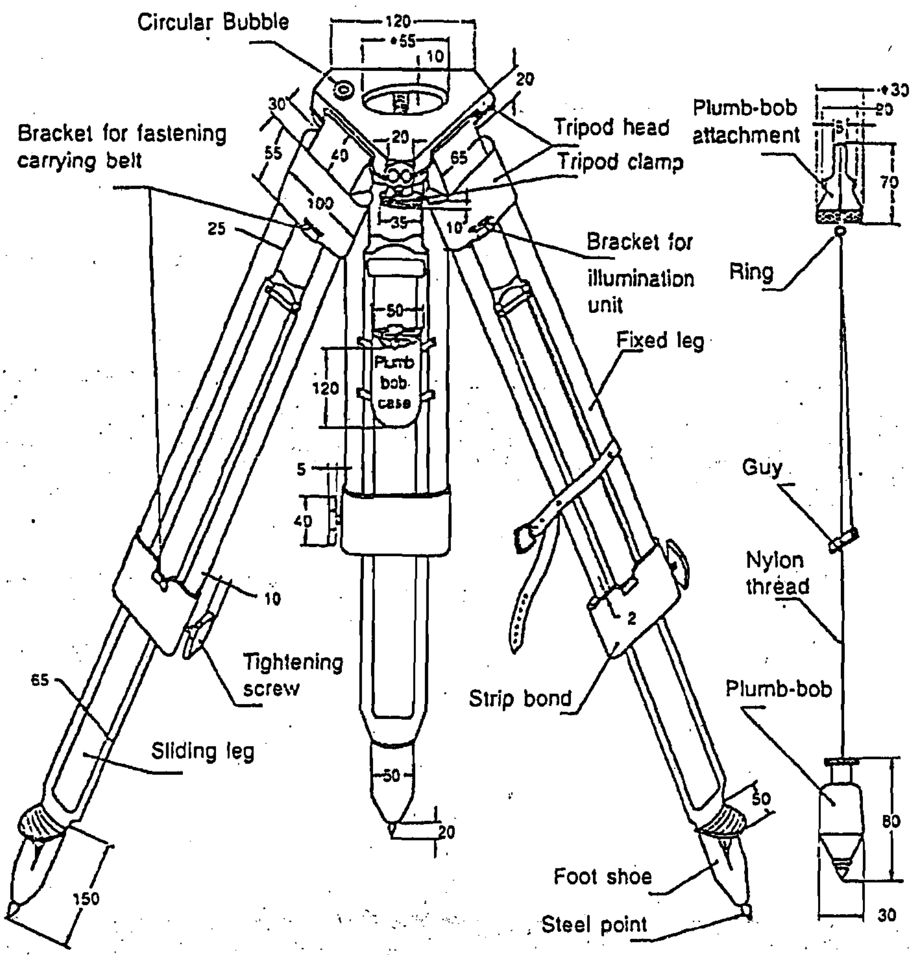


Fig. 10.5 Dimensions and nomenclature of tripod for theodolite (Telescopic type).

1. *Alidade*: The term alidade is applied to the whole of that part of the theodolite that rotates with the telescope.

2. *Centring*: Bringing the vertical axis of the theodolite immediately over a mark on the ground or under a mark overhead.

3. *Least count*: Measure of the smallest unit which a vernier will resolve.

4. *Limb*: It consists of the vertical axis, the horizontal circle and the illumination system.

5. *Standard*: Two vertical arms of the theodolite which bear the transit axis, telescope, vertical circle and vernier frame.

6. *Striding level*: A sensitive level mounted at right angles to the telescope axis and used mainly in astronomical observations for levelling the horizontal axis or measuring any error in the level of the axis.

7. *Transit, horizontal or trunnion axis*: The axis about which the telescope and vertical circle rotate.

8. *Tribrach*: The part of the theodolite carrying the levelling screws.

9. *Trivet*: An underpart of the theodolite which may be secured to the tripod top with which the toes of the levelling screws make contact.

10. *Vertical axis*: The axis about which the alidade rotates.

10.3 SOME BASIC DEFINITIONS

1. *Line of collimation*: It is an imaginery line joining the intersection of the cross hairs with the optical centre of the objective.

2. *Axis of the plate level*: It is the straight line tangential to the longitudinal curve of the plate level tube at its centre.

3. *Axis of the altitude level tube*: It is the straight line tangential to the longitudinal curve of the altitude level at its centre.

4. *Face left condition*: If the vertical circle is on the left side of the observer, it is known as face left condition. Since normally the vertical circle is on the left side, it is also known as normal condition.

5. *Face right condition*: If the vertical circle is on the right side of the observer, the theodolite is in the face right condition. The telescope is then in the inverted form and hence the condition is known reverse condition.

6. *Plunging the telescope*: This is also known as *transitting* or *reversing*. It is the process of rotating the telescope through 180° in the vertical plane. By this process the direction of objective and eyepiece ends are reversed.

7. *Swinging the telescope*: It is the process of turning the telescope clockwise or anticlockwise about its vertical axis. Clockwise rotation is called *swing right* and anticlockwise rotation is called *swing left*.

8. *Changing face*: It is the operation of changing face left to face right and vice versa.

9. *Double sighting or double centring*: It is the operation of measuring an angle twice, once with telescope in the normal condition and another in the reverse condition.

10.4 FUNDAMENTAL PLANES AND LINES OF A THEODOLITE

There are basically two planes and five lines in a theodolite. The planes are: (i) Horizontal plane containing the horizontal circle with vernier, and (ii) Vertical plane containing the vertical circle with vernier.

The lines are: (i) The line of collimation or line of sight. (ii) The transverse or horizontal axis of the telescope, (iii) The vertical axis, (iv) Altitude level axis, and (v) The plate level axis.

These lines and planes bear definite relation to one another in a well adjusted instrument. They are:

(a) The line of sight is normal to the horizontal axis.

(b) The horizontal axis is normal to the vertical axis.

(c) The vertical axis is normal to the plane containing the horizontal circle.
(d) The line of sight is parallel to the axis of the telescope bubble tube.
(e) The axes of the plate levels lie in a plane parallel to the horizontal circle.

Condition (a) ensures that line of sight generates a plane when the telescope is rotated about the horizontal axis.

Condition (b) ensures that the line of sight will generate a vertical plane when the telescope is plunged.

Condition (c) ensures that when the horizontal circle is horizontal (as indicated by the plate level) the vertical axis will be truly vertical.

Condition (d) ensures that when the telescope bubble is at the centre of its run, the line of sight is horizontal.

Condition (e) ensures that when the plate bubble is central, the horizontal circle is truly horizontal.

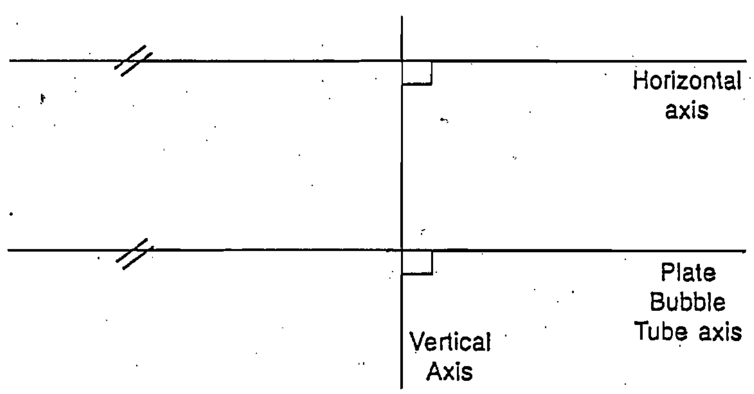
In addition to the above, for accurate reading some other requirements are:

(a) The movement of the focussing lens in and out when it is focussed is parallel to the line of sight.

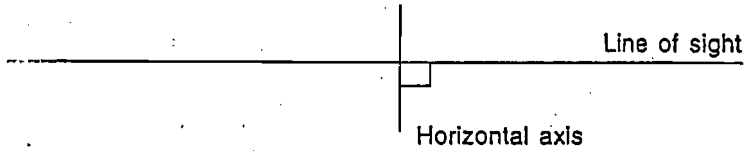
(b) The inner spindle and the outer spindle must be concentric.

(c) The line joining the indices of the *A* and *B* verniers must pass through the centre of the horizontal circle.

An instrument, however, is never in perfect adjustment and as such errors do occur when taking measurements. These errors can be greatly minimized by taking observations with double centring and also reading both verniers *A* and *B*. The fundamental lines and planes are shown schematically in Figs. 10.6 and 10.7.



(a) Front View



(b) Top View

Fig. 10.6 Fundamental lines of a theodolite.

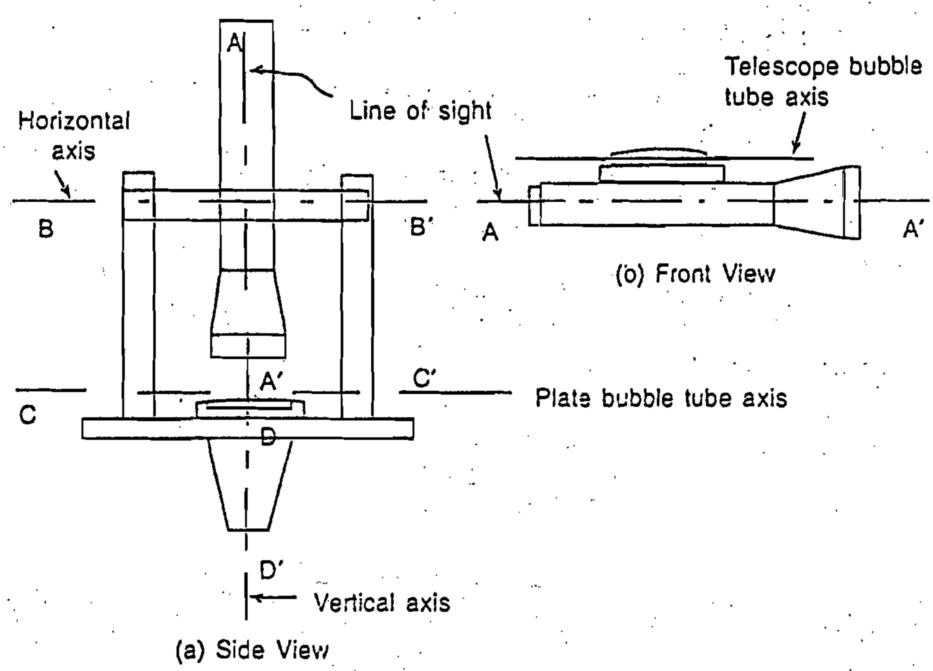


Fig. 10.7 Position of the fundamental lines in a schematic diagram of a theodolite.

10.5 FUNDAMENTAL OPERATIONS OF THE THEODOLITE

1. Vertical rotation of the telescope is controlled by the vertical motion clamp and vertical tangent screw.

2. The upper plate clamp locks the upper and lower circles together; the upper tangent screw permits a small differential rotation between the two plates.

3. A lower plate clamp locks it to the levelling head. The lower tangent screw rotates the lower plate in small increments relative to the levelling head.

4. If the upper plate clamp is locked and the lower one unlocked, the upper and lower one rotates as a unit, thereby enabling the sight line to be pointed at an object with a preselected angular value set on the plates.

5. With the lower clamp locked and the upper clamp loose, the upper plate can be rotated about the lower one to set a desired angular value. By locking the upper clamp, an exact reading or setting is attained by turning the upper tangent screw.

10.5.1 TEMPORARY ADJUSTMENTS OF A THEODOLITE

The following are the five temporary adjustments of a theodolite: (i) Setting up, (ii) Centring, (iii) Levelling up, (iv) Focussing the eye piece, and (v) Focussing the objective. These follow more or less the procedures explained for setting up a dumpy level in Section 5.14. In setting up a theodolite the tripod legs are spread and their points are so placed that the top of the tripod is approximately horizontal and the telescope is at a convenient height of sighting.

Next centring is done to place the vertical axis exactly over the station mark. Approximate centring is done by means of tripod legs. The exact centring is done by means of the shifting head or the centring device. The screw clamping ring of the shifting head is loosened and the upper plate of the shifting head is slid over the lower one until the plumb bob is exactly over the station mark. Tighten the screw clamping ring after the exact centring. Since angle measurement is involved, the instrument should be exactly over the station and hence exact centring is very important.

For levelling up and focussing, the procedures already explained in connection with levelling should be followed.

10.6 VERNIERS

In a theodolite there are two angular verniers for measuring horizontal and vertical angles. For horizontal measurements the main scale is on the lower plate, the vernier is on the upper plate. For vertical angles, the main scale is on the vertical circle, the vernier is on the *T* frame. Both the verniers have least count equal to 20″ which can be obtained as follows:

$$\text{L.C} = \frac{1}{n} \times \text{Smallest division of the main scale}$$

The main scale for horizontal circle is graduated from 0° to 360°. Each degree is divided into 3 parts, hence the smallest division of the main scale is 20′. n is equal to 60 as sixty divisions of the vernier coincides with 59 divisions of the main scale. Hence vernier constant is 1/60

and $$\text{L.C} = \frac{1}{60} \times 20' = 20''$$

Similarly for the vertical circle.

10.6.1 MEASURING A HORIZONTAL ANGLE

A horizontal angle is measured by first fixing the zero of the vernier in the upper plate to 0°00′00″ of the circular scale of the lower plate. For fixing to 0-0 the upper clamping screw and the upper tangent screw is to be used. Then loosening the lower clamping screw, the line of sight is back sighted along the reference line from which the angle is to be measured. The upper clamp is then loosened and the telescope rotated clockwise independently of the circle until the line of sight is on the foresight target. The fine adjustment in this operation is done by lower tangent screw which becomes operative when the lower clamping screw is tightened. This is explained with reference to Fig. 10.8.

The process can be systematized with the following steps.

(a) Loosen both clamps and bring the 0° circle mark roughly opposite the vernier index mark.

(b) Tighten the upper clamp and bring the zero (0°00′00″) mark of the circle into precise alignment with the vernier index, using the upper tangent screw. When upper clamp is tightened, circle and vernier (upper plate) are locked together as one rotating unit.

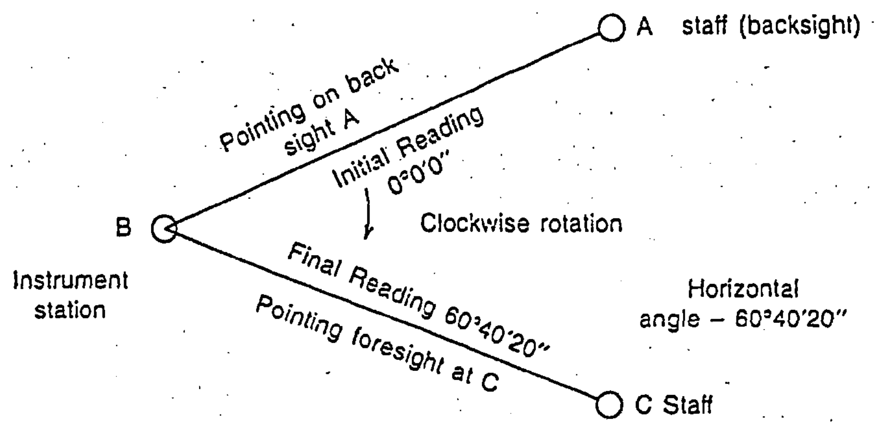


Fig. 10.8 Single measurement of a horizontal angle.

(c) With the lower clamp still loose, point telescope (rotating upper and lower plate unit by hand) to backsight.

(d) Tighten lower clamp and use lower tangent screw to align the vertical cross wire along the backsight.

(e) Loosen upper clamp and rotate upper plate until the telescope roughly points to the foresight.

(f) Tighten the upper screw and focus accurately the foresight by means of upper tangent screw.

(g) Read the value of the angle by taking the main scale reading and adding the value of the vernier reading.

10.6.2 LAYING A HORIZONTAL ANGLE

The following are the steps:

(a) Fix the 0–0 of the vernier with the 0–0 of the main scale with upper fixing screw and the upper tangent screw.

(b) Loosen the lower screw and rotate the two plates as a whole to point to *B*. The fine adjustment should be done by the lower tangent screw.

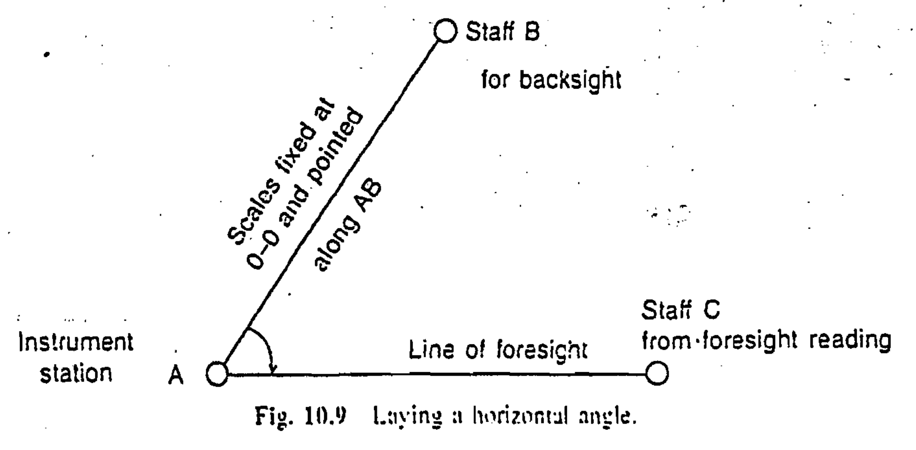


Fig. 10.9 Laying a horizontal angle.

(c) With lower clamp fixed loosen the upper fixing screw and set the vernier to the required angle.

(d) If the required angle is say 31°42′40″, the vernier should be roughly fixed at 31°40′ by the upper fixing screw and then finally adjusted to 31°42′40″ by the upper tangent screw.

(e) The line of sight is now along the required angle and point *C* can be established by depressing the line of sight.

10.7 ACCURATE MEASUREMENT OF AN ANGLE

A theodolite is never in perfect adjustment and the lines and planes are not ideally related to one another as required in Section 10.4. To minimize error as much as possible, an angle is measured a number of times with instrument: (i) Face left (vertical circle on the left of the telescope) swing right (clockwise movement) approach left (approaching the target from the left side); (ii) Face left, swing left, approach right; (iii) Face right, swing right, approach left; (iv) Face right, swing left, approach right.

The operation can be repeated with a different "zero" or initial value. Thus if the instrument is fitted with two verniers and three "zeros" are taken, the total number of angular readings will be $4 \times 3 \times 2 = 24$. The average of the above 24 readings will give a very good result.

The following are the advantages of the above procedure.

(a) The effect of swinging the telescope right and then left and of bringing the crosshair into coincidence approaching from left in the former case and from right in the latter case eliminates: (i) the error due to twist of the instrument support and back lash error, and (ii) slip due to defective clamping arrangement.

(b) "Face left" and "Face right" observations eliminate the errors due to the non-adjustment of the line of collimation and the trunnion axis.

(c) The "changing of zero" eliminates the errors due to defective graduation.

(d) The "reading of both verniers" also minimize error due to defective graduation. But it provides immediate check on the personal error in reading the verniers or micrometers.

(e) The "averaging" of all observed values minimizes the personal error.

However, the errors due to centring or non-levelling of the instrument cannot be eliminated by the above mentioned process.

10.7.1 MEASURING HORIZONTAL ANGLES BY REPETITION AND REITERATION

Horizontal angles can be measured accurately by either of the two methods. These are described below. (i) Method of repetition; (ii) Method of reiteration.

Method of Repetition

It involves the following steps.

1. Obtain the first reading of the angle following the procedure outlined in detail in Section 10.6.2. Read and record the value.

2. Loosen lower clamp, plunge (transit) the telescope, rotate upper/lower plate unit i.e. the whole instrument with angular reading fixed at initial value and point to backsight.

3. Tighten lower clamping screw and point accurately at the backsight with the help of lower tangent screw. The telescope is now inverted and aligned on backsight with the initial angle reading remaining set on the horizontal circle.

4. Loosen upper clamp, rotate upper plate, and point at foresight.

5. Tighten upper clamp and complete foresight pointing using the upper tangent screw.

6. The vernier reading now shows the angular measurement as the sum of first and second angle. Divide the sum by two (or the number of repetitions) to determine the average value of the angle.

The advantages of this method are as follows.

1. Since the average value of the final reading is taken, the angle can be read to a finer degree of subdivision than what can be read directly.

2. An error due to imperfect graduation of the scale is eliminated or reduced to a minimum as the reading is measured on several parts of the scale and then averaged.

3. Error due to inaccurate bisection is minimized as the average value of the final reading is taken.

4. Personal error in reading the vernier is reduced.

The disadvantages are:

1. Error due to slip and error due to the trunnion axis not being exactly horizontal.

2. When the number of angles are large more time is required by this method compared to the method of reiteration.

The precision attained by this method of measuring an angle is to a much finer degree than the least count of the vernier. Assume an angle of 84°27′14″ is measured with a 20″ transit. A single observation can be read correctly to within 20″. Hence the angle will be read as 84°27′20″ (error is + 6″ and possible error limit ± 10″). Measured twice the observed reading to within ± 10″ is 168° 54′20″. Divided by 2 , the average is 84°27′10″ correct to one half of the vernier's least count. error –4″ and has an error limit of ± 5″. Measured four times, the angular reading correct to 20″ is 337°49′00″, divided by 4, the average is 84°27′15″ and error + 1″ (within limit of ± 2.5″). Thus if no. of readings is n, precision attained will be ± 20/2n or least count/2 times no. of observations.

In short, measuring an angle by repetition (i) improves accuracy, (ii) compensates for systematic errors, and (iii) eliminates blunders.

10.7.2 LAYING OUT ANGLES BY REPETITION

Sometimes it is necessary to measure an angle more accurately than is available with the least count of the instrument. Suppose we want to set an angle at 24°30′47″. With 20″ transit available the instrument is set temporarily at 24°30′40″ ± 20″. The angle is then measured 4 times. Let the value of the angle be 24°30′44″ ± 05″. The difference between the required value of the angle and the value of the angle set out is 24°30′47″ − 24°30′44″ = 03″. Since this value is very small, it cannot be set out by means of an angular measurement. But it can be converted to linear distance if the length of the side of the angle is known. If the length is say 200 m, the linear distance is 200 × (radian measure of the difference). For 03″, this is equal to

$$200 \times \frac{03}{60 \times 60} \times \frac{\pi}{180} = .00\ 29088 \text{ m.}$$

This is shown graphically in Fig. 10.10.

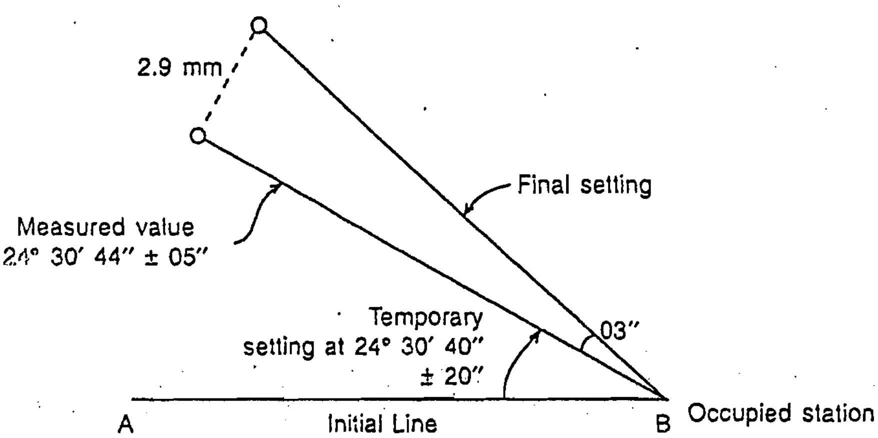


Fig. 10.10 Laying angle by repetition.

10.7.3 EXTENDING A STRAIGHT LINE

Suppose it is necessary to extend a line *AB* to *D* which is not directly visible from *A*. The instrument is then shifted to *B*. *A* is backsighted, the upper and lower plates clamped, telescope plunged and the new point *D′* is sighted which is on the prolongation of *AB*. This procedure is repeated until *D* is sighted. It is more accurate to plunge the telescope than turn 180° with the horizontal circle.

If the instrument is not in proper adjustment the above procedure will give erroneous result as shown in Fig. 10.11 and "Double Centring" should be adopted. Double centring or double sighting consists of making a measurement of a horizontal or vertical angle once with the telescope in the direct or erect position and once with the telescope in the reversed, inverted or plunged position. The act of turning the telescope upside down, that is, rotating it about the transverse axis is called "plunging" or "transitting" the telescope. As explained in Fig. 10.12, the instrument is set up at *B* and *A* is backsighted. The telescope is plunged and a point C_1 is

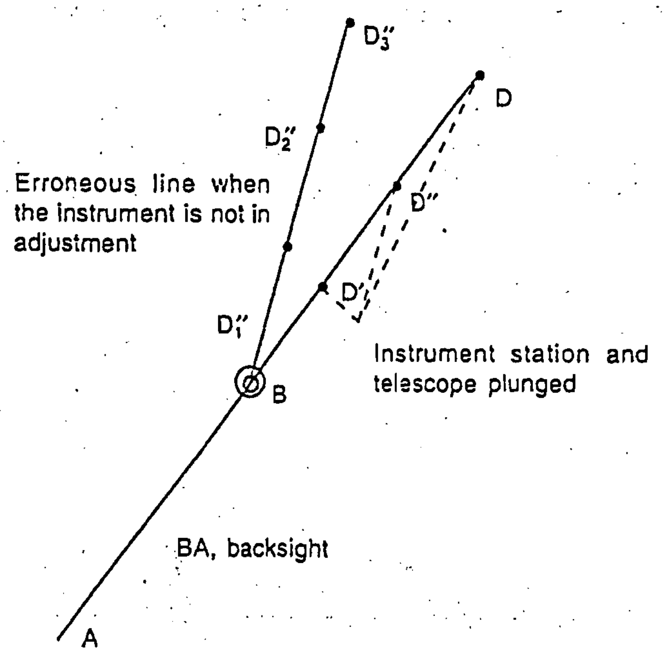


Fig. 10.11 Extending a straight line.

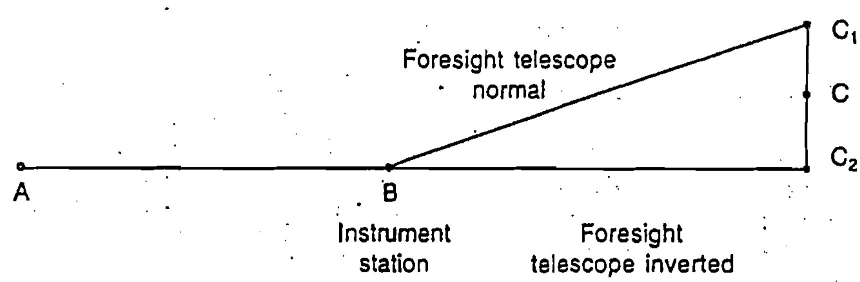


Fig. 10.12 Double centring. C_1C_2 = Twice the error, $C_1C = CC_2 = 1/2\ C_1C_2$.

obtained along BC_1 in continuation of AB. Then with the telescope remaining inverted, the instrument is rotated in azimuth through 180°. A is again backsighted the telescope is again plunged and a second mark C_2 is obtained. The two foresights will have equal and opposite errors if the instrument is not in perfect adjustment. The correct location of C will lie between C_1 and C_2.

10.7.4 METHOD OF REITERATION

This method is used when several angles are to be measured at the same station (Fig. 10.13). The instrument is placed at A and pointed towards B, the initial station with 0—0 vernier reading in one vernier. The reading of the other vernier is noted. With instrument "face left" the telescope is turned clockwise (swing right) to sight C. The upper fixing screw is clamped and the crosshair brought into coincidence approaching from the left (approach left). Both the vernier readings are recorded and their mean gives $\angle BAC$. Now unclamping the upper

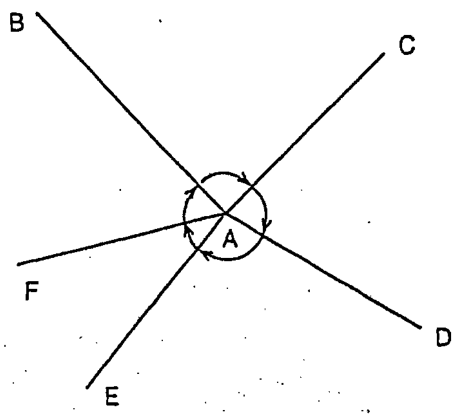


Fig. 10.13 Method of reiteration.

clamping screw, the instrument is further rotated to sight *D*. The mean of the vernier readings now give angle *BAD*. The difference of angles *BAD* and *BAC* give *CAD*. Other readings are taken with face right, swing left, approach right as explained before.

10.7.5 MEASUREMENT OF VERTICAL ANGLE

A vertical angle is an angle measured in a vertical plane from a horizontal line upward or downward to give a positive or negative value respectively. Positive or negative vertical angles are sometimes referred to as elevation or depression angles respectively. A vertical angle thus lies between 0° and ± 90°. A zenith angle is an angle measured in a vertical plane downwards from an upward directed vertical line through the instrument. It is thus between 0° and 180°. Some instruments such as the transit theodolite measure vertical angle while most optical theodolite measure zenith angle.

As the altitude bubble is more sensitive compared to the plate bubble, the former is used in measuring the vertical angle. The altitude bubble is usually fixed over the *T* frame as assumed in the following discussion. The steps for measuring vertical angles are given as follows:

1. Level the instrument initially with the help of plate bubbles.

2. Bring the altitude bubble parallel to one pair of foot screws. Bring the altitude bubble to the centre of its run by turning the two foot screws either both inwards or outwards.

3. Turn the telescope through 90° so that the altitude level is over the third foot screw. Bring the bubble to the centre of its run by turning the third foot screw.

4. Repeat the process till the bubble remains central in both positions.

5. If the permanent adjustments of the instrument are correct the bubble will remain central for all positions of the telescope. With correct permanent adjustments and no index error, the vertical circle verniers will read 0–0 when the line of sight is horizontal.

6. To measure vertical angle, then, loosen the vertical circle clamp and direct the telescope towards the object P whose vertical angle is required. Clamp the vertical circle and bisect P exactly by the fine adjustment tangent screw.

7. Read both verniers. The mean of both vernier readings gives the vertical angle.

8. Change the face of the instrument and again take mean of both the readings of the vernier.

9. The mean of the face left and face right readings will be the required angle.

Important points in measuring vertical angles:

1. The clip screw should not be touched while measuring the vertical angle. The clip screw is used for making the permanent adjustments.

2. Index error (vertical circle not reading 0–0 when line of sight is horizontal) cannot be eliminated by taking reading with only face left or face right. However, it can be eliminated when readings with both faces are taken and their mean is recorded.

10.8 ERRORS IN THEODOLITE ANGLES

The following errors occur in theodolite measurements.

10.8.1 INSTRUMENTAL ERRORS

Error due to eccentricity of inner and outer arms

As already explained, a theodolite has two spindles. The inner spindle carries the two verniers while the outer spindle carries the horizontal circle. The centres of these two spindles should coincide. Otherwise, errors will occur. Errors will also occur if the two verniers are not exactly 180° apart. This is known as eccentricity of verniers. This is examined with reference to Fig. 10.14. Let O_1 be centre of vertical axis and O_2 centre of graduated circle. Then O_1O_2 is eccentricity and A_1B_1, A_2B_2, A_3B_3 are different positions of the two verniers which are 180° apart i.e. there is no eccentricity of verniers. The observed angles are θ_1 and θ_2, the correct angle will be ϕ. Hence the error is

$$\alpha_1 = \theta_1 - \phi \quad \text{for vernier } A_2$$

$$\alpha_2 = \phi - \theta_2 \quad \text{for vernier } B_2.$$

$$\alpha_1 = \tan^{-1} \frac{O_2 E}{A_2 E}$$

$$= \tan^{-1} \frac{e \sin \phi}{r - e \cos \phi}$$

Since e is small and $\cos \phi$ lies between 0 and 1, $e \cos \phi$ can be neglected in comparison to r.

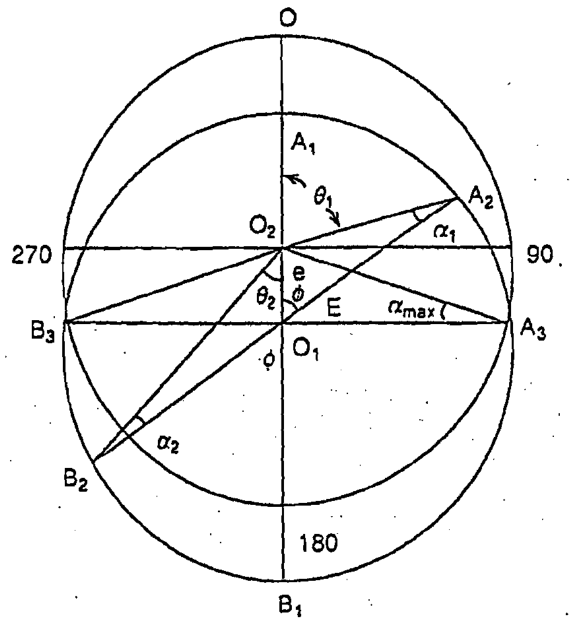


Fig. 10.14 Eccentricity of inner and outer axis.

Therefore $$\alpha_1 \approx \frac{e \sin \phi}{r}$$

Similarly $$\alpha_2 \approx \frac{e \sin \phi}{r}$$

If the verniers are 180° apart $A_1O_1B_1$ lie in one straight line. Rearranging

$$\phi = \theta_1 - \alpha_1 = \theta_2 + \alpha_2$$

Since $\alpha_1 \approx \alpha_2$, $2\phi = \theta_1 - \alpha_1 + \theta_2 + \alpha_2$

$$\approx = \theta_1 + \theta_2$$

or $$\phi = \frac{\theta_1 + \theta_2}{2}$$

Thus the average of two readings give the correct value

From Fig. 10.14 it can be seen that (i) along O_1O_2, $\alpha = 0$; (ii) at right angles to this line α is maximum.

When the instrument has both eccentricity of axes and eccentricity of verniers, error will occur due to both causes. This is shown in Fig. 10.15. From the figure, it can be seen that the total error of 2nd vernier $B_1 = \delta_1 = \lambda + 2\alpha$ where λ is the index error and index A_1 is at 0°.

When the index $B_2 = 0$, the total error of the other vernier $A_2 = \delta_2 = 2\alpha - \lambda$. When there is no eccentricity of verniers $\lambda = 0$. Hence when there is no eccentricity of axes or verniers $\delta_1 = \delta_2 = 0$. If there is eccentricity and A and B are 180° apart, then for the two positions A_1 at 0° and B_2 at 0°.

$$\delta_1 = \delta_2 = \text{constant}$$

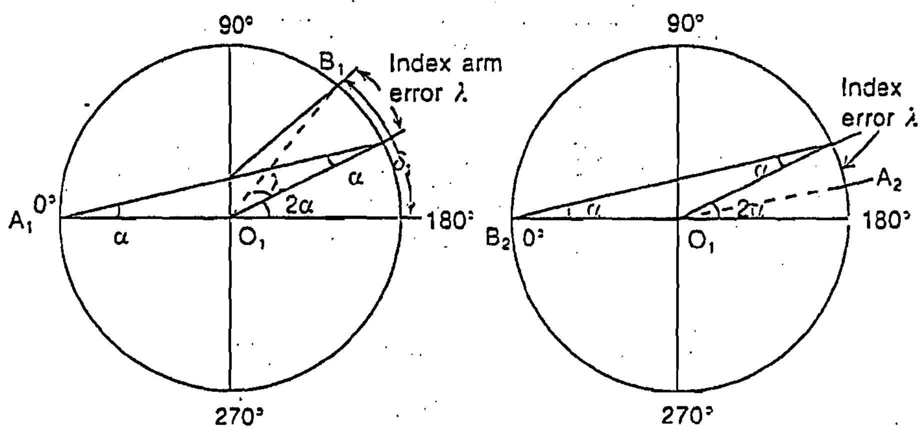


Fig. 10.15 Eccentricity of verniers and axes.

If there is no eccentricity of axes but *A* and *B* are not 180° apart, then $+\delta_1 = -\delta_2$, i.e. δ_1 and δ_2 are equal but opposite in sign. Finally if there is eccentricity of axes and *A* and *B* are not 180° apart, then δ_1 and δ_2 will vary in magnitude as zero setting is consecutively changed around the circle of centre O_2 but their difference will remain constant as $\delta_1 - \delta_2 = \lambda + 2\alpha - (2\alpha - \lambda) = 2\lambda$.

By reading both verniers and adopting a mean of both the readings, error due to both causes can be eliminated.

Error due to line of collimation not being perpendicular to the trunnion axis

If the line of collimation is not exactly perpendicular to the horizontal axis of the instrument, it would not revolve in a vertical plane when the telescope is raised or lowered. In fact, it will generate a cone, the axis of which coincides with the horizontal axis of the instrument (Fig. 10.16). *CE* is the line of sight which is not at right angles to the trunnion axis. *ZDE* is the horizontal plane and θ is the projection of angle ε on the horizontal plane. From the figure,

$$\tan\theta = \frac{DE}{ZD} \text{ but } ZD = CD\cos\alpha$$

that is,

$$\tan\theta = \frac{DE}{CD\cos\alpha} \text{ but } \frac{DE}{CD} = \tan\varepsilon$$

Hence

$$\tan\theta = \tan\varepsilon\sec\alpha.$$

As θ and ε are both small.

$$\theta = \varepsilon\sec\alpha$$

If for two observations on the same face, angles of elevations are α_1 and α_2 then the net error $\theta_1 - \theta_2 = \pm\varepsilon(\sec\alpha_1 - \sec\alpha_2)$. On changing face, the error will be of equal value but opposite in sign. Thus the mean of the face left and face right readings, i.e. double centring eliminates this error. The error increases with increase in angle of depression or elevation.

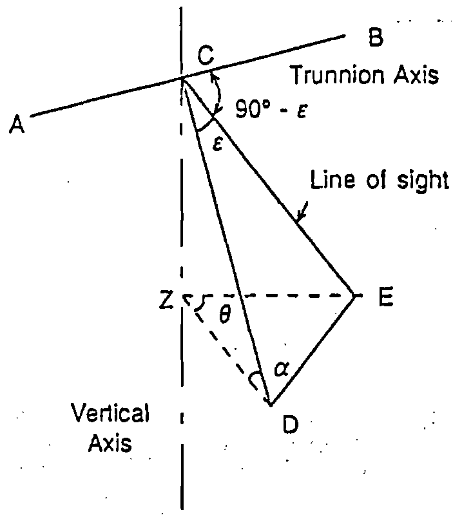


Fig. 10.16 Line of collimation not perpendicular to trunnion axis.

Trunnion axis not perpendicular to the vertical axis

If the trunnion axis or horizontal axis is not perpendicular to the vertical axis, when the telescope is rotated about the horizontal axis, it would not generate a vertical plane but would generate an inclined plane. An error will occur if the foresight and backsight are inclined at different angles to the horizontal.

From Fig. 10.17

ABCD—vertical plane swept out by the line of sight when the trunnion axis is truly horizontal.

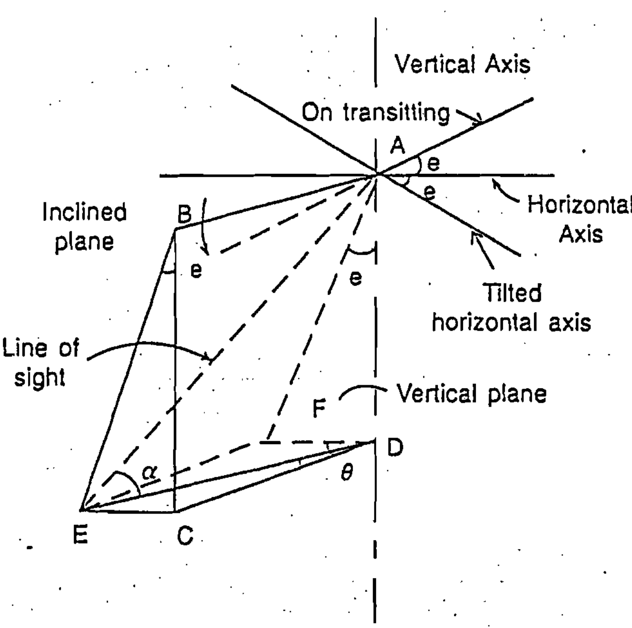


Fig. 10.17 Trunnion axis not perpendicular to vertical axis.

ABEF—inclined plane swept out by the line of sight when trunnion axis is inclined at an angle ε to the horizontal.

AE = line of sight.

α = angle of inclination of line of sight.

θ = Error in measuring horizontal angle

From the figure:

$$\sin\theta = \frac{EC}{ED}$$

But $$EC = BC\tan e$$

$$\sin\theta = \frac{BC\tan e}{ED}$$

But $$BC = AD \qquad \frac{BC}{ED} = \frac{AD}{ED} = \tan\alpha$$

or $\sin\theta = \tan\alpha\tan e$

since θ and e are small, $\theta = e\tan\alpha$.

On transitting the telescope, the inclination of the trunnion axis will be in the opposite direction as shown in Fig. 10.17 but of equal magnitude thus double centring eliminates this error.

Vertical axis not truly vertical

If the theodolite is out of adjustment, the vertical axis will not be truly vertical even if the plate bubble is at the centre of its run. If E is the angle by which the vertical axis is not truly vertical, the horizontal axis will not be truly horizontal by the same angle E. Thus the error in measuring horizontal angle will be $E\tan\alpha$. E, however, does not remain constant for all horizontal angles of the telescope. When the direction of pointing is in the same direction as the inclination of the vertical axis, the error is zero, no matter whatever is the vertical angle. The error is maximum when the pointing is at right angles to the direction of inclination of the vertical axis. This error is not eliminated by double centring as the vertical axis does not change in position or inclination as the change of face is done.

Figure 10.18 shows the difference in effect of change of face when the trunnion axis is not perpendicular to the vertical axis and when the vertical axis is not truly vertical.

Fig. 10.19 shows the error in measurement of an angle when the vertical axis is not truly vertical.

Observed angle A_2OB_2

Correct angle A_1OB_1

$= A_2OB_2 - C_1 + C_2$

$= A_2OB_2 - E_1\tan\alpha_1 + E_2\tan\alpha_2$

where E_1 and E_2 are the angles by which the vertical axis is not truly vertical and α_1, α_2 – direction of pointing of the line of sight.

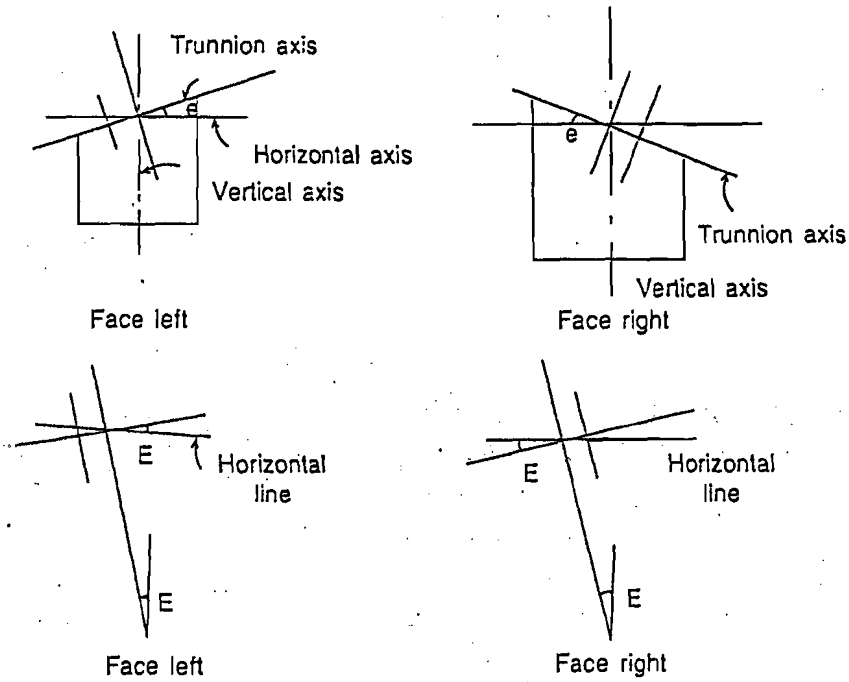


Fig. 10.18 Vertical axis not truly vertical.

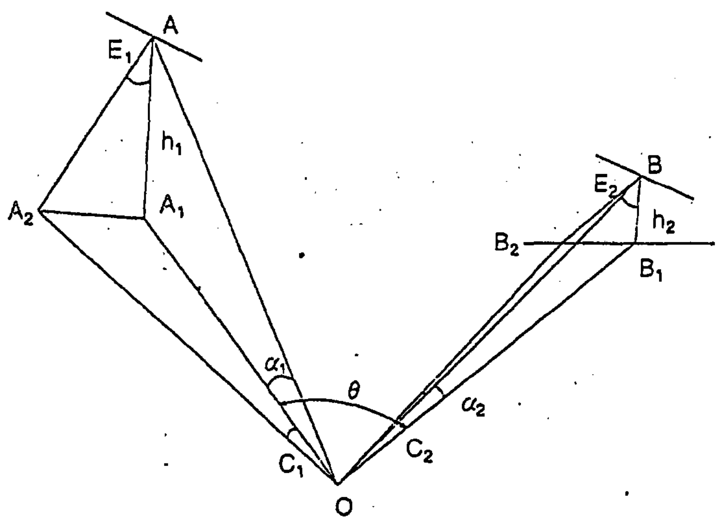


Fig. 10.19 Error in angle measurement.

Vertical circle index error

To obtain correct vertical angle, when the telescope is horizontal, i.e. altitude bubble is central, the line of collimation should be horizontal, and the vertical circle index should read 0–0. If not, error in measurement of vertical angle will occur. Let,

α = Correct altitude angle.

α_1, α_2 = Observed angles of altitude.

ϕ = Collimation error.

θ = Index error.

Figure 10.20 shows the different angles for both face left and face right observations when the bubble is made central by means of clip screws.

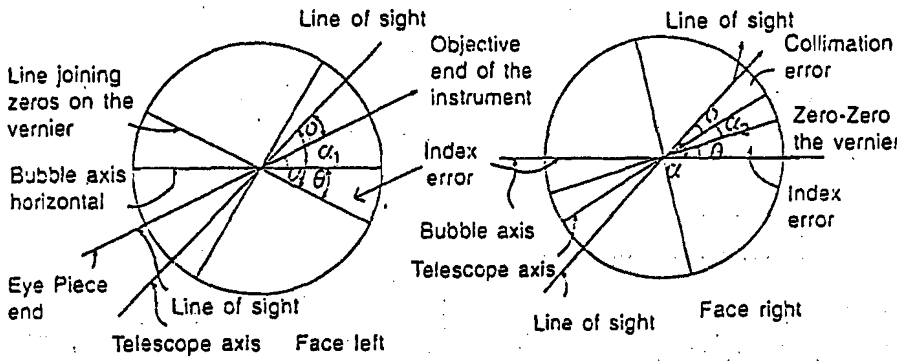


Fig. 10.20 Vertical circle index error.

From Face left observation

$$\alpha = \alpha_1 - \phi - \theta$$

From Face right observation

$$\alpha = \alpha_2 + \phi + \theta$$

Taking mean

$$\alpha = \frac{\alpha_1 + \alpha_2}{2}$$

If the altitude bubble is not central, the line connecting the vernier zeros would be inclined to that line obtained when the altitude bubble is central and any movement of the altitude bubble would cause an equivalent rotation of the line of vernier zeros in the same direction. If

O_R = Reading of the objective end of the bubble with face right

O_L = Reading of objective end of bubble with face left

E_R = Reading of eyepiece end with face right

E_L = Reading of eyepiece end with face left

Then rotation of the index frame $= \frac{O_R - E_R}{2}\theta''$ when face right

$= \frac{O_L - E_L}{2}\theta''$ when face left.

where θ'' = angular value of one division of the bubble tube.

If O_L is greater than E_L, then

$$\theta \text{ will be decreased by } \frac{1}{2}(O_L - E_L)\,\theta''$$

and $$\alpha = \alpha_1 - \phi - \left\{\theta - \frac{1}{2}(O_L - E_L)\,\theta''\right\}$$

When $O_R > E_R$. θ will be increased by $\frac{1}{2}(O_R - E_R)\,\theta''$

and $$\alpha = \alpha_2 + \phi + \left\{\theta + \frac{1}{2}(O_R - E_R)\,\theta''\right\}$$

Taking mean of the two readings

$$\alpha = \frac{1}{2}(\alpha_1 + \alpha_2) + \frac{\theta''}{4}(\Sigma O - \Sigma E)$$

Error due to imperfect graduations on horizontal scale

If any graduations on the horizontal circle are not uniformly spaced or if the scale is not properly centred the horizontal angle readings will not be correct. This error can be minimized by taking observations over different portions of the horizontal scale so that they are spaced over the entire scale and their mean come close to the correct value.

From the above discussion it is clear the effect of error is greatest when observations are taken with (i) line of sights at different vertical angles, (ii) of different lengths.

However, these errors can be almost eliminated by taking mean of two angles taken with face left and face right observations as then half the readings are too large while the other half too small. Averaging the sum gives the correct angle.

10.8.2 PERSONAL ERRORS

These errors result from limitations of human eye sight and are accidental in nature.

In accurate centring

If the instrument is not set exactly over the station point there will be error in measuring horizontal angles there. This is shown in Fig. 10.21 where B is the station point and A and C are the observed stations.

B = Required station point,

B_1 = Actual station point,

x = Displacement of B_1 from B,

θ_1 = Observed angle from B_1

θ = Required angle from B.

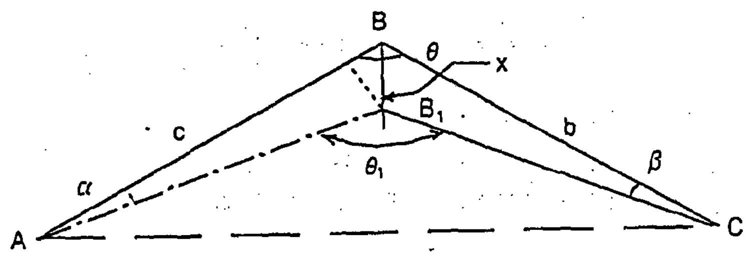


Fig. 10.21 Inaccurate centring.

Error in measurement $= \theta_1 - \theta = \alpha + \beta$

Let $ABB_1 = \phi$, then

$$\sin \alpha = \frac{x \sin \phi}{AB_1}$$

As α is small $\sin \alpha = \alpha$ radian

$$\alpha \text{ in second} = \frac{206265\, x \sin \phi}{AB_1}$$

$$= \frac{206265\, x \sin \phi}{c}$$

as $AB = AB_1 = c$.

Similarly

$$\beta \text{ in second} = \frac{206265\, x \sin (\theta - \phi)}{b}$$

Total error $\quad \alpha + \beta = 206265\, x \left(\frac{\sin \phi}{c} + \frac{\sin (\theta - \phi)}{b} \right)$

For maximum and minimum values

$$\frac{dE}{d\phi} = 206265\, x \left[\frac{\cos \phi}{c} - \frac{\cos (\theta - \phi)}{b} \right] = 0$$

or $$\frac{\cos \phi}{c} = \frac{\cos (\theta - \phi)}{b}$$

or $$\cos \phi = \frac{c}{b} (\cos \theta \cos \phi + \sin \theta \sin \phi)$$

Dividing by $\sin \phi$

$$\cot \phi = \frac{c}{b} (\cos \theta \cot \phi + \sin \theta)$$

or $$\cot \phi = \frac{c \sin \theta}{b - c \cos \theta}$$

when $\phi = 90°$, $\cot \phi = 0$. Hence $\sin \theta = 0$, or $\theta = 0°$ or $180°$.

If $b >> c$, then cot $\phi \rightarrow 0$ or $\phi \rightarrow 90°$, i.e. the maximum error exists when ϕ tends towards 90° relative to the shorter line.

Thus centring error depends on: (i) linear displacement x, (ii) direction of the instrument B_1 with respect to station B, (iii) length of lines b and c. It is maximum when the displaced direction is perpendicular to the shorter line. It is more when length of sight is small. Angular error is about 1′ when the error of centring is 1 cm and the length of sight is 35 m.

Error of pointing

This error has the same effect as shown in the previous paragraph. Greater care must be exercised on shorter sight distances and a narrower object.

Misreading a vernier

An accidental error occurs if the observer does not use a reading glass or if he does not look radially along the graduations when reading the verniers. Correct reading however, depends on experience.

Improper focussing (parallax)

While taking measurements, the cross wires should be carefully focussed. Then the image of the object should be brought in the plane of the cross wires. Horizontal and vertical angles suffer in accuracy when improper focussing causes parallax.

Level bubble not centred

The position of the bubble centre should be checked frequently and, if necessary, should be recentred. However, it should not be done in the middle of a measurement, i.e. between a backsight and foresight.

Displacement of tripod

The instrument man should be very careful in walking about the theodolite. The tripod is easily disturbed, particularly when it is set up in soft ground. In that case the instrument should be reset.

10.8.3 NATURAL ERRORS

These are due to environmental causes like wind, temperature change and other atmospheric conditions such as the following.

(a) Poor visibility resulting from rain, snowfall or blowing dust.

(b) Sudden temperature change causing unequal expansion of various components of a theodolite leading to errors. The bubble is drawn towards the heated end of the theodolite.

(c) Unequal refraction causing shimmering of the signals making accurate sighting difficult.

(d) Settlement of tripod feet on hot pavement or soft or soggy ground.

(e) Gusty or high velocity winds that vibrate or displace an instrument, move plumb bob strings and make sighting procedures difficult.

For accurate and precise work these errors can be minimized by (i) Sheltering the instrument from wind and rays of the sun, (ii) Driving stakes to receive the tripod legs in unstable ground, and (iii) Avoiding horizontal refraction by not allowing transit lines to pass close to such structures as buildings, smoke stacks, and stand pipes which radiate a great deal of heat.

10.9 MISTAKES IN THEODOLITE ANGLES

Mistakes occur due to carelessness of the observer. Some of them are:

(a) Forgetting to level the instrument.
(b) Turning the wrong tangent screw.
(c) Reading wrong numbers, say, 219° instead of 291°.
(d) Dropping one division of the main scale reading say 20′.
(e) Reading wrong vernier (in the case of a double vernier).
(f) Reading the wrong circle (clockwise or anticlockwise).
(g) Reading small elevation angle as depression angle or vice versa.
(h) Not centring the bubble tube before reading vertical angle.
(i) Sighting on the wrong target.
(j) Missing the direction in measuring deflection angle.

10.10 PERMANENT ADJUSTMENTS OF A VERNIER THEODOLITE

The primary function of a theodolite is to measure horizontal and vertical angles. As already explained in Section 10.5, there are two planes and five lines in a theodolite. These lines and planes bear definite relation to one another in a well adjusted instrument. They are:

(a) The line of sight is normal to the horizontal axis.
(b) The horizontal axis is normal to the vertical axis.
(c) The vertical axis is normal to the plane containing the horizontal circle.
(d) The line of sight is parallel to the axis of the telescope bubble tube.
(e) The axes of the plate levels lie in a plane parallel to the horizontal circle.

Based on above the principal adjustments of a vernier theodolite are: (i) Plate bubble tubes, (ii) Crosshairs and line of sight, (iii) Telescope bubble tube, (iv) Horizontal axis, (v) Vertical vernier, and (vi) Horizontal vernier.

10.10.1 PLATE BUBBLE TUBE

Purpose To make the axis of each plate level bubble perpendicular to the vertical axis.

Test The plate bubble tube is levelled by means of foot screws. It is then rotated in azimuth through 180°. If the bubble remains central the adjustment is correct.

Correction Bring the bubble half way back by the capstan headed screw at one end of the level and the other half by the foot screws.

Explanation Figure 10.22 (a) shows the initial condition when the bubble is levelled. The bubble level becomes horizontal but as it is not at right angles to the vertical axis, the vertical axis is not truly vertical. The plate is, however, at right angles to the vertical axis. Figure 10.22 (b) shows when the bubble is rotated about vertical axis through 180°. As the vertical axis and the angle between the vertical axis and the bubble tube remains unaltered, the bubble now makes an angle of $90° - \varepsilon$ on the left side of the vertical axis. Figure 10.22 (c) shows how the vertical axis is rotated through ε (half the deviation) by means of foot screws to make it truly vertical. The plate level which still remains at an angle ε is made horizontal by means of capstan headed screw.

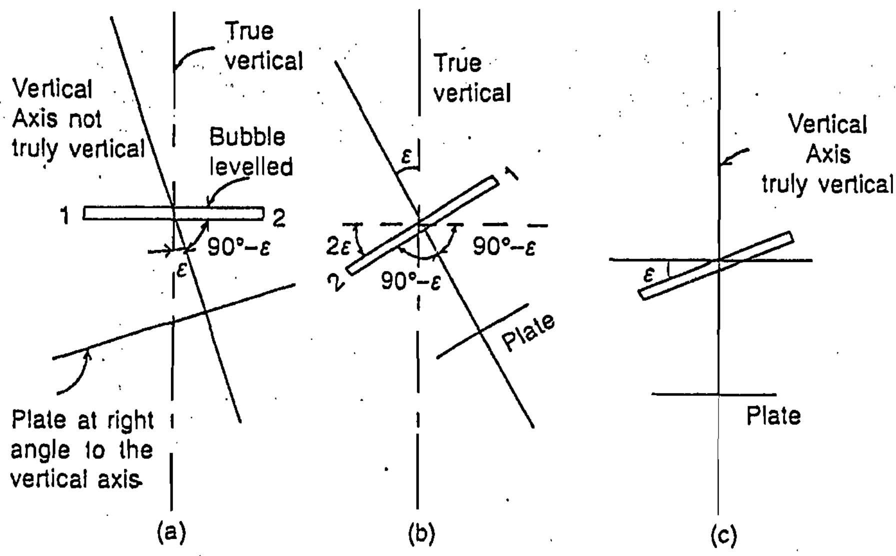


Fig. 10.22 Plate bubble test.

10.10.2 CROSSHAIRS AND LINE OF SIGHT

There are three adjustments:

Adjustment of vertical cross hair

Purpose To place the vertical crosshair in a plane perpendicular to the horizontal axis of the instrument.

Test The test is similar to that of horizontal crosshair in the adjustment of dumpy level. One end of the vertical hair is brought to some well defined point and the telescope is revolved on its transverse axis to see if the point appears to move along the hair. If it does not, the crosshair is not perpendicular to the horizontal axis.

Correction Loosen all the capstan screws and rotate the reticle carrying the crosshairs so that vertical hair becomes truly vertical and perpendicular to the horizontal axis. Repeat the test.

Adjustment of line of sight

Purpose The purpose of this adjustment is to make the line of sight perpendicular to the horizontal axis. This will allow true straight line extension when transitting the telescope.

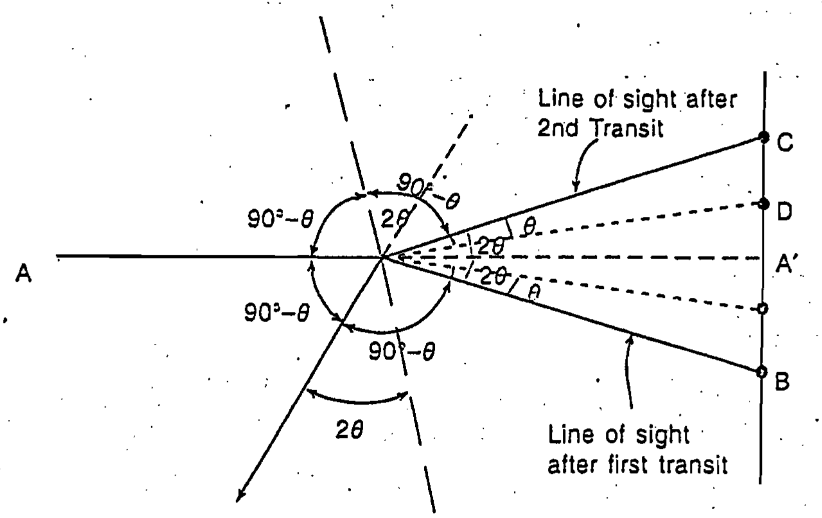


Fig. 10.23 Adjustment of line of sight.

Test Here double centring method of prolonging a line is applied. The steps are:

(i) Level the instrument and backsight carefully on a well defined point '*A*' about a chain away (Fig. 10.23).

(ii) *Transit* the telescope and see another point *B* at approximately the same elevation as '*A*' and atleast two chains away. If the instrument is in adjustment point *B* will be on the extension of the straight line.

(iii) With the telescope still in the inverted position, unclamp either plate, turn the instrument on the vertical axis, backsight on the first point *A* again and clamp the plate.

(iv) *Transit* the telescope again and set a point *C* beside the first foresight point *B*.

(v) Since two transittings are involved distance between *B* and *C* is *four* times the error of adjustment.

Correction Loosen one of the side capstan screw and tighten the other so that the vertical hair moves through 1/4th the distance *BC* to point *D*. Repeat the test till '*A*' is again seen after reversing backsight *A*.

Adjustment of horizontal crosshair

Purpose To bring the horizontal hair into the plane of motion of the optical centre of the object glass so that line of sight will be horizontal when the telescope bubble is in adjustment and the bubble is centred. This is necessary when the transit is used as a level or when it is used in measuring vertical angles.

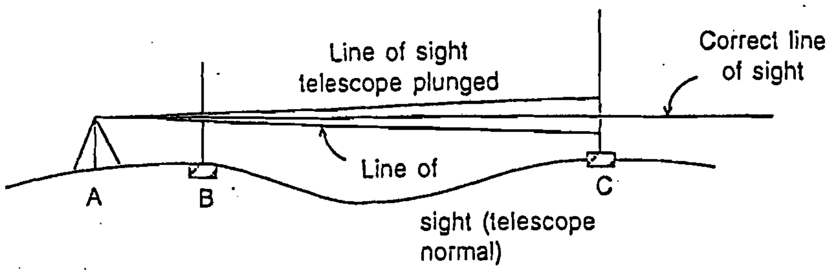


Fig. 10.24 Adjustment of horizontal crosshair.

Test Set up and level the instrument at *A* and mark two stations at *B* and *C*, the distance between them being at least 3 to 4 chains (Fig. 10.24). Take readings of horizontal crosshair at *B* and *C* with telescope both normal and inverted. If the difference in readings at *B* and *C* in both conditions is the same, then the line of sight is truly horizontal when the telescope bubble is at the centre of its run.

Correction If not at the centre, the reading is brought to mean of the two readings at *C* by means of capstan screws on the top and bottom of the telescope. Repeat the test and adjustment until the horizontal hair reading does not change for normal and plunged sights on the far point.

Adjustment of horizontal axis

Purpose The object of this adjustment is to make the telescope's horizontal axis perpendicular to the transit's vertical axis. In such a case when the plates are levelled, the horizontal axis is truly horizontal and the line of sight moves in a vertical plane as the telescope is raised or lowered.

Test Set up and level the instrument at a distance of 10 m from a tall vertical wall. Raise the telescope through a vertical angle of 30° and sight some distant point *A* on the wall. Plunge the telescope to 0° and mark a point *B* on the wall. Rotate the telescope through 180°, reverse the telescope and sight *A* again. Plunge again the telescope to 0° and mark a point *C* on the wall. If *B* and *C* do not coincide, the horizontal axis is not truly horizontal and needs adjustment, (Fig. 10.25).

Correction Set a point *D* half way between *B* and *C* and sight on it. With plates clamped elevate the telescope and bring it to point *A* by using the horizontal axis adjusting screw which raises or lowers the end of the cross arm until the crosshairs are brought to *A*. Tighten the clamp and check the adjustment by repeating the Test. This is known as *Spire Test*.

10.10.4 ADJUSTMENT OF TELESCOPE BUBBLE TUBE

Purpose To make the axis of the bubble tube parallel to the line of sight.

Test Same as the two peg test of dumpy level.

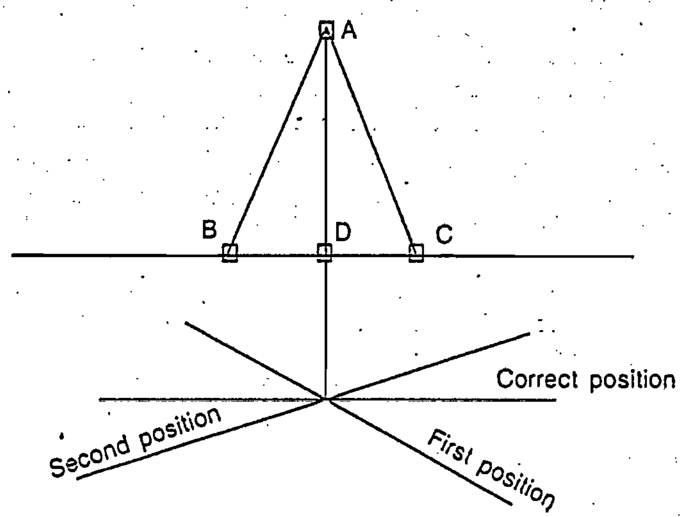


Fig. 10.25 Adjustment of horizontal axis.

Correction The correct reading for marking the line of sight horizontal is computed and it is set on the distant rod by means of vertical circle slow motion screw. The telescope bubble is then centred by turning the capstan screws at one end of the level vial.

10.10.5 ADJUSTMENT OF VERTICAL VERNIER

Purpose To ensure that vertical circle reads zero when the line of sight is horizontal.

Test Level the instrument with both plate level bubbles. Using the vertical circle slow motion screw, centre the telescope bubble. The vertical circle reading should now be zero. If not, there is index error.

Correction This adjustment is performed in two ways.

Method I This is used if there is no vertical vernier bubble tube. Set up and level the instrument and then using the vertical lock and slow motion. centre the telescope bubble. If the vertical vernier does not read exactly 0°, carefully loosen the vernier mounting screws, move it to a reading of exactly 0° and refix.

Method II This is used if the vertical vernier has a bubble tube. Set up and level the instrument, then, using the vertical lock and slow motion, centre the telescope bubble. Set the vernier to a reading of exactly 0° using the vernier slow motion screw and centre the vernier bubble using the bubble tube adjusting screws.

10.11 MICROMETER MICROSCOPE

Verniers in theodolites can read upto 20″. Precise verniers may read upto 10″. With micrometer microscopes readings may be taken on large geodetic theodolites to 1″ and on smaller theodolites direct to 10″ or 5″ and estimated to 2″ or 1″. Figures 10.26 (a) and (b) show the details of a micrometer microscope and how readings are taken. The low powered microscope is fitted with a small rectangular metal box at a point near where the image of the graduations formed by the objective will be situated. The box with windows in the top and bottom is fitted with a fixed mark or index and with a movable slide carrying a vertical hair or pair of parallel hairs placed very close together. These hairs are fitted so that they will lie parallel to the images of the division marks of the graduated arc. The slide can be moved by means of a milled head on the outside of the micrometer tube. The pitch of the screw is such that a complete revolution moves the slide through two successive divisions of the graduated arc. Fractional parts of a revolution of a drum, corresponding to fractional parts of a V division on the horizontal circle may be read on the graduated drum against an index mark fitted to the side of the box. The function of the eyepiece is to form a magnified image of the index, movable hairs and the image of the graduations formed by the objective.

The method of using and reading the micrometer will be understood from Fig. 10.26(b) which shows the view in the eyepiece together with graduations in the adjoining drum. When the drum reads zero, one of the graduations should be in the centre of the V, and this graduation should appear to be central between the two movable hairs.

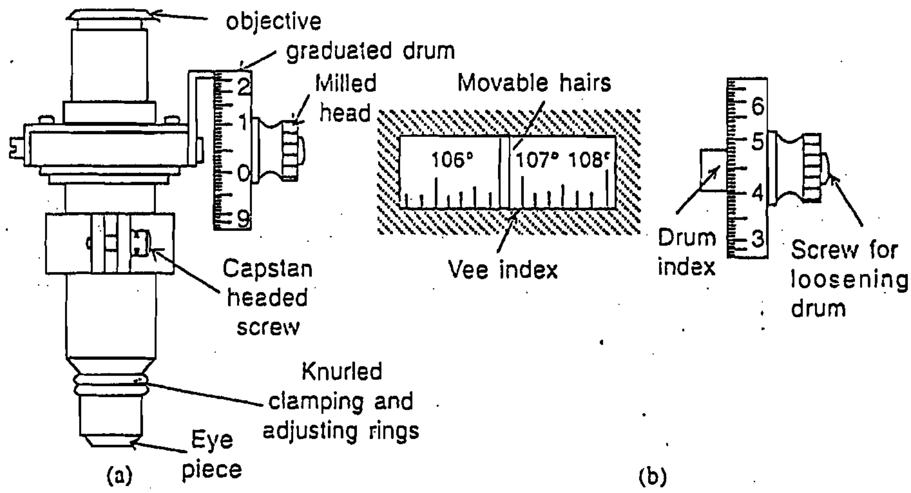


Fig. 10.26 (a) Micrometer microscope (b) Reading in a micrometer microscope.

In this example, the horizontal circle is graduated to 10′ of arc, the graduated drum is divided into 10 large intervals and each of the large intervals into 6 small ones. Therefore, each of the large divisons on the drum corresponds to 1′ of arc and each of the small divisions to 10″ of arc. To take a reading, note the divisions on either side of the V and take the lower one. From Fig. 10.26(b) it is 106°.50′. To measure the fractional part, the micrometer is turned until the graduation 106° 50′ lies midway between the movable hairs. The index beside the drum is now between the graduations 4′ 20″ and 4′30″ and by estimating tenths the reading on the drum may be taken as 4′ 27″. Hence the complete reading is 106° 54′27″.

10.12 OPTICAL THEODOLITES

These are used for precise survey and have many improved features compared to vernier theodolites. They can be both double centre and directional. Their main improved features are as follows:

1. The instrument is lightweight and compact and easy to operate weighing only about 5 kg.

2. Their vertical axis is cylindrical and rotates on precision ball bearings.

3. The circles and optical systems are completely enclosed and the instrument is dust proof and moisture proof.

4. The horizontal and vertical circles are made of glass and have precisely etched graduation lines and numerals.

5. Angles are read through an optical system consisting of a microscope and series of prism. An adjustable mirror on the outside of the instrument housing reflects light into the reading system, a battery powered light provides illumination for night work.

6. All circle readings and bubble position checks can be made from the eyepiece end it is not necessary to move round the instrument.

7. Telescope is short and internal focussing and equipped with a large objective lens to provide sharp views even at relatively short ranges. The alidade can be detached from its mounting or tribrach.

8. The horizontal line of sight is established by first centring the vertical circle control bubble and then setting off a 90° zenith angle using the vertical clamp and tangent screw. Some theodolites contain a pendulum compensator which minimizes "index error".

9. The inner spindle of most theodolites is hollow in order to provide a line of sight for the optical plummet which takes the place of the plumb bob used to centre the theodolite over the point to be occupied.

10. Optical reading repeating theodolite has an upper motion, a lower motion and a vertical motion together with appropriate clamps and tangent screws. Direction theodolite on the other hand, does not have the lower clamp and lower tangent screw. There is only one clamp screw and one tangent screw. The horizontal circle setting screw can be used for changing the position of the horizontal circle.

11. Vertical circles are graduated from 0° to 360°, 0° corresponding to the instrument's zenith. With the telescope level, in normal position, a zenith angle of 90° is read, in inverted position the angle is 270°. Optical reading systems of direction instruments permit an observer to simultaneously view the circle at diametrically opposite positions, thus compensating for any circle eccentricities.

10.12.1 PRINCIPLE OF OPTICAL MICROSCOPE AND OPTICAL PLUMMET

Figure 10.27 shows the schematic diagram of an optical microscope and optical plummet together. Light falling into the mirror after reflection and refraction through suitably placed prisms, passes partly through the graduations of the horizontal circle and partly through the graduations of the vertical circle. They then pass through a parallel sided glass block *C* which can rotate about a vertical axis. Finally they are focussed on the plate *D*. The block *C* is rotated by the micrometer setting knob and its position is indicated by the graduated sector scale *S*. Through the reading eyepiece at *D* the observer can see vertical circle graduations in the window *V* a portion of horizontal circle graduations in window *H* and a portion of the sector scale in window *S*. Each window has a fixed reference line or marker

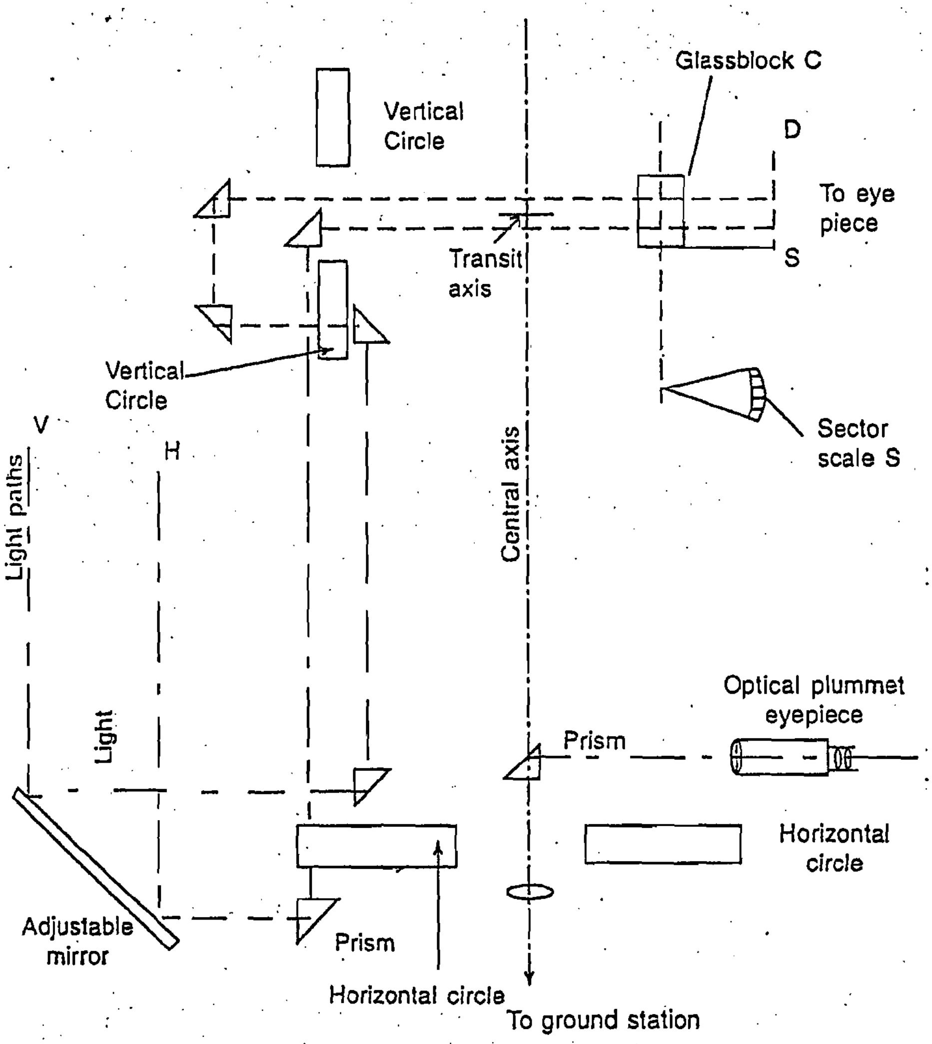


Fig. 10.27 Schematic diagram of an optical microscope and an optical plummet.

and as the block *C* is rotated, the pictures of the scales moves across the windows. To read the horizontal angle the window marked *H* is viewed. The micrometer screw is turned until the degree mark is exactly on the reference line. As the micrometer screw is turned, the micrometer scale reading also changes and it gives the fractional part in minutes and seconds. The total reading will, therefore, be equal to full degree reading plus a fractional part. Similarly for the vertical angle viewed in the window marked *V*.

However, to ensure the above, the thickness of the block *C* and the length of the scale *S* must be so adjusted that a movement of the scale from 00′00″ to 20′00″ causes the pictures in the windows *V* and *H* to move distance of exactly one division 20′ of the circle scales.

The optical plummet helps in centring the theodolite over the station. It is obtained by fitting lenses into the hollow central axis of the instrument so as to form a small telescope pointing vertically downwards. From the ground station line of sight passes through the lens and then through a reflecting prism to an eyepiece at the side of the instrument. Through the eyepiece the observer can view the ground station in relation to a diaphragm mark in the optical system.

The optical plummet sight is very useful for optical centring. But to be effective the sight line of the centring telescope should be vertical. This is ensured by doing simultaneously the levelling and centring operation which is time consuming. Its use, however, becomes obligatory when there is high wind or when the ground mark is at the bottom of a hole, or in some other special circumstances. Figure 10.27 shows the line of sight of an optical plummet.

10.12.2 CENTRING BY CENTRING ROD

This is another way of accurately centring the instrument over a ground station. The centring rod is a telescopic plumbing rod, the bottom of which is pointed and is set into the station mark. The verticality of the centring rod is ensured by bringing the bull's eye bubble attached to the centring rod to the centre. The top of the rod is moved laterally by means of the tribrach (with which it is attached) which moves over the top face of the tripod. The upper end of the rod and consequently, the tribrach is then locked into position by means of a knurled clamping nut at the upper end of the rod.

In this method, the theodolite can be removed and can be quickly interchanged with an EDM, reflector or a sight pole without disturbing integrity of the tripod/tribrach set up. This technique is referred to as "force centring". Advantages of forced centring are obvious—instead of three separate setups at every station (foresight, theodolite occupation, and backsight), only one placement of the tripod or tribrach is necessary. Two causes of accidental setting up errors have been eliminated.

10.13 ELECTRONIC THEODOLITES

Electronic theodolites use the principle of electronics to read, record and display horizontal and vertical angles. Generally, light-emitting diodes (LEDs) or liquid crystal diodes (LCDs) are used for display. The data obtained can be stored directly in an electronic data recorder for later retrieval, and computing by a microprocessor either in the field or in the office.

Sometimes the theodolite is equipped with an EDMI when it becomes a total station instrument or an electronic tacheometer. The instrument can then be used for measuring and displaying horizontal and vertical angles, horizontal distance, and elevation difference. With the help of in built computer slope distances can be reduced and horizontal distances can be corrected for curvature and refraction. Coordinates for the occupied station can be obtained when coordinates of other points are known.

10.14 MEASURING ANGLES WITH DIRECTION THEODOLITES

The direction theodolite reads "directions" or positions on its horizontal circle. It does not provide for a lower motion as is contained in a repeating instrument. For measuring the horizontal angle *ABC*, set up the instrument at *B*. With the horizontal clamp loose, make a rough pointing towards *A*, tighten clamp and make the perfect pointing with the horizontal tangent screw. The circular optical micrometer enables directions to be read to the nearest second or less. Let the reading be 21°15′27″ as shown in Fig. 10.28. Next loosen the horizontal clamp and observe the same procedure to point towards *C*. The reading, say, is 42°27′41″. The included angle is, then 42°27′41″ – 21°15′27″ or 21°12′14″.

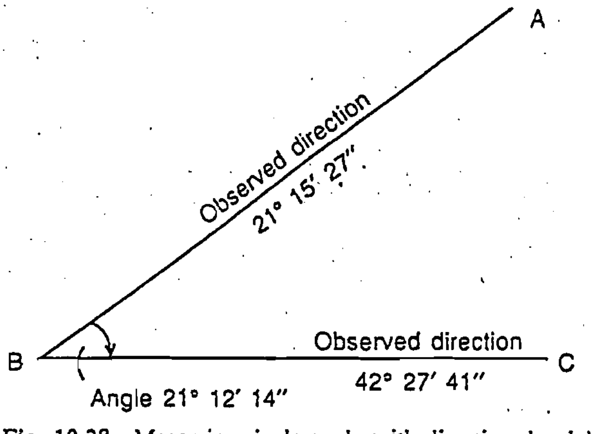


Fig. 10.28 Measuring single angle with direction theodolite.

Figure 10.29 shows how a number of angles can be more accurately measured with a direction theodolite. Set up the instrument at *A* and point at *B*, the left most station of the set. Initializing on *B* permits directions to be read in a clockwise sequence. Then as in single angle measurement observations should be taken on *B*, *C*, *D* and *E*. This is the first step. Next, loosen the horizontal clamp, rotate allidade through 180° and reverse the telescope and point again to *E*. The reading will differ by approximate 180° from the first reading. Sights are then taken to *D*, *C*, and *B* in the counter-clockwise direction. This completes the 1st position or set of angles. The mean of the two second values is suffixed to the direct reading of degree and minute to give the mean value. In this way sighting errors and instrumental

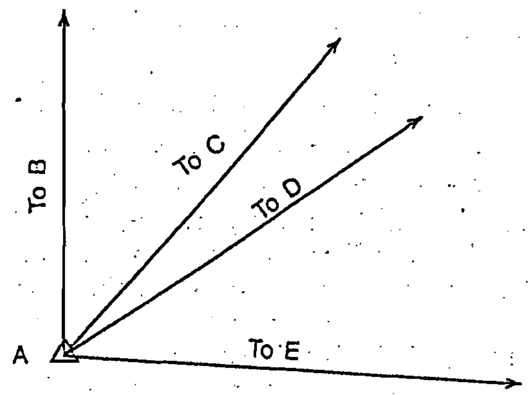


Fig. 10.29 Accurate measurements of angles by direction theodolite.

errors are eliminated but not random errors. From the directions the included angles can be obtained as before. A second set of readings can then be taken and a third set and so on. The initial reading for the second set will depend on the number of sets to be observed. The second set will start from the original reading plus 180/n where n is the number of sets. If there are two sets, added value will be 90° if three, it will be 60°. The final angles arc then the means obtained from all the sets.

Example 10.1 List the factors which determine the magnitude of the angular error due to defective centring of the theodolite.

The centring error in setting up a theodolite over a survey station is 1 mm. Compute the maximum and minimum errors in the measurement of clockwise angle $A\hat{B}C$ induced by the centring error if the magnitude of the angle is approximately 120° and the lengths of the lines AB and BC are approximately 5 m and 20 m respectively.

What conclusions can be drawn from this computation? [Eng. Council]

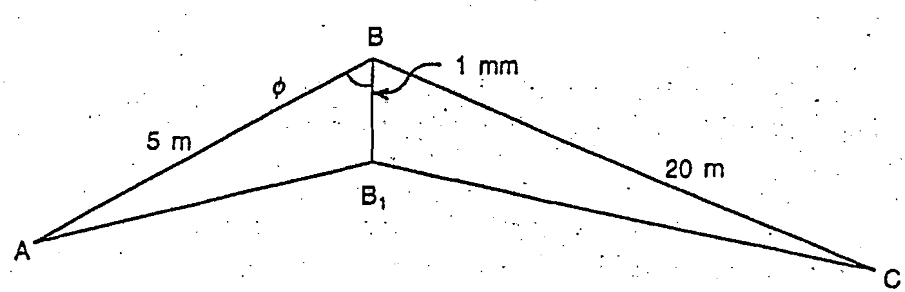


Fig. 10.30 Example 10.1.

Solution Theoretical portion has been explained in Section 10.8.
For maximum error

$$\cot \phi = \frac{c \sin \theta}{b - c \cos \theta}$$

$$= \frac{5 \sin 120°}{20 - 5 \cos 120°}$$

$$= 0.192$$

$$\phi = 79.106°$$

$$E = 206265\, x \left(\frac{\sin \phi}{c} + \frac{\sin (\theta - \phi)}{b}\right)$$

$$= \frac{206265 \times 1}{1000}\left(\frac{\sin 79.106}{5} + \frac{\sin (120 - 79.106)}{20}\right)$$

$$= 206.265\,(.1963957 + .032733)$$

$$= 47.26''$$

Minimum error is zero.

Conclusion: (i) Angular error E is maximum when the displacement tends to be perpendicular to the shorter line. (ii) Major contribution in angular error is from the shorter side.

Example 10.2 Derive an expression for the error in the horizontal circle reading of a theodolite (h) caused by the line of collimation not being perpendicular to the trunnion axis by a small amount C.

A theodolite under test for error in collimation and alignment of the trunnion axis is set with its axis truly vertical. Exactly 20.000 m away is a vertical wire carrying two targets at different levels. An accurate scale perpendicular to the line of sight is graduated from – 100 mm to + 100 mm and is mounted just touching the wire and in the same plane as the trunnion axis. The zero graduation coincides with the wire.

The theodolite is first pointed at a target, the telescope is then lowered to read the scale with the results given below. Determine the magnitude and sense of error in collimation of the theodolite and the inclination of the trunnion axis.

Target	Vertical Angle	Scale Reading
A	65°27′15″	+ 4.11 mm.
B	30°43′27″	– 6.24 mm.

(The error in horizontal circle reading caused by a trunnion axis misalignment t is $t \tan \alpha$ where α is the altitude). [Bradford]

Solution Let both the corrections to horizontal angle be positive. Then

1. Line of collimation bears to the right.

2. Trunnion axis is high on the left.

Let x be the angular collimation error.
e be the angular trunnion axis error

Then

$$e \tan \alpha_1 + x \sec \alpha_1 = -\frac{4.11}{20,000} \times 206265''$$

$$e \tan \alpha_2 + x \sec \alpha_2 = +\frac{6.24}{20,000} \times 206265''$$

Here

$$\alpha_1 = 65°27'15''$$
$$\tan \alpha_1 = 2.19$$
$$\sec \alpha_1 = 2.41$$
$$\alpha_2 = 30°43'27''$$
$$\tan \alpha_2 = 0.59$$
$$\sec \alpha_2 = 1.16$$

Therefore

$$2.19\, e + 2.41\, x = -42.39$$
$$0.59\, e + 1.16\, x = +64.35$$

Solving. $x = 147.62''$

Which is positive, hence collimation bears to the right.

$e = -181.74''$, hence trunnion axis is high on the right.

Theoretical portion has been covered in Section 10.8.

Example 10.3 If the horizontal axis of a theodolite makes an angle of $90° + \alpha$ with the vertical axis and if the instrument is otherwise in adjustment. show that the difference between circle left and circle right measurement of the horizontal angle subtended by two targets whose elevations are θ and ϕ above horizontal is $2\alpha(\tan\theta - \tan\phi)$. In a certain theodolite the horizontal axis is 0.025 mm out in 100 mm and the instrument is otherwise in correct adjustment. Find the difference. to the nearest second, between circle left and circle right values of the horizontal angle subtended by two targets whose elevations are 55°30′ and 22°00′. [London Univ.].

Solution For horizontal axis error α for each reading clockwise correction is {added/subtracted} if the axis is high on the {left/right} and its value is $\alpha \tan \theta$. Since in measurement, two sights corresponding to clockwise horizontal angles $\beta_1 < \beta_2$ will be taken at vertical angles θ and ϕ $\Delta_\beta = \delta\beta_1 - \delta\beta_2 = +\alpha(\tan\theta - \tan\phi)$ for face left reading when. say. axis is high on the left. On transiting for circle right reading axis is high on the right and correction

$$\delta\beta = \delta\beta_1 - \delta\beta_2 = -\alpha(\tan\theta - \tan\phi).$$

Hence on face left condition the observed reading will be correct value of the angle − $\alpha(\tan\theta - \tan\phi)$. Similarly for face right condition observed reading will be correct value + $\alpha(\tan\theta - \tan\phi)$.

Hence difference in reading $2\alpha(\tan\theta - \tan\phi)$.

Here $$\alpha \doteq \frac{0.025}{100} \times 206265''$$

$$\theta = 55°30' \quad \tan 55°30' = 1.455$$

$$\phi = 22°00', \quad \tan 22°00' = 0.404.$$

Hence $2\alpha(\tan\theta - \tan\phi)$

$$= 2 \times \frac{0.025 \times 206265}{100} \times (1.455 - 0.404)$$

$$= 108.39'' = 1'48.39''$$

Example 10.4 An angle of elevation was measured by vernier theodolite and it was noted that the altitude bubble was not in the centre of its run in either the face left or face right positions. Deduce the value of that angle from the data given below. *O* and *E* refer to the objective and eyepiece end respectively of the bubble, and one division of the altitude level is equivalent to 20 seconds.

[I. Struct. E]

Face	Vernier readings		Altitude level	
			O	*E*
Left	25°20'40"	25°21'00"	3.5 div.	2.5 div.
Right	20°21'00"	25°21'20"	4.5 div.	1.5 div.

Solution If the bubble is not central during observations (face left and face right) the apparent index error i is given by $(r_1 + r_2)/4$ where $r_1 = (a_1 - b_1)$ and $r_2 = (a_2 - b_2)$ a, b, being readings of the ends of the bubble. If α_1 and α_2 are the observed angles, the correct angle is

$$\alpha = \frac{1}{2}(\alpha_1 + \alpha_2) + \frac{\theta''}{4}(r_1 + r_2)$$

Here $\frac{1}{2}(\alpha_1 + \alpha_2) = \frac{1}{4}[25°20'40'' + 25°21'00'' + 25°21'00'' + 25°21'20'']$

$$= 25°21'00''$$

$$\frac{\theta''}{4}(r_1 + r_2) = \frac{20''}{4}(1 + 3) = 20''$$

Hence $$\alpha = 25°21'00'' + 20''$$

$$= 25°21'20''$$

PROBLEMS

10.1. (a) Define for a theodolite (i) Vertical axis, (ii) Bubble axis, (iii) Collimation axis, (iv) Horizontal axis.
(b) What relationships exist among the above principal axes of the theodolite
(c) Describe the 'spire test' for a theodolite explaining in detail the (i) Object, (ii) Necessity, (iii) Test (iv) Adjustment.
[AMIE Advanced Surveying Summer 1985]

10.2. (a) How is the principle of reversal applied while adjusting the axis of a plate bubble of a theodolite?
(b) Under what situation(s) can there be difference between the vernier readings of horizontal circles of a theodolite? How will you eliminate the error (s) in one or both of them?
(c) Bring out the difference in a theodolite, if any, between the (i) Horizontal axis and trunnion axis, (ii) Line of collimation and line of sight.
(d) What is index error in a theodolite? Briefly describe a method to remove it. [AMIE Advanced Surveying Winter 1985]

10.3. (a) What is the basic difference between temporary and permanent adjustments of a theodolite?
(b) Classify as temporary or permanent adjustment: (i) Focussing of eyepiece, (ii) Rendering trunnion axis horizontal, (iii) Centring a theodolite over ground mark, (iv) Bringing the image of an object exactly in the plane of diaphragm containing cross lines, (v) Making bubble axis horizontal, (vi) Adjusting vertical axes truly vertical.
(c) There are two vertical axes for the two horizontal plates top and bottom of a theodolite. Say what will happen and what you must do if the two vertical axes are: (i) Inclined to each other, (ii) Parallel to each other, (iii) Coincident.
(d) Why is modern theodolite called a transit in USA?
[AMIE Advanced Surveying Summer 1986]

10.4. (a) Bring out the main difference between the following pairs: (i) External and internal focussing telescopes. (ii) Keplar's and Galileo's types of telescopes. (iii) Transit and non-transit theodolites. (iv) Vernier and microptic theodolites.
(b) Explain the following misnomers: (i) The diaphragm of a surveyor's telescope is said to contain 'crosshairs' but there are no 'hairs'. (ii) The trunnion axis is also called 'horizontal axis' but then, it is not always 'horizontal' unless special efforts are periodically taken.
(c) Account for the following: (i) according to the principle of reversal, the apparent error on reversal is twice the real error, (say with reference to bubble axis of telescope) (ii) the error due to eccentricity of the verniers of a theodolite gets eliminated by averaging the verniers.
[AMIE Advanced Surveying Winter 1986]

10.5. (a) Mention the permanent adjustments of a common type of theodolite.
(b) Describe in detail the collimation adjustment of a transit.
[AMIE Advanced Surveying Winter 1987]

10.6. (a) Describe the method of repetition for measurement of horizontal angle theodolite.

(b) Explain the differences between, (i) Chain surveying and traverse surveying, (ii) Transitting and swinging, (iii) Free and Fast needle method of traversing. [AMIE Summer 1988]

10.7. Give a list of the permanent adjustments of a transit theodolite and state the object of each of the adjustment. Describe how you would make the trunnion axis perpendicular to the vertical axis.

[AMIE Advanced Surveying Summer 1988]

10.8. Differentiate between temporary and permanent adjustments of a vernier theodolite and name the temporary adjustments. Explain how you will carry out the adjustment of vernier theodolite for obtaining the relationship horizontal axis perpendicular to the vertical axis.

[AMIE Advanced Surveying Winter 1989]

10.9. (a) What are the permanent adjustments necessary in a vernier theodolite?

(b) Briefly discuss how (i) line of collimation and (ii) trunnion axis adjustments are made.

(c) What are the advantages of making 'face left' and 'face right' observations in the theodolite survey?

[AMIE Advanced Surveying Winter 1990]

10.10. (a) Describe the functions of the following parts of a theodolite: (i) Vernier (ii) Tangent screw (iii) Clip screw (iv) Tribrach plate (v) Foot screws (vi) Vertical circle.

(b) What are the basic differences between 'transit' and 'non-transit' theodolite?

(c) What is meant by 'face left', 'face right', 'swing left' and 'swing right' in theodolite operation.

(d) During a theodolite observation, if the 'crosshair' is not in its proper position, what error will occur? How would you bring the 'crosshair' to its proper positon? [AMIE Winter, 1979]

10.11. (a) Give a list of all permanent adjustments of a common type of theodolite.

(b) Describe the field operations for measuring a vertical angle when the available theodolite has perhaps a faulty trunnion (transit) axis.

[AMIE Advanced Surveying Winter 1979]

10.12. (a) Enumerate the permanent adjustments of a transit theodolite.

(b) What is the effect on an observed horizontal angle if the trunnion axis is not perpendicular to the vertical axis in a theodolite? Illustrate with a sketch.

(c) Explain how a transit theodolite is tested and if necessary adjusted so that it may be used to read vertical angles correctly.

[AMIE Advanced Surveying Winter 1980]

HINTS TO SELECTED QUESTIONS

10.2 (b) Difference in vernier readings can occur (i) if the centre of the graduated horizontal circle does not coincide with the centre of the vernier plate. Reading against either vernier will be incorrect. This can be eliminated by taking average of two readings. (ii) If there is imperfect graduations of the horizontal circle. This can be minimized by taking mean of several readings distributed over different portions of the horizontal circle. (iii) If the zeros of the vernier are not at the ends of the same diameter, this can be eliminated by taking mean of the two readings.

(c) (i) Trunnion axis of a theodolite is the axis about which the telescope and vertical circle rotate. It is the line passing through journals which fit into the bearings at the top of the standards. When this line is horizontal, it becomes the horizontal axis of the instrument. (ii) Line of sight is any line passing through the eyepiece and the optical centre of the objective of the telescope. Line of collimation is an imaginary particular line joining the intersection of the crosshairs of the diaphragm and the optical centre of the objective. This line should be perpendicular to the horizontal axis and should also be truly horizontal when the reading on the vertical circle is zero and the bubble on the telescope or on the vernier frame is at the centre of its run.

10.3. (a) Temporary adjustments are required to be made at each station before taking readings. Permanent adjustments which usually last for a long time put the fundamental lines, e.g. vertical axis, horizontal axis, plate level axis, etc. in proper relation to one another.

(b) (i) Temporary (ii) Permanent (iii) Temporary (iv) Temporary (v) Temporary (vi) Permanent.

(c) Non-parallelism of the two vertical axes is easily detected by carrying out the levelling up process round one of the axes and then rotating the instrument about the other. If the axes are not parallel the bubble will behave as if the instrument is not levelled.

If the axes are parallel but not coincident it leads to eccentricity. If the theodolite has two verniers placed at diametrically opposite points, the difference of the readings of the two will not remain constant but will vary periodically round the circle, if there is eccentricity. However mean of the two verniers will be free from eccentricity effect.

(d) The telescope in a modern theodolite can be rotated about its horizontal axis through a complete circle. This gives the name 'transit', the word 'transit' means to pass over or cross over and the line of sight of the transit can be made to cross over from one side to the other by rotating the telescope about its horizontal axis.

10.4. (b) (i) The crosshairs used in some surveying instruments are very fine threads taken from the cocoon of a brown spider. Many instrument makers use finely drawn platinum wires, some use fine glass threads and others use a glass diaphragm on which lines have been etched.

(ii) Trunnion axis is the axis about which the telescope rotates. This axis should be horizontal so that the telescope generates a vertical plane. Hence the name 'horizontal axis'.

10.5. (b) (ii) Transitting is the process of turning the telescope over its supporting axis through 180° in a vertical plane.

Swinging the telescope means turning the telescope in a horizontal plane. The swing is termed right or left accordingly as the telescope is rotated clockwise or anticlockwise.

10.9. (c) Face left and face right observations remove errors due to imperfect adjustments of (i) the horizontal axis or trunnion axis not being perpendicular to the vertical axis; (ii) the line of collimation not being perpendicular to the horizontal axis, (iii) the line of collimation not being parallel to the axis of altitude level or telescope level.

11

Traverse Survey and Computations

11.1 INTRODUCTION

A traverse is a series of connected lines whose lengths and directions are measured in the field. The survey performed to evaluate such field measurements is known as traversing.

There are two basic types of traverses: (i) open and (ii) closed. Both originate at a point of known location. An *open traverse* terminates at a point of unknown position. A *closed traverse* terminates at a point of known location. Figure 11.1 shows an open traverse. This is a typical layout for a highway or a pipe line. Figure 11.2 shows two closed traverses. In Fig. 11.2(a) *ABCDEF* represent a proposed highway route but the actual traverse begins at *A′* and ends at *F′*. This type of closed traverse is known as "geometrically open, mathematically closed". Figure 11.2(b) shows a traverse which covers a plot of land in the form *ABCDEA*. Note that the traverse originates and terminates at the same point.

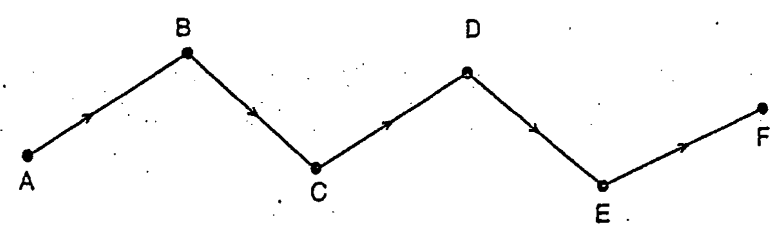


Fig. 11.1 Open traverse.

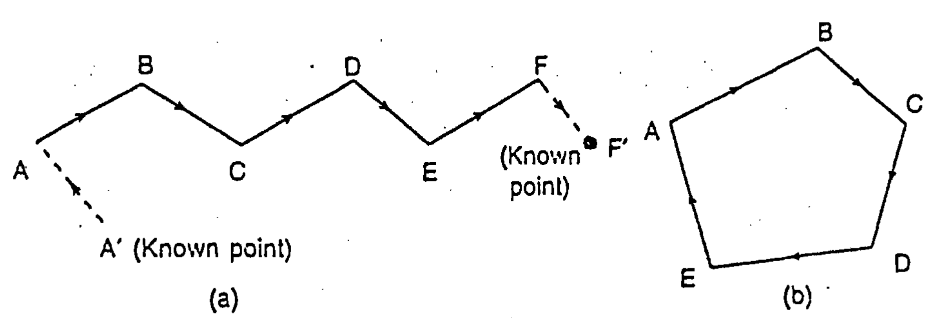


Fig. 11.2 Closed traverse.

This type of closed traverse is "geometrically and mathematically closed".

Traversing is used (i) to determine existing boundary lines, (ii) to calculate area within a boundary, (iii) to establish control points for mapping and also for photogrammetric work, (iv) to establish control points for calculating earth work quantities, and (v) for locating control points for railroads highways, and other construction work.

11.2 DEFICIENCIES OF OPEN TRAVERSE

An open traverse is usually run for preliminary survey. There is no arithmetical check for field measurements. Other deficiencies are:

(a) There is no check on summation of angles based on mathematical conditions.
(b) There is no check on position of intermediate points as there is no known or assumed position except the starting station.

The remedial steps are:

(a) Each distance should be measured in both directions and also should be roughly checked by using the stadia hairs of the theodolite.
(b) Angles should be measured by method of repetition and should also be checked by magnetic bearings.
(c) True azimuths or bearings of some of the lines should be determined with reference to the sun or stars depending on the importance of the survey.

In any case it is always desirable to avoid open traverse. Sometimes, it may be desirable to run a separate series of lines to close the traverse or to obtain coordinates of the starting and closing points by tying to marks of known positions.

11.3 CLOSED TRAVERSE

When a closed traverse originates and terminates at the same point and all the internal angles are measured we can utilize the mathematical condition that sum of the internal angles of a closed traverse is $(2n - 4)$ right angles where n is the number of sides. This affords a check on the accuracy of the measured angles. Moreover, by plotting the traverse or by mathematical calculations (to be explained later) it is possible to calculate the closing error which gives an indication of the accuracy of measurements. There is, however, no check on the systematic errors of measured length and hence, systematic errors should be detected and eliminated.

In case a traverse originates and closes on known points, there is check for both linear and angular measurements.

11.4 MEASUREMENT OF TRAVERSE ANGLES

Traverse angles can be (i) Interior angles, (ii) Deflection angles, (iii) Angles to the right, (iv) Azimuth angles, (v) Compass bearings.

Interior angles

Interior angles of a closed traverse should be measured either clockwise or anticlockwise. It is good practice, however, to measure all angles clockwise. Figure 11.3 shows measurement of interior angles.

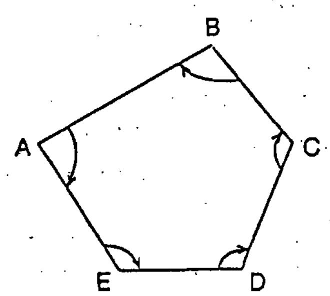


Fig. 11.3 Interior angles.

Deflection angles

Open traverses, e.g. route surveys are usually run by using deflection angles or angles to the right. A deflection angle is formed at a traverse station by an extension of the previous line and the succeeding one. The numerical value of a deflection angle must always be followed by *R* or *L* to indicate whether it was turned right or left from the previous traverse line extended. Figure 11.4 shows an open traverse and how a deflection angle is measured and noted.

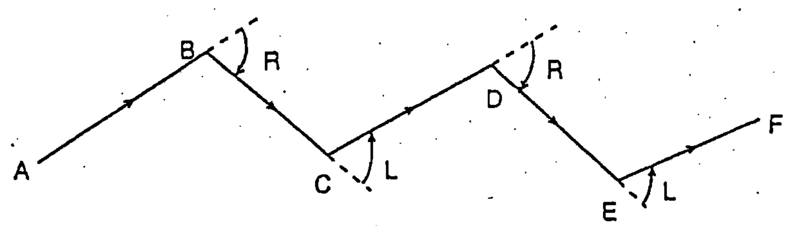


Fig. 11.4 Deflection angle.

Angles to the right

Angles measured clockwise from a backsight on the previous line are called angles to the right or azimuths from the backline. This can be used in both open or closed traverse. This is shown in Fig. 11.5. The angles can be improved by taking repeated readings and roughly checked by means of compass readings. Rotation should always be clockwise from the back sight. This conforms to the graduations of the scale in the theodolites which increase clockwise.

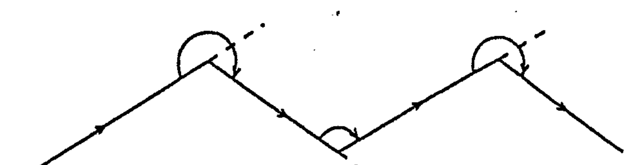


Fig. 11.5 Angles to the right.

Azimuth angles

A traverse can be run by reading azimuth angle directly. As shown in Fig. 11.6 azimuths are measured clockwise from the north end of the meridian through the angle points. At each station the transit is to be oriented by sighting the previous station with the back azimuth of the line as the scale reading.

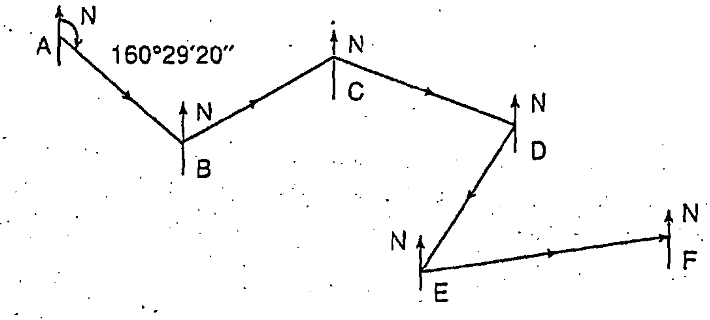


Fig. 11.6 Azimuth angles.

Suppose the azimuth of *AB* is 160°29′20″. The azimuth of *BA* is then 340°29′20″. Now if the theodolite with the reading 340°29′20″ is pointed towards *A* from *B*, the 0° reading will always point towards North. This pointing towards *A* should be adjusted by lower tangent screw when the reading will remain unchanged. If the upper screw is now unclamped and sight is taken along *C*, the clockwise circle reading will give the azimuth of *BC* directly. As a result it is not necessary to add or subtract angles to find the azimuth of a line. However, this method does not allow the use of double centring which eliminates most of the instrumental errors. The method, therefore, cannot be used where high precision is required.

Compass bearings

Here a compass can be used to get the bearings directly of all the lines of a traverse from which included angles can be obtained. However, the accuracy of a compass is very low and as such it is rarely used in a theodolite traverse. Sometimes, the theodolite is fitted with a compass. In such a case usually the bearing of the first line is measured with the help of the compass and all the angles are measured to obtain the bearing of the other lines. While the first method is known as loose needle method of bearings, the latter is known as fast needle method.

11.5 MEASUREMENT OF LENGTHS

Depending on the accuracy required, the length can be measured by chaining, taping, tacheometry or electronic distance measuring equipments. For low precision work chaining or tacheometry can be used. In tacheometry distance can be measured in both directions and average value taken.

11.6 SELECTION OF TRAVERSE STATIONS

Traverse station should be selected so that they facilitate the survey work. For property survey it will usually be a closed traverse with the traverse stations at the perimeter of the traverse. For route survey, it will, however, be an open traverse and the traverse stations are located at each angle point and at other important points along the centre line of the route. To improve precision, the length of the traverse lines should be long and number of stations minimum. The stations should be located on firm ground so that the instrument does not settle during taking of observations. The stations should be so located that from the traverse lines all the details of the area can be plotted.

11.6.1 MARKING AND REFERENCING OF TRAVERSE STATIONS

Traverse stations should be properly marked and referenced on the ground. Otherwise like benchmarks they will be lost. Usually a square wooden peg with a nail on the top is used as a traverse station. The top should be almost flush with the ground so that it is not knocked off. This is shown in Fig. 11.7(a). In Fig. 11.7(b) is shown a permanent station with a steel bolt fixed in a concrete block. For referencing a station it is measured with respect to three or more permanent objects as trees so that the point can be relocated as and when necessary, as shown in Fig. 11.7 (c).

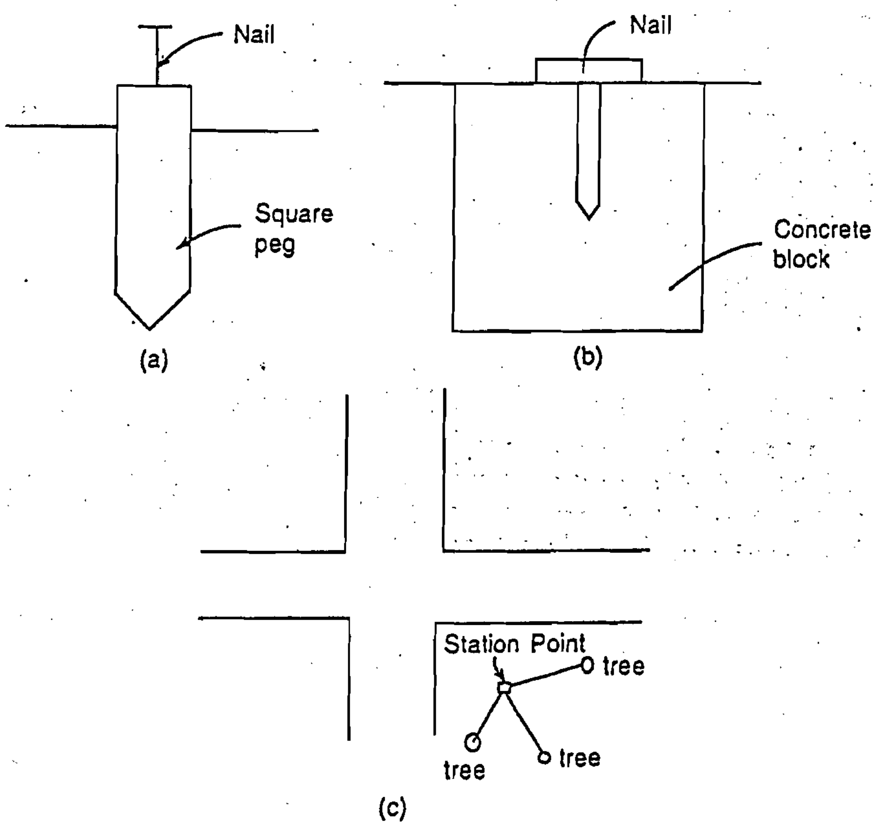


Fig. 11.7 (a), (b), (c) Marking and referencing a station.

11.7 ANGLE MISCLOSURE

For closed traverse, the sum of the interior angles should be equal to $(2n - 4)$ right angles where n is the number of sides. However, in practice, because of imperfections in equipment and errors made by surveyors the sum of the measured angles differ from the theoretical value. The permissible misclosure is based on the occurrence of random errors that may increase or decrease the sum of measured angles. It is given by the formula:

$$C = k\sqrt{n} \tag{11.1}$$

where n is the number of angles and k is a fraction of the least count of a transit vernier or smallest graduation of a theodolite scale. The fraction depends on the number of repetitions and the angular accuracy required. For ordinary theodolite traverse $k = 20''$ (least count of a theodolite). For a four sided traverse

$$C = \pm\, 20\sqrt{4} = \pm\, 40''.$$

Hence as the theoretical sum is 360°, the acceptable value will lie between 359° 59′20″ and 360°00′40″. If the angle misclosure is greater than the permissible value, the angles should be remeasured to get the acceptable value.

The algebraic sum of the deflection angles in a traverse is 360°, clockwise angles being termed positive and anticlockwise negative. This rule applies if the lines do not criss cross or cross an even number of times. When the lines in a traverse cross an odd number of times, the algebraic sum of deflection angles is zero. Whenever possible magnetic bearings of lines should be taken to act as a check on bearing computed from deflection angles. For a closed polygon traverse, the bearing of the first line should always be recomputed using the last angle as a check after progressing around the figure.

In an azimuth traverse, after starting from the initial station and going round the traverse, the initial station should be occupied again and azimuth of the initial line again measured. This should tally with the original azimuth.

11.7.1 BALANCING THE ANGLES OF A TRAVERSE

If the angle misclosure is within the allowable value, it should be distributed amongst the angles so that the sum is equal to the correct geometric total. Three methods of angle adjustments are: (i) arbitrary adjustments, (ii) average adjustments, and (iii) adjustments based on measuring conditions. For most ordinary traverses, the adjustment can be applied arbitrarily to one or more angles.

In the average adjustment method, the total misclosure is divided by the number of angles and applied to all the angles. However applying corrections equally sometimes give false impression of precision. For example, if the least count of the instrument is 20″ and 10″ correction is applied to all the angles, it will be inappropriate. Instead 20″ correction should be applied to one angle and no correction should be applied to the next angle.

Sometimes, it is possible to surmise that error has occurred in a particular angle due to adverse measuring conditions, e.g. obstructions in line of sight, making accurate sighting difficult. When two adjacent sides of a traverse are much shorter compared to other sides, error is more likely to occur in the angle containing the shorter sides. Hence larger correction should be applied to this angle. This is known as adjustments based on measuring conditions.

10.7.2 ANGLE DISTANCE RELATIONSHIP

Surveyors must strive to strike a balance in precision for angular and linear measurements. As the final error depends on error caused due to both angular and linear effects, it is no use making angular measurements very precise while not being able to maintain same precision in linear measurements. The relation between linear and angular measurements can be derived as follows:

Let θ be the angle measured and length l be the linear measurement to give the point P (Fig. 11.8). If there is angular error of $\delta\theta$, the point P shifts to P'. If there is linear error δl, the point P' shifts to give the final position P''.

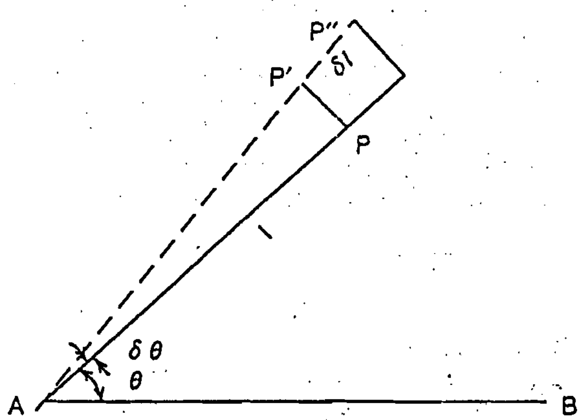


Fig. 11.8 Angle-distance relationship.

If the linear error is to be equal to the angular error

$$l\delta\theta = \delta l$$

or

$$\delta\theta = \frac{\delta l}{l}$$

If the angular measurement is made with an accuracy of 20″ (least count of an ordinary theodolite).

$$\frac{\delta l}{l} = 20''$$

$$= 20 \times \frac{1}{60 \times 60} \times \frac{\pi}{180} \text{ radian.}$$

$$= 9.6963 \times 10^{-5} = \frac{1}{10{,}300}$$

$$\approx \frac{1}{10{,}000}$$

This shows that with a 20″ theodolite, linear error is to be restricted to 1/10,000. Table 11.1 shows the compatible relation between angular error and linear error.

Table 11.1 Compatible Angular and Linear Error

Angular error	Linear error
05′	$\frac{1}{688}$
01′	$\frac{1}{3440}$
30″	$\frac{1}{6880}$
20″	$\frac{1}{10,300}$
10″	$\frac{1}{20,600}$
05″	$\frac{1}{41,200}$
01″	$\frac{1}{206,000}$

11.8 TRAVERSE BALANCING

Even if the angles of a traverse is balanced, if we try to plot the traverse either graphically or analytically by means of coordinates, it will not close as shown in Fig. 11.9. The amount by which it fails to close is known as closing error. The

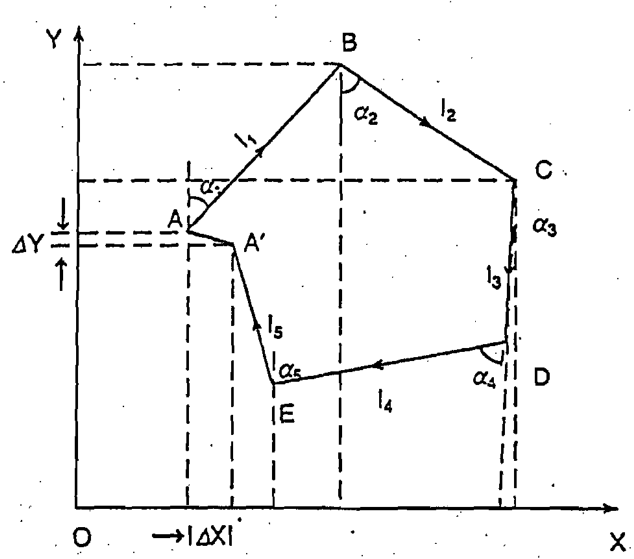


Fig. 11.9 Traverse balancing.

traverse will be balanced when the closing error is made zero. The latitude of a line is the distance it extends in the north or south direction. In terms of rectangular coordinates latitude is the Y coordinate of a line obtained by multiplying its length with cosine of the reduced bearing. From the Fig. 11.9 latitude of $AB = l_1 \cos \alpha_1$. Similarly for BC it is equal to $l_2 \cos \alpha_2$, and so on.

The positive direction of Y corresponds to North and hence the latitude of AB whose projection points towards the North is positive. Latitude of BC whose projection along the Y-Y axis points towards the South is negative. Departure of a line is its orthographic projection on the east-west axis of the survey. It is along the X-X axis of the rectangular coordinate survey and is found by taking sine of the reduced bearing of a side. For example, departure of $AB = l_1 \sin \alpha_1$, of $BC = l_2 \sin \alpha_2$ and so on. East departures are considered positive, west departures are negative. For a closed traverse with proper algebraic signs sums of both latitudes and departures must be zero. However, if the traverse does not close, the sum of the latitudes will not be zero, and there will be a small discrepancy in latitude known as latitude misclosure. Similarly, a small discrepancy in departure will be known as departure misclosure. From the figure it can be seen that latitude misclosure is ΔX and departure misclosure is ΔY and the misclosure in length $\Delta l = \sqrt{\Delta X^2 + \Delta Y^2}$.

If ΔX or ΔY is large it shows somewhere a mistake has been committed. Mistake may be in computation or in fieldwork. Fieldwork mistake may be in measuring angle or in length. By analysing the traverse closure it is sometimes possible to identify the line in which mistake has been committed. The ratio of the closure error to the perimeter of the traverse is known as closure precision. It is usually expressed in the form 1 in n, n will depend on the accuracy of the survey desired. The closing error is due to random errors rather than systematic error if the traverse begins or closes on the same point. However, if the traverse begins in a known point and closes at another known point, the closing error is due both to systematic error and random error.

11.9 CHECKS IN AN OPEN TRAVERSE

There is no reliable check in open traverse. As the traverse does not close, the mathematical condition of summation of angles cannot be applied. Except the starting station, other traverse stations cannot be checked as they are all unknown stations. To improve measurements of open traverse—(i) each distance should be measured in both directions and should be roughly checked using the stadia hairs of the theodolite, (ii) angles at the stations should be repeatedly measured by method of repetition, and as a further check magnetic bearings of lines should be observed, and (iii) the directions of some selected lines should be checked by astronomical observations, i.e. by observing the sun or stars.

The following checks can also be applied to part of the traverse.

Cut off lines method

The open traverse can be checked by running cut off lines between certain intermediate stations. In Fig. 11.10. AE is one cut off line, EJ is another. The

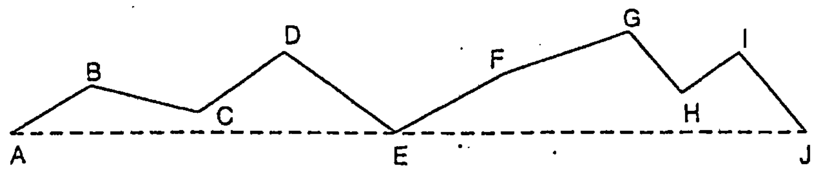


Fig. 11.10 Checking of traverse.

bearing of *AE* should be taken both at *A* and *E*. They should differ by 180°. The distance *AE* should be measured and checks for the closed traverse *ABCDE* should be applied. Similarly with the cut off line *EJ*.

Sighting a prominent object

A prominent object *O* is sighted from stations *A*, *D* and *G* of the traverse, i.e. bearings of *AO*, *DO* and *GO* are measured as also the lengths *AO*, *DO* and *GO* (Fig. 11.11). From the closed traverse *ABCDO*, coordinates of *O* can be computed. This should tally with the coordinates of *O* computed from the closed traverse *ODEFGO*.

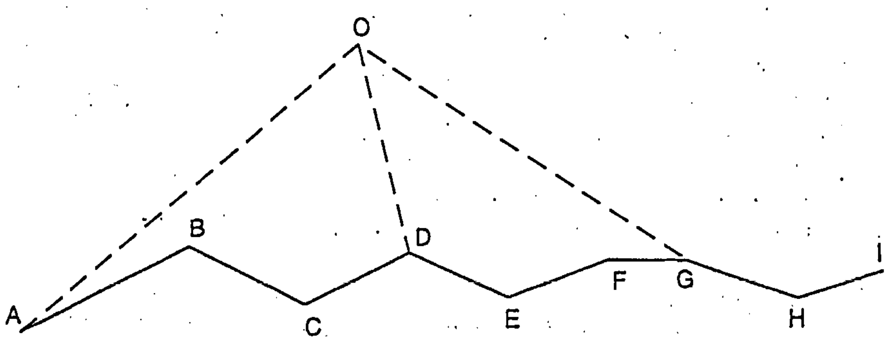


Fig. 11.11 Sighting a prominent object.

11.10 METHODS OF TRAVERSE ADJUSTMENTS

Before a traverse can be plotted, its closing error should be made zero. This is known as traverse adjustments. A traverse involves two types of measurements: (i) Measurement of length, and (ii) Measurement of angles.

We can have the following three basic conditions:

1. *The angular accuracy is higher than the linear accuracy* Such a case may occur in traversing in a hilly terrain if we measure angle by means of a theodolite which is expected to be accurate but measures distances horizontally by means of tapes. The accuracy of linear measurement will be low.

2. *The angular accuracy is the same as the linear accuracy* This is likely to occur when the angles are measured by means of theodolite but distances are measured with an EDM.

3. *The angular accuracy is lower than the linear accuracy* This is the condition when angles are measured with compass which gives less accurate values whreas lengths are measured by means of EDMs.

Depending on which category the traverse belongs appropriate methods should be applied for adjustment of the traverse.

Five basic methods of traverse adjustments are: (i) Arbitrary Method, (ii) Transit Rule, (iii) Compass or Bowditch Rule, (iv) Crandall Method, and (v) Least Square Method.

11.10.1 ARBITRARY METHOD

As the name suggests in this method there is no fixed rule for distributing the closing error in latitude and departure. However, engineering judgment is used. If there is reason to believe that because of field conditions or types of instruments used measurement of one line is less reliable than others, it would be reasonable to adjust only the latitude and departure of that line so that algebraic sums of latitudes and departures are made zero. This method of adjustment is very simple and in effect gives weightage to the expected accuracy of individual measurements.

11.10.2 TRANSIT RULE

In the transit rule the correction to latitude of a line is in proportion to the magnitude of latitude and departure correction is in proportion to the departure of the line. Symbolically,

$$\text{Adjustment in latitude of } AB = \frac{\text{Latitude of } AB \times \text{latitude misclosure}}{\text{Absolute sum of latitudes}}$$

$$\text{Adjustment in departure of } AB = \frac{\text{Departure of } AB \times \text{departure misclosure}}{\text{Absolute sum of departures}}$$

Theoretically, the transit rule should be used to balance a traverse where the angular measurements are more precise than the linear measurements. However, as different results are obtained for different meridians, this rule is seldom used.

11.10.3 COMPASS OR BOWDITCH RULE

Bowditch rule is used most frequently as this rule is applicable when the linear and angular measurements are equally precise. The probable error in linear measurement is taken to be equal to $\sqrt{l}$. Corrections are made by the following rules:

$$\text{Adjustment in latitude } AB = \frac{\text{length of } AB \times \text{latitude misclosure}}{\text{perimeter of traverse}}$$

$$\text{Adjustment in departure } AB = \frac{\text{length of } AB \times \text{departure misclosure}}{\text{perimeter of the traverse}}$$

The bearing of each line of the traverse will be altered after applying the Bowditch's rule. The method is most suitable for compass survey where the probable error of angular measurements and linear measurements tally. However, the method is most commonly used for an average engineering survey since (i) it is easy to apply (ii) The corrections do not alter the plottings significantly.

11.10.4 CRANDALL METHOD

In this method, it is assumed that linear measurements contain larger random errors than the angular measurements. Initially, the angle misclosure is distributed equally amongst all angles. The angles then remain unchanged. The linear measurements are then corrected by weighted least square procedure.

11.10.5 LEAST SQUARE METHOD

This is the most rigorous method of adjustment of traverse and is based on theory of errors developed in Chapter 2. By applying suitable weights difference in measurement accuracy of lengths and angles of a traverse can be taken into account. The adjustment is based on the principle of making the sum of the squares of the weighted residuals a minimum. Since large computation is involved, the method is computer based.

After the adjusted latitude and departure of a line has been determined, the new length and bearing of the line can be determined from the relation

$$L = \sqrt{(\text{latitude})^2 + (\text{departure})^2}$$

$$\text{Reduced bearing} = \tan^{-1}\left(\frac{\text{Departure}}{\text{Latitude}}\right)$$

The quadrant being obtained from the signs of latitude and departure.

11.11 RECTANGULAR COORDINATES

It is already seen from Section 11.8, that the rectangular coordinates are useful in computing the latitude and departure of a line and also the closing error of a traverse. Usually N–S line corresponds with the Y–Y axis and the East–West line with the X–X axis. The coordinates of the end point of a line with reference to its initial point are called consecutive coordinates or dependent coordinates of the end point of the line. The consecutive coordinates are equal to the latitudes and departures with proper signs. The coordinates of a point with respect to a common origin are known as independent coordinates of a point. They are also called total latitude or total departure of a point. Figure 11.9 is replotted in Fig. 11.12 with adjustment of closing error, i.e. closing error made zero. In that case A' will coincide with A.

The consecutive coordinates of

$$B = l_1 \cos \alpha_1,\ l_1 \sin \alpha_1$$

$$C = -\,l_2 \cos \alpha_2,\ l_2 \sin \alpha_2$$

and so on.

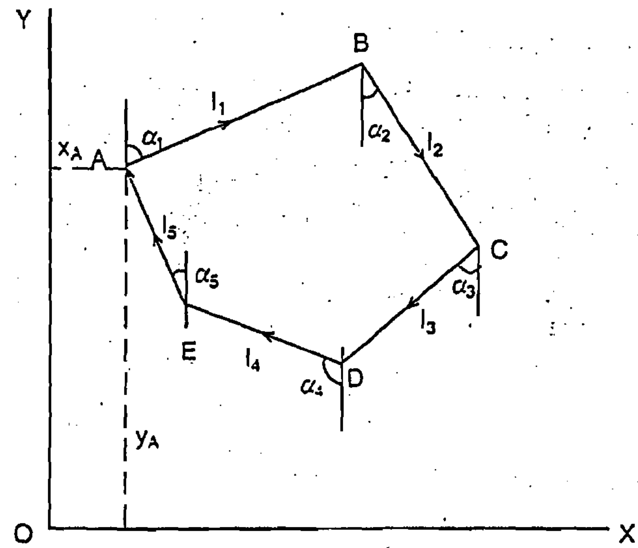


Fig 11.12 Coordinates of station points.

If the coordinates of A are x_A and y_A, the independent coordinates of B are $(y_A + l_1 \cos \alpha_1)$, $(x_A + l_1 \sin \alpha_1)$. The independent coordinates of $C = (y_A + l_1 \cos \alpha_1 - l_2 \cos \alpha_2)$ and $(x_A + l_1 \sin \alpha_1 + l_2 \sin \alpha_2)$ and so on.

Therefore total latitude of any point = Original latitude + algebraic sum of latitudes upto the point.

Total Departure = Original departure + algebraic sum of departures upto the point.

It is convenient to select the origin so that the whole of the traverse lies in the North East quadrant and all the points have positive independent coordinates as shown in Fig. 11.12. The rectangular coordinates are useful in (i) Calculating the length and bearing of a line, (ii) Calculating areas of the traverse, (iii) Making curve calculations, and (iv) Solving various problems of traverse calculations.

11.12 GALE'S TRAVERSE TABLE

In the field usually lengths and inward angles of a closed traverse are measured. In addition bearing of a line is taken. For adjustment of the traverse, the field data and computations are systematically recorded in a table known as *Gale's Traverse Table*. The steps involved are:

(a) Write the names of the traverse stations in column 1 of the table.

(b) Write the names of the traverse lines in column 2.

(c) Write the lengths of the various lines in column 3.

(d) Write the angles in column 4.

(e) Sum up all the angles and see whether they satisfy the geometric conditions as applicable, i.e. (i) sum of all interior angles = $(2n - 4)$ right angles, (ii) sum of all exterior angles = $(2n + 4)$ right angles.

(f) If not, adjust the discrepancy as shown in Section 11.7.1.

(g) Enter corrections in column 5.

(h) Write the corrected angles in column 6.

(i) Starting from the actual or assumed bearing of the initial line, calculate the whole circle bearings of all other lines from the corrected angles and enter in column 7.

(j) Convert the whole circle bearings to reduced bearings and enter in column 8.

(k) Enter the quadrants of the reduced bearings in column 9.

(l) Compute the latitudes and departures of the measured lines from lengths and bearings and put in proper columns 10, 11, 12 and 13 as applicable.

(m) Sum up the latitudes and departures to find the closing error.

(n) Calculate corrections by applying Transit rule or Bowditch's rule as desired.

(o) Enter the corrections in appropriate columns 14 to 17.

(p) Determine the corrected latitudes and departures and enter in appropriate columns 18 to 21. They will be corrected consecutive coordinates.

(q) Calculate the independent coordinates of all other points from the known or assumed independent coordinates of the first station. As a check the independent coordinates of the first point should be computed. It should tally with the known or assumed value.

All these details are shown by means of the example below.

Example 11.1 The mean observed internal angles and measured sides of a closed traverse *ABCDA* (in anticlockwise order) are as follows:

Angle	Observed value	Side	Measured length (m)
DAB	97°41′	*AB*	22.11
ABC	99°53′	*BC*	58.34
BCD	72°23′	*CD*	39.97
CDA	89°59′	*DA*	52.10

Adjust the angles, compute the latitudes and departures assuming that *D* is due *N* of *A*, adjust the traverse by the Bowditch method; and give the coordinates of *B*, *C*, and *D* relative to *A*. Assess the accuracy of these observations and justify your assessment. [I.C.E.]

Solution Solution is done with the help of Gales' Traverse Table. As *D* is due North of *A*, the approximate shape of the traverse *ABCDA* (in anticlockwise order) will be as shown in Fig. 11.13.

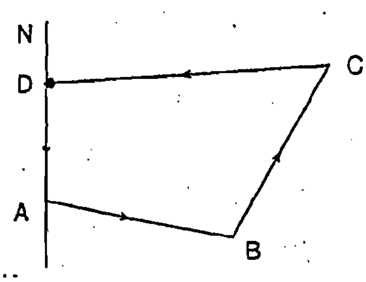


Fig. 11.13 Example 11.1.

Whole circle bearings of different lines can be computed after applying corrections to inward angles. Total discrepancy of inward angles is 4′ which is distributed equally amongst all the angles.

Computation of Bearings

Bearing of *AD*	=	0°0′00″
Bearing of *AB*	=	97°42′00″
		+ 180°00′00″
Bearing of *BA*	=	277°42′00″
∠*ABC*	=	99°54′00″
		377°36′00″
		− 360°00′00″
Bearing of *BC*	=	17°36′00″
Bearing of *CB*		197°36′00″
∠*BCD*		+ 72°24′00″
Bearing of *CD*		270°00′00″
		− 180°
		90°00′00″
		90°00′00″
Bearing of *BA*		180°00′00″

Total angular error is 4′. Since the angular reading is correct upto 1 minute, 4′ error is permissible in measurement of four angles at four stations. Rest of the calculations are shown in Gale's Traverse table (p. 256).

Example 11.2 The following lengths, latitudes, and departures were obtained for a closed traverse *ABCDEFA*.

	Length	Latitude	Departure
AB	183.79	0	+ 183.79
BC	160.02	+ 128.72	+ 98.05
CD	226.77	+ 177.76	− 140.85
DE	172.52	− 76.66	− 154.44
EF	177.09	− 177.09	0.00
FA	53.95	− 52.43	+ 13.08

Adjust the traverse by the Bowditch method. [L.U.B.Sc.]

Solution Computations are given in Table 11.3 (p. 256)

Table 11.2 Gale's Traverse Table (Example 11.1)

Station	Line	Length	Interior angles	Corrections	Corrected angles	Whole circle bearings	Reduced bearings	Quadrant	Consecutive Coordinates. Lat. N (+)	Lat. S (−)	Dep. E (+)	Dep. W (−)	Corrections (Bowditch Rule) Lat. +	Lat. −	Dep. +	Dep. −	Corrected Consecutive Coordinates Lat. N (+)	Lat. S (−)	Dep. E (+)	Dep. W (−)	Independent Coordinates	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
A			97°41′	+1′	97°42′																0.00	0.00
	AB	22.11				97°42′	82°18′00″	SE		2.96	21.91			−.07	+.05			3.03	21.96			
B			99°53′	+1′	99°54′																−3.03	+21.96
	BC	58.34				17°36′00″	17°36′00″	NE	55.61		17.64			−.18	+.14		55.43		17.78			
C			72°23′	+1′	72°24′																+52.40	+39.74
	CD	39.97				270°00′00″	90°00′00″	SW		0.00		39.97		−.13	+.10			0.13		39.87		
D			89°59′	+1′	90°00′																+52.27	−00.13
	DA	52.10				180°00′00″	00°00′00″	SE		52.10	0.00			−.17	+.13			52.27	00.13			
A																					00.00	00.00
Σ		172.52	359°56′	+4′	360°00′				55.61	55.06	39.55	39.97		−.55	+.42		55.43	55.43	39.87	39.87		

Σ Lat = + 0.55 Total closing error = $\sqrt{.55^2 + .42^2}$ = 0.69 m. Σ Dep = − 0.42. Precision = $\frac{172.52}{0.69}$ = 1 in 250.

Table 11.3 Example 11.2

Line	Length	Latitude +	Latitude −	Departure +	Departure −	Correction Latitude	Correction Departure	Corrected Latitude	Corrected Departure
AB	183.79	0.00		183.79		− 0.06	+ 0.07	− 0.06	+ 183.86
BC	160.02	128.72		98.05		− 0.05	+ 0.06	+ 128.67	+ 98.11
CD	226.77	177.76			140.85	− 0.07	+ 0.09	+ 177.69	−140.76
DE	172.52		76.66		154.44	− 0.05	+ 0.06	− 76.71	− 154.38
EF	177.09		177.09		0.00	− 0.05	+ 0.07	− 177.14	+ 0.07
FA	53.95		52.43	13.08		− 0.02	+ 0.02	− 52.45	+ 13.10
Σ	974.14	306.48	306.18	294.92	295.29	− 0.30	+ 0.37	00.00	00.00
				Σ latitude = + 0.30		Σ departure = − 0.37			

11.13 USE OF ANALYTICAL GEOMETRY IN SURVEY COMPUTATIONS

Since rectangular coordinates are used in computing latitude and departure of a line, rules of analytical geometry can be suitably applied in solving survey computational problems. Moreover, with computers they are highly efficient also. The following formulae of analytical geometry are useful in survey calculations.

1. Distance D between two points with rectangular coordinates (x_1, y_1) and (x_2, y_2).

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad (11.2)$$

2. Point (x, y) dividing the join of (x_1, y_1) and (x_2, y_2) in the ratio of $k : l$ is given by

$$x = \frac{lx_1 + kx_2}{l + k}, \quad y = \frac{ly_1 + ky_2}{l + k} \qquad (11.3)$$

where k and l have the same sign for internal division and opposite signs for external division.

3. The slope of a line (not parallel to the y-axis)

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} \qquad (11.4)$$

where θ is the inclination of the line with positive x-axis and (x_1, y_1) and (x_2, y_2) are any two points of the line (Fig. 11.14a).

4. The angle α, measured counter-clockwise from a line L_1 of slope m_1 to a line L_2 of slope m_2 is given by

$$\tan\alpha = \frac{m_2 - m_1}{1 + m_1 m_2} \qquad (11.5)$$

Two lines are parallel if $m_1 = m_2$ and perpendicular if $m_1 m_2 = -1$.

5. Area of a triangle joining three vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) in order is

$$\frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \qquad (11.6)$$

For polygon with n sides

$$\text{Area} = \frac{1}{2}\,[x_1(y_2 - y_n) + x_2(y_3 - y_1) + \ldots + x_{n-1}(y_n - y_{n-2}) + x_n(y_1 - y_{n-1})] \qquad (11.7)$$

6. Equations of a straight line are (Figs. 11.14b, c, d):

(a) Point-slope form; $y - y_1 = m(x - x_1)$ (11.8)

(b) Slope-intercept form, $y = mx + C$. (11.9)

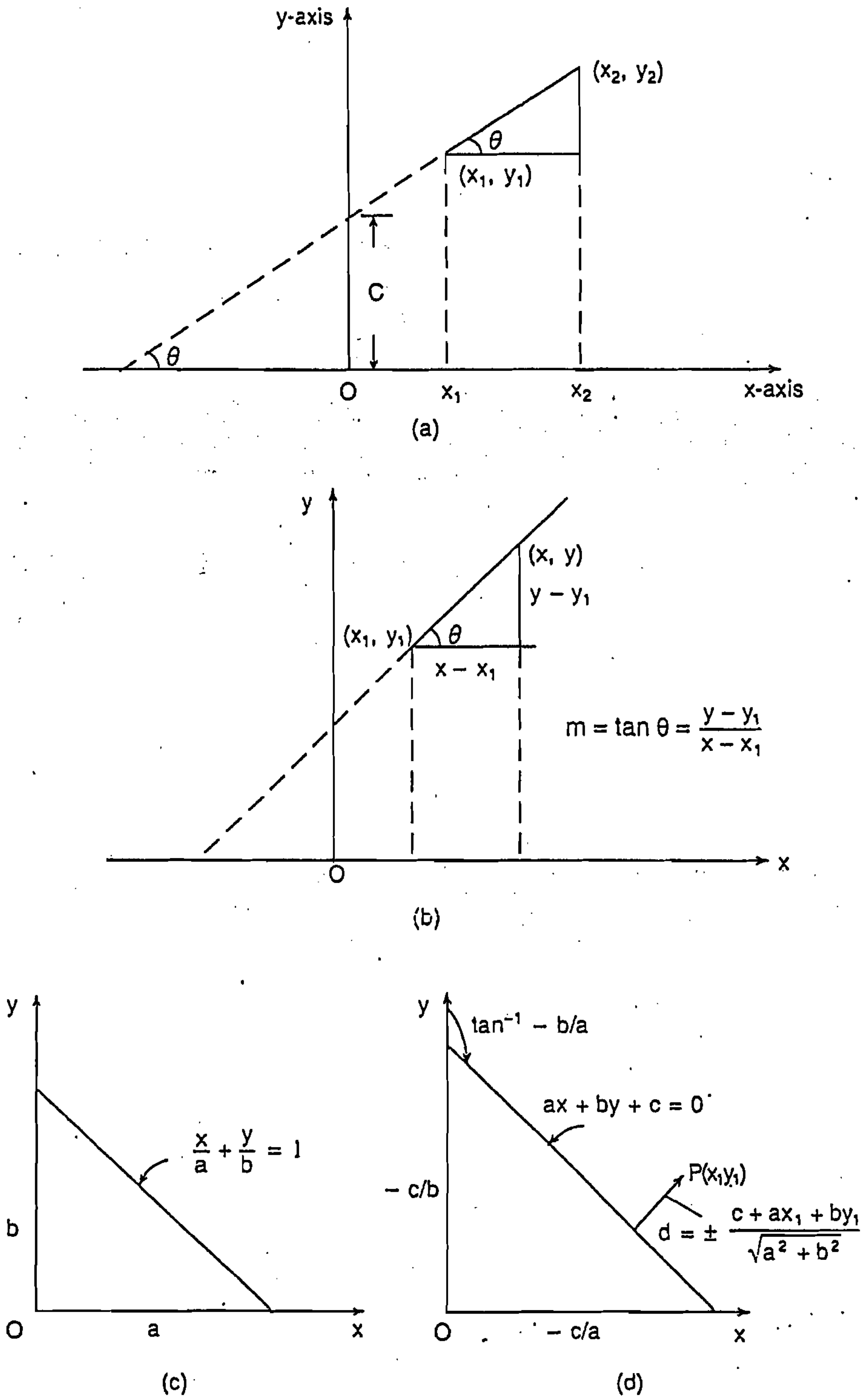


Fig. 11.14 Equations of a straight line.

(c) Intercept form $$\frac{x}{a} + \frac{y}{b} = 1. \tag{11.10}$$

(d) Normal form:

$$c > 0 \qquad \frac{c}{\sqrt{a^2 + b^2}} = \frac{-ax}{\sqrt{a^2 + b^2}} - \frac{by}{\sqrt{a^2 + b^2}}$$

$$c < 0 \qquad -\frac{c}{\sqrt{a^2 + b^2}} = \frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{a^2 + b^2}} \tag{11.11}$$

(e) General Equation: $ax + by + c = 0$ $(b \neq 0)$

$$\text{slope} = -a/b.$$

$$y \text{ intercept} = -c/b.$$

7. The distance of (x_1, y_1) from the line $ax + by + c = 0$ is given by

(i) $$c > 0 \qquad \pm\frac{c + ax_1 + by_1}{\sqrt{a^2 + b^2}}$$

according as (x_1, y_1) is on the origin or non-origin side of the line.

(ii) $$c < 0 \qquad \mp\frac{c + ax_1 + by_1}{\sqrt{a^2 + b^2}}$$

according as (x_1, y_1) is on the origin or non-origin side.

Example 11.3 The following traverse was run from station I to station V between which there occur certain obstacles.

Line	Length (m)	Bearing
I–II	351.3	N 82°28′E.
II–III	149.3	N 30°41′E.
III–IV	447.3	S 81°43′E.
IV–V	213.3	S 86°21′E.

It is required to peg the midpoint of I–V. Calculate the length and bearing of a line from station III to the required point. [I.C.E.]

Solution The solution is given in the tabular form below.

Coordinates of midpoint of I–V, $\frac{96.27}{2}, \frac{1079.97}{2}$ or 48.14, 539.99

Coordinates of III 174.42, 424.49

Length of the line = 171.13 m.

Table 11.4 Example 11.3

Point	Line	Length	Bearing	Latitude		Departure		Independent coordinates	
				N	S	E	W		
I								0	0
	I–II	351.3	N 82°28′E	46.04		348.27			
II								46.04	348.27
	II–III	149.3	N 30°41′E	128.38		76.22			
III								174.42	424.49
	III–IV	447.3	S 81°43′E		64.57	442.61			
IV								109.85	867.10
	IV–V	213.3	S 86°21′E		13.58	212.87			
V								96.27	1079.97

$$\theta \text{ with north} = \tan^{-1} \frac{424.49 - 539.99}{174.42 - 48.14}$$

$$= \tan^{-1} - \frac{115.50}{126.28} = - 42°26'41''$$

Hence reduced bearing of the line = S 42°26′41″E (Fig. 11.15).

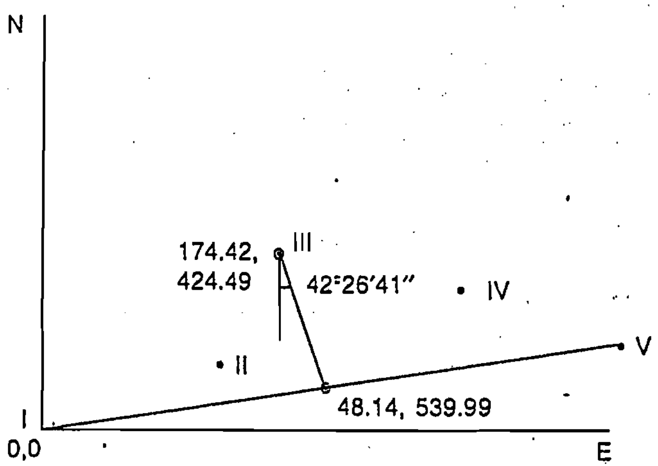


Fig. 11.15 Example 11.3

Example 11.4 It is proposed to extend a straight road *AB* in the direction of *AB* produced. The centre line of the extension passes through a small farm and in order to obtain the centre line of the road beyond the farm a traverse is run from *B* to a point *C*, where *A*, *B*, and *C* lie in the same straight line. The following angles and distances were recorded, the angles being measured clockwise from the back to the forward station.

$ABD = 87°42'$ $\quad BD = 29.02$ m

$BDE = 282°36'$ $\quad DE = 77.14$ m

$DEC = 291°06'$

Calculate (a) length of line *EC*; (b) angle to be measured at *C* so that the centre line of the road can be extended beyond *C*; (c) chainage of *C* taking the chainage of *A* as zero and *AB* = 110.34 m. [L.U.]

Solution Figure 11.16 gives a graphical presentation of the problem. Since *BDEC* is a closed traverse, projections of lines along and perpendicular to line *BC* will be zero. From the figure,

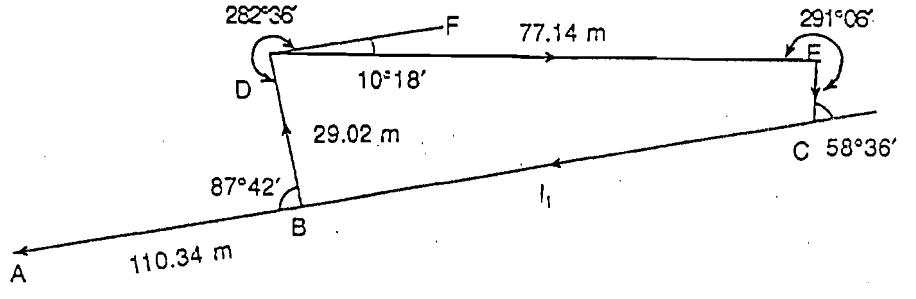


Fig. 11.16 Example 11.4.

$$\angle EDF = 87°42' - (360° - 282°36')$$
$$= 10°18'$$
$$\angle DEC = 360° - 291°06'$$
$$= 68°54'$$
$$\angle ECB = 360° - (77°24' + 68°54' + 92°18')$$
$$= 121°24'$$

Angle to be measured at *C* for prolongation of $BC = 180° - 121°24' = 58°36'$
Projecting along *BC*.

$$-29.02 \cos 87°42' + 77.14 \cos 10°18' - l_2 \cos 58°36' - l_1 = 0$$

Projecting perpendicular to *BC*.

$$29.02 \sin 87°42' - 77.14 \sin 10°18' - l_2 \sin 58°36' = 0$$

hence
$$l_2 = \frac{29.02 \sin 87°42' - 77.14 \sin 10°18'}{\sin 58°36'} = 17.82 \text{ m}$$

$$l_1 = 77.14 \cos 10°18' - 17.82 \cos 58°36' - 29.02 \cos 87°42'$$
$$= 65.71 \text{ m.}$$

Chainage of C = chainage of $A + AB + BC$

$$= 0.00 + 110.34 + 65.71 = 176.05 \text{ m.}$$

Example 11.5 A traverse *TABP* was run between the fixed stations *T* and *P* of which the coordinates are:

	E	N
T	+ 6155.04	+ 9091.73
P	+ 6349.48	+ 9385.14

The coordinate differences for the traverse legs and the data from which they are calculated are:

	Length	Adjusted Bearing	ΔE	ΔN
TA	354.40	210°41′40″	− 180.91	− 304.75
AB	275.82	50°28′30″	+ 212.75	+ 175.54
BP	453.03	20°59′50″	+ 162.33	+ 422.95

Applying the Bowditch's rule, calculate the coordinates of *A* and *B*.

Solution

Correct difference of

Easting between *T* and *P* = 6349.48 − 6155.04 = 194.44

Correct difference of Northing = 9385.14 − 9091.73 = 293.41

Observed difference of Easting = 212.75 + 162.33 − 180.91 = 194.17 m.

Observed difference of Northing = − 304.75 + 175.54 + 422.95 = 293.74 m

Closing error in Easting = 194.44 − 194.17 = + 0.27

Closing error in Northing = 293.41 − 293.74 = − 0.33.

Perimeter of the traverse = 1083.25.

Corrected ΔE

	ΔE	Correction	Corrected Value
TA	− 180.91	+ 0.09	− 180.82
AB	+ 212.75	+ 0.07	+ 212.82
BP	+ 162.33	+ 0.11	+ 162.44

Corrected ΔN

	ΔN	Correction	Corrected Value
TA	− 304.75	− 0.11	− 304.86
AB	+ 175.54	− 0.08	+ 175.46
BP	+ 422.95	− 0.14	+ 422.81

Coordinates of *A*

$$+\,6155.04 - 180.82 = +\,5974.22 \text{ E}$$
$$+\,9091.73 - 304.86 = +\,8786.87 \text{ N}$$

Coordinates of *B*

$$+\,5974.22 + 212.82 = +\,6187.04 \text{ E}$$
$$+\,8786.87 + 175.46 = +\,8962.33 \text{ N}$$

Check

Coordinates of *P*

$$+\,6187.04 + 162.44 = +\,6349.48 \text{ E}$$
$$+\,8962.33 + 422.81 = 9385.14 \text{ N}$$

Example 11.6 The coordinates of four points *A*, *B*, *C*, *D* are given below. Find the coordinates of the point of intersection of the line joining *A* and *D* with the line joining *B* and *C*.

Point	x	y
A	410.26	605.28
B	408.20	644.52
C	402.34	595.06
D	361.50	571.46

Solution Equation of the straight line *AD*

$$\frac{x - 410.26}{410.26 - 361.50} = \frac{y - 605.28}{605.28 - 571.46}$$

Equation of the straight line *BC*

$$\frac{x - 408.20}{408.20 - 402.34} = \frac{y - 644.52}{644.52 - 595.06}$$

Putting $x - 400 = x'$

and $y - 600 = y'$

the equations reduce to

$$\frac{x' - 10.26}{y' - 5.28} = \frac{48.76}{33.82} = 1.44$$

$$x' - 10.26 = 1.44y' - 7.60$$

$$x' - 1.44y' = 2.66$$

Similarly $$\frac{x' - 8.20}{5.86} = \frac{y' - 44.52}{49.46}$$

or $$x' - 8.20 = 0.118y' - 5.25$$

or $$x' - 0.118y' = 8.20 - 5.25 = 2.95$$

Solving $$x' = 2.975$$

$$y' = 0.219$$

giving $$y = 600.219$$
$$x = 402.975$$

Example 11.7 The coordinates of the three points *C*, *D* and *P* are given below.

Point	*x*	*y*
C	402.34 m	595.06 m
D	361.50 m	571.46 m
P	375.20 m	580.22 m

Find (i) the length of the line *CD*
(ii) Equation of the line *CD* and points at which it cuts the axis.
(iii) The length of the perpendicular dropped from *P* on the straight line *CD*.

Solution

(i) Length of line $CD = \sqrt{(595.06 - 402.34)^2 + (571.46 - 361.50)^2}$

$$= 284.99 \text{ m.}$$

(ii) Equation of a line in intercept form

$$\frac{x}{a} + \frac{y}{b} = 1.$$

where *a* and *b* are the intercepts on the axes

$$\frac{402.34}{a} + \frac{595.06}{b} = 1$$

$$\frac{361.50}{a} + \frac{571.46}{b} = 1$$

or $$\frac{1}{b}(1.478 - 1.580) = -2.8079 \times 10^{-4}$$

or $$b = 363.26 \text{ m.}$$

$$a = -630.52 \text{ m.}$$

Hence the equation of the line

$$-\frac{x}{630.52} + \frac{y}{363.26} = 1$$

It cuts the *x*-axis at (– 630.52, 0) and *y*-axis at (0, 363.26).

(iii) The equation of the line in the form $ax + by + c = 0$ is,

$$+ 363.26x - 630.52y + 229042.7 = 0$$

Equation in the normal form

$$\frac{363.26}{-\sqrt{363.26^2 + 630.52^2}}x - \frac{630.52}{-\sqrt{363.26^2 + 630.52^2}}y$$

$$+ \frac{229042.7}{-\sqrt{363.26^2 + 630.52^2}} = 0$$

The algebraic sign of the radical in the denominator is chosen to be the same as that of b.

Equation of the straight line then becomes

$$-0.4992x + 0.8665y - 314.7589 = 0$$

Substituting the coordinates of $P(375.20, 580.22)$ in the above form we get the distance of the point P from the straight line CD. Hence the distance is

$$-0.4992(375.20) + 0.8665(580.22) - 314.7589 = 0.7019 \text{ m}.$$

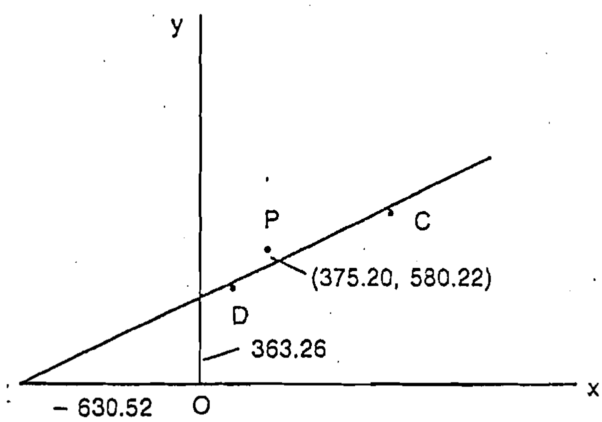


Fig. 11.17 Example 11.7.

Here c is negative and the point P is on the non-origin side. Hence the distance is positive.

Example 11.8 The following are the latitudes and departures of a series of survey lines in meters.

Line	Latitude in m	Departure in m
AB	99.405	298.095
BC	195.375	70.110
CD	154.840	119.520
DE	41.760	129.540

A line is to be set out from station E on a bearing of 342°. Calculate at which distance from A it intersects line AB. [AMIE, Nov. 1964]

Solution Taking coordinates of A as (0,0) coordinates of E

$$y \text{ coordinate} = 99.405 + 195.375 + 154.840 + 41.760 = 491.380$$

$$x \text{ coordinate} = 298.095 + 70.110 + 119.520 + 129.540 = 617.265 \text{ m}$$

The bearing of the line set out from E = 342°.
The equation of a line in point slope form,

$$y - y_1 = m(x - x_1).$$

$m = \tan\theta$ where θ is the angle with positive x axis. $\tan\theta$ is positive for acute angle and negative for obtuse angle. Here $\theta = 90° + 18° = 108°$ and $\tan\theta = -3.077$. As the line passes through the point (491.380, 617.265) the equation of the line

$$y - 491.380 = -3.077\,(x - 617.265)$$

$$= -3.077x + 1899.324.$$

Equation of the line AB

$$\frac{y - 99.405}{x - 298.095} = \frac{99.405}{298.095} = 0.3334$$

or $$y - 99.405 = 0.3334(x - 298.095)$$

$$= 0.3334x - 99.3764$$

Solving $$x = 700.994 \text{ m}$$

$$y = 233.740 \text{ m}$$

Distance from point $A = \sqrt{700.994^2 + 233.740^2} = 738.936$ m

11.14 PROBLEMS OF OMITTED MEASUREMENTS

Closed traverse is always preferable as they provide necessary checks. For such a traverse algebraic sum of latitude is zero. Similarly, algebraic sum of departures is also zero. Symbolically,

$$\sum L = 0 \quad \sum D = 0$$

or $$l_1 \cos\theta_1 + l_2 \cos\theta_2 + \ldots + l_n \cos\theta_n = 0.$$

and $$l_1 \sin\theta_1 + l_2 \sin\theta_2 + \ldots + l_n \sin\theta_n = 0.$$

where l is the length and θ the reduced bearing of a line.

With these two equations we can solve two unknowns or missing quantities. The unknowns may be lengths only, bearings only or length and bearing together. The following cases may occur

1. Bearing or length or both of one side omitted (Fig. 11.18). In the closed traverse *ABCDEA*, if the length, bearing, or both of the side *EA* has not been measured, they can be computed utilizing the two conditions of a closed traverse, i.e. $\sum D = 0$ and $\sum L = 0$.

2. Length of one side and bearing of an adjacent side omitted. This is shown in Fig. 11.19.

Here length of side *EA* and the bearing of adjacent side *AB* is unknown. Join *BE*. *BCDE* is now a closed traverse and therefore, length and bearing of *BE* can be determined. With known bearings of *BE* and *EA*, the inward angle α can be determined. Applying sine principle to triangle *ABE*, we can write

$$\frac{AB}{\sin\alpha} = \frac{BE}{\sin\gamma}$$

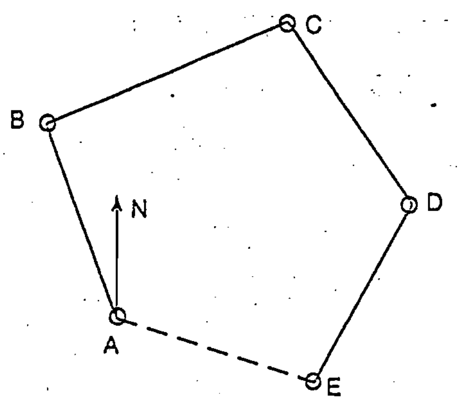


Fig. 11.18 Bearing or length omitted.

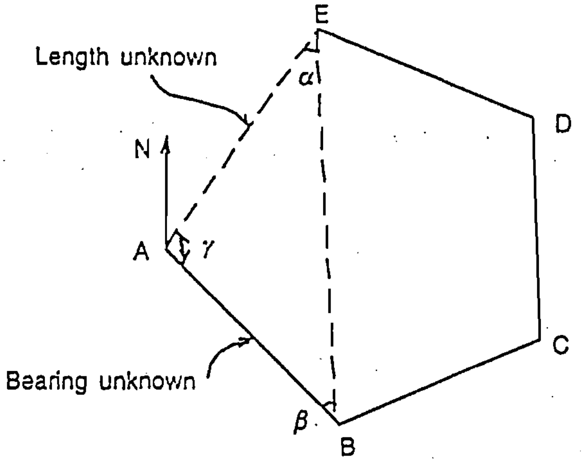


Fig. 11.19 Length and bearing omitted.

Hence γ can be determined.
Bearing of AB = Bearing of $AE + \gamma$.
With γ known, $\beta = 180° - (\alpha + \gamma)$

and
$$AE = \frac{\sin \beta}{\sin \gamma} BE.$$

Frequently, there will be two solutions and approximate shape of the figure must then be known.

3. Lengths of two adjacent sides omitted. Here lengths of AE and AB are unknown. From the conditions of closed figure $\sum L = 0$ and $\sum D = 0$, two algebraic equations involving l_1 and l_2 can be obtained and solving them simultaneously l_1 and l_2 can be found out. However, as before, length and bearing of BE can be found out (Fig. 11.20).

Since the bearings of BA, AE and BE are known, internal angles α, β, and γ can be computed. Applying sine rule,

$$\frac{AB}{\sin \alpha} = \frac{BE}{\sin \gamma} = \frac{AE}{\sin \beta}.$$

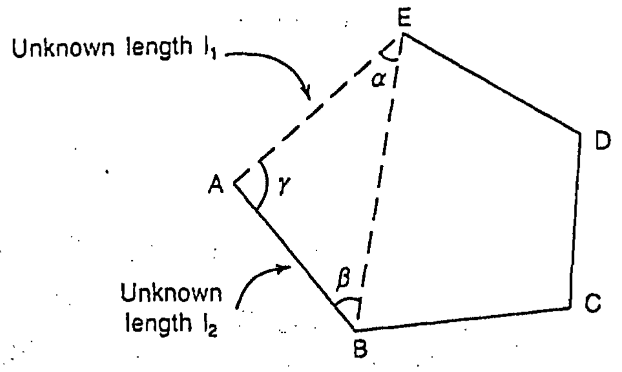


Fig. 11.20 Two lengths omitted.

or
$$AB = BE \frac{\sin \alpha}{\sin \gamma}$$

and
$$AE = BE \frac{\sin \beta}{\sin \gamma}$$

4. Bearings of two adjacent sides omitted. Suppose the bearings of AE and AB have not been determined. As before length and bearing of BE are known. In the triangle ABE lengths of the sides are known. Area of the triangle in terms of sides.

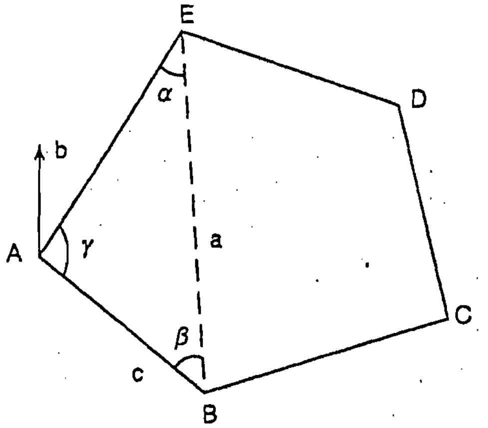


Fig. 11.21 Bearings.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where
$$s = \frac{a+b+c}{2}$$

The angles α, β and γ can be obtained from the relation

$$A = \frac{1}{2} cb \sin \gamma = \frac{1}{2} ca \sin \beta = \frac{1}{2} ab \sin \alpha.$$

5. When the two affected sides are not adjacent: In Fig. 11.22(a), the omitted

measurements are line *AB* and *CD*. Omissions may be in lengths, bearings or in both. Since the latitudes and departures of equal parallel lines are equal, this problem can be solved by shifting the line *CD* until it is adjacent to *AB*, so as to form the closed figure shown in Fig. 11.22(b). From the known sides *AF*, *FE*, *ED* and *DD'*, the length and bearing of *AD'* can be determined. Then this becomes a problem of omitted measurements of adjacent sides and can be solved as explained before.

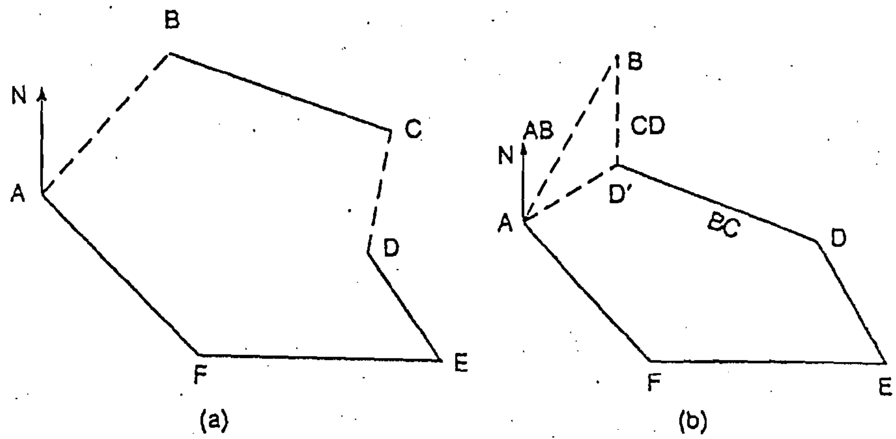


Fig. 11.22 Two non-adjacent sides omitted.

Example 11.9 An open traverse was run from *A* to *E* in order to obtain the length and bearings of the line *AE* which could not be measured direct with the following results:

Line	*AB*	*BC*	*CD*	*DE*
Length	102.5	108.7	92.5	125.0
W.C.B	261°41'	9°06'	282°22'	71°30'

Find by calculation the required information. [L.U.]

Solution This is a case of length and bearing of one line missing. The solution is done in tabular form given below.

Table 11.5 Example 11.9

Line	Length	W.C.B	R.B.	Northing	Southing	Easting	Westing
AB	102.5	261°41'	S81°41'W		14.796		101.426
BC	108.7	9°06'	N9°06'E	107.331		17.190	
CD	92.5	282°22'	N 77°38'W	19.815			90.352
DE	125.0	71°30'	N 71°30'E	39.663		118.540	
Σ				166.809	14.796	135.730	191.778

$$\text{Diff of North-South} = 152.013 \text{ m}$$

$$\text{Diff of East-West} = 56.048 \text{ m}$$

$$\text{Length of } AE = \sqrt{152.013^2 + 56.048^2} = 162.016 \text{ m}$$

$$\text{Reduced bearing of } AE = \tan^{-1} \frac{56.048}{152.013} = 20°13'12''$$

Bearing of *EA* is in South and East quadrant. Hence bearing of *AE* is in North and West quadrant.

Hence whole circle bearing of *AE* is $360° - 20°13'12'' = 339°46'48''$

Example 11.10 A clockwise traverse *ABCDEA* was surveyed with the following results:

AB	101.01 m	$\angle BAE = 128°10'20''$	$\angle DCB = 84°18'10''$
BC	140.24 m		
CD	99.27 m	$\angle CBA = 102°04'30''$	$\angle EDC = 121°30'30''$

The angle *AED* and the sides *DE* and *EA* could not be measured directly. Assuming no error in survey, find the missing lengths and their bearings if *AB* is due North. [L.U.]

Solution To obtain the angle *AED*, we have,

$$\angle AED = (2 \times 5 - 4)90° - [128°10'20'' + 84°18'10'' + 102°04'30'' + 121°30'30'']$$

$$= 540°00'00'' - 436°3'30''$$

$$= 103°56'30''$$

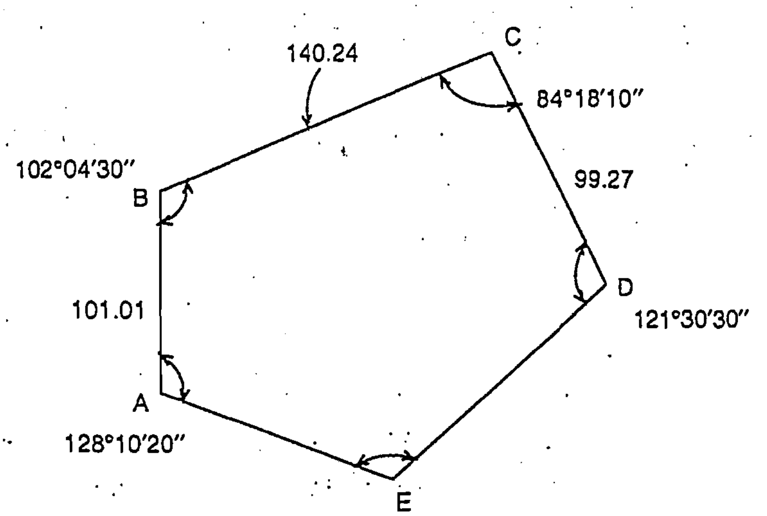


Fig. 11.23 Example 11.10.

The problem reduces to that of omitted measurements of two adjacent sides. The length and bearing of AD is determined from the table:

Table 11.6 Example 11.10

Line	Length	W.C.B	R.B.	North	South	East	West
AB	101.01	0°00′00″	N 0°00′00″ E	101.01			
BC	140.24	77°55′30″	N 77°55′30″E	29.34		137.14	
CD	99.27	173°37′20″	56°22′40″ E		98.66	11.03	
Σ				130.35	98.66	148.17	—

Difference of North and South = 31.69.

Difference of East and West = 148.17.

$$\text{Length of } AD = \sqrt{31.69^2 + 148.17^2}$$

$$= 151.52 \text{ m}$$

$$\text{Reduced bearing of } AD = \tan^{-1} \frac{148.17}{31.69}$$

$$= \text{N } 77°55'48''\text{E.}$$

Bearing of DA is in South and West quadrant and hence bearing of AD is in North and East quadrant.

$$\text{Whole circle bearing of } DE = 173°37'20'' + (180° - 121°30'30'')$$

$$= 232°06'50''$$

$$\text{Whole circle bearing of } DA = 257°55'48''$$

$$\angle ADE = 25°48'58''$$

$$\text{Whole circle bearing of } AE = 128°10'20''$$

$$\text{Whole circle bearing of } AD = 77°55'48''$$

$$\therefore \angle DAE = 50°14'32''$$

$$\text{Hence} \quad \angle AED = 180° - (25°48'58'' + 50°14'32'')$$

$$= 103°56'30''$$

Applying sine rule

$$\frac{151.52}{\sin 103°56'30''} = \frac{AE}{\sin 25°48'58''} = \frac{DE}{\sin 50°14'32''}$$

or

$$AE = \frac{151.52}{\sin 103°56'30''} \sin 25°48'58''$$

$$= 67.99 \text{ m.}$$

$$DE = \frac{151.52}{\sin 103°56'30''} \sin 50°14'32''$$

$$= 120.01 \text{ m}$$

Bearing of AE = 128°10′20″

Bearing of EA = + 180°00′00″

308°10′20″

Bearing of DE = 232°06′50″

Example 11.11 In a traverse the following lengths and bearings were measured.

Side	Length (m)	Bearing
AB	130.00	N 38°42′ W
BC	180.00	N 45°30′ E
CD	163.00	N 62°34′ E
DE	180.00	—
EA	—	S 75°00′ W

Compute the missing length and bearing.

Solution This is a case of omitted measurements of adjacent sides. Here bearing of DE and length of EA are missing (Fig. 11.24).

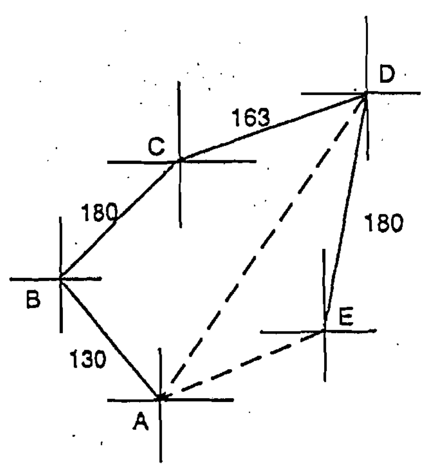


Fig. 11.24 Example 11.11.

Northing of AD = 130 cos 38°42′ + 180 cos 45°30′ + 163 cos 62°34′

= 101.46 + 126.16 + 75.10

= 302.72 m.

$$\text{Easting of } AD = -130 \sin 38°42' + 180 \sin 45°30' + 163 \sin 62°34'$$
$$= -81.28 + 128.39 + 144.67$$
$$= 191.78$$

$$\text{Length of } AD = 358.36 \text{ m}$$

$$\text{Bearing of } AD = \tan^{-1} \frac{191.78}{302.72}$$
$$= \text{N } 32°21°00''\text{E}$$

$$\text{Bearing of } AE = 75°00'00''$$
$$\text{Bearing of } AD = 32°21'00''$$
$$\angle DAE = 42°39'00''$$

Applying sine principle

$$\frac{AD}{\sin AED} = \frac{DE}{\sin DAE} = \frac{AE}{\sin ADE}$$

$$\sin AED = \frac{AD \cdot \sin DAE}{DE}$$
$$= \frac{358.36}{180} \sin 42°39'40''$$
$$= 0.914$$

$$\angle AED = 66°06'00''$$
$$\angle ADE = 180° - (42°39' + 66°06')$$
$$= 71°15'$$

$$AE = \frac{358.36}{0.914} \sin 71°15'$$
$$= 371.27 \text{ m}$$

$$\text{Bearing of } DA = 32°21'00'' + 180°00'00''$$
$$= 212°21'00''$$
$$\angle ADE = 71°15'00''$$
$$\text{Bearing of } DE = 141°06'00''$$

Example 11.12 In a traverse the following lengths and bearings were measured:

Side	Length (m)	Bearing	Side	Length	Bearing
AB	—	N 30°30′ E	*DE*	—	S 20°15′ E
BC	140 m	S 80°15′ E	*EF*	155 m	N 85°30′ W
CD	185 m	S 15°15′ W	*FA*	115 m	N 18°12′ W

Compute the missing sides.

Solution This is a problem where length of two non-adjacent sides of the closed traverse are missing. A rough sketch of the closed traverse is shown in Fig. 11.25 (a).

By moving parallely, the two non-adjacent sides *AB* and *DE* are made adjacent, (Fig. 11.25b).

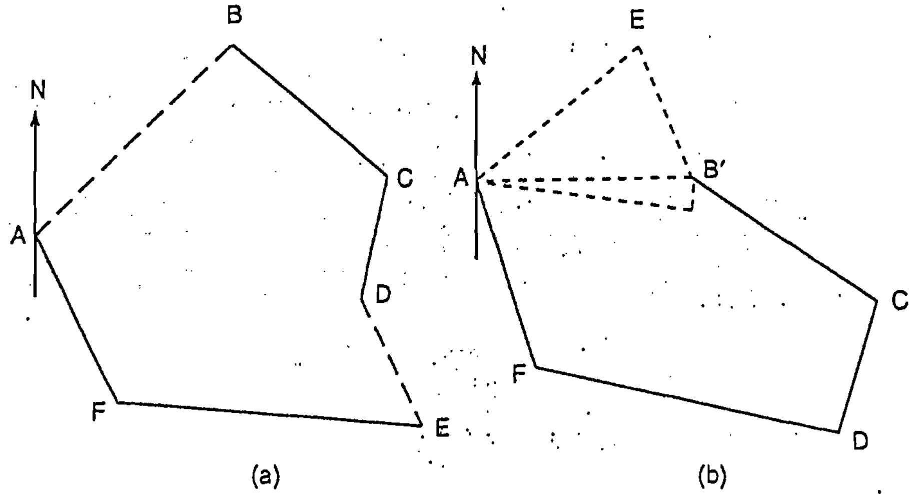


Fig. 11.25 Example 11.12.

Total latitude of AB' = + 140 cos 80°15′ + 185 cos 15°15′

− 155 cos 85°30′ − 115 cos 18°12′

= 140 × 0.169 + 185 × 0.96 − 155 × 0.078 − 115 × 0.949

= + 79.92 m.

Total departure of AB' = − 140 × sin 80°15′ + 185 sin 15°15′

+ 155 sin 85°30′ + 115 sin 18°12′

= − 137.97 + 48.66 + 154.22 + 35.92

= + 101.13

$$\text{Length } AB' = \sqrt{79.92^2 + 101.13^2} = 128.89 \text{ m.}$$

$$\text{Bearing of } AB' = \tan^{-1}\frac{101.13}{79.92} = 51°40'48''$$

$$\text{Bearing of } AE = \frac{-30°30'}{21°10'48''}$$

Bearing of $B'E$ = N 20°15′W

Whole circle bearing of $B'E$ = 339°45′

Whole circle bearing of $B'A$ = 231°40′48″.

$\angle AB'E$ = 108°04′12″

$$\angle AEB' = 180 - (21°10'48' + 108°04'12'')$$
$$= 180° - 129°15'$$
$$= 50°45'$$

Applying sine rule

$$\frac{AB'}{\sin AEB'} = \frac{B'E}{\sin B'AE} = \frac{AE}{\sin AB'E}$$

$$B'E = AB' \frac{\sin B'AE}{\sin AEB'}$$

$$= \frac{128.89 \times \sin 21°10'48''}{\sin 50°45'}$$

$$= 60.18 \text{ m.}$$

$$AE = \frac{AB' \times \sin AB'E}{\sin AEB'}$$

$$= \frac{128.89 \times \sin 108°04'12''}{\sin 50°45'}$$

$$= 158.231 \text{ m}$$

11.15 FINDING MISTAKE IN TRAVERSING

For a closed traverse, the closing error can always be computed, either analytically or graphically. If a line is found to be parallel to the closing error, it can be surmised that the closing error is due to faulty measurement along that line. The amount of closing error is the linear mistake and the measurement of the concerned line should be checked. However, when the closing error is not parallel to any line the perpendicular bisector of the closing error when produced passes through the opposite station. In that case, mistake is suspected at this angle and a correction to this angle will swing the traverse through an arc to eliminate the closing error. These are shown in Figs. 11.26(a) and (b).

Example 11.13 The field results of a closed traverse are:

Line	Whole circle bearing	Length (m)
AB	0°00′	50.60
BC	63°49′	91.08
CD	89°13′	67.06
DE	160°55′	61.57
EF	264°02′	41.15
FA	258°18′	121.62

The observed values of the included angles. Check satisfactorily but there is a mistake in the length of a line. Which length is wrong and by how much? As the

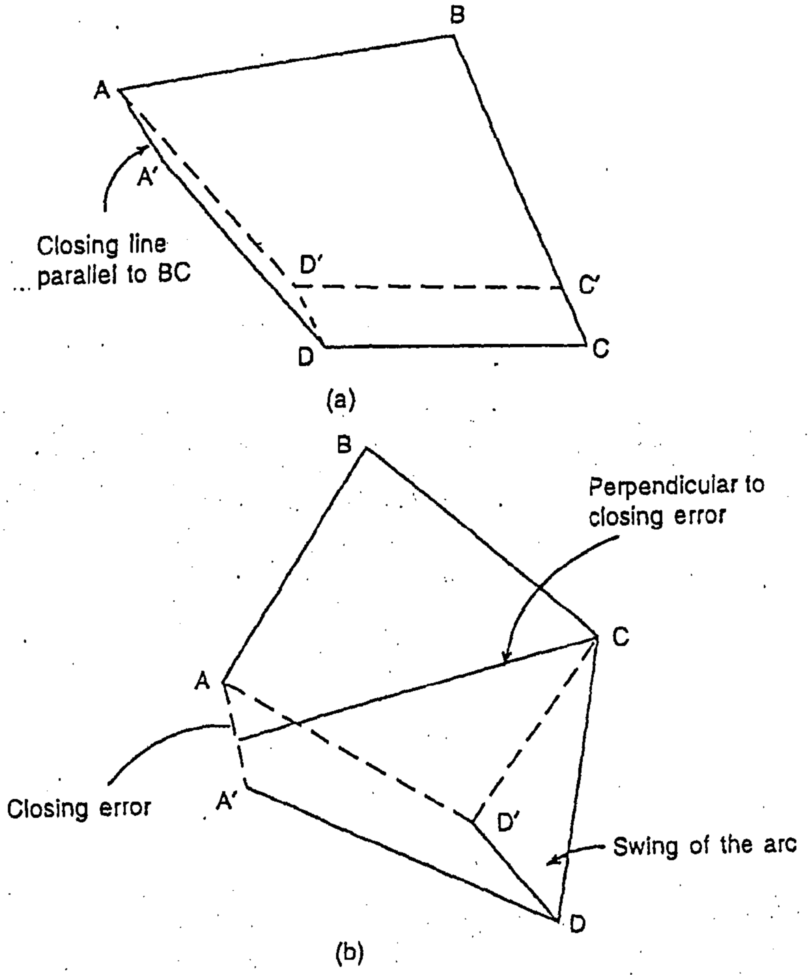


Fig. 11.26 Mistake in traversing.

lengths are measured by an accurate 30 m chain suggest how the mistake was made. [I.C.E.]

Solution Total Northing and Southing

$= 50.60 \cos 0°00' + 91.08 \cos 63°49' + 67.06 \cos 89°13'$

$- 61.57 \cos 19°05' - 41.15 \cos 84°02' - 121.62 \cos 78°18'$

$= 50.60 + 40.19 + 0.92 - 58.19 - 4.28 - 24.66$

$= 4.58$ m

Total of Easting and Westing

$= 50.60 \sin 0°00' + 91.08 \sin 63°49' + 67.06 \sin 89°13'$

$+ 61.57 \sin 19°05' - 41.15 \sin 84°02' - 121.62 \sin 78°18'$

$= 0.00 + 81.73 + 67.05 + 20.13 - 40.92 - 119.09$

$= 8.9$ m

$$\text{Closing error} = \sqrt{4.58^2 + 8.9^2} = 10.00 \text{ m}$$

$$\text{Bearing of closing error} = \tan^{-1} \frac{8.90}{4.58}$$

$$= 62°46'9.63''$$

which is close to bearing of BC. Hence it can be surmised that a mistake in length of 10 m has been made in the line BC. The mistake is due to tallies being similarly looking for 10 m and 20 m, a difference of 10 m.

Example 11.14 The table below gives the forward and back quadrantal bearings of a closed compass traverse. Tabulate the whole circle bearings corrected for local attraction indicating clearly your reasons for any corrections.

Line	Length (m)	Forward bearing	Back bearing
AB	130	N 55° E	S 54° W
BC	66	S $67^1/_2$° E	N 66° W
CD	65	S 25° W	N 25° E
DE	56	S 77° W	N $75^1/_2$° E
EA	88	N $64^1/_2$° W	S $63^1/_2$° E

A gross mistake has been made in the measurement or booking of one of the lines. State which line is in error. Using this corrected length adjust the departure and latitude of each line of the traverse to close, using Bowditch's method of corrections. [L.U.]

Solution From checking of forbearing and backbearing, it can be seen that line CD is free from local attraction. Hence the stations C and D are free from local attraction.

The whole circle bearing of the lines are:

	F.B.	B.B.
AB	55°	234°
BC	$112^1/_2$°	294°
CD	205°	25°
DE	257°	$75^1/_2$°
EA	$295^1/_2$°	$116^1/2$°

Bearing of CB = 294°

Bearing of BC = 114°

Observed bearing of BC = $112^1/_2$°

Local attraction at B = $1^1/_2$°

Bearing of BA = $235^1/_2$°

Bearing of AB = $55^1/_2$°

Observed bearing of AB = 55°

Local attraction at A = 1/2°

$$\text{Bearing of } DE = 257°$$
$$\text{Bearing of } ED = 77°$$
$$\text{Observed bearing of } ED = 75^1/_2°$$
$$\text{Local attraction at } E = 1^1/_2°$$
$$\text{Correct bearing of } EA = 295^1/_2° + 1^1/_2° = 297°$$
$$\text{Bearing of } AE = 117°$$
$$\text{Observed bearing of } AE = 116^1/_2°$$
$$\text{Local attraction at } A = 1/2°$$

Hence the lines and the corrected bearings are:

$$\text{Diff in latitude} = 15.23$$
$$\text{Diff in departure} = 6.99$$
$$\text{Closing error} = 16.75 \text{ m}$$
$$\text{Bearing} = \tan^{-1}\frac{6.99}{15.23} = \text{S } 24.65°\text{W}$$

Table 11.7 Example 11.14

Line	Length (m)	Whole circle bearing	Q.B.	Latitude		Departure	
				N	S	E	W
AB	130	55°30′	N 55°30′E	73.63		107.14	
BC	66	114°	S 66°E		26.84	60.29	
CD	65	205°	S 25°W		58.91		27.47
DE	56	257°	S 77° W		12.59		54.56
EA	88	297°	N 63°W	39.94			78.41
			Σ	113.57	98.34	167.43	160.44

Since the bearing of the closing error is almost parallel to line *CD*, the error has occurred in this line. Further the amount of closing error is very large close to a 20 m chain which may have been used. Hence it can be taken that length of *CD* should be 65 + 20 = 85 m.

Adjustment of the traverse.

Table 11.8 Example 11.14

Line	Length	W.C. bearing	Reduced bearing	Latitude		Departure	
				N	S	E	W
AB	130	55°30′	N 55°30′E	73.63		107.14	
BC	66	114°00′	S 66°00′E		26.84	60.29	
CD	85	205°00′	S 25°00′W		77.04		35.92
DE	56	257°00′	S 77°00′W		12.59		54.56
EA	88	297°00′	N 63°00′W	39.94			78.41
			Σ	113.57	116.47	167.43	168.89
				diff = 2.90		diff = 1.46	

Corrections

Table 11.9 Example 11.14

Line	N	S	E	W	Corrected values N	Corrected values S	Corrected values E	Corrected values W
AB	+ 0.89		+ 0.45		74.52		107.59	
BC		– 0.45	+ 0.23			26.39	60.52	
CD		– 0.58		– 0.29		76.46		35.63
DE		– 0.38		– 0.19		12.21		54.37
EA	+ 0.60			– 0.30	40.54			78.11
Σ	+ 1.49	– 1.41	+ 0.68	–0.78	115.06	115.06	168.11	168.11

PROBLEMS

11.1 (a) A man travels from a point *A* to due west and reaches the point *B*. The distance between *A* and *B* = 139.6 m. Calculate the latitude and departure of the line *AB*.

(b) What is 'closing error' in a theodolite traverse? How would you distribute the closing error graphically?

(c) In a closed traverse 'latitudes' and departures of sides were calculated and it was observed that

$$\sum \text{latitude} = 1.39$$

$$\sum \text{departure} = -2.17.$$

Calculate the length of bearing of closing error. [AMIE, Winter 1978]

11.2 From a common point *A*, traverses are conducted on either side of a harbour as follows:

Traverse (1)

Line	Length (m)	Bearing
AB	200	85°26′20″
BC	100	125°10′40″

Traverse (2)

Line	Length (m)	Bearing
AD	225	173°50′00″
DE	500	85°06′40″

Calculate the distance from *C* to a point *F* on *DE* due south of *C* and the distance *EF*.

[AMIE Winter 1979]

11.3 In order to fix a point *F*, exactly midway between *A* and *E*, a traverse was run as follows:

Line	Length	Bearing
AB	400 m	30°
BC	500 m	00°
CD	600 m	300°
DE	400 m	30°

Assuming point *A* as origin, calculate (a) the independent coordinates of points *C*, *E* and *F*; (b) the length and bearing of *CF*.

[AMIE Summer 1980]

11.4. An abstract from a traverse sheet for a closed traverse is given below:

Line	Length (m)	Latitude	Departure
AB	200	− 173.20	+ 100.00
BC	130	0.00	+ 130.00
CD	100	+ 86.00	+ 50.00
DE	250	+ 250.00	+ 0.00
EA	320	− 154.90	− 280.00

(a) Balance the traverse by Bowditch's method.
(b) Given the coordinates of *A*, 200 N, 00E, determine the coordinates of other points.
(c) Calculate the enclosed area in hectares by coordinate method.

[AMIE Winter 1980]

11.5. The bearings of two inaccessible stations *A* and *B* taken from station *C* were 225°00′ and 153°26′ respectively. The coordinates of *A* and *B* were as under:

Station	Easting	Northing
A	300	200
B	400	150

Calculate the independent coordinates of *C*. [AMIE Winter 1981]

11.6. It is not possible to measure the length and fix the direction of line *AB* directly on account of an obstruction between the stations *A* and *B*. A traverse *ACDB* was, therefore, run and following data were obtained:

Line	Length in m	Reduced bearing
AC	45	N 50°E
CD	66	S 70°E
DB	60	S 30°E

Find the length and direction of line *BA*. It was also required to fix a station *E* on line *BA*. It was also required to fix a station *E* on line *BA* such that line *DE* will be perpendicular to *BA*. If there is no obstruction between stations *A* and *E* calculate the data required for fixing the station. Graphical solution will not be accepted. [AMIE Summer 1982]

11.7. (a) Explain briefly the different methods of checking the correctness of angular observations in an open theodolite traverse.

(b) Following table give data of consecutive coordinates in respect of a closed theodolite traverse *ABCDA*.

Station	N	S	E	W
A	300.75			200.50
B	200.25		299.25	
C		299.00	199.75	
D		200.00		300.50

From the above data calculate:

(i) Magnitude and direction of closing error.

(ii) Corrected consecutive coordinates of station *B* using transit rule.

(iii) Independent coordinates of station *B* if those of *A* are 100,100.

[AMIE Winter 1982]

11.8. (a) Explain why transit rule is more suitable than Bowditch rule for adjustment of a closed theodolite traverse.

(b) Part of data and calculations in respect of a closed theodolite traverse *ABCDA* are as under:

Line	Length in m	R.B.	Northing.	Southing	Easting.	Westing.
AB		S 60°E		30.00		
BC		N 45°E			49.50	
CD						
DA				51.65		63.15

Complete the above table in all respects if there is no closing error for the traverse. [AMIE Summer 1983]

11.9. State the various methods of balancing a closed traverse. State under what circumstances each one is preferred.

11.10. A line *AC* of 2 km length was measured to be set out at right angles to a given line *AB*. This was done by traversing from *A* to *C* as follows:

Line	Length	Bearing
AB	—	360°
AD	750 m	120°
DE	500 m	80°
EF	600 m	105°

Compute the length and bearing of *FC*. [AMIE Summer 1985]

11.11. (a) What are the different mathematical methods of adjusting a closed traverse? Explain clearly where these methods are used and why.

(b) A and B are two stations of an open traverse and their independent coordinates are as follows:

Station	Latitude (m)	Departure (m)
A	27456.8	6007.2
B	26936.0	7721.6

It is proposed to construct a railway track from C roughly south of A, to D, roughly north of B. While C and D are not inter visible, the perpendicular offsets from the traverse stations to the railway track are measured to be $AC = 104$ m and $BD = 57.6$m. Determine the whole circle bearing of CD.

[AMIE Winter 1986]

11.12. (a) What is meant by closing error in a closed traverse and how is it adjusted graphically?

(b) The measured lengths and bearings of the side of a closed traverse $ABCDE$ run in the counter-clockwise direction and are tabulated below. Calculate the lengths CD and DE.

Line	Length (m)	Bearing
AB	298.7	0°0′
BC	205.7	N 25°12′W
CD	?	S 75°06′W
DE	?	S 56°24′E
EA	213.4	N 35°36′E

[AMIE Summer 1989].

11.13 The following notes refer to a closed traverse. Compute the missing quantities.

Line	Length (m)	Bearing
AB	725	S 60°00′E
BC	1050	?
CD	1250	?
DE	950	S 55°30′W
EA	575	S 02°45′W.

[AMIE Winter 1990]

11.14. To fix a station F, exactly midway between stations A and E, a traverse was run as follows:

Line	Length (m)	Bearing
AB	400	30°
BC	500	00°
CD	600	300°
DE	400	30°

Assuming station *A* as origin, calculate—

(a) the independent coordinates of stations *C*, *E* and *F*.

(b) the length and bearing of *CF*. [AMIE Summer 1991]

HINTS TO SELECTED QUESTIONS

11.1 (b) Graphical method based on Bowditch's rule has been explained in Chapter 9 for compass survey. Sometimes it is also used for a theodolite traverse of low accuracy.

11.8 (a) In the transit rule, the angles are changed less but the lengths are changed more. Compared to this in Bowditch's rule, the lengths are changed less and the angles are changed more. In theodolite traverse angles are accurately measured with theodolite whereas measurement of length with chain or tape is of lesser accuracy. Hence for adjustment transit rule is more appropriate.

12

Curves

12.1 INTRODUCTION

To avoid abrupt change of direction curves are introduced between two straights both in the horizontal and vertical plane. Curves can be broadly classified as follows.

- Curves
  - Horizontal curve
    - Circular
      - Simple
      - Compound
      - Reverse
    - Transition
      - Cubic Parabola
      - Clothoid
      - Cubic Spiral
      - Lemniscate
    - Combined
  - Vertical curve

12.2 BASIC DEFINITIONS

Figure 12.1 shows a simple circular curve.

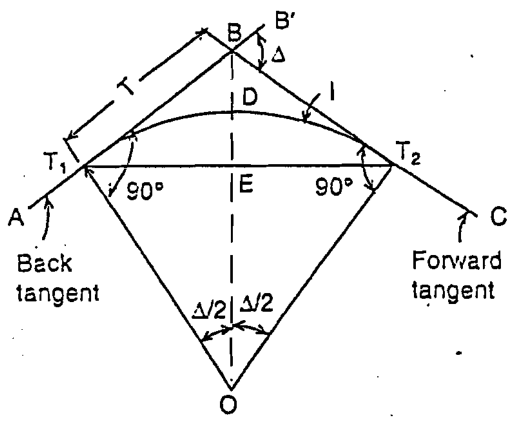


Fig. 12.1 Basic elements of a curve.

Two straight lines AB and BC intersect at the point B. The curve T_1DT_2 is inserted to make the change of direction from AB to BC smooth. AB is the back tangent or rear tangent. B is the vertex or point of inter section. Δ is the deflection angle. It is the angle by which the straight line BC deflects from the straight line AB. T_1 is the point of curvature. It is a point on the back tangent from where the curve starts. T_2 is the point of tangency. It is a point on the forward tangent at the end of the curve. T_1B and T_2B are tangent distances. They are equal. BD, the distance between the vertex B and midpoint D of the curve is called *external distance*. T_1T_2 is the long chord. E is the middle point of the long chord and DE is the mid ordinate. T_1DT_2 is the length of the curve. If the deflection angle is clock wise, the curve is a right hand curve. When the deflection angle is anticlockwise, the curve is a left hand curve.

12.2.1 DESIGNATION OF A CURVE

A curve can be designated either in terms of radius (R) or the degree of a curve. The degree of a curve (D) is defined as the angle subtended at the centre by an arc or chord of standard length which is usually the length of a chain. Arc definition is generally used in highway practice and the chord definition in railway practice.

Relationship between radius and degree of a curve: According to arc definition the degree of a curve is equal to the angle subtended at the centre by an arc of 30 m. From Fig. 12.2(a),

$$D_a = \left(\frac{360^\circ}{2\pi R}\right)(30) = \frac{1718.87}{R} \tag{12.1}$$

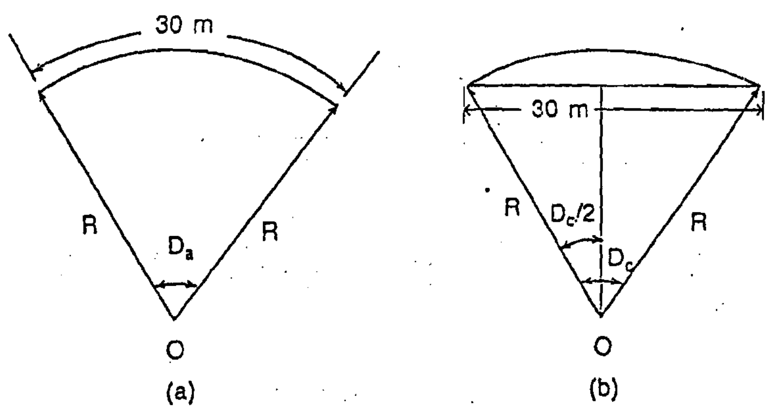


Fig. 12.2 Degree of a curve.

According to the chord definition the degree of a curve is equal to the angle subtended at the centre by a chord of 30 m length. From Fig. 12.2(b)

$$\sin\frac{D_c}{2} = \frac{15}{R}$$

For small angle $\sin(D_c/2) = D_c/2$ radian. Therefore,

$$\frac{D_c}{2} \times \frac{\pi}{180} = \frac{15}{R}$$

or

$$D_c = \frac{360}{\pi} \frac{15}{R}$$

$$= \frac{1718 \cdot 87}{R} \quad (12.2)$$

It is seen that the arc definition and the chord definition give identical results when the degree of curve is small.

12.2.2 ELEMENTS OF A SIMPLE CURVE

The following equations can be derived from Fig. 12.1.

1. *Length of curve* (l): Length of the circular curve T_1DT_2 is given by

$$l = \left(\frac{2\pi R}{360}\right)(\Delta) = \frac{\pi R \Delta}{180°}.$$

2. *Tangent length T*: Tangent length is given by

$$T_1B = T_2B = R \tan \Delta/2.$$

3. *Chainage of tangent points:* The chainage of intersection point B is generally known. Subtracting the tangent length T,

Chainage of T_1 = chainage of $B - T$.

Chainage of T_2 = chainage of T_1 + length of curve (l)

= chainage of $T_1 + \frac{\pi R \Delta}{180°}$

Chainge is usually expressed as number of full chains and part length of a chain. For example, a chainage of 4125.5 m with 30 m chain = 137 full chains + 15.5 m = 137 + 15.5.

To avoid confusion length of the chain should be clearly specified.

4. Length of long chord = $T_1ET_2 = 2 \times R \sin \Delta/2$

$= 2R \sin \Delta/2.$

5. External distance or apex distance = BD.

$= OB - OD = R \sec \Delta/2 - R.$

6. Mid ordinate = $OD - OE$

$= R - R \cos \Delta/2 = R(1 - \cos \Delta/2).$

12.2.3 SETTING OUT OF A CURVE

A circular curve can be set out by (i) Linear or chain and tape method when no angle measuring instrument is used; (ii) Instrument methods in which a theodolite, tacheometer or a total station instrument is used.

Chain and tape methods

1. *Offsets from the long chords* Usually the lines AB and BC (Fig. 12.3) are already plotted on the ground. The deflection angle Δ may be set out very accurately

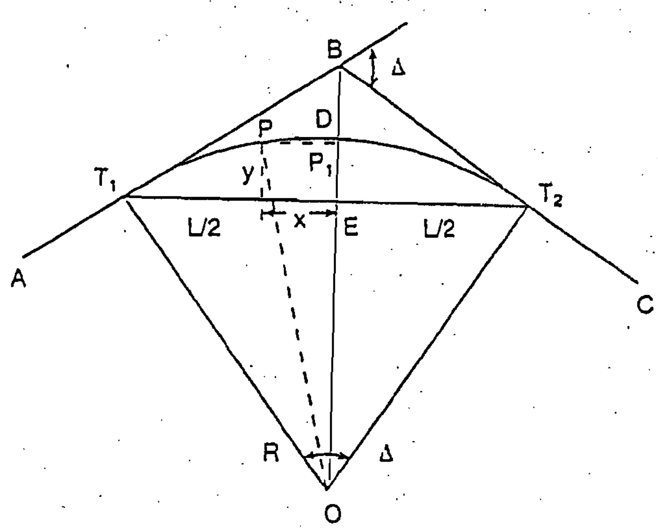


Fig. 12.3 Offsets from long chords.

by means of a theodolite. Lengths BT_1, BT_2 and T_1T_2 are calculated. Points T_1, T_2 and the midpoint E of T_1T_2 are obtained on the field. If L is the length of long chord $T_1E = L/2$

and $$OE = \sqrt{R^2 - (L/2)^2}$$

and $$DE = R - \sqrt{R^2 - (L/2)^2}$$

To get y at any distance x from E, from the triangle OPP_1

$$x^2 + OP_1^2 = R^2$$

or $$OP_1^2 = R^2 - x^2$$

$$(OE + y)^2 = R^2 - x^2$$

$$OE + y = \sqrt{R^2 - x^2}$$

$$y = \sqrt{R^2 - x^2} - OE$$

$$= \sqrt{R^2 - x^2} - \sqrt{R^2 - (L/2)^2} \quad (12.3)$$

Dividing the long chord into an even number of parts, points on the curve can be obtained with corresponding values of x.

2. *Offsets from the tangents* Curves can also be set out by measuring offsets from the tangents. The offsets from the tangent can be either radial or perpendicular to the tangent.

Radial offset (Fig. 12.4): From the triangle OT_1Q,

$$OT_1^2 + T_1Q^2 = OQ^2$$

or $$R^2 + x^2 = (R + y)^2$$

or $$R + y = \sqrt{R^2 + x^2}$$

or $$y = \sqrt{R^2 + x^2} - R$$

$$= R\left(1 + \frac{x^2}{R^2}\right)^{1/2} - R$$

$$\approx R\left(1 + \frac{x^2}{2R^2}\right) - R$$

$$\approx \frac{x^2}{2R} \qquad (12.4)$$

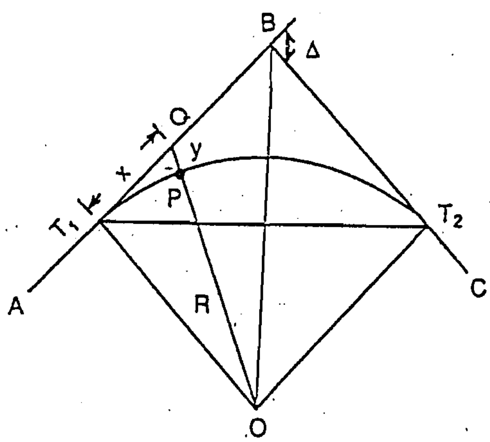


Fig. 12.4 Offsets from tangents.

Perpendicular offset: From the triangle OPP_1 (Fig. 12.5)

$$OP^2 = OP_1^2 + PP_1^2$$

$$R^2 = (R - y)^2 + x^2$$

$$(R - y)^2 = R^2 - x^2$$

$$R - y = \sqrt{R^2 - x^2}$$

$$y = R - \sqrt{R^2 - x^2} \qquad (12.5)$$

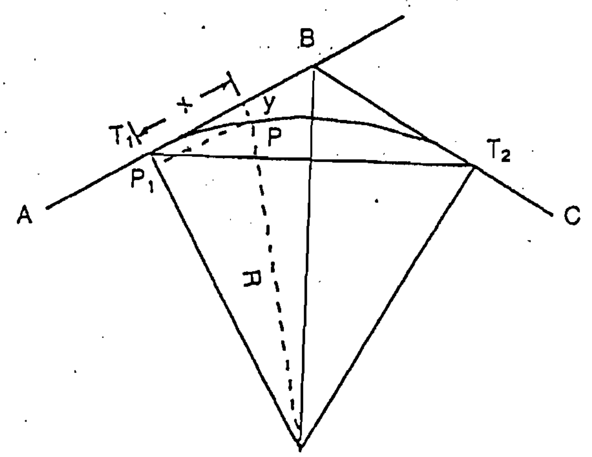


Fig. 12.5 Perpendicular offset.

$$\approx R - R\left(1 - \frac{x^2}{R^2}\right)^{1/2}$$

$$\approx R - R + \frac{x^2}{2R}$$

$$\approx \frac{x^2}{2R} \qquad (12.6)$$

3. *Offsets from the chord produced* This method has the advantage that not all the land between T_1 and T_2 (Forward Tangent point, not shown in Fig.) need be accessible. However, to have reasonable accuracy the length of the chord chosen should not exceed $R/20$. The method has a drawback that error in locating is carried forward to other points. The method is based on the premise that for small chords, the chord length is small and approximately equal to the arc length (Fig. 12.6).

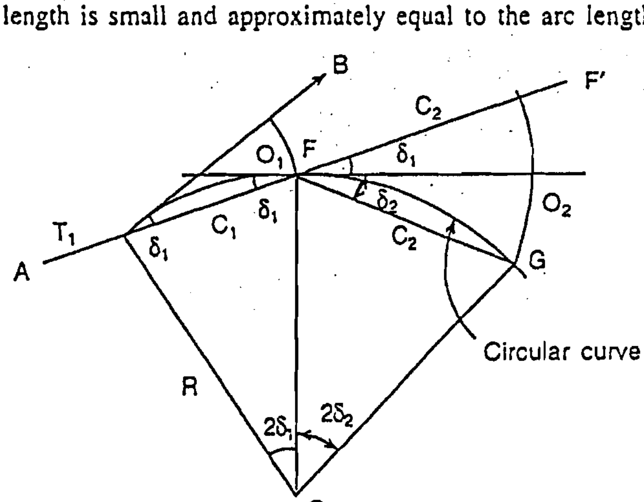


Fig. 12.6 Offsets from chord produced.

From the property of circle if the angle $\angle BT_1F = \delta_1$
The angle at the centre $\angle T_1OF = 2\delta_1$

$$C_1 = \text{chord } T_1F \approx \text{arc } T_1F = 2\delta_1 R.$$

or $$\delta_1 = \frac{C_1}{2R}$$

The first offset $$O_1 = C_1\delta_1$$

$$= C_1 \frac{C_1}{2R} = \frac{C_1^2}{2R}$$

The first chord C_1 is called a subchord. The length of the subchord is so adjusted that the chord length when added to the chainage of T_1 makes the chainage of F a full chain. Subsequent chord lengths C_2, C_3, ... are all full chains. T_1F is now produced to F' such that $FF' = C_2$, a full chain.

The second offset $$O_2 = C_2(\delta_1 + \delta_2)$$

$$= C_2\left(\frac{C_1}{2R} + \frac{C_2}{2R}\right)$$

$$= \frac{C_2}{2R}(C_1 + C_2)$$

Similarly $$O_3 = \frac{C_3}{2R}(C_2 + C_3)$$

but $$C_2 = C_3 = C_n - 1$$

hence $$O_3 = \frac{C_2}{2R}(2C_2) = \frac{C_2^2}{R}$$

The last offset $$O_n = \frac{C_n}{2R}(C_{n-1} + C_n) \quad (12.7)$$

where C_{n-1} is a full chain and C_n is the last subchord which is normally less than one chain.

Instrumental Methods

1. *Tape and Theodolite method* In this method a tape is used for making linear measurements and a theodolite is used for making angular measurements. The method is also known as the Rankine method, the tangential angle method or the deflection angle method. The method is accurate and is used in railways and highways (Fig. 12.7).

Let T_1FH be part of a circular curve with T_1, the initial tangent point. Then T_1F is the first subchord which is normally less than 1 chain.

Then from property of circle

$$C_1 = 2\delta_1 R$$

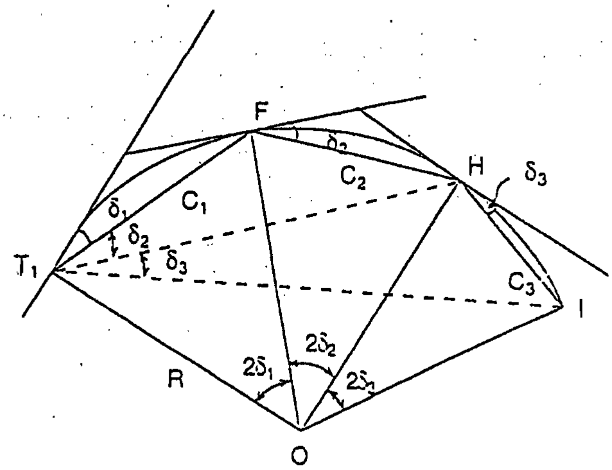


Fig. 12.7 Tape and theodolite method.

or
$$\delta_1 = \frac{C_1}{2R} \text{ radian}$$
$$= \frac{C_1}{2R}\frac{180°}{\pi}$$
$$= \frac{C_1}{2R}\frac{180 \times 60}{\pi} \text{ minute}$$
$$= 1718.87 \frac{C_1}{R} \text{ minute} \qquad (12.8)$$

Therefore, to locate the point F with the help of a theodolite and tape, the instrument is set at T_1 and the line of sight is put at an angle δ_1 as computed above. Then with the help of a tape and ranging rod, the tape is put along the line of sight and distance C_1 is then measured to locate F along the line of sight. Similarly,

$$\delta_2 = 1718.87 \frac{C_2}{R} \text{ minute}$$

Since the theodolite remains at T_1, H is sighted from T_1 by measuring $\delta_1 + \delta_2 = \Delta_2$ from the tangent line. The point H is located with the help of a tape and ranging rod. The tape with the ranging rod is so adjusted that the tape measures $FH = C_2$ and the ranging rod lies along the line of sight T_1H.
Similarly,

$$\Delta_3 = \delta_1 + \delta_2 + \delta_3$$
$$\ldots$$
$$\Delta_n = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n$$

In practice C_1 is the first subchord and C_n the last subchord. $C_2 = C_3 = \ldots = C_{n-1}$ are full chain lengths. As a check the deflection angle Δ_n for the last point T_2 is equal to $\Delta/2$ where Δ is the angle of intersection.

2. *Two-theodolites method* In this method two theodolites are used one at T_1 and the other at T_2 (Fig. 12.8). From the geometry of a circle if the tangential angle BT_1F is δ_1, then the angle at the circumference T_1FT_2 is also δ_1. δ_1 can be computed as before. One theodolite is placed at T_1 and the angle δ_1 is set out with respect to tangent T_1B. Similarly the second theodolite is placed at T_2 and the angle δ_1 is measured with respect to T_2T_1. A ranging rod is moved such that it intersects both the lines of sights and locates the point F. To locate the point G, the instrument at T_1 is set out at $\delta_1 + \delta_2 = \Delta_2$. Similarly instrument at T_2 is set at $\Delta_2 = \delta_1 + \delta_2$ from T_2T_1. The intersection of the two lines of sight locates the point G which is found with the help of a ranging rod. The method is accurate though expensive as two theodolites with two instrument men are involved. It is specially convenient when the ground is rough and accurate chaining is not possible. The error is not carried forward as each point is fixed independently.

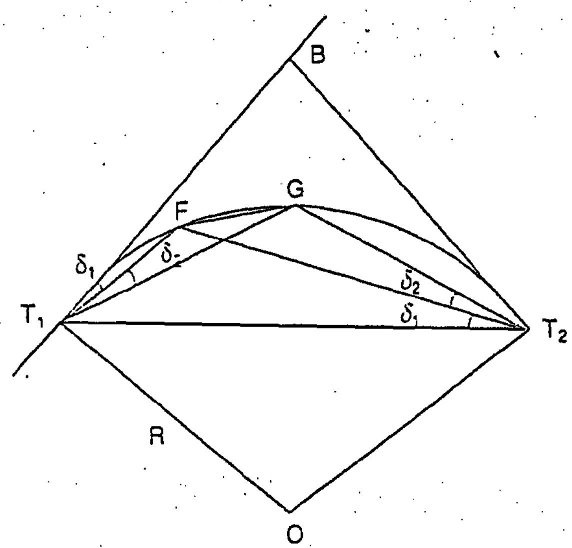


Fig. 12.8 Two-theodolites method.

3. *Total station instrument method* A total station instrument consists of an optical theodolite to measure angle and an E.D.M to measure distance. The radial stake out technique is used for the setting out of a curve (Fig. 12.9).

The following are the steps:

(a) The instrument is set up at A whose coordinates are known from previous control survey.

(b) Take a back'sight to another point B of known coordinates. The reference azimuth can then be calculated as:

$$\alpha_{AB} = \tan^{-1} \frac{E_B - E_A}{N_B - N_A} \tag{12.9}$$

(c) Coordinates for curve points are determined by the deflection angles from the tangent which are computed before hand. Let the coordinates of the intersection of the tangents be N_{PI}, E_{PI}, azimuth of the back tangent α_1, intersection angle Δ and curve radius R, then the coordinates of the point of curvature are:

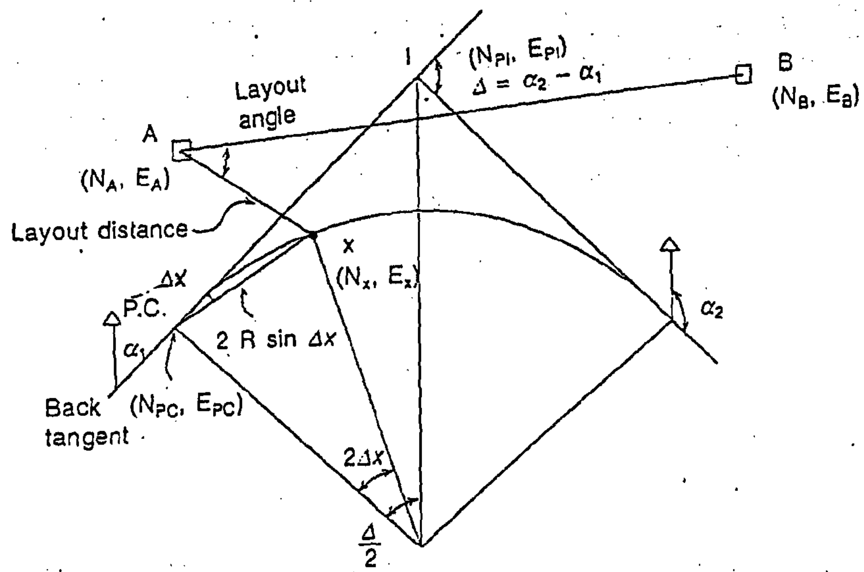


Fig. 12.9 Total station instrument method.

$$N_{PC} = N_{PI} - R \tan \Delta/2 \cos \alpha_1 \tag{12.10}$$

$$E_{PC} = E_{PI} - R \tan \Delta/2 \sin \alpha_1 \tag{12.11}$$

(d) Curve point coordinates are then calculated as:

$$N_x = N_{PC} + (2R \sin \Delta x) \cos (\alpha_1 + \Delta x) \tag{12.12}$$

$$E_x = E_{PC} + (2R \sin \Delta x) \sin (\alpha_1 + \Delta x) \tag{12.13}$$

Here Δx is the total deflection angle for point x. It is positive for clockwise and negative for counter-clockwise rotation.

(e) The required layout angle is then given by

$$\text{Layout angle} = \tan^{-1}\left(\frac{E_x - E_A}{N_x - N_A}\right) - \alpha_{AB} \tag{12.14}$$

where the measurement is clockwise from the reference backsight.

(f) The horizontal distance is computed from coordinates.

$$\text{Layout distance} = [(N_x - N_A)^2 + (E_x - E_A)^2]^{1/2} \tag{12.15}$$

12.2.4 PROBLEMS IN SETTING OUT CURVES

The following difficulties may occur in setting out a curve: (i) point of intersection of tangents not visible, (ii) initial tangent point not accessible, and (iii) final tangent point not accessible.

Point of intersection of tangents not accessible (Fig. 12.10)

1. Locate points M and N on AB and BC respectively.

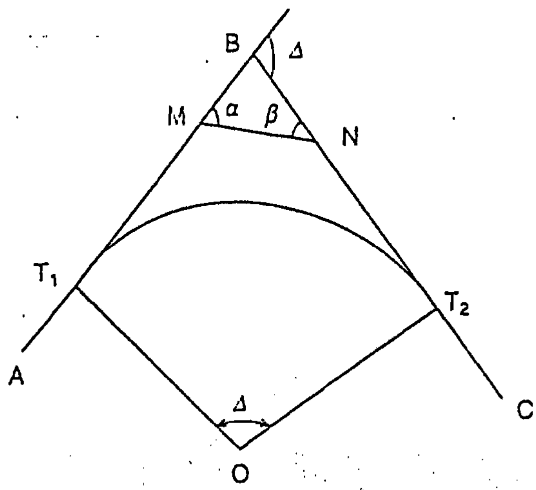


Fig. 12.10 Point of intersection of tangent not visible.

2. Measure angles α and β with a theodolite and length MN with a chain or tape.

3. Then
$$\frac{BM}{\sin\beta} = \frac{MN}{\sin\Delta}$$

or
$$BM = \frac{MN\sin\beta}{\sin\Delta}$$

Similarly
$$BN = \frac{MN\sin\alpha}{\sin\Delta}$$

4. Calculate
$$MT_1 = BT_1 - BM$$
$$NT_2 = BT_2 - BN$$

5. Thus T_1 and T_2 can be located from M and N respectively and the curve can be plotted from T_1.

Initial tangent point not accessible (Fig. 12.11)

In this case, the curve has to be set out from the second tangent point T_2. Let the first chord be T_2E. The angle subtended at $T_1 = \Delta/2 - \alpha_E$.

This is equal to BT_2E as the angle between the chord and tangent is equal to the angle at the circumference. Since the scale graduations read clockwise the vernier should be set at 0–0 with pointing towards T_2B and should be set out at $360° - (\Delta/2 - \alpha_E)$ or $360° - \Delta/2 + \alpha_E$ for pointing towards T_2E. Similarly to locate D angle reading should be $360° - (\Delta/2 - \alpha_D)$.

Final tangent point not accessible (Fig. 12.12)

1. Two points M and N are selected on BC which are accessible. By measuring MQ and QN and if MQN is made a right angle then $MN = \sqrt{MQ^2 + QN^2}$

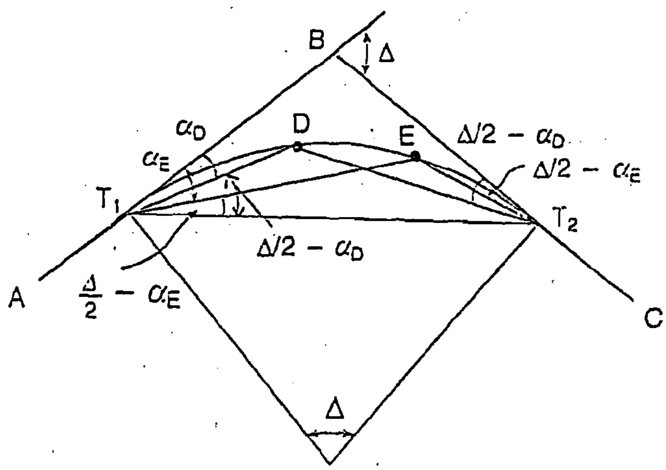


Fig. 12.11 Inaccessible initial tangent point.

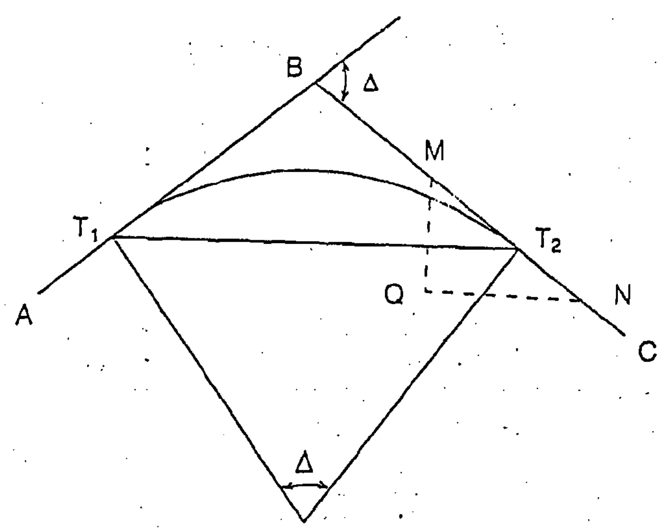


Fig. 12.12 Inaccessible final tangent point.

2. BT_2 is known and BM is measured, hence MT_2 is known.
3. T_2N then is equal to $MN - MT_2$.
4. The chainage of N is equal to chainage of T_1 plus length of the curve T_1T_2.

12.2.5 SETTING OUT CURVE FROM AN INTERMEDIATE POINT

Two cases may arise:

CASE I The instrument is set up at a point D which is visible from T_1, the initial tangent point (Fig. 12.13).

1. Let point D be visible from T_1 but E point is not visible.
2. Shift the instrument to D and make the reading 0–0.

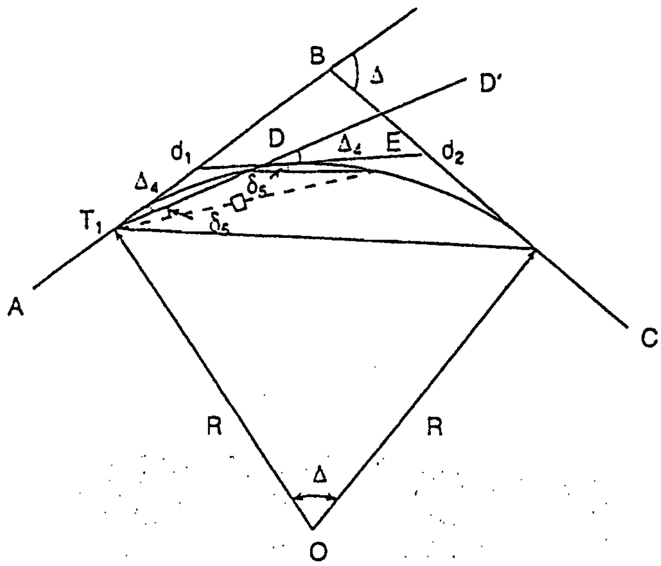


Fig. 12.13 Setting out from intermediate visible point.

3. Point towards T_1 and then plunge the telescope so that DD' is obtained.

4. The direction of DE is then obtained by measuring clockwise $\Delta_5 = \Delta_4 + \delta_5$ as originally obtained.

CASE II When the instrument is set up at a point E which is not visible from T_1 (Fig. 12.14).

1. Set up the instrument at E and fix vernier at Δ_D.

2. Point towards D and reverse the line of sight. The pointing is now along ED'.

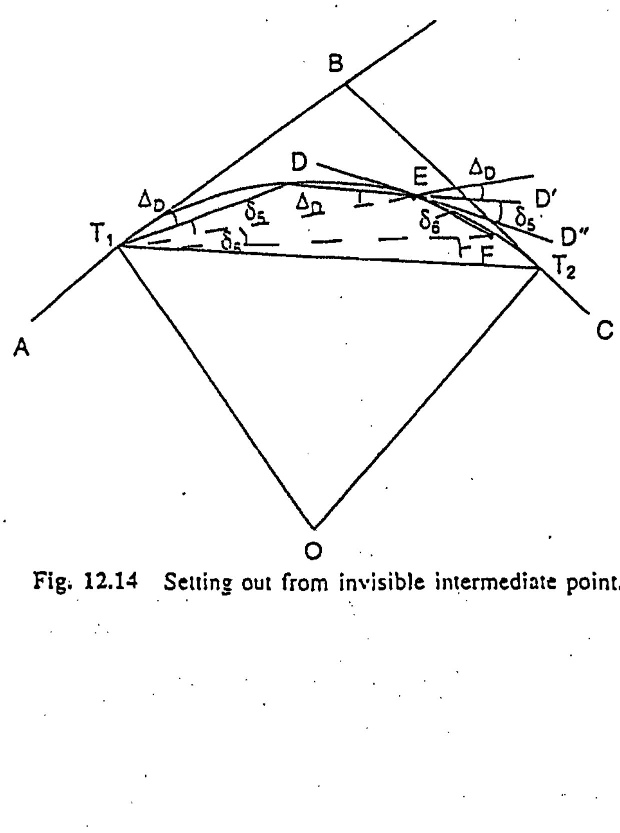


Fig. 12.14 Setting out from invisible intermediate point.

3. Rotate the upper plate through $\delta_5 + \delta_6$, i.e. make the total reading Δ_6 to locate point F.

12.2.6 SETTING OUT THE CURVE FROM THE POINT OF INTERSECTION

The procedure is based on the following geometrical relations. Let D be the point which is to be located (Fig. 12.15). DD_1 is perpendicular on the tangent BT_1. GD is drawn parallel to BT_1. In the triangle BD_1D

$$\tan\alpha = \frac{DD_1}{BD_1} = \frac{T_1G}{BT_1 - D_1T_1}$$

$$= \frac{T_1G}{BT_1 - GD}$$

$$\therefore \quad \tan\alpha = \frac{R - R\cos\theta}{R\tan\Delta/2 - R\sin\theta}$$

$$= \frac{1-\cos\theta}{\tan\Delta/2 - \sin\theta}$$

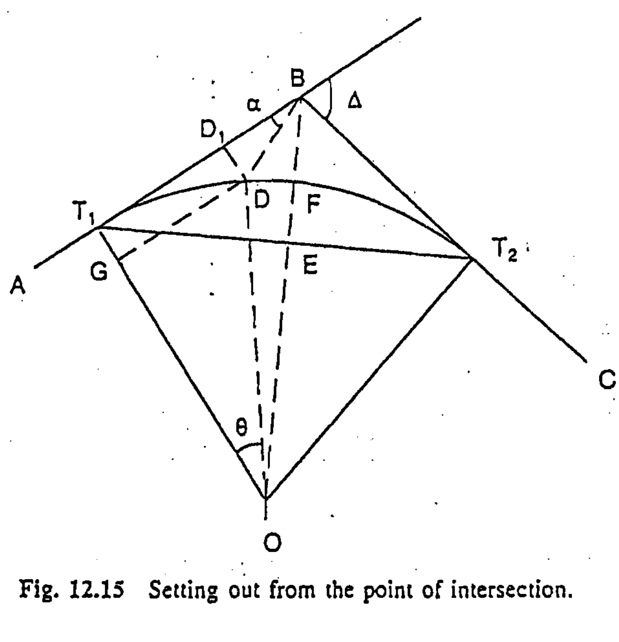


Fig. 12.15 Setting out from the point of intersection.

If the curve is divided into ten equal parts

$$\theta_1 = \Delta/10,\ \theta_2 = \frac{2\Delta}{10},\ \ldots,\ \theta_n = \Delta$$

To locate the point D, the instrument is set up at B and making an angle α_1 calculated corresponding to θ_1. As the rotation is anti-clockwise if the reading is zero-zero along BT_1, the reading along BD will be $(360° - \alpha_1)$. The length T_1D will be $l/10$. With zero of tape at T_1, and ranging rod at $l/10$, the tape is swung till the angle α_1 is bisected. Similarly for other points.

12.2.7 PASSING A CIRCULAR CURVE THROUGH A FIXED POINT

It is necessary to find the radius of a circular curve tangential to AB and BC and passing through the point P. Let the origin of the coordinate system be at B, the intersection point, X axis along the back tangent and Y axis at right angles to it as shown in Fig. 12.16. Then the coordinates of the point O are $-R \tan \Delta/2$ and $-R$. If the length BP and the angle θ are known, the coordinates X_P and Y_P of the point P are known. Here

$$X_P = -BP \cos \theta = -Z \cos \theta$$

$$Y_P = -BP \sin \theta = -Z \sin \theta$$

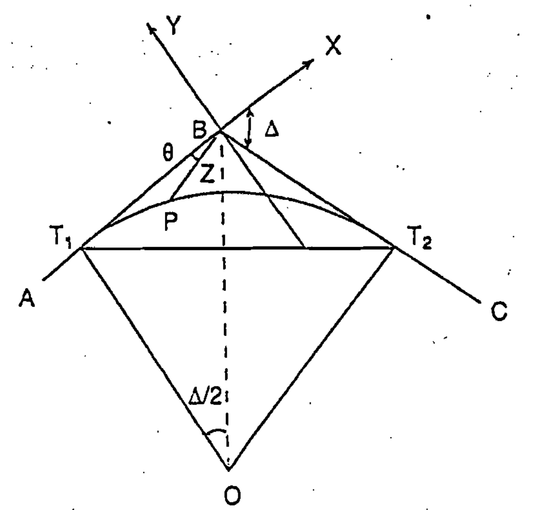


Fig. 12.16 Passing a circular curve through a fixed point.

Equation of a circle having origin at O $(-R \tan \Delta/2, -R)$ and passing through the point P $(-Z \cos \theta, -Z \sin \theta)$ is:

$$R^2 = (-Z \cos \theta + R \tan \Delta/2)^2 + (-Z \sin \theta + R)^2$$

As Z, θ, Δ are known a solution for R can be found.

12.3 INTERSECTION OF A LINE AND CIRCLE

This can be determined if the equation of the straightline and the equation of the circle are known. Let coordinates of A and B be (X_A, Y_A) and (X_B, Y_B) respectively (Fig. 12.17). The equation of the straightline passing through A and B is

$$\frac{Y_B - Y_A}{X_B - X_A} = \frac{Y_P - Y_A}{X_P - X_A}$$

Similarly the equation of a circle of radius R, centre O (X_O, Y_O) and passing through point P (X_P, Y_P) is given by

$$R^2 = (X_P - X_O)^2 + (Y_P - Y_O)^2$$

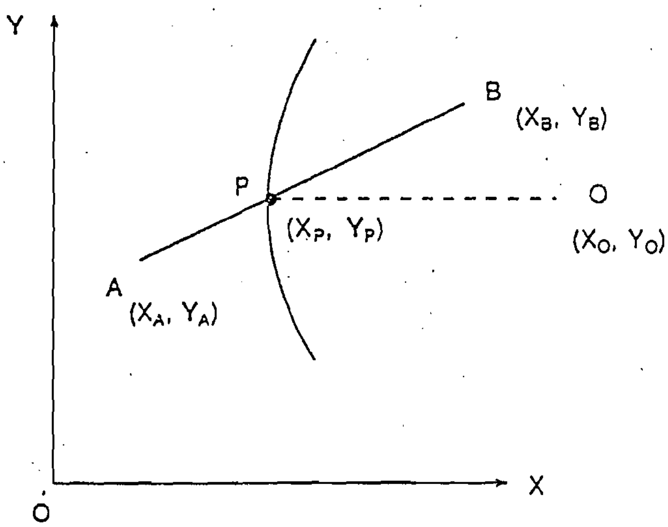


Fig. 12.17 Intersection of a line and a circle.

Solving the above two equations simultaneously a quadratic equation of the form

$$aY_P^2 + bY_P + c = 0$$

is obtained from which Y_P and then X_P can be obtained.

12.3.1 INTERSECTION OF TWO CIRCULAR CURVES

This can be solved by writing equations when coordinates of the centres of the curves and radii of the curves are known (Fig. 12.18).

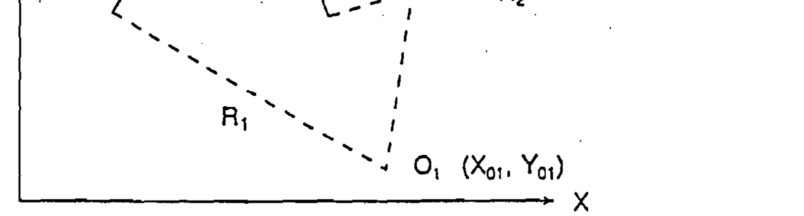


Fig. 12.18 Intersection of two circles.

The equation of a circle passing through (X_P, Y_P) with radius R_1 and centre $O_1(X_{01}, Y_{01})$

is $$(X_P - X_{01})^2 + (Y_P - Y_{01})^2 = R_1^2$$

Similarly with O_2 at centre and radius R_2

$$(X_P - X_{02})^2 + (Y_P - Y_{02})^2 = R_2^2$$

From the above two equations two unknowns X_P, Y_P can be obtained.

12.3.2 CURVE PASSING TANGENTIAL TO THREE LINES

Let *AD*, *DE* and *EC* be three lines (Fig. 12.19). It is required to draw a circle touching the three lines. If the circle touches the lines at T_1, T_3 and T_2, then from Fig. 12.19

$$T_1D = DT_3 = R \tan \alpha/2$$

$$T_3E = ET_2 = R \tan \beta/2$$

Then,

$$R \tan \alpha/2 + R \tan \beta/2 = DE$$

or $$R = \frac{DE}{\tan \alpha/2 + \tan \beta/2}$$

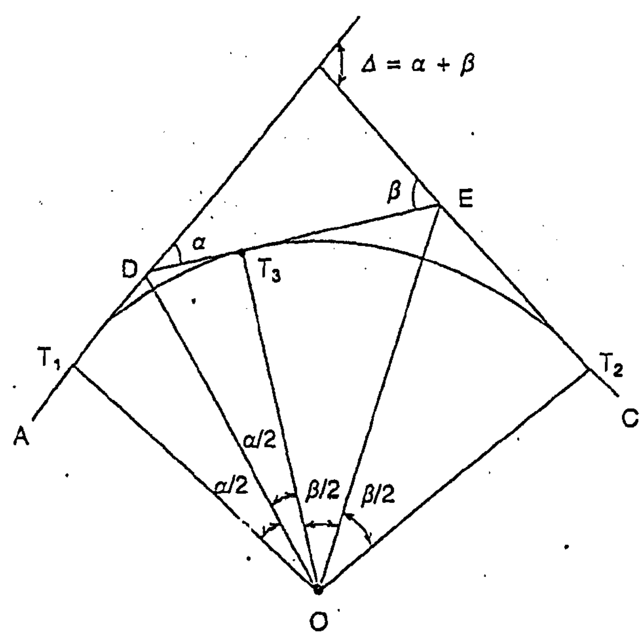


Fig. 12.19 Curve tangential to three lines.

Example 12.1 For the circular curves having radius *R* (a) 250 m, (b) 500 m, (c) 1200 m, what is their degree of curve by (i) arc definition; (ii) chord definition?

Solution

(a) $$D_a = \frac{360°}{2\pi R} \times 30 = \frac{360 \times 30}{2\pi \times 250} = 6.875°$$

$$D_c = 2 \sin^{-1} \frac{15}{R} = 2 \sin^{-1} \frac{15}{250} = 6.879°$$

(b) $$D_a = \frac{360° \times 30}{2\pi \times 500} = 3.438°$$

$$D_c = 2 \sin^{-1} \frac{15}{500} = 3.438°$$

(c) $$D_a = \frac{360 \times 30}{2\pi \times 1200} = 1.432°$$

$$D_c = 2 \sin^{-1} \frac{15}{1200} = 1.432°$$

From the above it is clear that the degree of a curve by arc definition and chord definition is practically the same for curves with large radius, i.e. flat curves but differ when the radius of the curve is small, that is, steeper curves.

Example 12.2 Two roads having a deviation angle of 45°48′ are to be joined by a 180 m radius curve. Calculate the necessary data if the curve is to be set (a) by chain and offsets only; (b) a theodolite is available.

Solution

(a) By offsets from the long chords (Fig. 12.20)

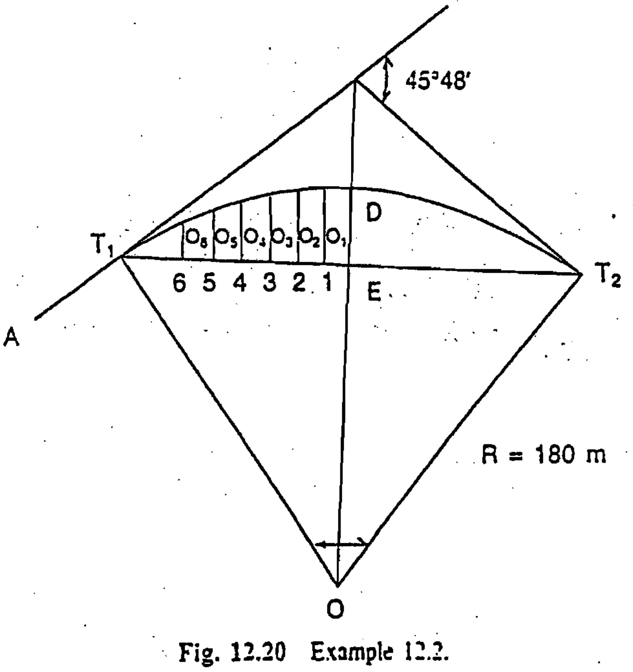


Fig. 12.20 Example 12.2.

Length of tangent = $R \tan \Delta/2$.

$$= 180 \tan \frac{45°48'}{2}$$

$$= 76.03 \text{ m} \approx 76 \text{ m}$$

Chainage of T_1 = 3123.8 − 76 = 3047.8 m

$$\text{Length of the curve} = \frac{R \times \Delta \times \pi}{180} = \frac{180 \times 45.8 \times \pi}{180} = 143.88 \text{ m}$$

Chainage of T_2 = 3047.8 + 143.88 = 3191.68 m

$$T_1T_2 = 2R \sin \frac{45°48'}{2} = 2 \times 180 \sin 22°54'$$

$$= 140.08 \approx 140 \text{ m}$$

$$T_1E = 70 \text{ m}$$

Central offset $ED = R\,(1 - \cos \Delta/2) = 180\,(1 - \cos 22.9°) = 14.187$ m

Starting from E offset distances at 10 m interval,

$$y = \sqrt{R^2 - x^2} - \sqrt{R^2 - (L/2)^2}$$

$$O_1 = \sqrt{180^2 - 10^2} - \sqrt{180^2 - 70^2} = 179.722 - 165.831 = 13.89$$

$$O_2 = \sqrt{180^2 - 20^2} - 165.831 = 13.05$$

$$O_3 = \sqrt{180^2 - 30^2} - 165.831 = 11.65$$

$$O_4 = \sqrt{180^2 - 40^2} - 165.831 = 9.67$$

$$O_5 = \sqrt{180^2 - 50^2} - 165.831 = 7.09$$

$$O_6 = \sqrt{180^2 - 60^2} - 165.831 = 3.87$$

$$O_7 = \sqrt{180^2 - 70^2} - 165.831 = 0.00$$

These offsets are shown in Fig. 12.20.

(b) By offsets from the tangents:

$$y = \sqrt{R^2 + x^2} - R$$

chainage at T_1 = 3047.8 m = 101 ch + 17.8 m

Taking $x_1 = 30 - 17.8 = 12.2$ $\quad y_1 = \sqrt{180^2 + 12.2^2} - 180 = 0.41$ m

$x_2 = 30 + 12.2$ $\quad y_2 = \sqrt{180^2 + 42.2^2} - 180 = 4.88$ m

$x_3 = 60 + 12.2$ $\quad y_3 = \sqrt{180^2 + 72.2^2} - 180 = 13.94$ m

$x_4 = 76$ $\quad y_4 = \sqrt{180^2 + 76^2} - 180 = 15.38$ m

(c) By perpendicular offsets:

$$y = R - \sqrt{R^2 - x^2}$$

Taking $x_1 = 12.2 \quad y_1 = 180 - \sqrt{180^2 - 12.2^2} = 0.41$ m

$x_2 = 42.2 \quad y_2 = 180 - \sqrt{180^2 - 42.2^2} = 5.02$ m

$x_3 = 72.2 \quad y_3 = 180 - \sqrt{180^2 - 72.2^2} = 15.11$ m

$x_4 = 76.0 \quad y_4 = 180 - \sqrt{180^2 - 76^2} = 16.83$ m

(d) By offsets from the chord produced:
Length of 1st subchord = 12.2 m

$$O_1 = \frac{C_1^2}{2R} = \frac{12.2^2}{2 \times 180} = 0.413 \text{ m}$$

$$O_2 = \frac{C_2}{2R}(C_1 + C_2) = \frac{30}{2 \times 180}(12.2 + 30) = 3.516 \text{ m}$$

$$O_3 = \frac{C_2^2}{R} = \frac{30^2}{180} = 5 \text{ m}$$

chainage of T_2 = 3191.68 m = 106 ch + 11.68 m.

Length of last subchord = 11.68 m

$$\text{Last offset} = O_n = \frac{C_n}{2R}(C_{n-1} + C_n)$$

$$= \frac{11.68}{2 \times 180}(30 + 11.68)$$

$$= 1.352 \text{ m}$$

(e) With the help of tape and theodolite Length of 1st subchord = 12.2 m

$$\delta_1 = 1718.87 \frac{C_1}{R} \text{ minute} = \frac{1718.87 \times 12.2}{180} \text{ minute}$$

$$= 116.50' = 1°56'30''$$

$$\delta_2 = \frac{1718.87 \times 30}{180} \text{ minute} = 286.48' = 4°46'29''$$

$$\delta_3 = \delta_4 = \ldots = \delta_{n-1} = \delta_2$$

last subchord = 11.68 m

$$\delta_n = \frac{1718.87 \times 11.68}{180} \text{ minute}$$

$$= 1°51'30''$$

Readings from the tangent point

$\Delta_1 = \delta_1 = 1°56'30''$

$\Delta_2 = \delta_1 + \delta_2 = 1°56'30'' + 4°46'29'' = 6°42'59'' = 6°43'00''$

$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = 6°42'59'' + 4°46'29'' = 11°29'28'' = 11°29'20''$

$\Delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4 = 11°29'28'' + 4°46'29''$

$= 16°15'57'' = 16°16'00''$

$\Delta_5 = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 16°15'57'' + 4°46'29''$

$= 21°02'26'' = 21°02'20''$

$\Delta_6 = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 = 21°02'26'' + 1°51'30''$

$= 22°53'56'' = 22°54'00''$

$\Delta_6 = 22°54' = \Delta/2.$

The last reading indicates the values which can be obtained with a 20″ theodolite.

Example 12.3 The intersection point *C* of two railway straights *ABC* and *CDE* is inaccessible and so convenient points *B* and *D* in the straights are selected giving *BD* = 183 m angle *CBD* = 9°24′, angle *CDB* = 10°36′ and the forward chainage of *B* = 2750.00 m. The conditions of the sight are such that it is decided to make *B* the first tangent point. Determine the radius of a circular curve to connect the straights, tabulate all the data necessary to set out pegs at 20 m intervals of through chainage and show your calculations check. (L.U.)

Solution From (Fig. 12.21), Deflection angle $\Delta = 9°24' + 10°36' = 20°$

$$\frac{183}{\sin 160°} = \frac{CB}{\sin 10°36'} = \frac{CD}{\sin 9°24'}$$

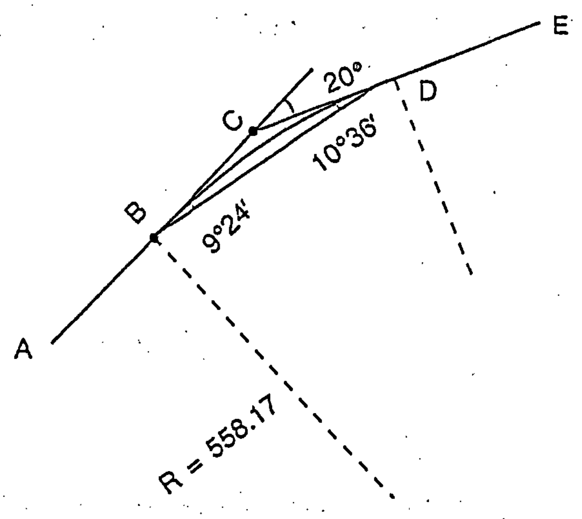


Fig. 12.21 Example 12.3.

Therefore $$CB = \frac{183 \times \sin 10°36'}{\sin 160°} = 98.42 \text{ m}$$

$$CD = \frac{183 \times \sin 9°24'}{\sin 160°} = 87.39 \text{ m}$$

$$R \tan \Delta/2 = CB = 98.42$$

or $$R = \frac{98.42}{\tan 10°} = 558.167 \text{ m}$$

Chainage of the 1st tangent point = 2750.00 m. With 20 m chain this is equal to 137 ch + 10 m.

Hence the length of 1st subchord = 10 m.

$$\text{Length of curve} = \frac{\pi R}{180} \times 20 = \frac{\pi R}{9}$$

$$= \frac{\pi \times 558.167}{9}$$

$$= 194.837 \text{ m}$$

$$= 9 \text{ ch} + 14.837 \text{ m}$$

Length of last subchord = 14.837 m.

$$\Delta_1 = \delta_1 = \frac{1718.87 \times 10}{558.167} = 30'48''$$

$$\Delta_2 = \delta_1 + \delta_2 = \frac{1718.87}{558.167}(20 + 10) = 1°32'23''$$

... ...

$$\Delta_n = \delta_1 + \delta_2 + \ldots + \delta_n = \frac{1718.87}{558.167}(194.837) = 10°$$

$$\Delta_n = \frac{20}{2} = \Delta/2.$$

Example 12.4 In improving an existing railway curve by inserting transition curves 4 chains in length, 6 chains of the existing 25 chains radius curve are taken up at each end and replaced in part by curves of sharper radius. Determine the radius of the sharpened curves, also the total centre line length of track to be relaid. [L.U.]

Solution Let R_o and R_n (Fig. 12.22) be the original and amended radii respectively and let the shift $s = \frac{L^2}{24R_o} = \frac{16}{24 \times 25} = 0.0267$ ch (approximately as here R_o has been substituted for R_n)

From Fig. 12.22

$$s = (R_o - R_n) \text{ vers } \beta \qquad [\text{vers } \beta = 1 - \cos \beta] \qquad \text{(i)}$$

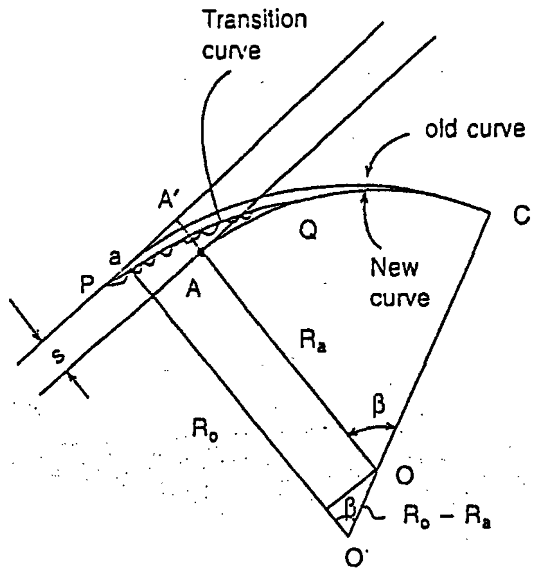


Fig. 12.22 Example 12.4.

But $\beta = \frac{aC}{R_o}$ rad $= \frac{6}{25}$ rad $= 0.24$ rad $= 13°45'$

and $1 - \cos\beta = 1 - \cos 13°45' = 0.02865$

But actual $s = \frac{L^2}{24R_a} = \frac{4^2}{24R_n} = \frac{2}{3R_a}$

Substituting in (i) $\frac{2}{3R_n} = (R_o - R_n)$ vers $\beta = (R_o - R_n)\ (0.02865)$

or $R_n^2 - 25R_n + 23.261 = 0$

or $R_n = 24.032$ ch

Now $\frac{AC}{aC} = \frac{R_n}{R_o}$ and

$$AC = \frac{24.032 \times 6}{25} = 5.768 \text{ ch}$$

The transition replaces an equal amount of circular curve. Hence $AQ = 2.000$ and since the shift AA' bisects the transition PQ; the tracks to be replaced at each end is 7.768 ch.

Total centre line length of track = 15.536 ch.

Example 12.5 A single circular highway curve will join tangents XV and VY and also be tangent to BC. Calculate R, L and stations PC and PT in Fig. 12.23.

$$BC = 190 \text{ m}$$

$$= R \tan\frac{34}{2} + R \tan\frac{21.5}{2}$$

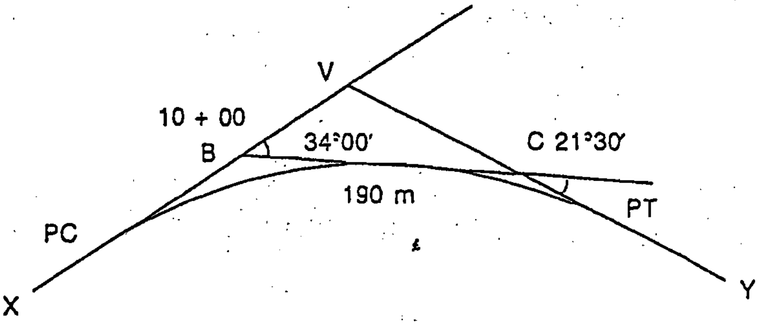


Fig. 12.23 Example 12.5.

or $$R = \frac{190}{\tan 17^\circ + \tan 10.75^\circ} = \frac{190}{0.305 + 0.189}$$

$$= 384.61 \text{ m}$$

$$\text{Length of curve} = \frac{\pi R \times 55.5}{180}$$

$$= \frac{\pi \times 384.61 \times 55.5}{180}$$

$$= 372.55 \text{ m}$$

$$\text{Chainage of } PC = 10 \times 30 - 384.61 \times 0.305$$

$$= 182.7 \text{ m}$$

$$= 6 \text{ ch} + 2.7 \text{ m}$$

$$\text{Chainage of } PT = 372.55 + 182.7$$

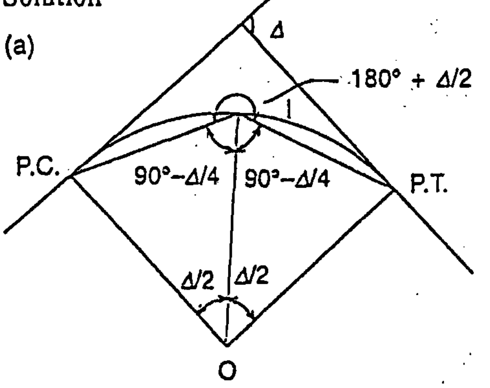
$$= 555.25 \text{ m} = 18 \text{ ch} + 15.25 \text{ m}$$

Example 12.6 After a back sight on the *PC* with 0°00″ set on the instrument, what is the deflection angle to the following curve points (Fig. 12.24)?

(a) Setup at midpoint, deflection to *PT*.
(b) Instrument at mid point of curve, deflection to 3/4 point.
(c) Setup at 1/4 point of curve, deflection at 3/4 point.

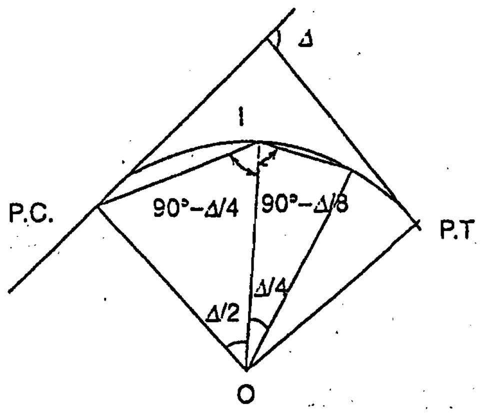
Solution

(a)



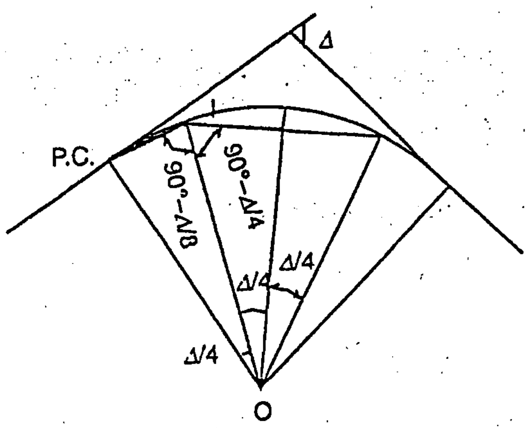
The deflection angle
= 180° + Δ/2
Fig. 12.24 (i)

(b)



The deflection angle
$= 180° + \Delta/4 + \Delta/8$
$= 180° + \frac{3\Delta}{8}$
Fig. 12.24 (ii)

(c)



The deflection angle
$= 180° + \Delta/8 + \Delta/4$
$= 180° + \frac{3\Delta}{8}.$
Fig. 12.24 (iii)

Fig. 12.24 Example 12.6.

Example 12.7 In Fig. 12.25 the coordinates of points A and O are $X_A = 80.00$, $Y_A = 130.00$ and $X_O = 210.00$, $Y_O = 250.00$. If the azimuth of line AB is 39°28′ and the circular curve radius 60.00 m, calculate the coordinates of intersection point P.

Solution Let the coordinates of the point of intersection be X_P and Y_P Fig. 12.25. We get

$$\tan 39°28' = \frac{X_P - X_A}{Y_P - Y_A} = \frac{X_P - 80.00}{Y_P - 130.00}$$

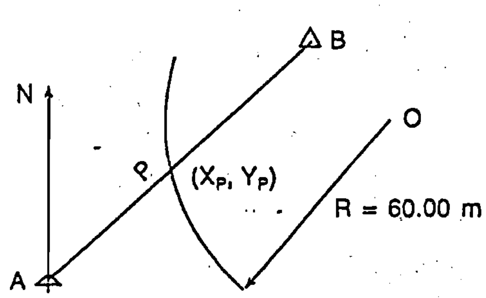


Fig. 12.25 Example 12.7.

or $$0.823 = \frac{X_P - 80.00}{Y_P - 130}$$

or $$X_P = 0.823\ Y_P - 26.99$$

For circle with origin at O.

$$(X_P - X_O)^2 + (Y_P - Y_O)^2 = R^2$$

or $$(X_P - 210.00)^2 + (Y_P - 250.00)^2 = 60^2$$

Substituting

$$(0.823\ Y_P - 26.99 - 210.00)^2 + (Y_P - 250.00)^2 = 60^2$$

or $$1.677\ Y_P^2 - 890.08\ Y_P + 115064.26 = 0$$

or $$Y_P = 307.95,\ 408.176$$

$$X_P = 226.45,\ 308.93$$

Example 12.8 Coordinates of a circle of centre O_1 are $XO_1 = 330.00$ m and $YO_1 = 330.00$ m and for circle with centre O_2. $XO_2 = 470.00$ m and $YO_2 = 200.00$ m. $R_1 = 100$ m and $R_2 = 120$ m. Compute the coordinates of interesection point P shown in the Fig. 12.26.

Solution (Fig. 12.26)

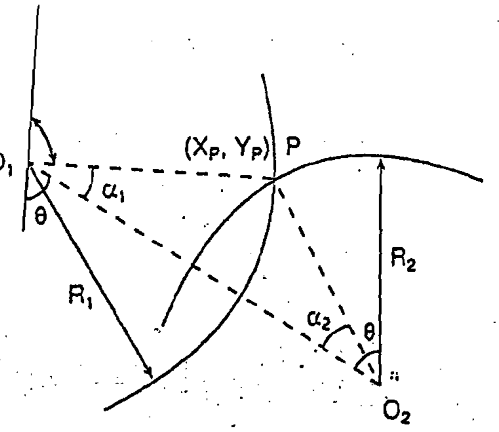


Fig. 12.26 Example 12.8.

$$O_1O_2 = \sqrt{(XO_1 - XO_2)^2 + (YO_1 - YO_2)^2}$$

$$= \sqrt{(330.0 - 470.0)^2 + (330 - 200)^2}$$

$$= 191.05 \text{ m}$$

$$\tan\theta = \frac{XO_2 - XO_1}{YO_1 - YO_2} = \frac{470.00 - 330.00}{330.00 - 200.00} = \frac{140}{130}$$

$$\theta = \tan^{-1}\frac{14}{13} = 47.12°$$

From the law of cosines

$$\alpha_1 = \cos^{-1} \frac{100^2 + 191.05^2 - 120^2}{2(100)(191.05)}$$

$$= 32.85°$$

$$\alpha_2 = \cos^{-1} \frac{120^2 + 191.05^2 - 100^2}{2(120)(191.05)}$$

$$= 26.87°$$

Azimuth of O_1P = 180° − (47.12 + 32.85)

= 100.03°

Azimuth of O_2P = 360° − (47.12 − 26.87)

= 339.35°

Coordinates of P from O_1

$$X_P = 330.00 + 100 \sin 79.97°$$

$$= 428.47 \text{ m}$$

$$Y_P = 330.00 - 100 \cos 79.97°$$

$$= 312.48 \text{ m}$$

Check (coordinates from O_2)

$$X_P = 470.00 + 120 \sin 20.25°$$

$$= 428.46 \text{ m}$$

$$Y_P = 200.00 + 120 \cos 20.25°$$

$$= 312.58 \text{ m}$$

12.4 COMPOUND CURVE

A compound curve consists of a number of circular curves of different radii joined together with centres of the curves all lying on one side of the curve. The point of curvature of the next curve is the point of tangency of the previous one. Figure 12.27 shows a compound curve. The equations of the compound curve can be derived by considering the Figs. 12345 as a closed traverse and applying the usual conditions of a closed traverse, i.e. algebraic sum of departures and latitudes are zero. Assuming the direction of 1–2 as the North line, the azimuth and length of other lines can be tabulated as follows:

The following three equations can then be derived:

$$T_1 + T_2 \cos \Delta - R_2 \sin \Delta + (R_2 - R_1) \sin \Delta_1 = 0 \qquad (12.16)$$

$$T_2 \sin \Delta + R_2 \cos \Delta - (R_2 - R_1) \cos \Delta_1 - R_1 = 0 \qquad (12.17)$$

$$\Delta_1 + \Delta_2 = \Delta \qquad (12.18)$$

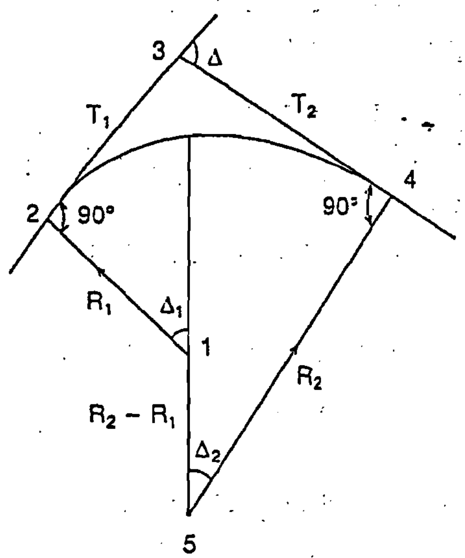


Fig. 12.27 Elements of a compound curve.

Table 12.1 Data of Traverse 1-2-3-4-5.

Side	Azimuth	Length	Departure		Latitude	
			E	W	N	S
1–2	0°	R_1	0		R_1	
2–3	90°	T_1	T_1			
3–4	90° + Δ	T_2	$T_2 \cos \Delta$			$T_2 \sin \Delta$
4–5	180° + Δ	R_2		$R_2 \sin \Delta$		$R_2 \cos \Delta$
5–1	Δ_1	$R_2 - R_1$	$(R_2 - R_1)$ $\sin \Delta_1$		$(R_2 - R_1)$ $\cos \Delta_1$	

There are seven unknowns T_1, T_2, R_1, R_2, Δ_1, Δ_2 and Δ.

Since there are three equations, out of seven unknowns four must be known before the equations can be fully solved.

Example 12.9 A railway siding is to be curved through a right angle and in order to avoid buildings. The curve is to be compound, and radii of the two branches are 240 m and 360 m. The distance from the intersection point of the end straight to the tangent point at which the 240 m radius curve leaves the straight is 300 m (Fig. 12.28). Obtain the second tangent length of whole curve.

Solution The three equations of compound curve are:

$$T_1 + T_2 \cos \Delta - R_2 \sin \Delta + (R_2 - R_1) \sin \Delta_1 = 0 \quad (1)$$

$$T_2 \sin \Delta + R_2 \cos \Delta - (R_2 - R_1) \cos \Delta_1 - R_1 = 0 \quad (2)$$

$$\Delta_1 + \Delta_2 = \Delta \quad (3)$$

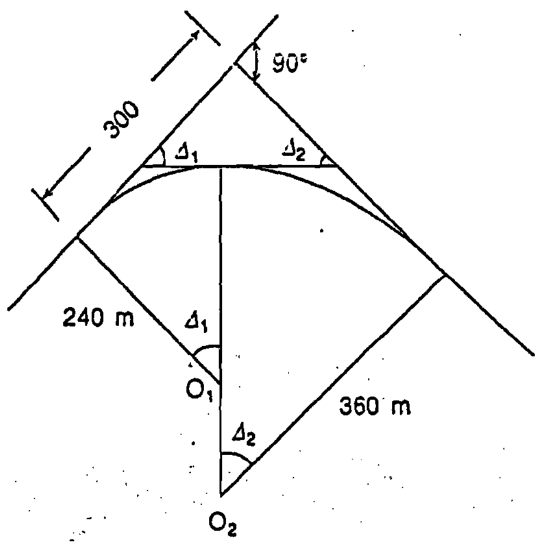


Fig. 12.28 Example 12.9.

Here

$$T_1 = 300 \text{ m}$$
$$\Delta = 90°$$
$$R_1 = 240 \text{ m}$$
$$R_2 = 360 \text{ m}$$

Equation (1) then reduces to

$$300 + 0 - 360 + 120 \sin \Delta_1 = 0$$
$$\sin \Delta_1 = \frac{60}{120} = 0.5$$
$$\Delta_1 = \sin^{-1} 0.5 = 30°$$
$$\Delta_2 = 60°$$

Equation (2) gives

$$T_2 + 0 - 120 \cos 30° - 240 = 0$$
$$T_2 = 240 + 120 \cos 30°$$
$$= 343.92 \text{ m}$$

Example 12.10 Referring to Fig. 12.29, if $T_1 = 100$ m, $R_1 = 140$ m, $\Delta_1 = 18°15'$, $\Delta = 42°10'$ and the chainage of the point of intersection is at station 50 + 19.70. Using the arc definition of degree of curve, compute T_2, R_2 and Δ_2 and the chainages of the point of compound curvature and the point of tangency.

Solution The three equations of compound curve are:

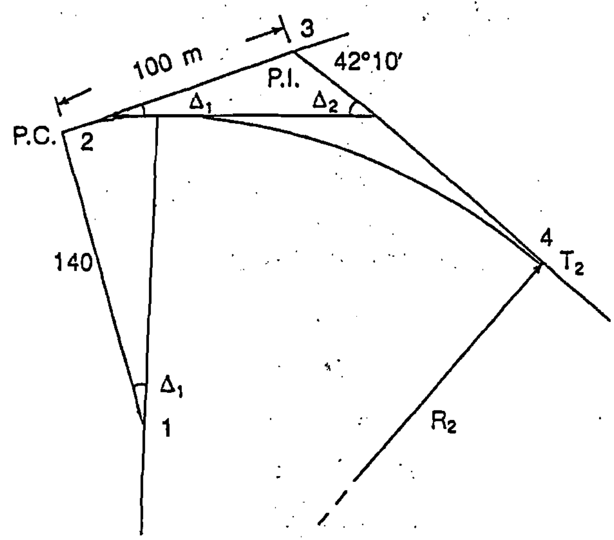


Fig. 12.29 Example 12.10.

$$T_1 + T_2 \cos \Delta - R_2 \sin \Delta + (R_2 - R_1) \sin \Delta_1 = 0 \quad (1)$$

$$T_2 \sin \Delta + R_2 \cos \Delta - (R_2 - R_1) \cos \Delta_1 - R_1 = 0 \quad (2)$$

$$\Delta_1 + \Delta_2 = \Delta \quad (3)$$

Here $T_1 = 100$ m

$R_1 = 140$ m

$\Delta_1 = 18°15'$

$\Delta = 42°10'$

Equation (1) then reduces to

$$100 + T_2 \cos 42°10' - R_2 \sin 42°10' + (R_2 - 140) \sin 18°15' = 0 \quad (4)$$

Equation (2) reduces to

$$T_2 \sin 42°10' + R_2 \cos 42°10' - (R_2 - 140) \cos 18°15' - 140 = 0 \quad (5)$$

Equation (3) reduces to

$$18°15' + \Delta_2 = 42°10' \quad (6)$$

From (6) $\Delta_2 = 23°55'$

From (4)

$$100 + T_2(0.74) - R_2(0.67) + (R_2 - 140)(0.31) = 0$$

From (5)

$$T_2(0.67) + R_2(0.74) - (R_2 - 140)(0.95) - 140 = 0$$

or $0.74\, T_2 - 0.36\, R_2 = -56.6$

$$0.67\ T_2 - 0.21\ R_2 = 7.0$$

Solving $T_2 = 167.90$ m

$R_2 = 502.35$ m

Chainage of point of intersection = 50 + 19.70

= 1500 + 19.70

Chainage at the beginning of curve (assuming 30 m chain) = 1500 + 19.7 – 100

= 47 + 9.7

Length of first curve = 140 × Δ_1 (radian)

$$= 140 \times \frac{18.25 \times \pi}{180}$$

$T_1 = 44.593$ m

Chainage of point of compound curvature = 47 + 9.7 + 44.593

= 47 + 54.293

= 48 ch + 24.293 m

Chainage of the point of tangency = 48 + 24.293 + Length of second curve

$$\text{Length of second curve} = \frac{502.35 \times 42.17 \times \pi}{180} = 369.732$$

Chainage of point of tangency = 48 + 24.293 + 369.732

= 48 + 13 + 4.025

= 61 ch + 4.025 m

Example 12.11 The following data refer to a compound circular curve which bears to the right (Fig. 12.30).

Angle of intersection (or total deflection) = 60°
Radius of 1st curve = 20 chains.
Chainage of point of intersection = 164 ch + 15.2 m.

Determine the running distances of the tangent point and the point of compound curvature given that the latter point is 4.25 chain from the point of intersection at a back angle of 294°30′ from the 1st tangent. Assume 30 m chain.

Solution Angle *ABD* = 360° – 294°30′

= 65°30′

Radius of 1st curve = 20 × 30 = 600 m

Chainage of intersection point = 164 ch + 15.2 m

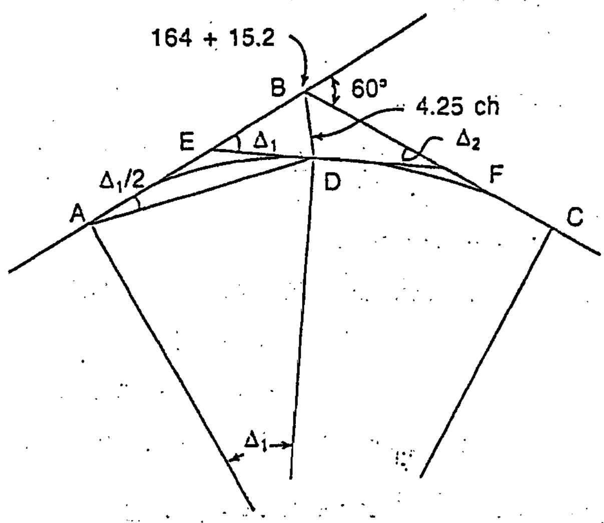


Fig. 12.30 Example 12.11.

$$= 164 \times 30 + 15.2$$

$$= 4935.20 \text{ m}$$

$$4.25 \text{ ch} = 4.25 \times 30 = 127.50 \text{ m}$$

From triangle *ABD*

$$\frac{\sin \Delta_1/2}{127.5} = \frac{\sin 65°30'}{AD} = \frac{\sin 65°30'}{2 \times 600 \sin \Delta_1/2}$$

or

$$(\sin \Delta_1/2)^2 = \frac{127.5 \times \sin 65°30'}{1200} = 0.09$$

$$\Delta_1/2 = 18.115°$$

$$\Delta_1 = 36.23°$$

$$= 36°\ 13'48''$$

$$\Delta_2 = 60° - 36°13'48''$$

$$= 23°46'12''$$

$$AE = ED = R_1 \tan \Delta_1/2$$

$$= 600 \tan 18.115$$

$$= 196.284 \text{ m}$$

$$\frac{BE}{\sin 78.27} = \frac{196.284}{\sin 65.5}$$

$$BE = \frac{196.284 \sin 78.27}{\sin 65.5}$$

$$= 211.176 \text{ m}$$

$$EF = \frac{211.176 \times \sin 120}{\sin 23.77}$$

$$= \frac{211.176 \times 0.866}{0.403}$$

$$= 453.79$$

Chainage of A = Chainage of $B - BE - AE$ = 4935.2 − 211.176 − 196.28

= 4527.74

$EF = 453.79 \qquad DE = 196.284 \qquad DF = 257.51 = R_2 \tan 11.885$

$R_2 = 1223.56$ m

$$\text{Length of curve } AD = 600 \times \frac{36.23}{180} \times \pi$$

$$= 379.399 \text{ m}$$

Chainage of point of compound curvature = 4527.74 + 379.40

= 4907.14

$$\text{Length of 2nd curve} = \frac{1223.56 \times 23.77}{180°} \times \pi$$

$$= 507.61$$

Chainage of PT = 4907.14 + 507.61

= 5414.75 m

Example 12.12 A 200 m length of straight connects two circular curves both of which deflect to the right. The radius of the 1st curve is 250 m and that of the 2nd is 200 m. The central angle for the second curve is 30°. The combined curve is to be replaced by a single circular curve between the same tangent points. Find the radius of the curve. Assume that the two tangent lengths of the earlier set are equal. Also determine (a) central angle of the new curve (b) central angle of 1st curve of radius 200 m.

Solution Since the combined curve is to be replaced by a single circular curve between the same tangent points, the tangent lengths must be equal. Figure 12.31 shows the original curve with a straight portion in between. The dotted line shows the proposed circular curve.

Since the tangent lines remain the same, the straight lines AO_1 and DO_2 when produced will intersect at O, the centre of the new curve.

Draw O_2C_1 perpendicular to O_1B.

$$O_1C_1 = O_1B - BC_1$$

But $\quad O_1B = O_1A$ = radius of 1st curve = 250 m

$BC_1 = O_2C$ = radius of 2nd curve = 200 m

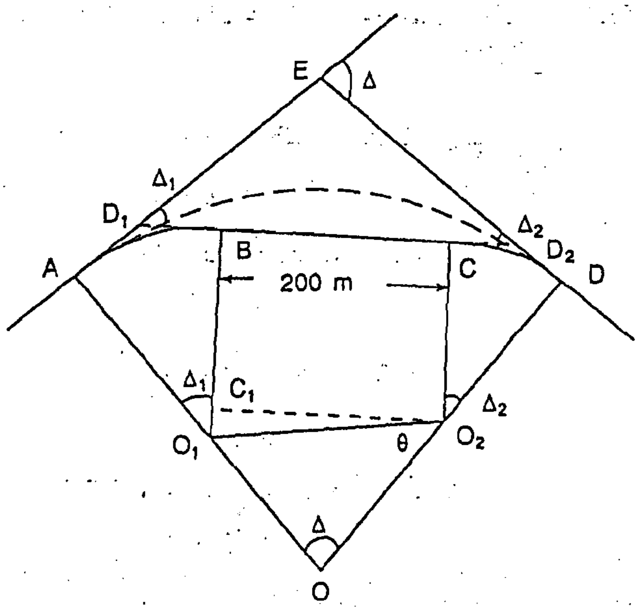


Fig. 12.31 Example 12.12.

Hence $\quad O_1C_1 = 250 - 200 = 50$ m

and $\quad O_1O_2 = \sqrt{200^2 + 50^2} = 206.2$ m

$$\theta = \tan^{-1}\frac{O_1C_1}{O_2C_1} = \tan^{-1}\frac{50}{200} = 14.03°$$

$$= 14°2'$$

At point O_2, $O_1O_2O + \theta + 90° + \Delta_2 = 180°$

Therefore, $\quad O_1O_2O = 180° - (\theta + 90° + \Delta_2)$

$$= 180° - (14°2' + 90° + 30°)$$

$$= 45°58'$$

From triangle O_1O_2O

$$OO_1^2 = O_1O_2^2 + OO_2^2 - 2O_1O_2OO_2 \cos 45°58'$$

$$(R - 250)^2 = 206.2^2 + (R - 200)^2 - 2(R - 200)(206.2)(0.695)$$

or $\quad R^2 - 500R + 250^2 = 206.2^2 + R^2 - 400R + 200^2 - 286.61R + 57323.6$

Solving, $R = 414.458$ m.

In the triangle OO_1O_2 $\quad \dfrac{OO_1}{\sin O_1O_2O} = \dfrac{OO_2}{\sin OO_1O_2}$

$$OO_1 = R - 250 = 164.458 \text{ m}$$

$$OO_2 = R - 200 = 214.458 \text{ m}$$

Hence
$$\frac{164.458}{\sin 45°58'} = \frac{214.458}{\sin OO_1O_2}$$

$$\sin OO_1O_2 = \frac{214.458 \times \sin 45°58'}{164.458}$$

$$= 0.9375$$

$$\angle OO_1O_2 = 69.63°$$

$$\Delta = 180° - (69°37'48'' + 45°58') = 64°24'12''$$

$$\Delta_1 = \Delta - \Delta_2 = 64°24'12'' - 30° = 34°24'12''$$

12.5 REVERSE CURVE

A reverse curve is composed of two simple curves turning in opposite directions as shown in Fig. 12.32.

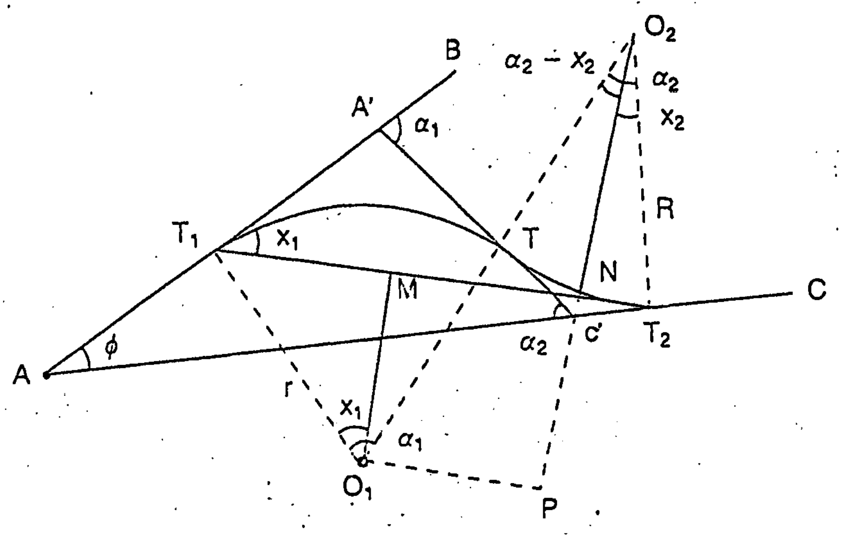


Fig. 12.32 Reverse curve.

Reverse curves are used when the straights are parallel or intersect at a very small angle. The use of reverse curve is limited to unimportant places like sidings and cross overs. Sometimes reverse curves are provided on roads and railways designed for low speeds. High speeds cannot be provided on reverse curves for the following reasons:

1. They involve sudden change of super elevation (cross slopes) at the junction of two branches of the curve.
2. Steering is dangerous in case of highways.

12.5.1 GENERAL EQUATION OF REVERSE CURVE

AB and AC are two straights meeting at an angle ϕ. T_1 and T_2 are the two tangent points. T is the point of common tangency. The common tangent $A'TC'$ makes an angle of α_1 and α_2 with AB and AC respectively. α_1 and α_2 are also angles subtended at the centres by arcs of radii r and R respectively.

Let $\angle A'T_1T_2$ be x_1 and $\angle AT_2T_1$ be x_2. Then $\angle T_1O_1M = x_1$ and $\angle T_2O_2N = x_2$ as O_1M and O_2N are drawn perpendiculars to T_1T_2. O_2N is produced and O_1P is drawn parallel to MN so that they intersect at P. From Fig. 12.32, the following derivations can be made:

$$\alpha_1 = \phi + \alpha_2$$

or $$\phi = \alpha_1 - \alpha_2$$

Similarly $$x_1 = \phi + x_2$$

or $$\phi = x_1 - x_2$$

Hence, $$\alpha_1 - \alpha_2 = x_1 - x_2$$

or $$\alpha_1 - x_1 = \alpha_2 - x_2$$

From Fig. 12.32,

$$T_1M = r \sin x_1$$

$$MN = O_1P = (R + r) \sin (\alpha_2 - x_2)$$

$$NT_2 = R \sin x_2$$

Hence $$T_1T_2 = T_1M + MN + NT_2$$

$$= r \sin x_1 + (R + r) \sin (\alpha_2 - x_2) + R \sin x_2$$

Similarly $$O_1M = PN = r \cos x_1$$

$$O_2N = R \cos x_2$$

$$O_2P = O_2N + O_1M = R \cos x_2 + r \cos x_1$$

$$= (R + r) \cos (\alpha_2 - x_2)$$

or $$\cos (\alpha_1 - x_1) = \cos (\alpha_2 - x_2) = \frac{R \cos x_2 + r \cos x_1}{R + r}$$

When the two straights are parallel

$$\phi = \infty \text{ (infinity) and } \alpha_1 = \alpha_2 = \alpha \text{ (say)}$$

Then $$T_1T_2 = T_1T + TT_2 = 2(R + r) \tan \alpha/2$$

$$\text{Common tangent length} = A'C' = (R + r) \tan \alpha/2$$

Example 12.13 Two parallel railway lines are to be connected by a reverse curve, each section having the same radius. If the lines are 10 m apart and the maximum distance between tangent points measured parallel to the straights is 40 m, find the maximum allowable radius. If, however, both the radii are different, calculate the radius of the second branch if that of the 1st branch is 50 m. Also calculate lengths of both branches.

Solution From Fig. 12.33, we have

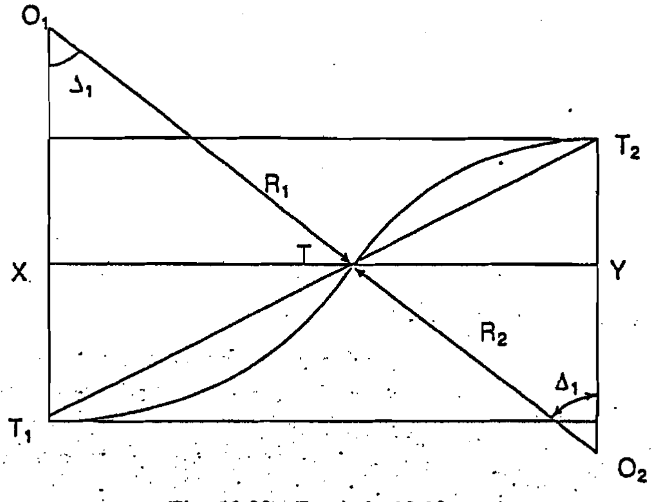


Fig. 12.33 Example 12.13.

(i)

$$\tan \Delta_1/2 = \frac{10}{40} = 0.25$$

$$\Delta_1/2 = \tan^{-1} 0.25 = 14°02'$$

$$\Delta_1 = 28°04'$$

When the radii are equal, let the common radius be R. Then

$$XT = R \sin \Delta_1 = TY$$

$$XY = XT + TY = 2R \sin \Delta_1 = 40$$

$$R = \frac{40}{2 \sin \Delta_1} = \frac{40}{2 \times 0.47} = 42.50 \text{ m}$$

(ii) Let the radii be R_1 and R_2, then

$$XY = XT + TY = R_1 \sin \Delta_1 + R_2 \sin \Delta_1$$

$$= (R_1 + R_2) \sin \Delta_1$$

$$= 40$$

$$R_1 + R_2 = \frac{40}{\sin \Delta_1} = \frac{40}{0.47} = 85.02$$

If $R_1 = 50$, $R_2 = 85.02 - 50 = 35.02$

$$\text{Length of 1st branch} = \frac{\pi R_1 \Delta_1}{180} = \frac{\pi \times 50 \times 28°4'}{180}$$

$$= 24.49 \text{ m}$$

$$\text{Length of 2nd branch} = \frac{\pi R_2 \Delta_1}{180} = \frac{\pi \times 35.02 \times 28°4'}{180}$$

$$= 17.15 \text{ m}$$

12.6 TRANSITION CURVE

A transition curve is a curve of varying radius introduced between a straight and a circular curve or between two circular curves to facilitate change over from straight to curve or from one curve to another. As soon as a vehicle or a train enters a curve, it experiences a centrifugal force which tends to cause derailment, overturning, or side slipping of vehicles. To avoid this, super elevation is provided which means raising the outer edge of a curve over the inner one. Transition curve helps in (i) providing super elevation. (ii) increase or decrease in curvature gradually.

12.6.1 SUPER ELEVATION

It is the raising of the outer edge of the railway or road surface above the inner one as shown in Fig. 12.34. When a vehicle moves on a curve there are two forces acting: (i) Weight of the vehicle W, (ii) Centrifugal force P.

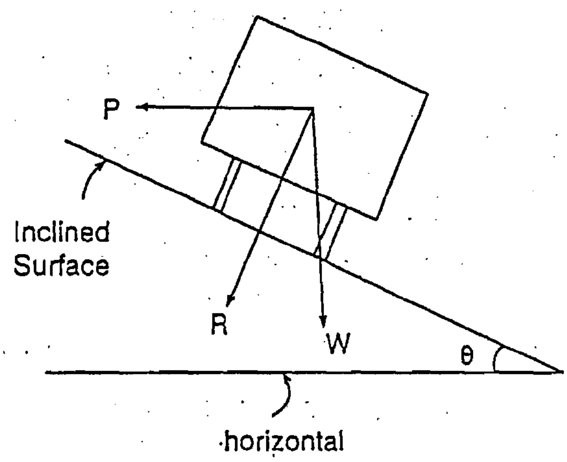


Fig. 12.34 Super elevation.

In order that there is no lateral thrust the resultant force must be normal to the inclined surface.

$$\text{The centrifugal force } P = \frac{Wv^2}{gR}$$

Hence

$$\tan\theta = \frac{\text{Centrifugal force}}{\text{Weight of the vehicle}} = \frac{Wv^2}{gR} \times \frac{1}{W}$$

$$= \frac{v^2}{gR}$$

where v = velocity

g = acceleration due to gravity

R = radius of the curve

If B = width of the road in m

G = gauge of the railway in m.

then $$\tan\theta = \frac{h}{B} \text{ for roadways}$$

$$= \frac{G}{B} \text{ for railways}$$

or $$h = B\tan\theta = \frac{Bv^2}{gR} \text{ on roads}$$

$$= \frac{Gv^2}{gR} \text{ on railways}$$

In railways if the cant or super elevation is provided by the above relation, it is known as *equilibrium cant.* If less cant is provided the track will have *cant deficiency*. In railways cant is usually restricted to 15 cm.

12.6.2 SUPER ELEVATION ON HIGHWAYS

In highways the friction between the tyre of the vehicle and the road surface comes into play when the vehicle negotiates a curve. This frictional force acts parallel to the pavement of the highway and is given in terms of the side friction factor (f) which is the ratio of the sum of forces due to friction acting parallel to the pavement to sum of forces normal to the pavement.

From Fig. 12.34,

$$\text{Forces normal to the pavement} = P\sin\theta + W\cos\theta$$

$$\text{Forces parallel to the pavement} = P\cos\theta - W\sin\theta$$

Hence side friction factor $f = \dfrac{P\cos\theta - W\sin\theta}{P\sin\theta + W\cos\theta}$

or $$P\cos\theta - W\sin\theta = f(P\sin\theta + W\cos\theta)$$

or $$\frac{P}{W} = \frac{\sin\theta + f\cos\theta}{\cos\theta - f\sin\theta}$$

$$= \frac{\tan\theta + f}{1 - f\tan\theta}$$

Now $$\frac{P}{W} = \frac{v^2}{gR}$$

Hence $$\frac{v^2}{gR} = \frac{\tan\theta + f}{1 - f\tan\theta}$$

for safe design. $$\frac{v^2}{gR} \le \frac{\tan\theta + f}{1 - f\tan\theta} \tag{12.19}$$

As θ is small, $\tan\theta$ is small and $f\tan\theta$ is still smaller and hence can be neglected. The usual value of f is taken as 0.15 for speeds greater than 50 km/hr and 0.18 for speeds less than 50 km/hr.

Equation (12.16) shows that centrifugal force is balanced partly by friction and partly by super elevation. If we want to balance P entirely by super elevation f is taken to be zero and we get

$$\frac{v^2}{gR} = \tan\theta$$

If maximum super elevation is taken so that $\tan\theta = 1/4$, then

$$\frac{v^2}{gR} = 1/4$$

and $$R = \frac{4v^2}{g}$$

If the entire centrifugal force is to be balanced only by the friction then $\theta = 0$ and

$$\frac{v^2}{gR} = f$$

and $$R = \frac{v^2}{gf}$$

If the maximum value of f is taken as 0.25

$$R = \frac{v^2}{0.25g} = \frac{4v^2}{g}$$

12.7 CENTRIFUGAL RATIO

The ratio of the centrifugal force and weight is called the centrifugal ratio.

$$\text{Thus centrifugal ratio} = P/W = \frac{Wv^2}{gRW} = \frac{v^2}{gR}.$$

The maximum value of centrifugal ratio is 1/4 for road and for railway it is 1/8. Thus for roads

$$\frac{v^2}{gR} = 1/4$$

or $$v = \sqrt{\frac{gR}{4}} \quad (12.20a)$$

for railways $$v = \sqrt{\frac{gR}{8}} \quad (12.20b)$$

12.8 LENGTH OF TRANSITION CURVE

Length of a transition curve may be taken in the following manner:

(a) As an arbitrary value from past experience say 50 m.
(b) When super elevation is applied at a uniform rate say 0.1 m in 100 m.

For cant of h meter length of curve = 1000 h

$$= 1000 \times \frac{Gv^2}{gR}$$

If

$$G = 1.5 \text{ m}$$

$$V = \text{km/hr}$$

$$= \frac{1000\ V}{60 \times 60} \text{ m/sec}$$

$$\text{Length } L = \frac{1000 \times 1.5 \times (1000 \times V)^2}{(60 \times 60)^2 \times 9.81} \cdot \frac{1}{R}$$

$$= 424.7 \times \frac{V^2}{R}$$

$$= 424.7\ \frac{V^2}{R}$$

(c) When super elevation is applied at an arbitrary time rate of r units per second, let us say,

L = length of transition curve in meters
h = amount of super elevation.
v = speed of vehicle in m/sec.
r = time rate cm/sec.
V = speed in km/hr.

Time taken by the vehicle to pass over the transition curve

$$t = \frac{L}{v} \text{ sec}$$

Super elevation attained in this time

$$t \times r \text{ cm} = \frac{L}{v} \cdot r \text{ cm}$$

$$= \frac{Lr}{v \times 100}$$

But

$$h = \frac{Gv^2}{gR}$$

when $G = 1.5$ m $\quad V$ = km/hr

$$h = \frac{L \cdot r}{v100} = \frac{1.5}{9.81} \times \left(\frac{V \times 1000}{60 \times 60}\right)^2 \frac{1}{R}$$

or

$$L = \frac{v \times 100 \times 1.5}{r \times 9.81 \times R} \times \left(\frac{V \times 1000}{60 \times 60}\right)^2$$

$$= \frac{V \times 1000}{60 \times 60} \times \frac{100 \times 1.5}{9.81 \times Rr} \times \left(\frac{V \times 1000}{60 \times 60}\right)^2$$

$$= 0.3277 \frac{V^3}{Rr}$$

(d) By the rate of change of radial acceleration: In this method the length of the transition curve is decided so that the passenger does not experience any discomfort due to sudden application of centrifugal force. W.H Shortt gives a rate of change of radial acceleration of about 1/3 m/sec^3 as a *comfort limit* above which side throw will be noticed.

If v = speed in m/sec, time taken to travel over the transition curve is L/v sec. If

$$\alpha = \frac{1}{3} \text{ m/sec}^3$$

is the permissible rate of change of radial acceleration.
Acceleration attained during this time is

$$\frac{1}{3} \times \frac{L}{v} \text{ m/sec}^2$$

This should be equal to v^2/R, acceleration at the beginning of circular curve. Hence

$$\frac{L}{3v} = \frac{v^2}{R}$$

or

$$L = \frac{3v^3}{R} \text{ m}$$

If V is expressed in km/hr

$$L = 3\left(\frac{V \times 1000}{60 \times 60}\right)^3 \frac{1}{R}$$

$$= 6430 \frac{V^3}{R} \text{ m}$$

12.9 IDEAL TRANSITION CURVE

In a transition curve super elevation is gradually provided. It is zero at the straight and reaches its maximum value at the beginning of the circular curve. Thus

$$h \propto l$$

but $h = \frac{bv^2}{gr}$ hence $\frac{bv^2}{gr} \propto l$

If b, v and g are constants

$$\frac{1}{r} \propto l$$

or $$lr = \text{constant} = LR$$

where L is the total length of the transition curve and R, radius of the curve at its end (i.e. minimum radius). Thus the fundamental requirement of a transition curve is that its radius of curvature r at any point shall vary inversely as the distance l from the beginning of the curve. Such a curve is the clothoid or the Glover's spiral and is known as the ideal transition curve.

12.9.1 INTRINSIC EQUATION OF THE IDEAL TRANSITION CURVE

Fig. 12.35 shows an ideal transition curve. Here

T = tangent point = beginning of the transition curve
Tc = initial tangent
A = any point on the transition curve at a distance l from the origin T
r = radius of curve at point A
ϕ = angle which the tangent at A to the curve makes with the initial tangent Tc
ϕ_S = spiral angle, i.e. the angle between the initial tangent and the tangent to the transition curve at the junction point D.
R = radius of circular curve
L = total length of the transition curve.
X, Y = coordinates of the junction point D.
x, y = coordinates of any point A of the transition curve.

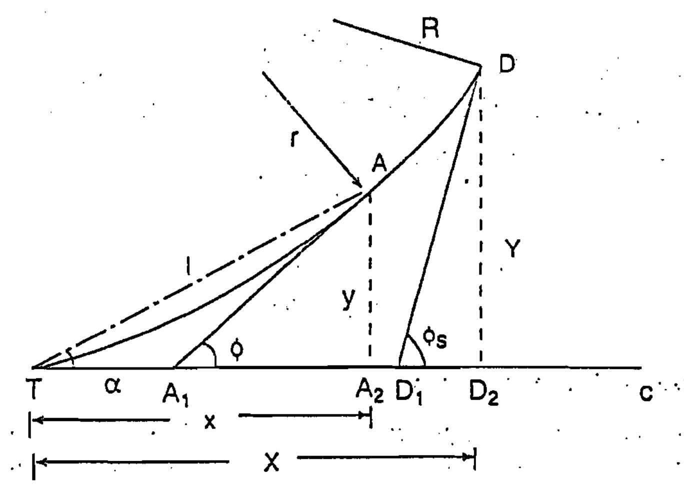


Fig. 12.35 Ideal transition curve.

It has already been shown that for an ideal transition curve

$$lr = \text{constant} = LR$$

or $$\frac{1}{r} = \frac{l}{LR}$$

But $$\frac{1}{r} = \text{curvature} = \frac{d\phi}{dl}$$

$\therefore$ $$\frac{d\phi}{dl} = \frac{l}{LR}$$

or $$d\phi = \frac{l\,dl}{LR}$$

Integrating, we get

$$\phi = \frac{l^2}{2LR} + c$$

at $l = 0$, $\phi = 0$, $c = 0$. Therefore

$$\phi = \frac{l^2}{2LR} \tag{12.21}$$

This is the *intrinsic equation* of the ideal transition curve. It can be further deduced

$$l = \sqrt{2RL\phi} = k\sqrt{\phi} \tag{12.22}$$

where $$k = \sqrt{2RL}$$

when $l = L$, i.e. at the junction

$$\phi = \phi_S = \frac{L}{2R}$$

12.9.2 EQUATIONS OF THE CURVE IN TERMS OF CARTESIAN COORDINATES

From calculus,

$$dx = dl\cos\phi$$

$$= dl\left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \ldots\right)$$

But $$l = k\sqrt{\phi}$$

$\therefore$ $$dl = \frac{k}{2}\cdot\frac{1}{\phi^{1/2}}\,d\phi$$

Substituting

$$dx = \frac{k}{2}\left(\phi^{-1/2} - \frac{\phi^{3/2}}{2!} + \frac{\phi^{7/2}}{4!} - \ldots\right)d\phi$$

Integrating $$x = k\left(\phi^{1/2} - \frac{\phi^{5/2}}{10} + \frac{\phi^{9/2}}{216} - \ldots\right)$$

The constant $c = 0$ as $x = 0$ when $\phi = 0$ or

$$x = l\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\right) \tag{12.23}$$

But $$\phi = \frac{l^2}{k^2} \text{ and } k^2 = 2RL$$

$$x = l\left(1 - \frac{l^4}{10\,k^4} + \frac{l^8}{216\,k^8} - \ldots\right)$$

or $$x = l\left(1 - \frac{l^4}{40\,R^2L^2} + \frac{l^8}{3456\,R^4L^4} - \ldots\right)$$

Similarly,

$$dy = dl \sin\phi$$

$$= dl\left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots\right)$$

Substituting, $$dl = \frac{k}{2}\frac{1}{\phi^{1/2}}\,d\phi$$

$$dy = \frac{k}{2}\left(\phi^{1/2} - \frac{\phi^{5/2}}{6} + \frac{\phi^{9/2}}{120} - \ldots\right)d\phi$$

Integrating $$y = k\left(\frac{\phi^{3/2}}{3} - \frac{\phi^{5/2}}{42} + \frac{\phi^{11/2}}{1320} - \ldots\right) \tag{12.24}$$

Putting $$l = k\sqrt{\phi} \quad \text{or} \quad k = \frac{l}{\sqrt{\phi}}$$

$$y = l\left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots\right) \tag{12.25}$$

Putting $$\phi = \frac{l^2}{k^2}$$

$$y = \frac{l^3}{3k^2}\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440} - \ldots\right)$$

$$= \frac{l^3}{3k^2}\left(1 - \frac{l^4}{14k^4} + \frac{l^8}{440k^8} - \ldots\right)$$

$$= \frac{l^3}{6RL}\left(1 - \frac{l^4}{56R^2L^2} + \frac{l^8}{7040R^4L^4} - \ldots\right) \tag{12.26}$$

From Fig. 12.35

$$\tan\alpha = \frac{y}{x} = \frac{k\left(\frac{\phi^{3/2}}{3} - \frac{\phi^{7/2}}{42} + \frac{\phi^{11/2}}{1320} - \ldots\right)}{k\left(\phi^{1/2} - \frac{\phi^{5/2}}{10} + \frac{\phi^{9/2}}{216} - \ldots\right)}$$

$$= \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5997} \qquad (12.27)$$

This is very closely approximated by the expression

$$\tan\phi/3 = \frac{\phi}{3} + \frac{\phi^3}{81} + \frac{\phi^5}{18225}$$

Hence $\tan\alpha = \tan\phi/3$

or $\alpha = \phi/3$ as both α and ϕ are small

$$= \frac{1}{3}\cdot\frac{l^2}{2RL} = \frac{l^2}{6RL} \text{ radian} \qquad (12.28)$$

$$= \frac{1800\, l^2}{\pi RL} \text{ minute} \qquad (12.29)$$

Considering only the 1st term the following approximate expressions can be obtained.

$$x = l$$

$$y = \frac{l^3}{6RL}$$

and finally

$$y = \frac{x^3}{6RL}$$

The expression $y = l^3/6RL$ is the equation of a cubic spiral. Here only one approximation has been made, i.e. $\sin\phi = \phi$. But in the form

$$y = \frac{x^3}{6RL}$$

which is expression for a cubic parabola, two approximations i.e.

$$\sin\phi = \phi \quad \text{and} \quad \cos\phi = 1$$

have been made. The cubic parabola is, therefore, inferior to cubic spiral.

The expression $\alpha = \phi/3$ is approximate. For layout of true spirals or for applications requiring more accuracy, a small correction must be subtracted to get an exact relation,

$$\alpha = \phi/3 - c''$$

where c'' is a correction expressed in seconds and found by $c'' = 0.00309\,\phi^3 + 0.00228\,\phi^{-5}$

where ϕ is in decimal degrees. The correction is small and can be neglected in most work. Table 12.1 shows the typical values, c'' is a nonlinear function of ϕ, increasing at a greater rate for larger ϕ's.

Table 12.1 Corrections to ϕ

ϕ (deg.)	c''
5	0.4
10	3.1
15	10.46
20	24.83
30	84.11
35	133.88
40	200.39

12.9.3 MINIMUM RADIUS OF CURVATURE OF A CUBIC PARABOLA

The equation of a cubic parabola $y = \dfrac{x^3}{6RL}$

$$= Mx^3$$

$$\text{radius of curvature } r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

For cubic parabola

$$\frac{dy}{dx} = 3\,Mx^2.$$

$$\frac{d^2y}{dx^2} = 6\,Mx.$$

But

$$\frac{dy}{dx} = \tan\phi$$

$\therefore$

$$3\,Mx^2 = \tan\phi$$

$$x = \sqrt{\frac{\tan\phi}{3M}} \tag{12.30}$$

$$\frac{d^2y}{dx^2} = 6\,Mx = 6\,M\sqrt{\frac{\tan\phi}{3M}}$$

$$= \sqrt{12\,M\tan\phi}$$

$$\therefore \quad r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1 + (\tan\phi)^2]^{3/2}}{\sqrt{12\,M\tan\phi}}$$

$$= \frac{\sec^3\phi}{\sqrt{12\,M\tan\phi}} = \frac{1}{\sqrt{12\,M\sin\phi\cdot\cos^5\phi}} \qquad (12.31)$$

The radius r will be minimum when the denominator is a maximum. Then

$$\frac{d}{d\phi}(\sin\phi\cdot\cos^5\phi) = 0$$

or $$\cos^6\phi - 5\sin\phi\cdot\cos^4\phi\cdot\sin\phi = 0.$$

or $$\cos^2\phi - 5\sin^2\phi = 0$$

or $$\tan^2\phi = 1/5$$

or $$\tan\phi = \frac{1}{\sqrt{5}}$$

or $$\phi = 24°5'41''$$

Substituting the value of ϕ in Eq. (12.31)

$$r_{min} = \frac{1}{\sqrt{12M \times 0.04082 \times (0.91287)^5}}$$

$$= \frac{1}{1.762\sqrt{M}}$$

with $$M = \frac{1}{6RL}$$

$$r_{min} = \frac{1}{1.762\sqrt{\frac{1}{6RL}}}$$

$$= 1.39\sqrt{RL} \qquad (12.32)$$

The derivation shows that the radius of curvature of a cubic parabola decreases from $\phi = 0$ when it is infinity to $\phi = 24°5'41''$ when it is $1.39\sqrt{RL}$. Beyond that it increases and violates the fundamental equation of a transition curve. This is due to the approximations made in deriving the equation of a cubic parabola from the intrinsic equation of a transition curve. So beyond $\phi = 24°05'41''$, the cubic parabola cannot be used as a transition curve.

12.10 CHARACTERISTICS OF A TRANSITION CURVE

To accommodate a transition curve, the original circular curve is usually shifted slightly inwards as shown in Fig. 12.36. Sometimes, however, in an old track, the main curve is either sharpened or sharpened and shifted in order to accommodate the transition curve.

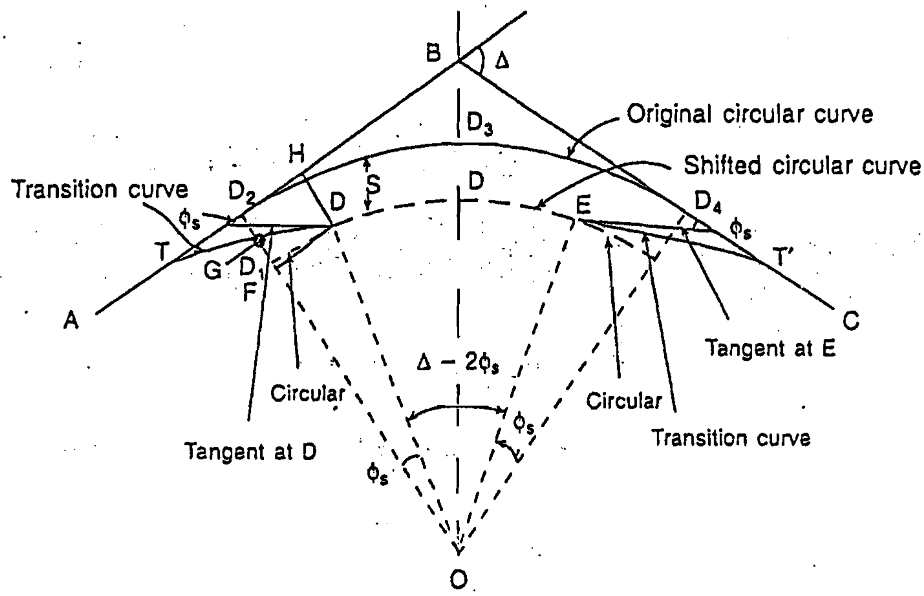


Fig. 12.36 Characteristics of transition curve.

In the Fig. 12.36, *TB* is the original tangent and $D_2D_3D_4$ is the original circular curve. *DE* is the shifted circular curve and *TD* is the transition curve.

D is the point of junction of the circular curve with the transition curve and ϕ_s is the angle which the tangent to the curves at *D* makes with the original tangent *TB*.

Some equations are derived in the following lines below

Shift of the circular curve

GD, part of the transition curve is approximately equal to the circular portion. D_1D.
From Fig. 12.36,

$$D_1D = R \cdot \phi_s$$

But $$\phi_s = \frac{L}{2R}, \quad \therefore\ D_1D = R \cdot \frac{L}{2R} = \frac{L}{2}$$

where *L* is the length of the transition curve. This means at *G*, the transition curve is bisected. Further,

$$TG = TD_2 = \frac{L}{2}$$

$$y = \frac{x^3}{6RL} = \frac{(L/2)^3}{6RL} = \frac{L^3}{48RL} = \frac{L^2}{48R} = D_2G$$

$$DH = \frac{L^3}{6RL} = \frac{L^2}{6R} = FD_2$$

where DF is perpendicular to OD_2

$$FD_1 = OD_1 - OF = R - R\cos\phi_s$$

$$= R - R\left(1 - \frac{\phi_s^2}{2!}\right)$$

$$= R - R + R\cdot\frac{\phi_s^2}{2!}$$

$$= R\cdot\frac{L^2}{4R^2}\cdot\frac{1}{2!}$$

$$= \frac{L^2}{8R}$$

$$D_2D_1 = FD_2 - FD_1$$

$$= \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R} = \text{shift of the curve.}$$

As $D_2G = \frac{L^2}{48R}$, the shift gets bisected at the point G.

More accurate values of the shift can be obtained as follows:

$$\text{shift } S = FD_2 - FD_1$$

$$= DH - FD_1$$

$$= k\left[\frac{\phi_s^{3/2}}{3} - \frac{\phi_s^{7/2}}{42} + \frac{\phi_s^{11/2}}{1320} - \ldots\right] - R\,[1 - \cos\phi_s]$$

$$= k\left[\frac{\phi_s^{3/2}}{3} - \frac{\phi_s^{7/2}}{42} + \frac{\phi_s^{11/2}}{1320}\right] - R\left[1 - \left(1 - \frac{\phi_s^2}{2!} + \frac{\phi_s^4}{4!}\right)\right]$$

Taking $k = 2R\sqrt{\phi_s}$ and simplifying

$$S = \frac{L^2}{24R}\left(1 - \frac{\phi_s^2}{48} + \frac{\phi_s^4}{1320} - \ldots\right) \tag{12.33}$$

where ϕ_s is in radian

Total tangent length

(a) True spiral: $BT = BD_2 + TD_2$

$= (R + S)\tan\Delta/2 + (TH - HD_2)$

$= (R + S)\tan\Delta/2 + (X - DF)$

Now $$X = L\left(1 - \frac{\phi_s^2}{10}\right)$$

But $$\phi_s = \frac{L}{2R}$$

$$X = L\left(1 - \frac{L^2}{4R^2}\frac{1}{10}\right)$$

$$= L\left(1 - \frac{L^2}{40R^2}\right)$$

$$DF = R \sin \phi_s$$

$$= R\left(\phi_s - \frac{\phi_s^3}{6}\right) = R\left(\frac{L}{2R} - \frac{L^3}{48R^3}\right)$$

∴ $$BT = (R + S) \tan \Delta/2 + L\left(1 - \frac{L^2}{40R^2}\right)$$

$$- \frac{L}{2}\left(1 - \frac{L^2}{24R^2}\right)$$

$$= (R + S) \tan \Delta/2 + \frac{L}{2}\left(1 - \frac{L^2}{120R^2}\right)$$

$$= (R + S) \tan \Delta/2 + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$$

In the above expression $S \tan \Delta/2$ is called the *shift increment* and $(X - R \sin \phi_s)$ or $(L/2)(1 - S/5R)$ is called as the *spiral extension.*

(b) Cubic parabola. As before

$$BT = (R + S) \tan \Delta/2 + (X - R \sin \phi_s)$$

$$= (R + S) \tan \Delta/2 + \left(L - R\frac{L}{2R}\right)$$

$$= (R + S) \tan \Delta/2 + L/2$$

Length of combined curve. Angle subtended at the centre of the circular curve = $(\Delta - 2\phi_s)$ degree.

$$\text{Length of circular curve} = \frac{\pi R(\Delta - 2\phi_s)}{180}$$

Hence combined length of curve = $\dfrac{\pi R(\Delta - 2\phi_s)}{180} + 2L$

12.11 SETTING OUT THE COMBINED CURVE

The following data are required for computations of various quantities required for setting out the transition curve:

1. Deflection angle Δ between the original tangents.
2. Radius R of the circular curve.
3. Length L of the transition curve.
4. Chainage of the point of intersection.

The following calculations should then be made.

(a) Shift of circular curve $S = L^2/24R$
(b) Spiral angle $\phi_s = L/2R$ radian.
(c) Total tangent length = $(R + S)\tan \Delta/2 + L/2$.
(d) Length of the combined curve = $\dfrac{\pi R(\Delta - 2\phi_s)}{180} + 2L$

The following are the subsequent steps:

1. From the chainage of the *PI* subtract the length of the tangent to get the chainage of *T*, the beginning of the transition curve.

2. Add the length of the transition curve to get the chainage of the junction point *D*.

3. Add the length of the circular curve to get the chainage of the other junction point *E*.

4. Add again the length of the transition curve to get the chainage of the 2nd tangent point *T'*

5. The transition curve should be set out from the point *T* by the deflection angle method when α = deflection angle = $573l^2/RL$ minutes.

For setting out the curve by linear method any of the following formulae should be used depending on accuracy desired:

(a) $y = \dfrac{l^3}{6RL}\left(1 - \dfrac{l^4}{56R^2L^2}\right)$

(b) $y = \dfrac{l^3}{6RL}$

(c) $y = \dfrac{x^3}{6RL}$

For transition curve chord length is taken as 10 m while the 1st and last chord will usually be subchords.

The circular curve is to be set out with respect to tangent at *D* by the formula

$$\delta = 1719\,\frac{C}{R} \text{ minutes for angular method}$$

or $$\theta_n = \frac{b_n(b_{n-1} + b_n)}{2R} \text{ for linear method as explained before.}$$

12.12 THE LEMNISCATE CURVE

The form of transition curve usually used in modern roadways is the *Bernoulli's Lemniscate*. The curve is symmetrical to the major and minor axes and is very much suitable when the deflection angle between the straights is large due to the following reasons:

(a) The radius of curvature decreases more rapidly.

(b) The rate of increase of curvature reduces towards the end of the transition curve.

(c) It is close to the *'autogenous curve'* (i.e. the path traced out by an automobile when moving on the curve).

12.12.1 EQUATION OF BERNOULLI'S LEMNISCATE

Figure 12.37 shows the different elements associated with Bernoulli's Lemniscate.

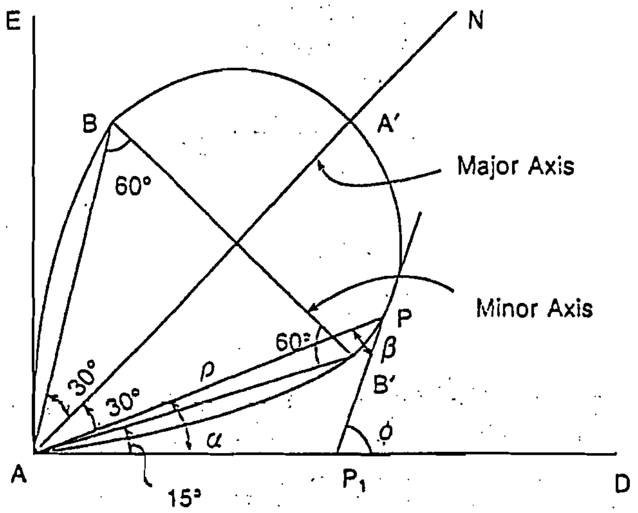


Fig. 12.37 Lemniscate curve.

In the figure AD and AE = tangents at the origin A.

AA' = major axis
BB' = minor axis

P = any point on the curve.
PP_1 = tangent to the curve at P
ϕ = angle which the tangent at P makes with the straight line AD
$\rho = AP$ = polar ray of AP
α = deflection angle of P, i.e. angle between AP and AD
β = angle between polar ray AP and the tangent PP_1 at P

The polar equation of Bernoulli's Lemniscate is

$$\rho = k\sqrt{\sin 2\alpha} \qquad (12.34)$$

From the properties of polar coordinates

$$\tan \beta = \rho \cdot \frac{d\alpha}{d\rho}$$

By differentiating Eq. (12.34)

$$\frac{d\rho}{d\alpha} = \frac{k \cos 2\alpha}{\sqrt{\sin 2\alpha}}$$

or
$$\tan \beta = \frac{k \cdot \sqrt{\sin 2\alpha}\,\sqrt{\sin 2\alpha}}{k \cos 2\alpha}$$

$$= \tan 2\alpha$$

or
$$\beta = 2\alpha.$$

Again
$$\phi = \alpha + \beta$$

$$= \alpha + 2\alpha = 3\alpha$$

Thus for the lemniscate curve, derivation angle ϕ is exactly equal to three times the polar deflection angle α. In case of clothoid or cubic parabola, this is approximately true.

For polar coordinates, the radius of curvature (r) at any point is given by

$$r = \frac{\left\{\rho^2 + \left(\frac{d\rho}{d\alpha}\right)^2\right\}^{3/2}}{\left\{\rho^2 + 2\left(\frac{d\rho}{d\alpha}\right)^2 - \rho\frac{d^2\rho}{d\alpha^2}\right\}}$$

substituting the values of $d\rho/d\alpha$ and $d^2\rho/d\alpha^2$ in the above equation and simplifying we get

$$r = \frac{k}{3\sqrt{\sin 2\alpha}}$$

As
$$k = \frac{\rho}{\sqrt{\sin 2\alpha}}$$

$$r = \frac{\rho}{3 \sin 2\alpha}$$

Also
$$k^2 = \frac{\rho^2}{\sin 2\alpha}$$

$$= \frac{\rho^2 \cdot 9r^2}{k^2}$$

$$k^4 = 9\rho^2 r^2$$

$$k^2 = 3\,\rho r$$

$$k = \sqrt{3\rho r}$$

At the end of the curve $r = R$, $l = L$ and $\phi = \phi_1 = 3\alpha_{max}$. The major axis AA' makes an angle of 45° to AD. The polar ray to B makes an angle of 15° with AD. The triangle ABB' is equilateral.

Example 12.14 A road curve of 180 m radius is to be setup to connect two tangents. The maximum speed on this part of the road will be 13.2 m/s. Transition curves are to be introduced at each end of the curve. Find a suitable length of the transition curve and calculate (1) the necessary shift of the circular curve, (2) the chainage at the beginning and at the end of the combined curve and (3) the value of the first two deflection angles of the transition curve assuming a peg interval of 10 m.

Angle of intersection 62°30′; rate of change of radial acceleration = 0.3 m/sec^3 and chainage of intersection point = 1092.18 m. [AMIE, May, 1971]

Solution

(i) Length of transition curve $= \dfrac{v^3}{\alpha R}$

$$= \frac{(13.2)^3}{0.3 \times 180} = \underline{42.59\text{ m.}}$$

(ii) Shift $= \dfrac{L^2}{24R} = \dfrac{42.59^2}{24 \times 180} = \underline{0.42\text{ m}}$.

(iii) Angle of intersection = 62°30′

Deflection angle = 62°30′

Usually deflection angle is 180° – intersection angle. But, since the intersection angle is less than 90°, here it means the deflection angle.

$$\text{Length of circular curve} = \frac{(\Delta - 2\phi_s)\cdot\pi}{180}\,180$$

$$\phi_s = \frac{L}{2R}\text{ radian} = \frac{42.59}{2 \times 180} \times \frac{180}{\pi} = 6.778°$$

$$\text{Length of circular curve} = (62.5 - 13.556)\,(\pi)$$

$$= 153.76\text{ m}$$

(iv) Total tangent length $= (R + S)\tan \Delta/2 + L/2$

$$= (180 + 0.42)\tan\frac{62.5}{2} + \frac{42.59}{2}$$

$$= 130.78$$

(v)

Chainage of intersection point	=	1092.18 m
Deduct tangent length	=	130.78 m

Chainage of 1st tangent point	=	961.40 m
Add length of transition	=	42.59 m
		1003.99 m
Add length of circular curve	=	153.76 m
Chainage of 2nd tangent point between circular and transition curve	=	1157.75 m
Add length of transition		42.59
Chainage of end of transition curve		1200.34 m

(vi) Deflection angles

Peg interval of transition curve	=	10 m
Full chainage of 1st point of curve	=	970 m
Chainage of 1st tangent point	=	961.40 m
Length of 1st subchord		8.60 m
Chainage of 2nd point on curve	=	980 m
Length of 2nd chord	=	18.60 m

Deflection angle from T_1 to 1st point on curve

$$= \frac{573\, l^2}{RL} \text{ min}$$

$$= \frac{573 \times 8.60^2}{180 \times 42.59}$$

$$= \underline{5.53'}$$

Deflection to 2nd tangent point

$$= \frac{573 \times 18.60^2}{180 \times 42.59}$$

$$= \underline{25.86'}$$

Example 12.15 Show how the computed values of Example 12.14 change when more accurate formulae are used.

Solution

(i) $$\text{Shift } S = \frac{L^2}{24R}\left(1 - \frac{\phi_s^2}{48} + \frac{\phi_s^4}{1320}\right)$$

$$= 0.42\left(1 - \frac{(0.1183)^2}{48} + \frac{(0.1183)^4}{1320}\right)$$

$$= 0.42\ (1 - 2.9156 \times 10^{-4} + 1.49376 \times 10^{-7})$$

$$= 0.42\ (0.9997) = 0.419874 \text{ m}$$

(ii) Total tangent length for true spiral.

$$= (R + S) \tan \Delta/2 + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$$

$$= (180 + 0.419874) \tan \frac{62.5}{2} + \frac{42.592}{2}\left(1 - \frac{0.419874}{5 \times 180}\right)$$

$$= 109.48146 + 21.286065$$

$$= 130.76753.$$

It can, therefore, be seen that change is very very small and as such use of accurate formula is not necessary.

Example 12.16 A curve connecting two straights which deflects through an angle of 12° is transitional throughout (Fig. 12.38). If the junction of the two transition curves is 5.00 m from the intersection point of the straights determine (to the nearest meter) the minimum radius of curvature of the curve and the length of each tangent. [Salford]

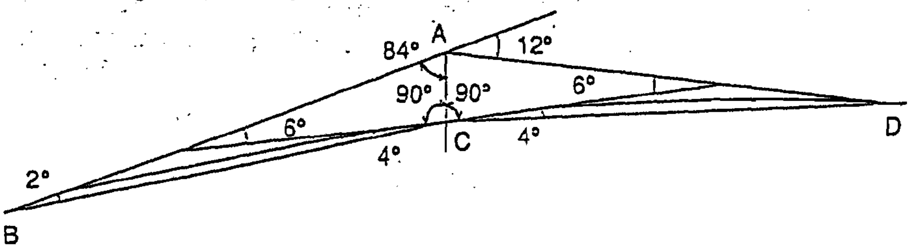


Fig. 12.38 Example 12.16.

Solution

$$\phi \text{ radian} = \frac{6 \times \pi}{180}$$

$$\alpha = \frac{1}{3} \cdot \phi = \frac{6}{3} = 2°$$

In the triangle *ABC*

$$\frac{\sin 94°}{T} = \frac{\sin 2°}{5}$$

or

$$T = \frac{\sin 94°}{\sin 2°} \cdot 5$$

$$= 146.69 \text{ m}$$

Similarly

$$\frac{BC}{\sin 84°} = \frac{AC}{\sin 2°} = \frac{5}{\sin 2°}$$

or

$$BC = 5 \cdot \frac{\sin 84°}{\sin 2°} = 142.88 \text{ m.}$$

Taking curve length approximately equal to the chord length as 2° is a small angle.

$$R = \frac{L \times 180}{2 \times 6 \times \pi} = \frac{142.88 \times 180}{12 \times \pi} = 682.2 \text{ m}$$

Example 12.17 A symmetrical highway curve joining two straights having a total deflection angle of 28°30′ is to be transitional throughout. Show that an angle of 04°45′ is required to locate the intersection of the two transition curves from the tangent point on one of the straights (Fig. 12.39). If the design velocity is 100 km/hr and the rate of change of radial acceleration is 1/3 m/s³, determine (a) the length and minimum radius of curvature of each spiral; (b) the tangent lengths. [Salford]

Solution

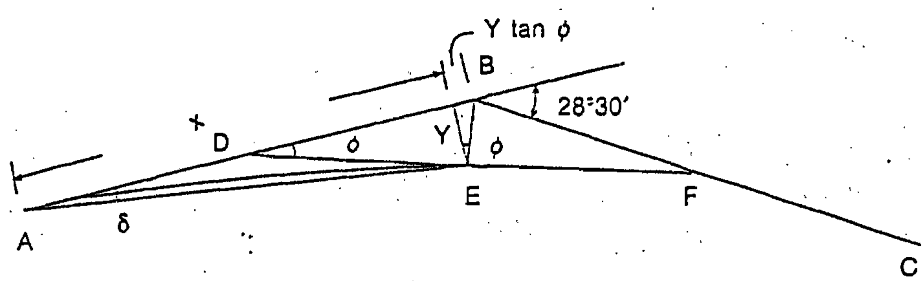


Fig. 12.39 Example 12.17.

Total deflection angle = 28°30′

$$\therefore \quad \phi = 14°15'$$

$$\phi \text{ radian} = 0.2487$$

$$\delta = \phi/3 = 4°45'$$

(a) From the condition for rate of change of radial acceleration

$$\frac{1}{3} = \frac{v^3}{RL} = \left(\frac{1000}{36}\right)^3 / LR$$

or

$$LR = \left(\frac{1000}{36}\right)^3 (3) = 64300.412$$

As the curve is spiral,

$$L = 2R\phi$$

$$= 2R\,\frac{14.25 \times \pi}{180}$$

$$= 0.4974R$$

Therefore,

$$0.4974\,R^2 = 64300.412$$

$$R = 359.55 \text{ m}$$

$$L = 0.4974\ R = (0.4974)\ (359.55)$$

$$= 178.84 \text{ m}$$

(b) From the figure

$$\text{tangent length} = X + Y \tan \phi$$

$$X = L\left(1 - \frac{\phi^2}{10}\right)$$

$$= 178.84\left(1 - \frac{0.2487^2}{10}\right)$$

$$= 177.73$$

$$Y = \frac{L^2}{6R}\left(1 - \frac{\phi^2}{14}\right)$$

$$= \frac{178.84^2}{6 \times 359.55}\left(1 - \frac{0.2487^2}{14}\right)$$

$$= 14.76 \text{ m.}$$

Therefore, tangent length $= 177.73 + 14.76 \times \tan 14.25$

$= 181.48$ m

Example 12.18 The curve connecting two straights is to be wholly transitional without intermediate circular arc and the junction of the two transitions is to be 5 m from the intersection point of the straights which deflect through an angle of 18° (Fig. 12.40). Calculate the tangent distances and the minimum radius of curvature. If the super elevation is limited to 1 vertical in 16 horizontal, determine the correct velocity for the curve and the rate of gain of radial acceleration.

[L.U.]

Solution

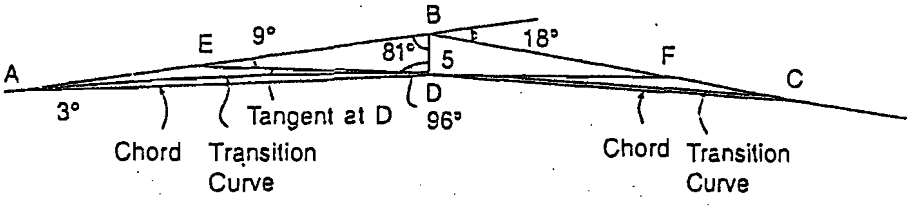


Fig. 12.40 Example 12.18.

Angle at $E = 18/2 = 9°$.

AD makes $1/3 \times 9 = 3°$ at A with tangent

Tangent distance $\dfrac{AB}{\sin 96°} = \dfrac{5}{\sin 3°}$

or $$AB = \frac{5 \cdot \sin 96°}{\sin 3°}$$

$$= 95.01 \text{ m}$$

$L = 2R\phi$ where R = minimum radius of curvature

From $\Delta\, ADB$

$$\frac{AD}{\sin 81°} = \frac{5}{\sin 3°}$$

or $$AD = \frac{5 \cdot \sin 81°}{\sin 3°}$$

$$= 94.36$$

$$\approx L$$

$$R = \frac{L}{2\phi} = \frac{94.36 \times 180}{2 \times 9 \times \pi}$$

$$= 300.35 \text{ m}$$

$$\tan \alpha = \frac{1}{16} = \frac{v^2}{gR}$$

or $$v^2 = \frac{9.806 \times 300.35}{16}$$

or $$v = 13.567 \text{ m/sec}$$

or $$v = 48.84 \text{ km/hr}$$

$$a' = \frac{v^3}{LR} = \frac{(13.567)^3}{300.35 \times 94.36}$$

$$= 0.088 \text{ m/sec}^3$$

Example 12.19 Two straights deflecting at an angle of 48°40′ are to be connected by an arc of radius 300 m with clothoid transition curves of the form $\delta = m\sqrt{\phi}$ and 75 m long at each end (Fig. 12.41). Employing the 1st two terms only of the expansions for sin ϕ and cos ϕ, calculate (a) the cartesian coordinates of the first junction point taking the tangent point as origin and the straight as the x-axis,

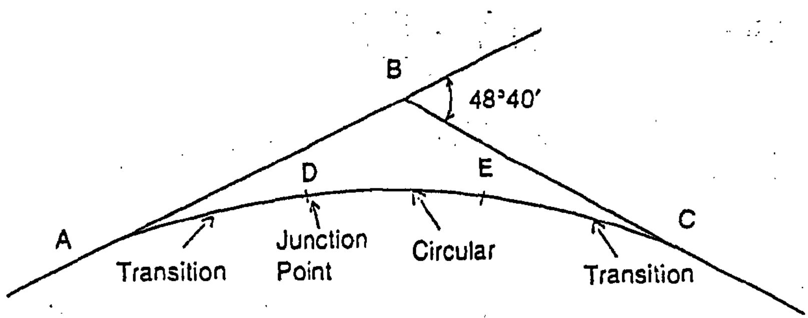


Fig. 12.41 Example 12.19.

(b) the shift; (c) the tangent length from the intersection point; (d) the total length of the curve.
You are not required to prove any of the expressions used in the solution.

[L.U.]

Solution

(a) ϕ, at junction point $D = \frac{L}{2R}$ radian $= \frac{75}{2 \times 300}$

$= 0.125$ radian

$= 7.16°$

x, at junction point $= L\left(1 - \frac{\phi^2}{10}\right)$

$= 75\left(1 - \frac{0.125^2}{10}\right)$

$= 74.8828$

$y = \frac{L^2}{6R}\left(1 - \frac{\phi^2}{14}\right)$

$= \frac{75^2}{6 \times 300}\left(1 - \frac{0.125^2}{14}\right)$

$= 3.1215$ m.

(b) Shift $S = \frac{L^2}{24R}\left(1 - \frac{\phi_s^2}{48} + \frac{\phi_s^4}{1320}\right)$

$= \frac{75^2}{24 \times 300}\left(1 - \frac{0.125^2}{48} + \frac{0.125^4}{1320}\right)$

$= 0.7825 - .000253 + 1.4375 \times 10^{-7}$

$= 0.781$ m

(c) Tangent length $= (R + S) \tan \Delta/2 + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$

$= (300 + 0.781) \tan 24°20' + \frac{75}{2}\left(1 - \frac{0.781}{5 \times 300}\right)$

$= 136.016 + 37.480$

$= 173.496$ m.

(d) Total length of the curve $= \frac{\pi R(\Delta - 2\phi_s)}{180} + 2L$

$$= \frac{\pi(300)(48.67 - 2 \times 7.16)}{180} + 2 \times 75$$

$$= 329.856 \text{ m}$$

Example 12.20 A circular curve of radius 500 m deflects through an angle of 35°. This curve is to be replaced by one of smaller radius so as to admit transitions 100 m long at each end. The deviation of the new curve from the old at their midpoints is 0.5 m towards the intersection point (Fig. 12.42).

Determine the amended radius assuming that the shift can be calculated with sufficient accuracy on the old radius. Calculate the lengths of the track to be lifted and of new track to be laid.

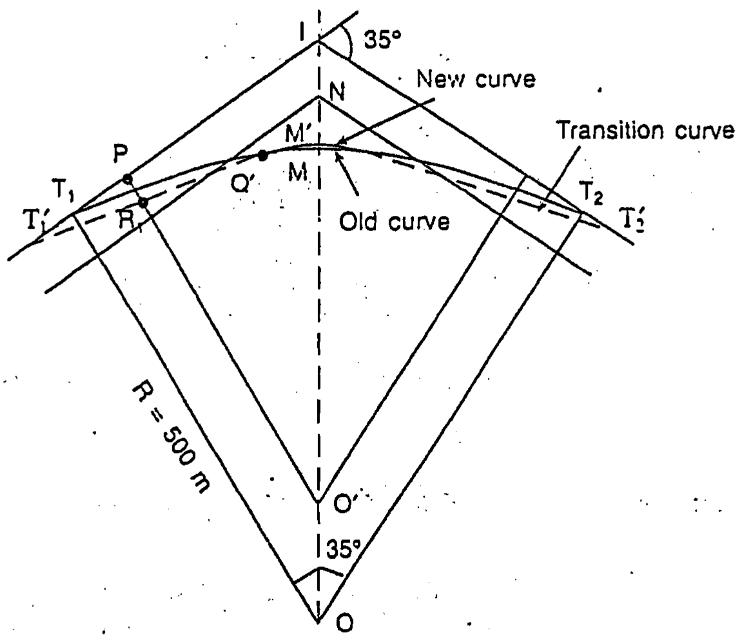


Fig. 12.42 Example 12.20.

Solution

Old curve

$$R = OT_1 = 500 \text{ m.}$$

$$\text{Tangent length} = R \tan \Delta/2 = 500 \tan 35/2 = 157.649 \text{ m.}$$

$$IM = R(\sec \Delta/2 - 1) = 500(\sec 17.5° - 1) = 24.264 \text{ m.}$$

$$\text{Length of curve } T_1T_2 = 500 \times \Delta = \frac{500 \times 35 \times \pi}{180} = 305.43 \text{ m.}$$

New curve

$$\text{Shift (using old radius)} = \frac{L^2}{24R}$$

$$= \frac{100^2}{24 \times 500}$$

$$= 0.833 \text{ m.}$$

$$IN = \text{shift} \times \sec \Delta/2 = 0.833 \times \sec 17.5°$$

$$= 0.873 \text{ m.}$$

$$NM' = IM - MM' - IN$$

$$= 24.264 - 0.500 - 0.873.$$

$$= 22.891 \text{ m}$$

But

$$NM' = R'(\sec \Delta/2 - 1) = R' \times (0.04852)$$

$$= 22.891$$

or

$$R' = 471.696 \text{ m}$$

$$\text{Shift using new radius} = \frac{L^2}{24R'}$$

$$= \frac{100^2}{24 \times 471.696}$$

$$= 0.883 \text{ m.}$$

The length of the new curve $T_1'T_2'$

$$= 2T_1'M'$$

Making suitable approximation

$$= 2(T_1'Q' + Q'M')$$

$$= 2(T_1'R_1 + R_1Q' + Q'M')$$

As the transition curve gets bisected at R_1

$$T_1'R_1 = L/2$$

and R_1Q' is approximately equal to the corresponding circular portion.

Hence the length of the new curve

$$= L + \text{Length of circular portion}$$

$$= 100 + R' \times \Delta$$

$$= 100 + 471.696 \times \frac{35 \times \pi}{180}$$

$$= 388.142 \text{ m}$$

which is the length of the new track to be laid

Length of old track to be lifted

$$= T_1'T_1 + T_1M$$

$$PI = (R' + \text{shift}) \tan \Delta/2$$

$$= (471.696 + 0.883) \tan 17.5°$$

$$= 149.00 \text{ m}$$

$$T_1'I = T_1'P + PI$$

$$= \frac{100}{2} + 149.00$$

$$= 199.00 \text{ m.}$$

$$T_1'T_1 = T_1'I - T_1I$$

$$= 199.000 - 157.649$$

$$= 41.351 \text{ m}$$

Hence length of old track to be lifted

$$= 2(T_1'T_1 + T_1M)$$

$$= 2 \times 41.351 + 305.43$$

$$= 388.132 \text{ m}$$

PROBLEMS

12.1 Two straights meet at an apex angle 126°48′ and are to be joined by a circular curve of 300 m radius. Calculate the data necessary to set out the curve using a 30 m chord. Tabulate the data properly for field use.

[AMIE Winter 1978]

12.2 Two straights AB and BC meet in an inaccessible point B. To connect them by a 3° simple circular curve (based on 30 m chord) two points M and N were selected on AB and BC respectively and the following data were recorded:

Bearing of AB = 60°
Bearing of MN = 90°
Bearing of CB = 300°
Distance MN = 200 m.

(a) Compute the necessary details for setting out the curve by method of deflection angles with a 20″ vernier theodolite and a unit chord of 30 m.

(b) Tabulate the requisite data showing the readings to be set on the theodolite for setting out the curve.

[AMIE Summer 1980]

12.3 (a) Supplementing with neat sketches differentiate between simple, compound and reverse curves.

(b) Two parallel railway tracks were to be connected by a reverse curve, each section having the same radius. The distance between their centre lines was 20 m. The distance between tangent points measured parallel to the track was 80 m. Determine the radius of the curve. If the radii were to be different, calculate:

(i) the radius of the second if that of the first was 90 m.

(ii) The lengths of both branches of the curve. [AMIE Winter 1980]

12.4 A 200 m length of straight connects two circular curves deflecting to the right. The radius of the 1st curve was 250 m and that of the second curve was 200 m. The central angle for the second curve was 15°58′. The combined curve is to be replaced by a single circular curve between the same tangent points. Find the radius of the curve. [AMIE Winter 1981]

12.5 (a) Show the various elements of a compound curve on a neatly drawn sketch. Also state the formulae to calculate various quantities in this case when necessary data are known.

(b) Prove that the shift bisects the transition curve and transition curve bisects the shift. [AMIE Winter 1982]

12.6 (a) State the conditions to be fulfiled by a transition curve introduced between the tangent and circular curve.

(b) Draw a neat sketch of a reverse curve provided to join two parallel straights. Using the usual notations state the relationship between the several elements of the curve.

(c) Two straights *AB* and *BC* meet at an in accessible point *B*. They are to be joined by a circular curve of 450 m radius. Two points *P* and *Q* were selected respectively on *AB* and *BC*. The following observations were made: $\angle APQ = 160°$, $\angle CQP = 145°$, distance $PQ = 125$ m and chainage of $P = 1500.00$ m. Calculate the chainages of intersection point *B*, point of curvature and point of tangency.

[AMIE Winter 1983]

12.7 (a) List the requirements to be satisfied in setting out a transition curve.

(b) Derive an expression for the super elevation to be provided in a transition curve.

(c) A 10 m wide road is to be deflected through an angle of 35°30′. A transition curve is to be used at each end of the circular curve of 500 m radius. It has to be designed for a rate of gain of radial acceleration of 0.2 m/s^2 and a speed of 60 km/hr. Calculate the suitable length of the transition curve and super elevation. [AMIE Winter 1985]

12.8 (a) Draw a neat sketch and show the various elements of a simple circular curve.

(b) What is a transition curve and where is it used? How will you determine the length of a transition curve and the amount of super elevation to be provided?

(c) Two straights T_1P and PT_2 are intersected by a third line AB such that $\angle PAB = 46°24'$, $\angle PBA = 32°36'$ and distance $AB = 312$ m. Calculate the radius of the simple circular curve which will be tangential to the three lines T_1P, AB, and TP_2 and the chainage of the point of the curve (T_1) and point of tangency (T_2) if the chainage of the point P is 2857.5 m. [AMIE Summer 1986]

12.9 (a) Give any five general requirements of a transition curve.

(b) A road bend which deflects 80° is to be designed for a maximum speed of 120 km/hr, a maximum centrifugal ratio of 1/4 and a maximum rate of change of acceleration of 30 cm/sec^2. The curve consisting of a circular arc combined with two cubic spirals. Calculate (i) the radius of the circular arc; (ii) the requisite length of the transition; (iii) total length of the composite curve. [AMIE Winter 1987]

12.10 (a) Explain the following terms for a simple circular curve:
(i) Back and forward tangents. (ii) Point of intersection, curve and tangency. (iii) Deflection angle to any point. (iv) External distance. (v) Degree of curve.

(b) The two tangents of a simple circular curve intersect at chainage 59 + 60, the deflection angle being 50°30′. It is proposed to set out the curve by offsets from chords by taking peg intervals equal to 100 links. The length of the chain consisting of 100 links is 20 m. Determine the lengths of all offsets to set out a curve of 15 chains radius.
[AMIE Winter 1988]

13

Vertical Curves

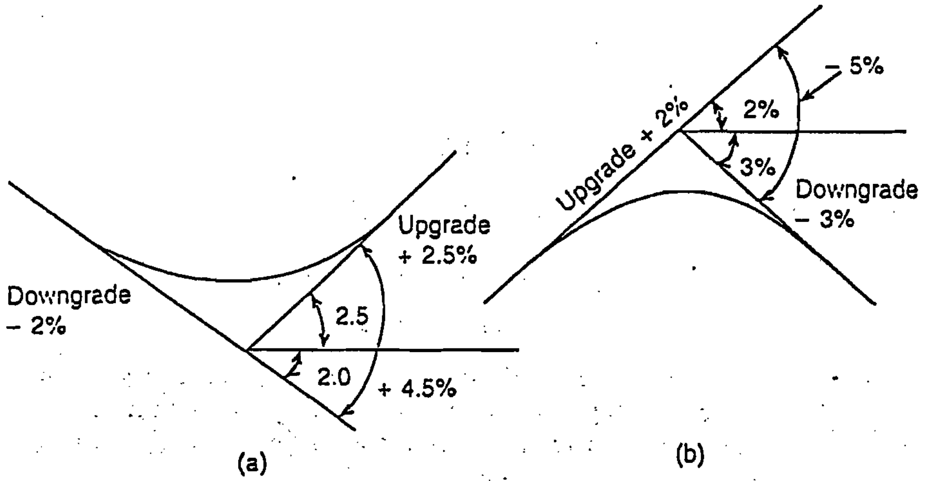
13.1 INTRODUCTION

For highways and railways whenever there is a change of grade in the vertical plane, a vertical curve is required to smoothen the change. It is usually parabolic as parabolic curves provide a constant rate of change of grade. A vertical curve should be so designed that

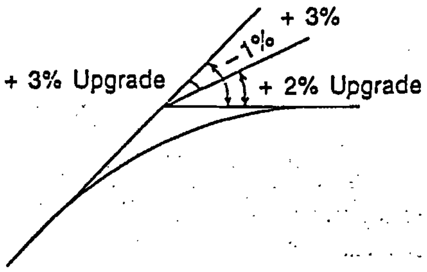
(i) it gives smooth riding qualities which again will occur if (a) there is a constant change of gradient, (b) uniform rate of increase of centrifugal force; and (ii) adequate sighting distance is available before the vehicle reaches the summit.

There are four types of vertical curves as shown in Fig. 13.1—(a) Sag curve; (b) Crest or summit curve; (c) Rising curve; (d) Falling curve.

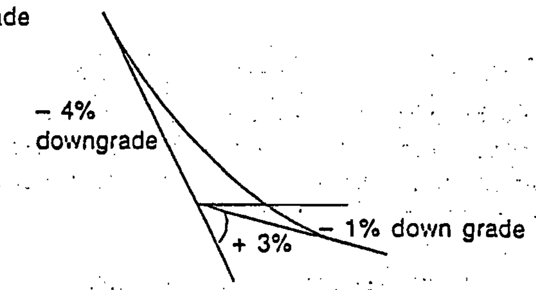
In a sag curve, a down grade is followed by an upgrade. In a summit curve an upgrade is followed by a downgrade. In a rising curve an upgrade is followed by another upgrade. In a falling curve a down grade is followed by another downgrade.



(a) (b)



(c)



(d)

Fig. 13.1 Different types vertical curves: (a) Sag curve. (b) Crest or summit curve. (c) Rising curve. (d) Falling curve.

13.2 GENERAL EQUATION OF A PARABOLIC CURVE

The general equation of a parabolic curve is,

$$y = Ax^2 + Bx + C$$

$$\text{at } x = 0 \quad y = C$$

Differentiating $$\frac{dy}{dx} = 2Ax + B$$

$$\text{at } x = 0 \quad \left(\frac{dy}{dx}\right)_0 = B$$

which means B is the slope of the tangent at origin

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \text{rate of change of slope}$$

$$= 2A = \text{a constant.}$$

Hence the statement that *for a parabolic curve rate of change of slope is a constant*. This is shown in Fig. 13.2. If an upward slope of g_1 is followed by another slope g_2 the change of grade is $g_2 - g_1$. If the change occurs over a length L, the rate of change of grade is $(g_2 - g_1)/L$: Therefore

$$2A = \frac{g_2 - g_1}{L}$$

or $$A = \frac{g_2 - g_1}{2L}$$

Figure 13.3 shows the nomenclature of a vertical curve when inserted between two grades. Here point A is the beginning of vertical curve (BVC), point B is the end of the vertical curve (EVC), g_1 is the upgrade and g_2 is the downgrade. The equation of a parabola

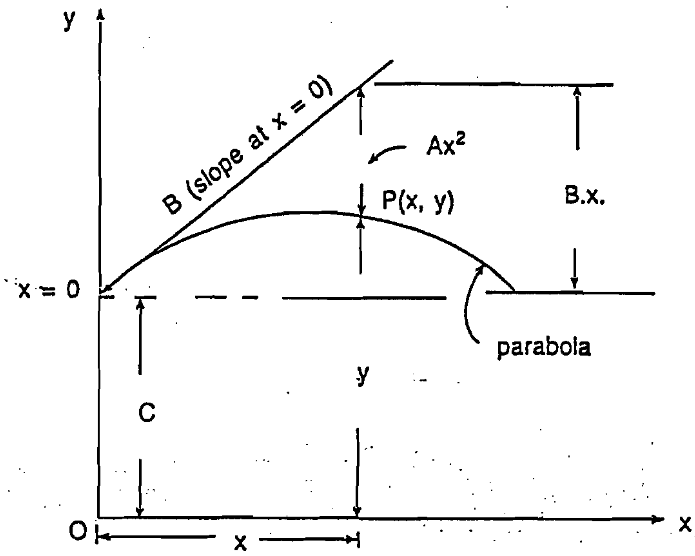


Fig. 13.2 Parabolic vertical curve.

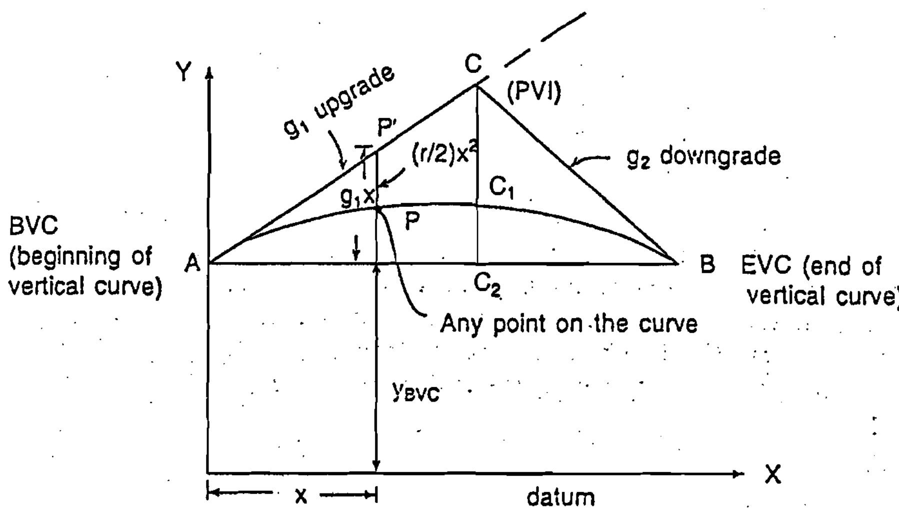


Fig. 13.3 Equation of a vertical curve.

$$y = Ax^2 + Bx + C$$

can now be transformed to

$$y = Ax^2 + g_1x + y_{BVC}$$

as $g_1 = B$

and $C = y_{BVC}$.

with $A = \dfrac{g_2 - g_1}{2L}$

$$y = \left(\frac{g_2 - g_1}{2L}\right)x^2 + g_1x + y_{BVC} \qquad (13.1)$$

If
$$r = \frac{g_2 - g_1}{L}$$
$$y = \left(\frac{r}{2}\right)x^2 + g_1 x + y_{BVC} \tag{13.2}$$

r should be used with proper sign.

Taking upward grade as positive and downward grade as negative, r has the following signs.

Fig. 13.1:

(a) sag curve r is positive.
(b) summit curve r is negative.
(c) rising curve r is negative.
(d) Falling curve r is positive.

When a vertical curve is laid out such that the point of intersection of the grade lines, i.e. C (PVI) lies midway between the two ends of the curve measured horizontally then it is an equal tangent parabolic curve. The points on the vertical curve can therefore be plotted by using the parabolic equations of the curve. The points on the vertical curve can also be plotted from the grade lines by utilizing the following properties of an equal tangent vertical parabola:

1. The offsets from the tangent to the curve at a point are proportional to the squares of the horizontal distances from the point. This is shown in Fig. 13.3 where $PP' = (r/2)x^2$.

2. The offsets from the two grade lines are symmetrical with respect to the point of intersection of the two grade lines.

3. The curve lies midway between the point of intersection of the grade lines and the middle point of the chord joining the BVC and EVC, i.e. $CC_1 = C_1C_2$.

The method of plotting the points on the curve from the corresponding points on the grade is known as method of *tangent offsets* or *tangent corrections*. These methods are explained in detail through illustrative examples.

13.3 COMPUTATIONS FOR AN UNEQUAL TANGENT CURVE

An unequal tangent curve is simply a combination of two equal tangent curves. Figure 13.4 shows such a curve.

Here BVC to CVC is one equal tangent curve and from CVC to EVC is another equal tangent curve. A and B are midpoints of $BVCV$ and $VEVC$ respectively. From the elevations of A and B, the gradient of the line AB can be computed. The two equal tangent curves should be computed as explained before.

13.4 HIGH OR LOW POINT ON A VERTICAL CURVE

It is often necessary to locate the highest point (in the case of summit) and the lowest point (in the case of a valley) for a vertical curve. This helps in investigating

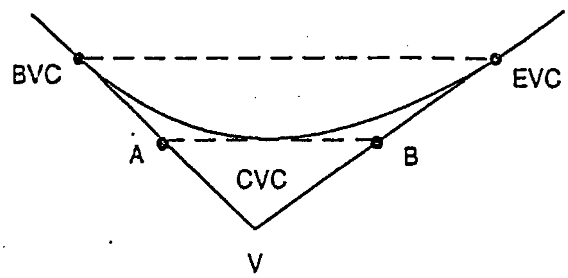


Fig.13.4 Unequal tangent curve.

drainage conditions, in computing clearance beneath overhead structures, or cover over pipes or sight distance. At the highest or lowest point, the tangent to the curve will be horizontal and slope will be zero.

Now $$\frac{dy}{dx} = 2Ax + B.$$

With $$B = g_1 \quad \text{and } A = \frac{g_2 - g_1}{2L}$$

$$\frac{dy}{dx} = \frac{g_2 - g_1}{L}x + g_1 = 0$$

or $$x = -\frac{g_1 L}{g_2 - g_1} = \frac{g_1 L}{g_1 - g_2}$$

where x is the distance from the origin to the point.

g_1 is tangent grade through *BVC*

g_2 is tangent grade through *EVC*, and L is the curve length.

13.5 VERTICAL CURVE PASSING THROUGH A FIXED POINT

While designing a vertical curve, it is often necessary that it passes through a fixed point of known elevation. This is required when a new grade line must meet an existing railroad or highway crossing or a minimum vertical distance must be maintained between the grade line and the underground structures.

From Fig. 13.5

$$y_1 = \frac{r}{2}\left(\frac{L}{2} - x_0\right)^2 \text{ with respect to tangent } AB.$$

$$y_2 = \frac{r}{2}\left(\frac{L}{2} + x_0\right)^2 \text{ with respect to tangent } BC.$$

Therefore $$\frac{y_1}{y_2} = \frac{\left(\frac{L}{2} - x_0\right)^2}{\left(\frac{L}{2} + x_0\right)^2}$$

When y_1, y_2 and x_0 are known L can be found out from the quadratic equation.

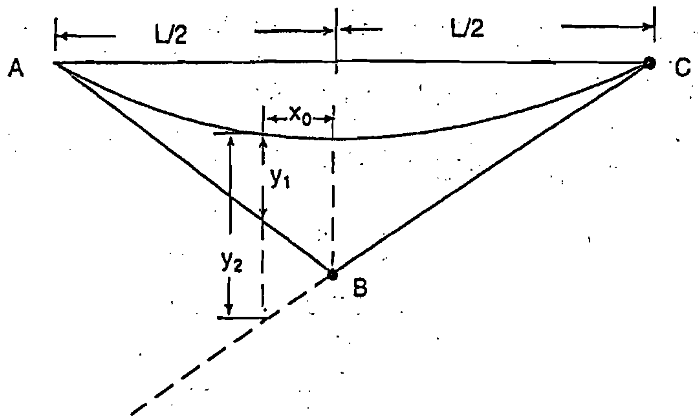


Fig. 13.5 Vertical curve passing through a fixed point.

13.6 DESIGN OF VERTICAL CURVE

The length of a vertical curve is based on two considerations. (i) Centrifugal effect; (ii) Visibility.

In sags and in summits of flat gradients, centrifugal effect is the main consideration. At summits visibility is usually the determining factor.

Centrifugal effect

Acceleration due to moving in a curve

$$\alpha = \frac{v^2}{R}$$

If the permissible centrifugal acceleration = 0.75m/sec^2

$$R = \frac{v^2}{\alpha}$$

If $V = 100$ km/hr,

$$R = \frac{\left(\frac{100 \times 1000}{60 \times 60}\right)^2}{0.75}$$

$$= 1028.8 \approx 1000 \text{ m}$$

$$\text{Curvature} = \frac{d^2y/dx^2}{\{1 + (dy/dx)^2\}^{3/2}}$$

As vertical curves are usually flat curves $(dy/dx)^2$ is very small and can be neglected. Hence

$$\text{Curvature} = \frac{d^2y}{dx^2} = \frac{1}{R} = 2A$$

$$= \frac{g_2 - g_1}{L}$$

or $$R = \frac{L}{g_2 - g_1}$$

$$L_{min} \text{ with } R = 1000 \text{ m}$$

$$= 1000\,(g_2 - g_1)$$

If two gradients 1 in 50 meet in a sag

$$\therefore \quad L_{min} = 1000\left[\frac{1}{50} - \left(-\frac{1}{50}\right)\right]$$

$$= \frac{1000 \times 2}{50} = 40 \text{ m}$$

Sight distance

At summits the length of the curve is controlled by sight distance. A minimum sight distance is required to avoid accident.

Two cases may occur: (i) when sight distance S is entirely on the curve, i.e. $S < L$, and (ii) when sight distance S overlaps the curve and extends on to the tangent $S > L$.

CASE I $S < L$

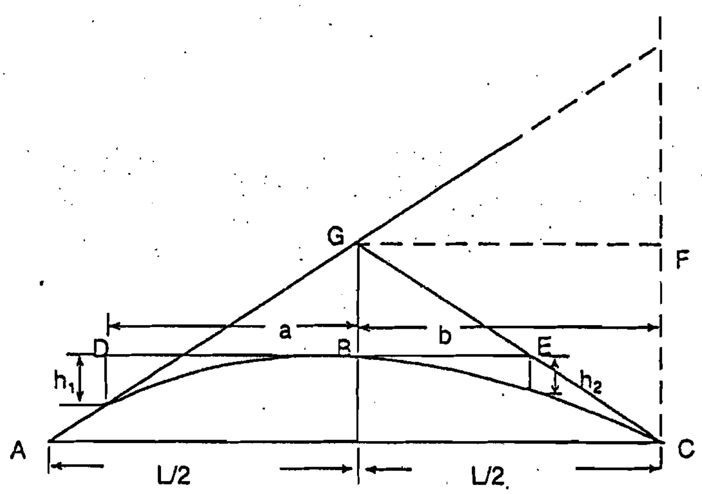


Fig. 13.6 Sight distance $S < L$.

Let h_1 = height of driver's eye above the roadway.

h_2 = height of object or hazard on the travelled road.

Line of sight DBE is tangential to the summit of the curve at B.

It is already shown $y = \frac{r}{2}x^2$

$$= \frac{g_2 - g_1}{2L}x^2$$

When $x = a$, $y = h_1$. Since h_1 is positive, while r is negative in summit

$$h_1 = -\frac{g_2 - g_1}{2L} a^2$$

or
$$a = \sqrt{\frac{2L}{g_1 - g_2}} \sqrt{h_1}$$

Similarly
$$b = \sqrt{\frac{2L}{g_1 - g_2}} \sqrt{h_2}$$

But sight distance $S = a + b$

$$= \sqrt{\frac{2L}{g_1 - g_2}} (\sqrt{h_1} + \sqrt{h_2})$$

Usually g_2, g_1 are expressed in %, e.g. 1%, 3% so that g becomes $g/100$ and the expression reduces to

$$S = \sqrt{\frac{200L}{g_1 - g_2}} (\sqrt{h_1} + \sqrt{h_2}) \qquad (13.3)$$

The value of h_1 is usually taken as 1.05 m and h_2 as 0.15 m.

If $h_1 = h_2 = h$, Eq. (13.3) reduces to

$$S = \frac{28.28\sqrt{Lh}}{\sqrt{g_1 - g_2}}$$

CASE II: $S > L$

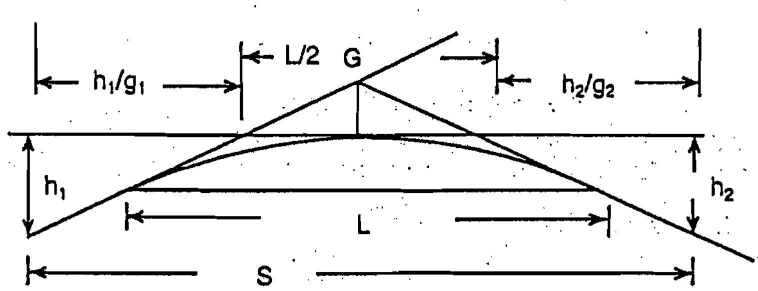


Fig. 13.7 Sight distance $S > L$.

Assuming scalar value for g_2

$$S = L/2 + h_1/g_1 + h_2/g_2$$

Setting the final derivative of S to zero, we get

$$-\frac{h_1}{g_1^2} dg_1 - \frac{h_2}{g_2^2} dg_2 = 0$$

To make S a minimum, the rate of change of g_2 will be equal and opposite to that of g_1.

or $$dg_1 = -dg_2$$

or $$\frac{h_1}{g_1^2} - \frac{h_2}{g_2^2} = 0$$

or $$g_2 = \sqrt{\frac{h_2}{h_1}}\, g_1$$

or $$g_1 = \sqrt{\frac{h_1}{h_2}}\, g_2$$

or $$g_1 + g_2 = A = g_1 + \sqrt{\frac{h_2}{h_1}}\, g_1$$

$$= \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_1}}\, g_1$$

or $$g_1 = \frac{\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \times A$$

$$g_2 = \frac{\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \times A$$

Substituting the values of g_1 and g_2

$$S = L/2 + \frac{h_1}{\sqrt{h_1} \times A}(\sqrt{h_1} + \sqrt{h_2}) + \frac{h_2(\sqrt{h_1} + \sqrt{h_2})}{\sqrt{h_2} \times A}$$

$$= L/2 + \frac{(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

$$= L/2 + \frac{(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 + g_2}$$

or $$L = 2S - \frac{2(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 + g_2} \qquad (13.4)$$

At the summit g_2 is negative, hence with proper algebraic sign the expression becomes:

$$L = 2S - \frac{2(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 - g_2} \qquad (13.4a)$$

13.7 SIGHT DISTANCE OF VERTICAL CURVES AT A SAG

Design of vertical curve in sag is based on minimum stopping sight distance. It is stipulated that the head light of a vehicle which is normally 0.75 m above the road surface with the beam of light inclined at an angle of 1° to the horizontal will illuminate this distance.

CASE I: $S < L$

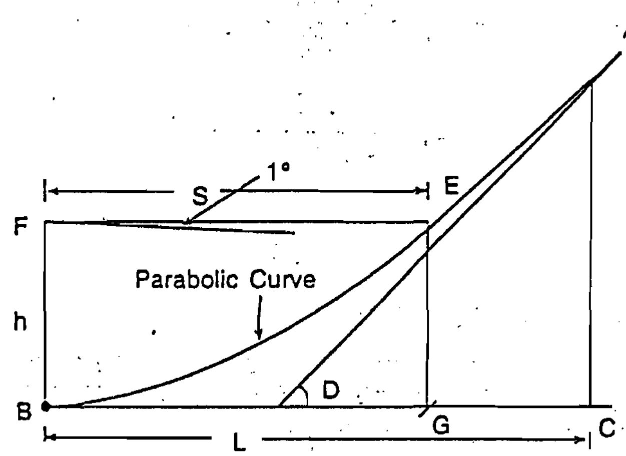


Fig. 13.8 Sight distance at sag $S < L$.

It is already deduced

$$EG = \frac{g_2 - g_1}{2L} S^2$$

and

$$AC = \frac{g_2 - g_1}{2L} L^2$$

But

$$EG = h + S \tan 1°$$

As

$$h = 0.75 \text{ m}$$

$$\therefore \quad EG = 0.75 + S \tan 1°$$

Equating,

$$\frac{g_2 - g_1}{2L} S^2 = 0.75 + S \tan 1°$$

or

$$L = \frac{(g_2 - g_1) S^2}{2(0.75 + S \tan 1°)}$$

$$= \frac{(g_2 - g_1) S^2}{2(0.75 + 0.0174\, S)}$$

$$= \frac{(g_2 - g_1) S^2}{1.5 + 0.035\, S} \qquad (13.5)$$

CASE II. When $S > L$

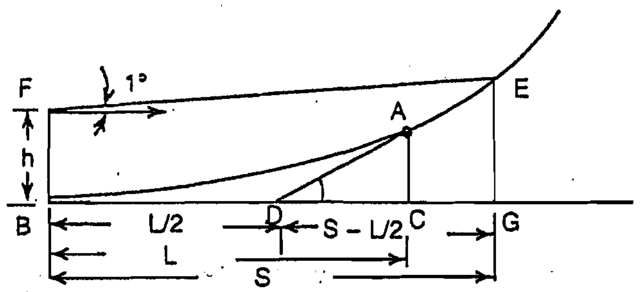


Fig. 13.9 Sight distance at sag $S > L$.

In this case $EG = (g_2 - g_1)(S - L/2)$
But $EG = h + S \tan 1°$

$$\therefore \qquad h + S \tan 1° = \frac{(g_2 - g_1)}{2}(2S - L)$$

with $h = 0.75$ m.

$$0.75 + S\,(0.0174) = \frac{g_2 - g_1}{2}(2S - L)$$

or
$$L = 2S - \frac{(1.5 + 0.035S)}{g_2 - g_1} \qquad (13.6)$$

Example 13.1. A + 3.5% grade meets a – 1.5% grade at station 60 + 15 and elevation 250 m (Fig. 13.10). An equal tangent parabolic curve 300 m long has been selected to join the two tangents. Compute and tabulate the curve for stakeout at full stations. Check by second differences. Assume 30 m chain.

Solution

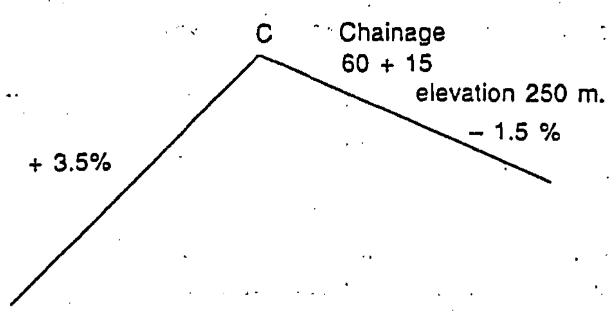


Fig. 13.10 Example 13.1.

$$r = \frac{(-1.5 - 3.5)}{300}\,\frac{1}{100} = -0.000167.$$

Chainage of C = 60 + 15	=	1815 m
$- L/2$	=	– 150 m
Beginning of Vertical Curve (BVC)	=	1665 m

$+ L$	=	300 m
EVC	=	1965 m
	=	65 + 15

$$\text{Elev}_{BVC} = 250 - \frac{3.5}{100}\,150 = 244.75 \text{ m}$$

Table 13.1 Example 13.1

Station	x	g_1x	$rx^2/2$	Curve elevation (m)	First difference (m)	Second difference (m)
55 + 15 (*BVC*)	0.00	0.000	0.000	244.750		
56 + 00	15.00	0.525	− 0.018	245.257		
57 + 00	45.00	1.575	− 0.169	246.156	− 0.899	− 0.149
58 + 00	75.00	2.625	− 0.469	246.906	− 0.750	− 0.151
59 + 00	105.00	3.675	− 0.920	247.505	− 0.599	− 0.151
60 + 00	135.00	4.725	− 1.522	247.953	− 0.448	
60 + 15	150.00	5.250	− 1.878	248.122		
61 + 00	165.00	5.775	− 2.273	248.252		
62 + 00	195.00	6.825	− 3.175	248.400	− 0.148	− 0.150
63 + 00	225.00	7.875	− 4.227	248.398	+ 0.002	− 0.151
64 + 00	255.00	8.925	− 5.430	248.245	+ 0.153	− 0.149
65 + 00	285.00	9.975	− 6.782	247.943	+ 0.302	
65 + 15	300.00	10.500	− 7.515	247.735		

A check on curve elevations is obtained by computing the first and second differences between the elevations of full stations as shown in the right hand columns of the table. Unless disturbed by rounding off all second differences (rate of change) should be equal.

The elevation of the curve's central point

$$= 244.75 + 0.035 \times 150 - \frac{.000167}{2}(150)^2 = 248.12$$

This can be checked by the property of the parabola, that the line joining the vertex and the midpoint of the long chord gets bisected by the curve.

Elevation of midpoint of the

$$\text{chord of the parabola} = \frac{244.75 + 247.735}{2} = 246.2425$$

$$\text{Elevation of the curve centre} = \frac{246.2425 + 250.00}{2} = 248.12 \text{ m. (check)}$$

Example 13.2 A grade of – 3.5% meets another grade of + 0.50%. The elevation of the point of intersection is 267 m and chainage is 780 m (Fig. 13.11). Field coordinates require that the vertical curve should pass through a point of elevation 268 m at chainage 780 m. Compute a suitable equal tangent vertical curve and fullstation elevations.

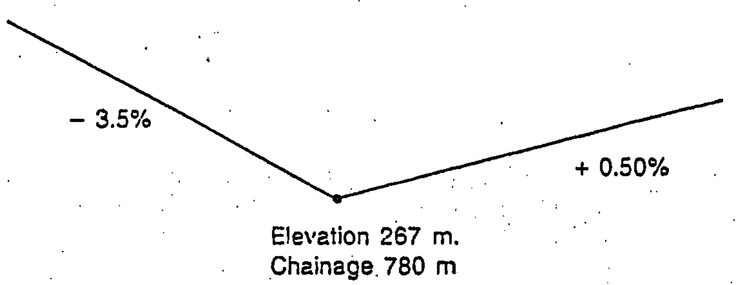


Fig. 13.11 Example 13.2.

Solution The general equation of a parabola

$$y = y_{BVC} + g_1 x + \left(\frac{r}{2}\right) x^2$$

$$(268) = 267 + \frac{3.5}{100}\frac{L}{2} + \frac{3.5 + 0.5}{2L(100)} \cdot \left(\frac{L}{2}\right)^2 - \frac{3.5}{100} \cdot L/2$$

or $$268 = 267 + .0175L + .005L - .0175L$$

$$= 267 + .005L$$

or $$L = \frac{1}{.005}$$

$$= 200 \text{ m}$$

Full Station Elevations

Chainage at the beginning of the curve = 780 − 100 = 680 m

= 22 + 20 m

Elevation at the beginning of the curve $= 267 + \frac{3.5}{100} \times 100$

= 270.5 m

The rest of the calculations are given in Table 13.2.

Table 13.2 Example 13.2

Station	x	g_1x	$\frac{rx^2}{2}$	Curve elevation	First diff.	Second diff.
22 + 20 (BVC)	0	0	0	270.50		
23 + 00	10	− 0.35	+ 0.01	270.16		
					+ 0.90	
24 + 00	40	− 1.40	+ 0.16	269.26		+ 0.18
					+ 0.72	
25 + 00	70	− 2.45	+ 0.49	268.54		+ 0.18
					+ 0.54	
26 + 00	100	− 3.50	+ 1.00	268.00		+ 0.18
					+ 0.36	
27 + 00	130	− 4.55	+ 1.69	267.64		+ 0.18
					+ 0.18	
28 + 00	160	− 5.60	+ 2.56	267.46		
					+ 0.00	+ 0.18
29 + 00	190	− 6.65	+ 3.61	267.46		
29 + 10	200	− 7.00	+ 4.00	267.50		

Example 13.3 A + 4.00% grade meets *a* − 2.00% grade at station 50 + 00 and elevation 400 m. Length of 1st curve is 180 m and that of 2nd curve is 120.00 m. Compute and tabulate full station elevations.

Solution

(i) Elevation of $BVC = 400 - \frac{180 \times 4}{100} = 392.8$ m

Elevation of $A = 392.8 + \frac{90 \times 4}{100} = 396.4$ m

Elevation of $EVC = 400 - \frac{2}{100} \times 120 = 397.6$ m

Elevation of $B = 400 - \frac{2}{100} \times 60 = 398.8$ m

Grade $AB = \frac{396.4 - 398.8}{150} = 1.6\%$

These elevations are shown in Fig. 13.12

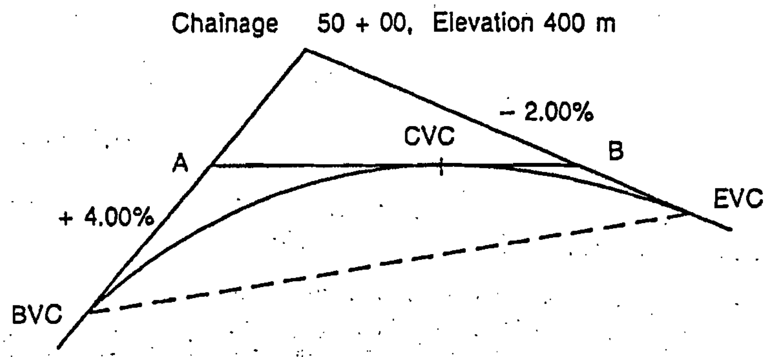


Fig. 13.12 Example 13.3.

(ii) For the first curve $g_1 = +4.00\%$

$$g_2 = +1.6\%$$

$$\frac{r}{2} = \frac{1.6 - 4.0}{2(180)} \times \frac{1}{100}$$

$$= -6.67 \times 10^{-5}$$

For the second curve

$$g_1 = +1.6\% \quad r_2 = \frac{-2 - 1.6}{2(1 \times 20)} \times \frac{1}{100}$$

$$g_2 = -2.0\% \quad = -1.50 \times 10^{-4}$$

The rest of the calculations are given in Table 13.3.

Table 13.3 Example 13.3

Station	x	g_1x	$rx^2/2$	Curve elevation	First difference	Second difference
44 + 0	0	0	0	392.80		
					1.14	
45 + 0	30	+ 1.2	– 0.06	393.94		0.12
					1.02	
46 + 0	60	+ 2.4	– 0.24	394.96		0.12
					0.90	
47 + 0	90	+ 3.6	– 0.54	395.86		0.12
					0.88	
48 + 0	120	+ 4.8	– 0.96	396.64		0.12
					0.66	
49 + 0	150	+ 6.0	– 1.50	397.30		0.12
					0.54	
50 + 0	180	+ 7.2	– 2.16	397.84		
					+ 0.34	
51 + 0	30	0.48	– 0.14	398.18		+ 0.26
					+ 0.08	
52 + 0	60	0.96	– 0.54	398.26		+ 0.28
					– 0.20	
53 + 0	90	1.44	– 1.22	398.06		+ 0.26
					– 0.46	
54 + 0	120	1.92	– 2.16	397.60		

Example 13.4 What is the minimum length of vertical curve to provide a required sight distance for the following conditions?

(i) Grades of + 3.4% and – 2.4%, sight distance 230 m.
(ii) Grades of + 4.8% and – 3.4%, sight distance 270 m.
(iii) Grades of + 0.5% and – 1.2%, sight distance 400 m.

Solution At summit $h_1 = 1.05$ m, $h_2 = 0.15$ m.

If $S < L$

$$S = \sqrt{\frac{2L}{g_1 - g_2}}(\sqrt{h_1} + \sqrt{h_2})$$

or
$$\sqrt{\frac{2L}{g_1 - g_2}} = \frac{S}{(\sqrt{h_1} + \sqrt{h_2})}$$

or
$$\frac{2L}{g_1 - g_2} = \frac{S^2}{(\sqrt{h_1} + \sqrt{h_2})^2}$$

or
$$L = \frac{S^2}{2}\frac{(g_1 - g_2)}{(\sqrt{h_1} + \sqrt{h_2})^2}$$

For case (i)

$$L = \frac{230^2}{2}\frac{(3.4 - (-2.4))}{(\sqrt{1.05} + \sqrt{0.15})^2}\frac{1}{100}$$

$$= \frac{230^2 \times 5.8}{2 \times 2.15} \times \frac{1}{100}$$

$$= 720.23 > 230.$$

For case (ii)
$$L = \frac{270^2[4.8 - (-3.4)]}{100 \times 2 \times 2.13}$$

$$= 1403.24 > 270.$$

For case (iii)
$$L = \frac{400^2[0.5 - (-1.2)]}{100 \times 2 \times 2.13}$$

$$= 638.49 > 400 \text{ m}.$$

with $h_1 = h_2 = 1.05$ m the formula reduces to

$$L = \frac{S^2(g_1 - g_2)}{8h}$$

For case (i)
$$L = \frac{230^2(5.8)}{(8)(1.05)(100)} = 333.5 > 230$$

For case (ii)
$$L = \frac{270^2(8.2)}{(100)(8)(1.05)} = 649.76 > 270$$

For case (iii) $L = \dfrac{400^2(1.7)}{(100)(8)(1.05)} = 295.65 < 400$

For case (iii) the result is not valid as $L < S$ whereas the assumption was $L > S$. Hence the formula to be used.

$$L = 2S - \frac{2(\sqrt{h_1} + \sqrt{h_2})^2}{g_1 - g_2}$$

with $h_1 = h_2$ $$L = 2S - \frac{8h}{g_1 - g_2}$$

$$= 2(400) - \frac{8 \times 1.05 \times 100}{1.7}$$

$$= 258.82 \text{ m} < 400 \text{ m}.$$

Example 13.5 A parabolic vertical curve of length 100 m is formed at a summit between grades of 0.7 per cent up and 0.8 per cent down. The length of the curve is to be increased to 120 m, retaining as much as possible of the original curve and adjusting the gradients on both sides to be equal. Determine this gradient.

[L.U. BSc]

Solution Let L' be the length of the new vertical curve. Taking axes as shown (Fig. 13.13), the equation of the parabola may be written as:

$$y = Ax^2$$

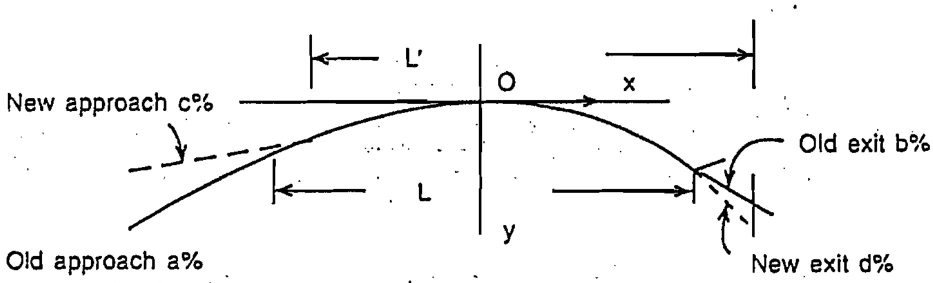


Fig. 13.13 Example 13.5.

As change of slope is very small chord lengths are assumed to be the same lengths as the distance along the curve, i.e. assumed to be L and L' respectively

$$y = Ax^2$$

$\therefore$ $$\frac{dy}{dx} = 2Ax$$

or $$x = \frac{dy}{dx} \times \frac{1}{2A}$$

Equating this successively to the gradients $(a + b)$ and then $(c + d)$, we get

$$L = \frac{1}{2A}(a+b)$$

or $$\frac{1}{2A} = \frac{L}{a+b}$$

Similarly $$L' = \frac{1}{2A}(c+d)$$

$$= \frac{L}{(a+b)}(c+d)$$

Here $L = 100$ m $a = 0.7\%$ $b = 0.8\%$ $L' = 120$ m
Let g be the new grade both up and down

$$100 = \frac{1}{2A}\frac{(0.7+0.8)}{100}$$

$$120 = \frac{1}{2A}\left(\frac{2g}{100}\right)$$

or $$\frac{100}{120} = \frac{15}{100} \times \frac{100}{2g}$$

or $$g = 0.9\%$$

Example 13.6 On a straight portion of a new road an upward gradient of 1 in 100 was connected to a downward gradient of 1 in 150 by a vertical parabola. Summit curve of length 150 m (Fig. 13.14). A point P, at chainage 5910.0 m, on the first gradient, was found to have a reduced level of 45.12 m and a point Q at a chainage of 6210.0 m on the second gradient of 44.95 m.

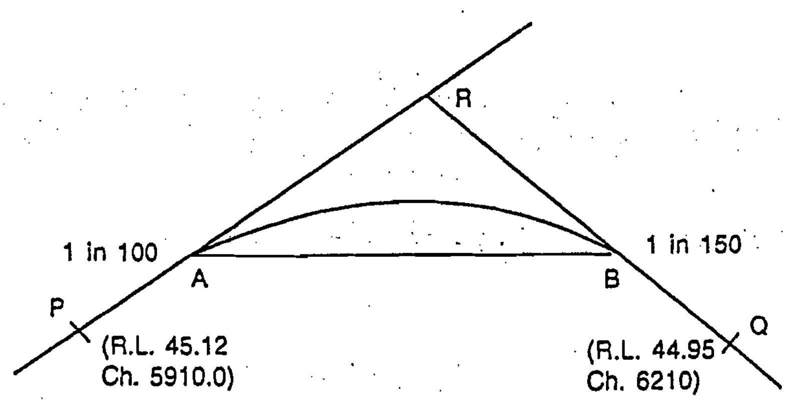


Fig. 13.14 Example 13.6.

(i) Find the chainages and reduced levels of the tangent points to the curve.

(ii) Tabulate the reduced levels of the points on the curve at intervals of 20 m from P and of its highest point.

Find the minimum sighting distance to the road surface for each of the following cases:

(iii) The driver of a car whose eye is 1.05 m above the surface of the road
(iv) The driver of a lorry for whom the similar distance is 1.80 m.

Solution Let the point of intersection R has chainage x.

Hence
$$45.12 + (x - 5910.0) \times \frac{1}{100}$$
$$= 44.95 + (6210.00 - x) \times \frac{1}{150}$$

or
$$x = 6019.8 \text{ m}$$

(i) Chainage at the 1st tangent point
$$= 6019.8 - 75$$
$$= 5944.8 \text{ m}$$

Chainage at the 2nd tangent point
$$= 6019.8 + 75$$
$$= 6094.8 \text{ m}$$

$$\text{R.L of 1st tangent Point} = 45.12 + \frac{(5944.8 - 5910.0)}{100}$$
$$= 45.12 + 0.348$$
$$= 45.468 \text{ m}$$

R.L of 2nd tangent point
$$= 44.95 + (6210.0 - 6094.8)\frac{1}{150}$$
$$= 45.718 \text{ m.}$$

(ii) The R.L's of different points on the curve at 20 m from P are given in tabular form below.

$$\text{Highest point occurs at a distance } x = \frac{g_1 L}{g_1 - g_2}$$
$$= \frac{\frac{1}{100} \times 150}{\frac{1}{100} - \left(-\frac{1}{150}\right)}$$
$$= 90 \text{ m}$$

Chainage of points at 20 m interval of P = 5910, 5930, 5950 and so on chainage of 1st tangent point = 5944.80

(iii) When $S < L$
$$S = \sqrt{\frac{(200)(L)}{g_1 - g_2}}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

Table 13.4 Example 13.6

Chainage	x	g_1x	$rx^2/2$	Curve elevation	Remarks
5944.80	0.00	0.00	0.0000	45.468	BVC
5950.00	5.20	0.052	0.00150	45.519	
5970.00	25.20	0.252	0.03528	45.685	
5990.00	45.20	0.452	0.11350	45.807	
6010.00	65.20	0.652	0.23616	45.884	
6030.00	85.20	0.852	0.40328	45.917	
6034.80	90.00	0.900	0.44995	45.918	Highest Point
6050.00	105.20	1.052	0.61483	45.905	
6070.00	125.20	1.252	0.87083	45.849	
6090.00	145.00	1.450	1.16806	45.750	
6094.80	150.00	1.500	1.25000	45.718	EVC

$$= \sqrt{\frac{(200)(150)}{1-(-0.67)}}(\sqrt{1.05}+\sqrt{0.15})$$

$$= \sqrt{\frac{(200)(150)}{1.67}}(1.024 + 0.387)$$

$$= 189.11 > L \text{ hence invalid}$$

(iv) When $S > L$

$$S = \frac{L}{2} + \frac{(\sqrt{h_1}+\sqrt{h_2})^2}{g_1 - g_2}$$

$$= \frac{150}{2} + \frac{(1.411)^2}{.01 + .0067}$$

$$= 75 + 119.22$$

$$= 194.22 > 150 \text{ m.}$$

when $h_1 = 1.80$ m

$$S = 75 + \frac{(\sqrt{1.80}+\sqrt{0.15})^2}{.01 + .0067}$$

$$= 75 + 178.93$$

$$= 253.93 > 150 \text{ m.}$$

PROBLEMS

13.1 Why a parabola is used as a vertical curve? Why not a circle?

13.2 What is meant by rate of change of grade on vertical curves and why it is important?

13.3 Tabulate station elevations for an equal tangent parabolic curve for the data given below:

A + 3% grade meets *a* – 1.5% grade at station 60 + 15 and elevation 300 m. 350 m curve. stake out at full stations.

13.4 Field conditions require a highway curve to pass through a fixed point. Compute a suitable equal-tangent vertical curve and full station elevations.
Grades of – 3.5% and + 0.5%, PVI elevation 260.00 m at station 26 + 00. Fixed point elevation 261.00 m at station 26 + 00

13.5 Compute and tabulate full-station elevation for an unequal tangent vertical curve to fit the requirements below:
A + 4.00% grade meets *a* – 2% grade at station 40 + 00 and elevation 400.00 m. Length of 1st curve 200 m, second curve 133 m.

13.6 (a) Explain why the second differences of curve elevations are equal for a parabolic curve.
(b) Why are parabolic curves not generally used for horizontal highway curves?

13.7 In determining sight distances on vertical curves, how does the designer determine whether the cars or objects are on the curve or tangent?

13.8 Calculate the minimum length of curve to provide required sight distance in the following cases.

(a) + 3.5% and – 2.5% sight distance 200 m.
(b) + 4.5% and – 3.5% sight distance 300 m.
(c) + 0.5% and – 1.20% sight distance 400 m.

HINTS TO SELECTED QUESTIONS

13.1 Because parabolas provide a constant rate of change of grade, they are ideal for vertical alignments used for vehicular traffic. Circle does not provide constant change of grade.

13.6 (a) Second difference indicates rate of change. As rate of change of a parabolic curve is constant, second difference is constant.
(b) On highways, parabolas are seldom used because drivers are able to overcome abrupt directional changes at circular curves by steering a parabolic path as they enter and exit the curves.

14

Areas and Volumes

14.1 INTRODUCTION

It is often necessary to compute the area of a tract of land which may be regular or irregular in shape. Land is ordinarily bought and sold on the basis of cost per unit area. To compute volumes of earthwork to be cut or filled in planning a highway, it is necessary to compute the areas of cross sections. In S.I units the area is measured in square meters, hectares or square kilometers. 1 hectare = 10,000 m^2. The relationship between acres and hectares is: 1 hectare = 2.471 acre or 1 acre = 0.4047 hectare.

14.2 METHODS OF MEASURING AREA

There are many methods for measuring area. They are: (1) geometrical methods when the area is divided into a number of triangles, rectangles or trapeziums; (2) by taking offsets from a straight line; (3) double meridian distances; (4) coordinates. When the plan or map of an area is available, however irregular it may be, planimeter can be run over the enclosing lines to compute the area of the plot. The area of a triangle whose sides are known can be computed by the formula,

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \tag{14.1}$$

where a, b, c are sides of the triangle and $s = \frac{1}{2}(a+b+c)$

14.2.1 AREA OF A TRACT WITH IRREGULAR BOUNDARIES

If the boundaries of a tract are irregular, it is not possible to run the traverse along the boundaries. The traverse is usually run at a convenient distance from the actual boundaries. The offsets from the traverse to the irregular boundary are taken at regular intervals or if necessary at irregular intervals. The area between the traverse line and the irregular boundary is determined by

1. Mid ordinate rule.
2. Average ordinate rule.
3. Simpson's rule.

Mid ordinate rule

Figure 14.1 explains the application of this rule.

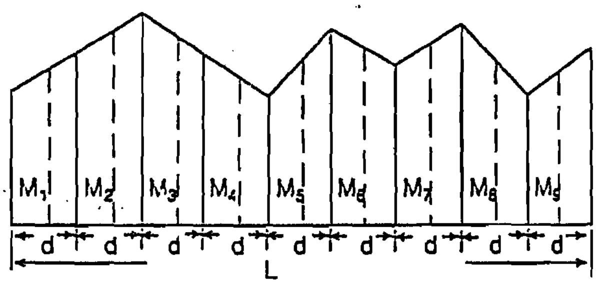


Fig. 14.1 Mid ordinate rule.

Total area of the irregular plot:

$$A = M_1d + M_2d + M_3d + M_4d + M_5d + M_6d + M_7d + M_8d + M_9d$$

$$= d(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 + M_9)$$

$$= \frac{L}{n}(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 + M_9) \qquad (14.2)$$

Average ordinate rule

Figure 14.2 explains this method.

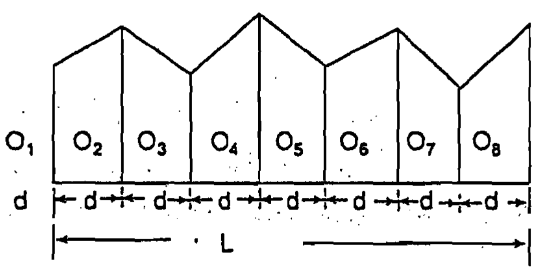


Fig. 14.2 Average ordinate rule.

If $O_1, O_2, \ldots, O_8$ are the ordinates to the boundary from the baseline

$$\text{Average ordinate} = \frac{O_1 + O_2 + O_3 + \ldots + O_7 + O_8}{8}$$

and area = average ordinate × length

$$= \frac{O_1 + O_2 + O_3 + \ldots + O_8}{8} \times L \qquad (14.3)$$

The area can also be computed by applying the trapezoidal rule which is obtained by considering each part as a trapezium and then adding the part mass together.

Therefore, total area

$$A = \frac{O_1 + O_2}{2}d + \frac{O_2 + O_3}{2}d + \ldots + \frac{O_6 + O_7}{2}d + \frac{O_7 + O_8}{2}d$$

$$= d\left(\frac{O_1 + O_8}{2} + O_2 + O_3 + O_4 + O_5 + O_6 + O_7\right).$$

If the number of segments is n, $d = L/n$ and no. of ordinates $= n + 1$. Therefore

$$A = \frac{L}{n}\left(\frac{O_1 + O_{n+1}}{2} + O_2 + O_3 + \ldots + O_n)\right). \qquad (14.4)$$

The trapezoidal rule can, therefore, be stated as:

> Area is equal to product of the common interval d and sum of intermediate ordinates plus average of the first and last ordinates. If the intervals are not equal the areas of the trapeziums have to be computed separately and added together.

Simpson's rule

In the rules stated above the irregular boundary consists of a number of straight lines. If the boundary is curved, it can be approximated as a series of straight lines. Alternatively, Simpson's rule is applied which assumes that the short lengths of boundaries between the ordinates are parabolic arcs. Figure 14.3 shows an area with a curved boundary.

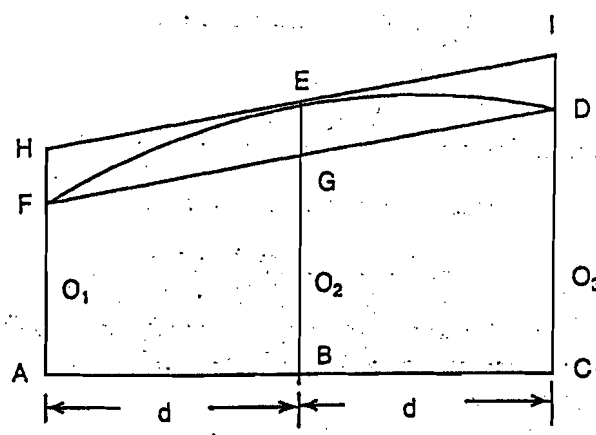


Fig. 14.3 Derivation of Simpson's rule.

The area $ABCDEF$ consists of two parts:

(a) Area $ABCDGF$ which is equal to

$$\frac{O_1 + O_3}{2} \times 2d = d(O_1 + O_3).$$

(b) Area $DEFGD$

$$= \frac{2}{3} \times \text{area of enclosing parallelogram } FHEIDG.$$

$$= \frac{2}{3} \times (2d)(DEF)$$

$$= \frac{2}{3} \times (2d) \times \left(O_2 - \frac{O_3 + O_1}{2}\right)$$

$$= \frac{2}{3} \cdot d \times [2O_2 - (O_3 + O_1)].$$

$$\text{Total area} = \frac{O_1 + O_3}{2} \times 2d + \frac{2d}{3}[2O_2 - (O_1 + O_3)]$$

$$= \frac{d}{3}[3O_1 + 3O_3 + 4O_2 - 2O_1 - 2O_3]$$

$$= \frac{d}{3}[O_1 + 4O_2 + O_3].$$

Area of the next two segments

$$= \frac{d}{3}[O_3 + 4O_4 + O_5]$$

Taking all the areas together, total area becomes

$$A = \frac{d}{3}[(O_1 + 4O_2 + O_3) + (O_3 + 4O_4 + O_5) + (O_5 + 4O_6 + O_7) + \ldots + (O_{n-1} + 4O_n + O_{n+1})]$$

$$= \frac{d}{3}[O_1 + O_{n+1} + 4(O_2 + O_4 + O_6 + \ldots) + 2(O_3 + O_5 + O_7 + \ldots)] \quad (14.5)$$

Since we are taking 2 segments at a time, the number of segments n should always be even and the number of ordinates odd for Simpson's rule to be applicable. In words, the rule is "To get the area by Simpson's rule, add 1st and last ordinates to four times the even ordinates and two times the odd ordinates and multiply the sum by one third the common interval." The accuracy of Simpson's rule is more than the trapezoidal rule for curved boundary. Whether the area computed is more or less than the actual value depends on whether the area is concave or convex to the baseline.

There exists another formula (known as Simpson's 3/8 formula) which assumes a third degree polynomial passing through four consecutive points of the ground profile as shown in Fig. 14.4. It takes the following form

$$A = \frac{3d}{8}(O_0 + 3O_1 + 3O_2 + O_3)$$

To apply Simpson's 3/8 formula to a sequence of intervals, the number of intervals must be divisible by three.

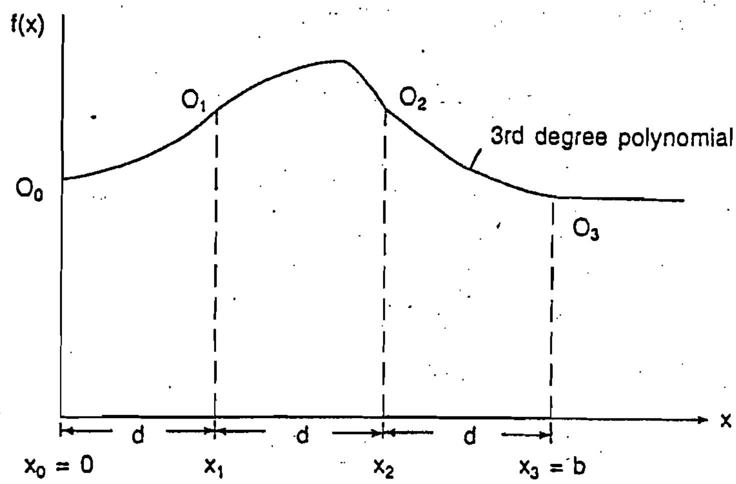


Fig. 14.4 Simpson's 3/8th formula.

14.2.2 AREA OF A CLOSED TRAVERSE

The corrected latitude and departure of a line is known after adjustment of the traverse. These are utilized in computing the area of the closed traverse for which four methods are available:

1. Meridian distance method.
2. Double meridian distance method.
3. Double parallel distance method.
4. Departure and total latitude method.

Meridian distance method

The meridian distance of a line is the perpendicular distance from the line's midpoint to a reference meridian (North-South line). To avoid negative signs, the reference meridian is generally chosen as passing through the most westerly corner of the traverse or further away from it. Figure 14.5 shows the different associated terms. *EF* is the meridian distance of *AB*.

GH is the meridian distance of *BC*. Mathematically meridian distance of *BC* is equal to meridian distance of *AB* plus half the departure of *AB* plus half the departure of *BC*. Similarly, meridian distance of *CD* is equal to the meridian distance of *BC* plus half the departure of *BC* plus half the departure of *CD*. Thus the meridian distance of any line is equal to the meridian distance of the preceding line plus half the departure of the preceding line plus half the departure of the line itself. For applying this rule, the sign of the departure should be considered, Eastern departure being positive and Western departure negative. Similarly, latitude is positive towards North and negative towards South.

Taking the North-South line passing through '*A*', (Fig. 14.6)

EF = meridian distance of *AB*.
AM = latitude of *AB*.
HG = meridian distance of *BC*.

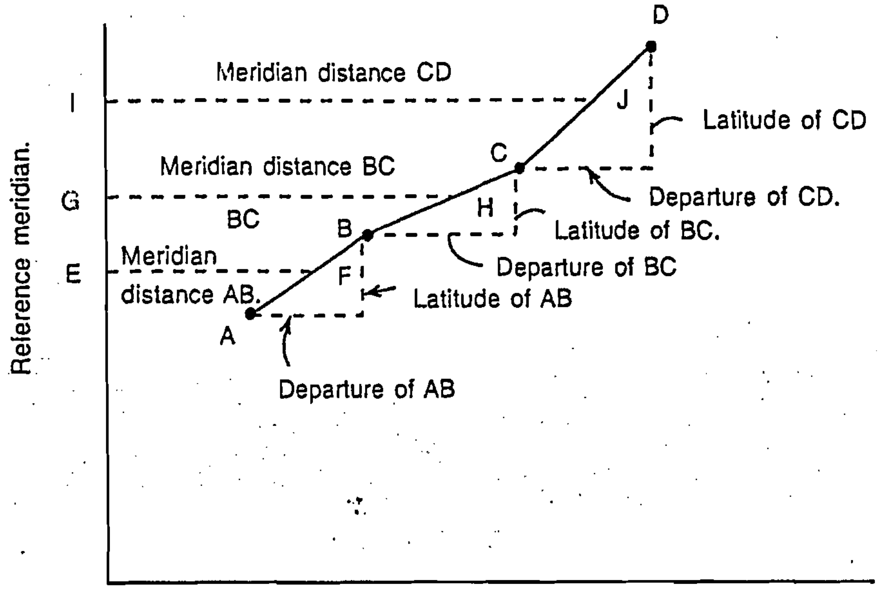


Fig. 14.5 Meridian distance method.

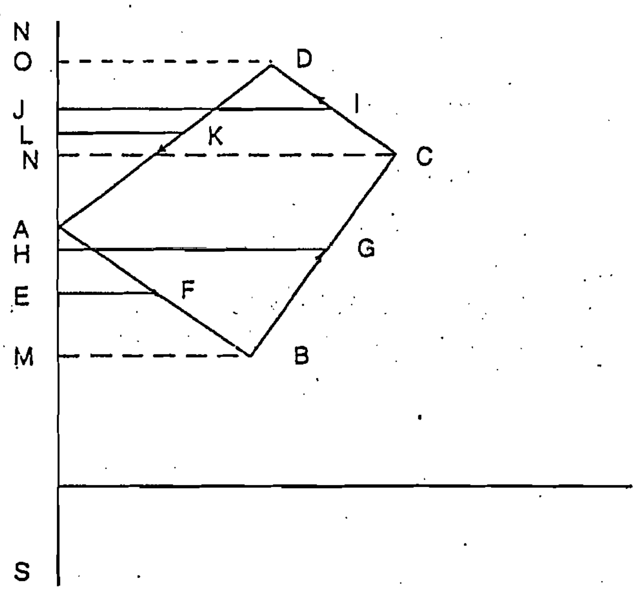


Fig. 14.6 Derivation of area by meridian distance method.

MN = latitude of BC.
IJ = meridian distance of CD.
ON = latitude of CD.
LK = meridian distance of DA.
OA = latitude of DA.

$$\text{Area } ABCD = -AMB + NMBC + NCDO - ODN$$
$$= (-AM) \times EF + NM \times HG + JI \times ON + LK \times (-OA) \tag{14.6}$$

Here all the departures are positive as A, the most westerly station has been chosen as origin. As regards sign of latitude, from the direction of arrows it is clear that latitudes of AB and DA point downwards, i.e. towards south, and hence is negative.

$$\text{Symbolically } A = \sum L \times M_d$$

where L = latitude

and M_d = meridian distance.

Double meridian distance method

In order to avoid working with half departures, surveyors use the double meridian distance, i.e., twice the meridian distance in making computations. Thus the *DMD* of *BC* is equal to the *DMD* of *AB* plus the departure of *AB* plus the departure of *BC*. The following are the rules for computing *DMD*s for a closed traverse.

1. The *DMD* of the first line is equal to the departure of the 1st line. If the 1st line is chosen as the one that begins at the western most corner, negative *DMD*s can be avoided.

2. The *DMD* of each succeeding line is equal to the *DMD* of the previous line plus the departure of the previous line plus the departure of the line itself.

3. The *DMD* of the last line of a balanced closed traverse is equal to the departure of the line but with opposite sign.

It has already been shown

$$\text{Area } ABCD = (-AM) \times EF + NM \times HG + JI \times ON + LK \times (-OA).$$

This can be rewritten as:

$$= \frac{1}{2}[(-AM) \times 2EF + NM \times 2HG + ON \times 2JI + (-OA) \times 2LK)]$$

$$= \frac{1}{2}[(-AM) \times DMD_{AB} + NM \times DMD_{BC}$$
$$+ ON \times DMD_{CD} + (-OA) \times DMD_{DA}]$$

$$= \frac{1}{2}[\Sigma L \times DMD]$$

or $2ABCD = \Sigma L \times DMD$.

Here summation of products of *DMD* and latitudes of lines of a closed traverse with proper sign gives twice the area of the traverse. If the traverse is covered clockwise, the area will be negative, if counter clockwise, the area will be positive.

Double parallel distance method

In this method perpendicular distances of mid points of different lines are measured from a reference parallel. The reference parallel is usually taken through the most southerly point of the traverse along the east-west line, i.e. perpendicular to the reference meridian. The double parallel distance is twice the parallel distance of a line. The *DPD* for any traverse line is equal to the *DPD* of the previous line plus the latitude of the previous line plus latitude of the line itself. The traverse area can be computed by multiplying the *DPD* of each line by its departure, summing the products and taking half the absolute value of the total.

Departure and total latitude method

Total latitude of a point is equal to its distance from the reference station measured parallel to the reference meridian.

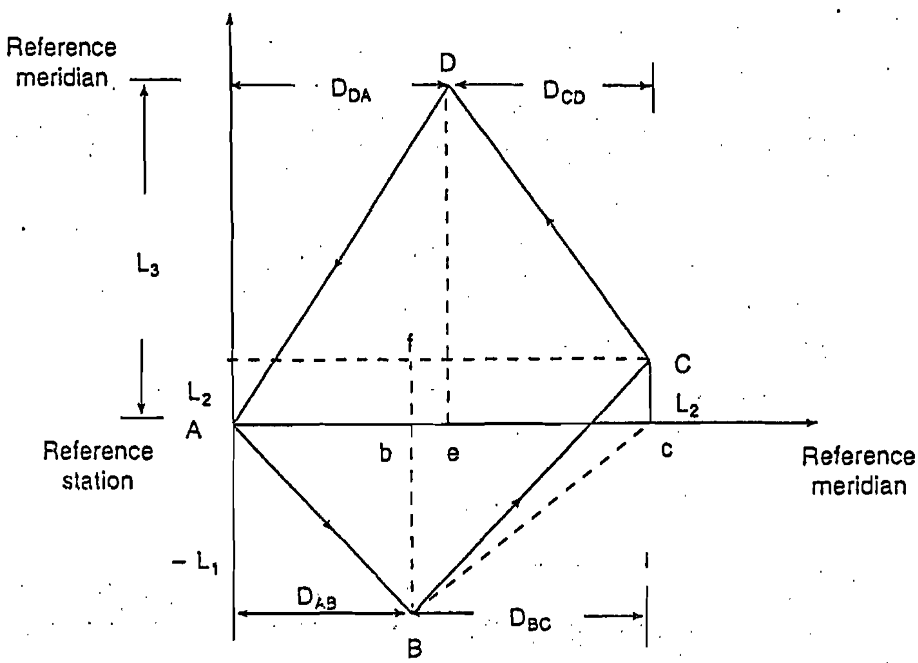


Fig. 14.7 Total latitude method.

Area of the closed traverse *ABCD*,

$$A = ABb + Bbc + Dec\,C + ADe - CcB.$$

$$= \frac{1}{2}(-L_1)\,D_{AB} + \frac{1}{2}(-L_1)\,bc + \frac{1}{2}(L_2 + L_3)(-D_{CD})$$

$$+ \frac{1}{2}(L_3)(-D_{DA}) - 1/2\,L_2 bc$$

$$= 1/2(-L_1)D_{AB} + \frac{1}{2}(-L_1)D_{BC}$$

$$+ \frac{1}{2}(L_2 + L_3)(-D_{CD}) + \frac{1}{2}(L_3)(-D_{DA}) - 1/2\,L_2(-D_{BC})$$

$$= \frac{1}{2}(-L_1)(D_{AB} + D_{BC}) + \frac{1}{2}L_2(D_{BC})$$

$$+ \frac{1}{2}L_3(-D_{CD} - D_{DA}) - \frac{1}{2}L_2(D_{CD})$$

$$= \frac{1}{2}\{(-L_1)(D_{AB} + D_{BC}) + (L_2)(+D_{BC} - D_{CD})$$

$$+ L_3(-D_{CD} - D_{DA})\}$$

or $$2A = \{(-L_1)(D_{AB} + D_{BC}) + (L_2)(D_{BC} - D_{CD}) + (L_3)(-D_{CD} - D_{DA})\}$$

$$= \Sigma \text{ total latitude of a point}$$

$$\times \text{ (algebraic sum of two adjacent departures).}$$

Hence following steps should be followed in computing area by this method:

1. Compute the total latitude of each station from a reference station.
2. Compute the algebraic sum of departures of lines meeting at this station.
3. Find the product of total latitude of each station and the corresponding algebraic sum of departures.
4. Half the algebraic sum of these products gives the required area.

14.2.3 COORDINATES METHOD

In this method independent coordinates of the points are used in the computation of areas.

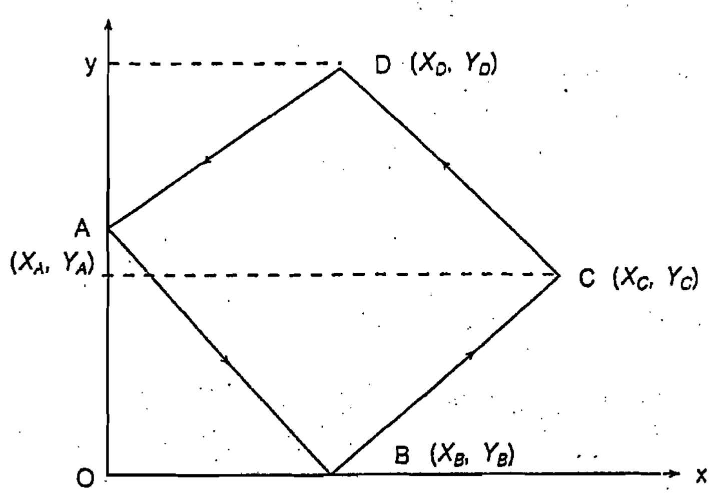


Fig. 14.8 Coordinates method.

To avoid negative sign, the origin O is chosen at the most southerly and westerly point.

Total area of the traverse

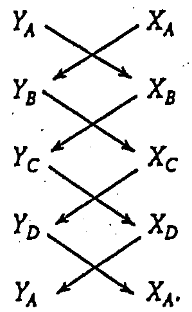
$$A = \frac{1}{2}(X_C + X_B)(Y_C - Y_B) + \left(\frac{X_D + X_C}{2}\right)(Y_D - Y_C)$$

$$- \left(\frac{X_B + X_A}{2}\right)(Y_A - Y_B) - \left(\frac{X_D + X_A}{2}\right)(Y_D - Y_A)$$

or

$$\begin{aligned}2(A) &= X_A Y_B + X_B Y_C + X_C Y_D + X_D Y_A \\ &\quad - X_B Y_A - X_C Y_B - X_D Y_C - X_A Y_D \\ &= (X_A Y_B - X_B Y_A) + (X_B Y_C - X_C Y_B) + (X_C Y_D - X_D Y_C) \\ &\quad + (X_D Y_A - X_A Y_D). \end{aligned} \tag{14.7}$$

The above relation can be expressed as follows for easy remembrance.



The coordinates can also be listed in the following form:

$$\frac{X_A}{Y_A}\,\frac{X_B}{Y_B}\,\frac{X_C}{Y_C}\,\frac{X_D}{Y_D}\,\frac{X_A}{Y_A} \tag{14.8}$$

Two sums of products should be taken.

1. Product of all adjacent terms taken down to the right, i.e.

$$X_A Y_B,\ X_B Y_C,\ X_C Y_D,\ X_D Y_A$$

2. Products of all adjacent terms up to the right.

$$Y_A X_B,\ Y_B X_C,\ Y_C X_D,\ Y_D X_A$$

The traverse area is equal to half the absolute value of the difference between these two sums. In applying this procedure, it is to be observed that the first coordinate listed must be repeated at the end of the list.

14.2.4 MEASUREMENT OF AREA BY PLANIMETER

The planimeter is a mechanical instrument. It measures the area of a plan or map, however irregular its shape may be, very accurately. There are two types of planimeters:

1. Amsler polar planimeter;
2. Roller planimeter

The Amsler polar planimeter is more frequently used and hence explained here in detail.

Figures 14.9(a) and (b) show the essential parts of an Amsler polar planimeter which are described as follows:

1. *Pole block or anchor* It is a heavy block. This is fixed on the plan by a fine retaining pin called the anchor point.

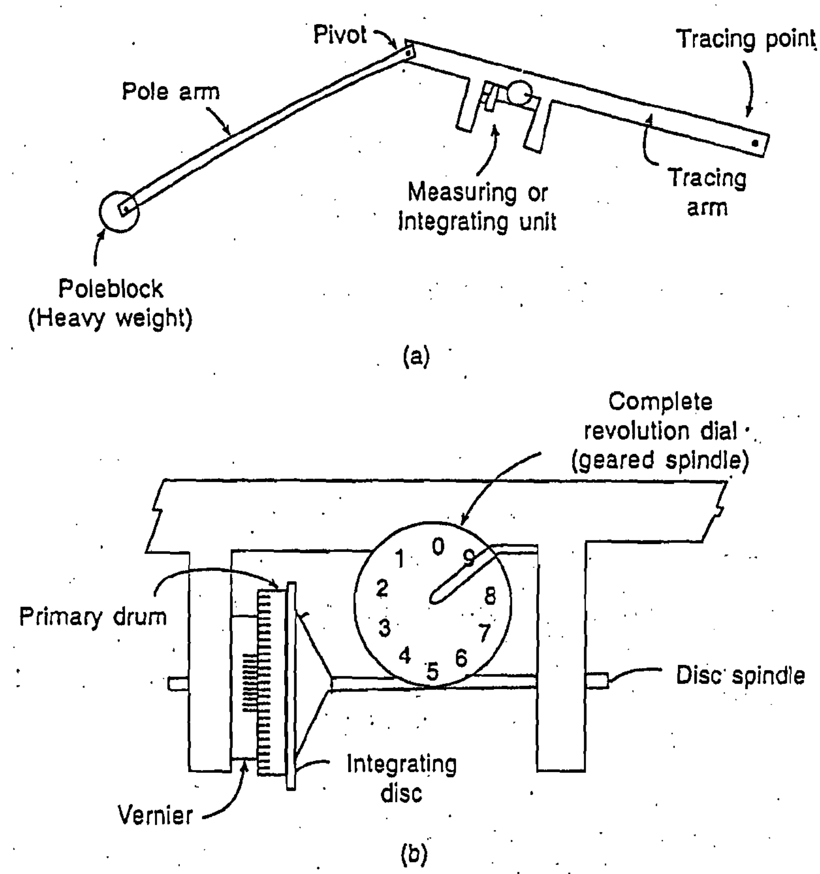


Fig. 14.9 (a) Schematic diagram of Amsler polar planimeter. (b) The integrating unit.

2. *Pole arm or anchor arm* This is a bar whose one end is pivoted about the pole block and the other end about the integrating unit.
3. *Tracing arm* This may be either fixed or variable in length. Its one end is attached to the integrating unit while the other end carries the tracing point or optical tracer.
4. *The integrating unit or measuring unit* It consists of a hardened steel integrating disc carried on pivots. The disc spindle is connected to a primary drum or roller which is divided into 100 parts. By means of a vernier, readings upto 1/1000th of a revolution of the roller is obtained. The roller is so designed that when it completes one revolution, another disc connected with the roller shows one dimension. This disc is divided into *ten* divisions. The disc, therefore completes one revolution after every 10 revolutions of the roller. The reading of a planimeter is in 4 digits. If the reading is say, 3456 it shows: (i) 3 on the disc indicating 3 full rotations of the roller, (ii) 45 on the roller which means out of 100 divisions roller has moved through 45 divisions, and (iii) 6 indicates the vernier reading of the roller which is in thousandth.

The planimeter when placed over a plan or map whose area is to be measured rests on three points: (i) anchor point, (ii) drum or roller, and (iii) tracing point or tracer. The area is measured by moving down the tracer over the outline of the plan or map in a clockwise direction. The area is then obtained as:

$$A = M(F.R. - I.R. \pm 10\,N \pm C) \tag{14.9}$$

where M = Multiplying constant or planimeter constant and is equal to the area per revolution of the roller. This value is marked on the tracing arm.

$F.R.$ = Final reading.

$I.R.$ = Initial reading.

N = Number of full revolutions of the disc. As one revolution of the disc is 10 units, it is multiplied by 10 in the above expression. Plus sign is to be used when the rotation is clockwise and minus sign when anticlockwise.

C = Constant of the instrument usually marked on the tracing arm just above the scale divisions. This constant when multiplied by M gives the zero circle. It is to be added when the anchor point is within the circle and is taken to be zero when the anchor point is outside the circle.

While using a planimeter the following points should be observed:

1. The tracer point should be guided by a triangle or straight edge though usually it is steered free hand.

2. The anchor point should preferably be placed outside the traverse as this will avoid the additive constant C.

3. The movement of the disc should be carefully watched and clockwise or anticlockwise rotation of the zero mark of the disc against the index mark should be noted.

4. The tracing point should always be moved clockwise.

5. Since area obtained by planimeter is not necessarily an exact value, it is good practice to trace a figure several times and take an average of the results thus obtained. It is also desirable to trace the figure one or more times in the opposite directions and average these values also. The different values should agree within a limit of 2 to 5 units.

The length of the tracing arm of the planimeter can be adjusted and accordingly value of M will vary. Since the bar setting may not be perfect it is best to check the planimeter constant by running over the perimeter of a carefully laid out square 5 cm on a side with diagonals 7.07 cm. The area should be 25 cm^2. If the difference in reading is, say, 100,

$$1 \text{ unit} = \frac{25}{100} \text{ cm}^2 = 0.25 \text{ cm}^2$$

If the observed difference in reading is 1125, area is $0.25 \times 1125 = 281.25$ cm^2. If the scale of the map is 1 cm = 100 m.

or

$$1 \text{ cm}^2 = 100 \times 100 \text{ m}^2.$$

$$\text{Area} = 281.25 \times 10^4 \text{ m}^2.$$

The planimeter is useful in measuring irregular areas. It is often used in measuring cross-sectional areas of highways and in computing areas of property surveys.

Partitioning land

It is often necessary to partition land into two or more pieces for sale or distribution to family members, heirs and so on. Initially a boundary survey is to be made, latitudes, departures computed and after proper balancing of the traverse total area of the traverse is calculated. For regular shapes and in some cases, analytical solution is possible for division of an area into required parts. Some problems and their solutions are outlined below.

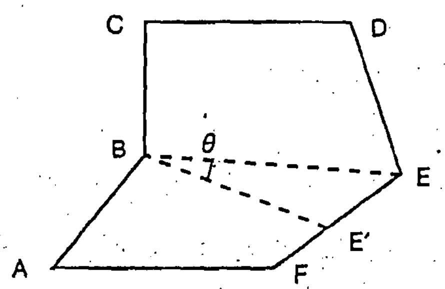


Fig. 14.10 Partitioning by a line between two points.

1. Area cut off by a line *BE* between two points. It is necessary to calculate the areas *BCDE* and *ABEF*. Knowing the coordinates of *B* and *E*, length and bearing of *BE* can be obtained. Hence latitude and departure of *BE* are known. Now by applying *DMD* methods areas of the parts *BCDE* and *BEFA* can be obtained.

If instead of line *BE*, the area is divided by the line *BE'*, either θ or *EE'* must be known. If *EE'* and hence *FE'* are known, considering *BE'FA* as a closed traverse, length and bearing of *BE'* can be obtained. If θ is known, length *EE'* can be obtained by applying the sine rule to the triangle *BEE'* and then area *BCDEE'* or *ABE'F* can be obtained.

2. To cut off a required area by a line from a closed traverse through a fixed point.

Let *ABCDEFA* be a closed traverse (Fig. 14.11). Let *G* be the given point on *DE*. Find out a station point which will approximately divide the traverse into required areas. Let it be *A*. Compute the area *ABCDG*. This will not be equal to the required area A_1. Let the area to be added be A_2 and is equal to *AGH*.

Applying sine rule the area of the triangle is

$$\frac{1}{2}\, AH \cdot AG \cdot \sin GAH$$

Since bearing of *AG* is known angle *GAH* can be determined. Therefore

$$\frac{1}{2}\, AH \cdot AG \cdot \sin GAH = A_2$$

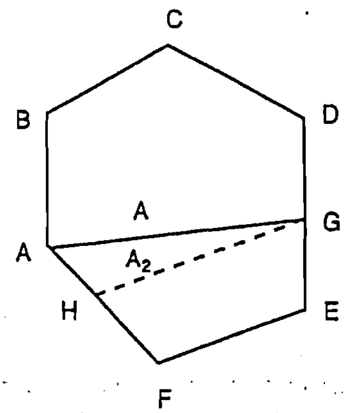


Fig. 14.11 Partitioning by a line through a fixed point.

or
$$AH = \frac{2A_2}{AG \cdot \sin GAH}.$$

Knowing the angle AGH, bearing of GH can be determined.

3. To cut off a required area by a line running in a given direction: As before the line should initially pass through a station point say E, to give an area close to the required area A_1. If the discrepancy is A_2 which is to be added to the initial area $BCDEF$ the line EF is to be shifted parallely to GH such that the area $EFIJ$ is equal to the required area A_2 (Fig. 14.12)

$$\text{The area } EFIJ = FE \times h - \frac{1}{2} h^2 \tan \theta - \frac{1}{2} h^2 \tan \phi$$

In the above equation the only unknown is h as other values, viz. area $EFIJ$, tan θ and tan ϕ are all known. Further,

$$FI = h \sec \phi, \quad JE = h \sec \theta$$

and
$$IJ = HG - h(\tan \phi + \tan \theta)$$

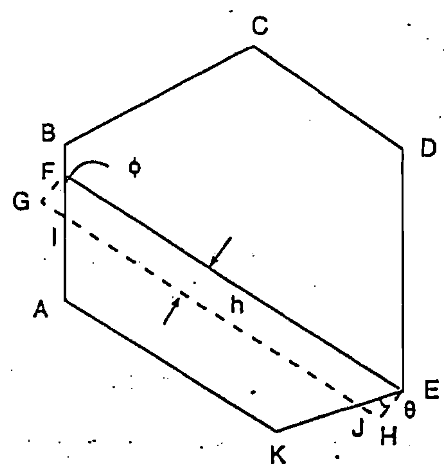


Fig. 14.12 Partitioning by a line running in a given direction.

The line *IJ* can be located from the distances *IF*, *FE*, and *EJ*.
A general direct method for land subdivisions has been suggested by Easa.

14.3 VOLUMES

Surveyors are often required to compute volumes of earthwork either in cut or in fill when planning a highway system. To compute stockpiles of coal, gravel or other materials knowledge of volume computation is required. There are basically three methods for this: (i) Cross section method, (ii) Unit area or borrow pit method, (iii) Contour area method.

14.3.1 CROSS SECTION METHOD

This is employed for computation of volumes for highways, railways and canals. Here a series of cross sections are taken along the length of the line at regular intervals. These are obtained by measurements in the field. They can also be obtained by photogrammetry. The cross sections are plotted on a sheet and over the cross section design templates are superimposed. The difference in the two areas will be the amount of cut or fill. This is shown in Fig. 14.13.

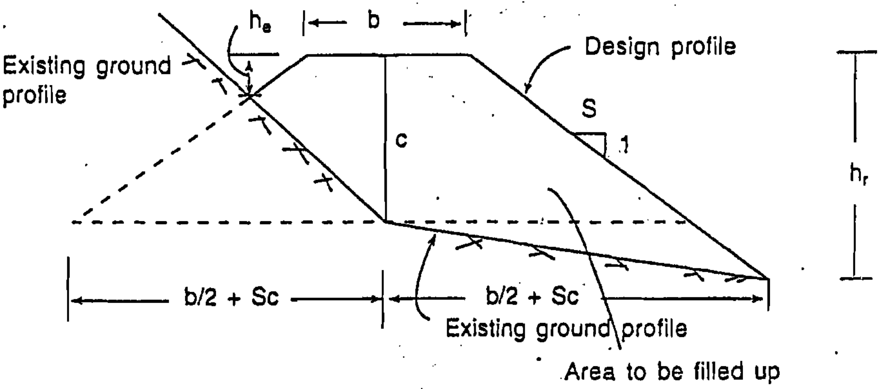


Fig. 14.13 Cross section of existing ground profile.

The following five types of cross sections generally occur in practice:

1. Level Section
2. Two level Section.
3. Side hill two level Section.
4. Three level Section.
5. Multi level Section.

While different formulae can be derived for different types of cross sections, it is useful if all the areas are derived by using one method only, i.e. by considering the figure as a closed traverse. The coordinate axes are the finished grade and the centre line of the cross section. The general formula for area is

$$A = \frac{1}{2}[X_1(Y_2 - Y_n) + X_2(Y_3 - Y_1) + X_3(Y_4 - Y_2) + \dots + X_n(Y_1 - Y_{n-1})]$$

or

$$A = \frac{1}{2}[Y_1(X_2 - X_n) + Y_2(X_3 - X_1) + Y_3(X_4 - X_2) + Y_4(X_5 - X_3) + \dots + Y_n(X_1 - X_{n-1})].$$

Since for a cross section in earth work, Y coordinates of two points are zero, the computation will be shortened if the second equation is used.

Level section

Coordinates of A, B, C and D, from Fig. 14.14,

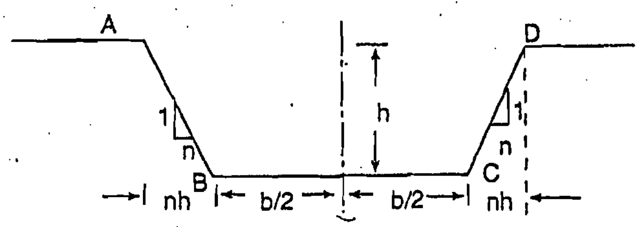


Fig. 14.14 Level section.

Point	x	y
A	$-(b/2 + nh)$	h
B	$-b/2$	0
C	$b/2$	0
D	$(b/2 + nh)$	h

$$\text{Area } A = \frac{1}{2}[h\{-b/2 - (b/2 + nh)\} + 0\{b/2 - (-(b/2 + nh))\} + 0\{b/2 + nh - (-b/2)\} + h\{-(b/2 + nh) - (b/2)\}]$$

$$= \frac{1}{2}[h(-b - nh) + h(-b - nh)]$$

$$= -h[b + nh]. \tag{14.10}$$

The negative sign is immaterial and should be ignored.

Two-level section in cutting

coordinates of

	x	y
A	$-b/2$	0
B	$+b/2$	0
C	$+w_1$	h_1
E	$-w_2$	h_2

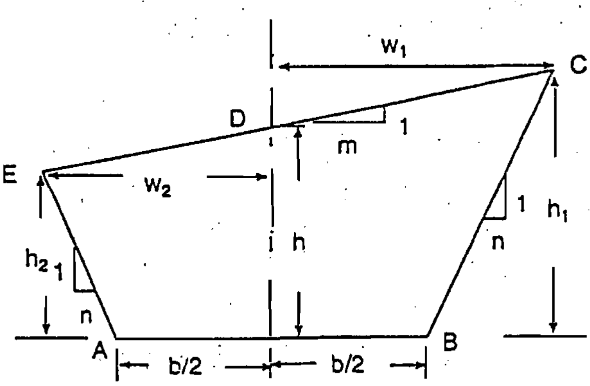


Fig. 14.15 Two level section in cutting.

From the geometry of Fig. 14.15

$$h + w_1/m = h_1$$

$$b/2 + h_1 n = w_1$$

or $$b/2 + \left(h + \frac{w_1}{m}\right) \cdot n = w_1$$

or $$b/2 + nh = w_1(1 - n/m)$$

$$= w_1 \left(\frac{m - n}{m}\right)$$

or $$w_1 = (b/2 + nh)\,\frac{m}{m - n}$$

Similarly $$w_2 = (b/2 + nh)\,\frac{m}{m + n}$$

Area in terms of coordinates

$$= \frac{1}{2}\,[0 + 0 + (h_1)(-\,w_2 - b/2) + (h_2)(-\,b/2 - w_1)]$$

$$= -\,1/2\,[b/2(h_1 + h_2) + h_1 w_2 + h_2 w_1]$$

$$= -\,1/2\left[b/2(h_1 + h_2) + \left(h + \frac{w_1}{m}\right) w_2 + \left(h - \frac{w_2}{m}\right) w_1\right]$$

$$= -\,1/2\,[b/2(h_1 + h_2) + h(w_1 + w_2)] \qquad (14.11)$$

Two-level section in filling

From geometry of the Fig. 14.16

$$w_1 = (b/2 + nh)\,\frac{m}{m + n}$$

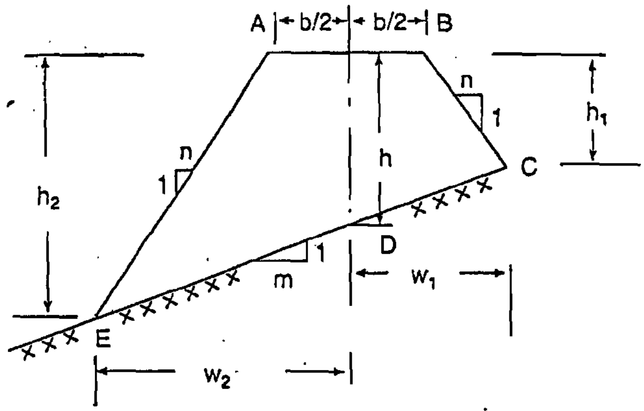


Fig. 14.16 Two level section in filling.

$$w_2 = (b/2 + nh)\,\frac{m}{m - n}$$

$$h_1 = h - w_1/m \quad h_2 = h + w_2/m.$$

The same formula as derived above is applicable. Hence if b, h, n, m are given h_1, h_2, w_1 and w_2 can be computed and the area obtained as shown above.

Side hill two-level section

From the geometry of Fig. 14.17

$$w_2 = n_2h_2 + b/2$$

From similar triangles HGF and FIC

$$h_2/h = \frac{n_2h_2 + b/2 - mh}{mh}$$

$$mh_2 = n_2h_2 + b/2 - mh$$

$$h_2(m - n_2) = b/2 - mh$$

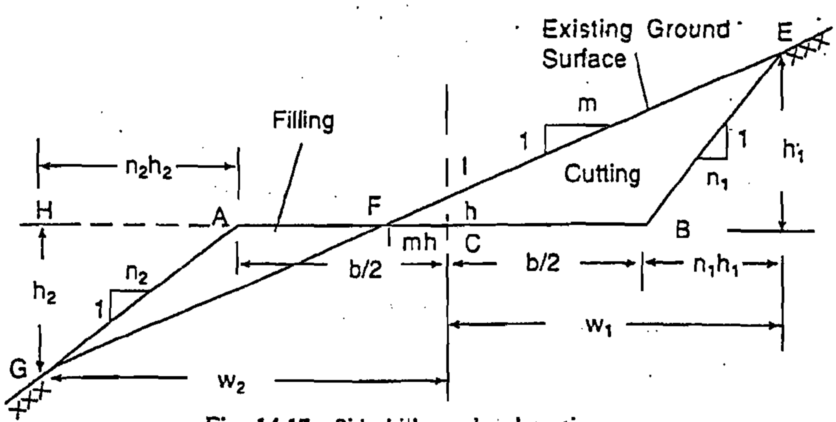


Fig. 14.17 Side hill two-level section.

$$h_2 = \frac{b/2 - mh}{m - n_2}$$

or
$$w_2 = \frac{n_2}{m - n_2}[b/2 - mh] + b/2$$

$$= \frac{n_2 \cdot b/2 - n_2 mh + b/2 \cdot m - b/2 \cdot n_2}{m - n_2}$$

$$= \frac{m}{m - n_2}(b/2 - n_2 h)$$

Similarly
$$w_1 = \frac{m}{m - n_1}(b/2 + n_1 h)$$

$$\text{Area of cutting} = \frac{1}{2} h_1(b/2 + mh)$$

But
$$b/2 + n_1 h_1 = w_1$$

or
$$h_1 = \frac{1}{n_1}(w_1 - b/2)$$

Hence
$$\text{Area} = \frac{1}{2} \cdot \frac{1}{n_1}(w_1 - b/2)(b/2 + mh).$$

Substituting the values of w_1, we have

$$\text{Area of cutting} = \frac{1}{2}\frac{(b/2 + mh)^2}{m - n_1} \qquad (14.12)$$

Similarly,
$$\text{Area of filling} = \frac{1}{2}\frac{(b/2 - mh)^2}{m - n_2} \qquad (14.13)$$

Three level section

Here at least three levels are required to define the ground slope. The shapes in cutting and filling are shown in Fig. 14.18(a).

Points	Coordinates x	y
A	$-b/2$	0
B	$+b/2$	0
C	w_1	h_1
D	0	h
E	$-w_2$	h_2

$$\text{Area} = \frac{1}{2}[0(b/2 + w_2) + 0(w_1 + b/2) + h_1(0 - b/2) + h(-w_2 - w_1) + h_2(-b/2 - 0)]$$

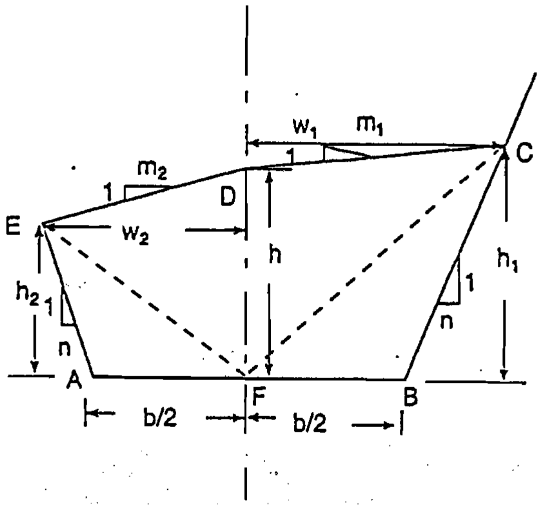


Fig. 14.18 (a) Three level section.

$$= \frac{1}{2}\left[-\frac{bh_1}{2} - h(w_1 + w_2) - \frac{bh_2}{2}\right]$$

$$= -\frac{1}{2}\left[\frac{b}{2}(h_1 + h_2) + h(w_1 + w_2)\right]$$

As before

$$w_1 = b/2 + nh_1$$

$$= m_1(h_1 - h)$$

or $$h_1 - h = \frac{w_1}{m_1}$$

or $$h_1 = \frac{w_1}{m_1} + h$$

$$w_1 = b/2 + n\left(\frac{w_1}{m_1} + h\right)$$

or $$w_1\left(\frac{m_1 - n}{m_1}\right) = b/2 + nh$$

or $$w_1 = \frac{m_1}{m_1 - n}(b/2 + nh)$$

Similarly $$w_2 = \frac{m_2}{m_2 + n}(b/2 + nh)$$

$$h_1 = h + \frac{w_1}{m_1}$$

$$h_2 = h - w_2/m_2.$$

Hence if h and slopes m_1, m_2 and n are given, h_1, h_2, w_1 and w_2 can be computed and hence the area.

Multilevel section

Figure 14.18(b) shows a multilevel section where more than three levels are required to define the transverse slope of the ground. For a multilevel section the coordinate method of determining area is convenient. This has already been explained in Section 14.2.3.

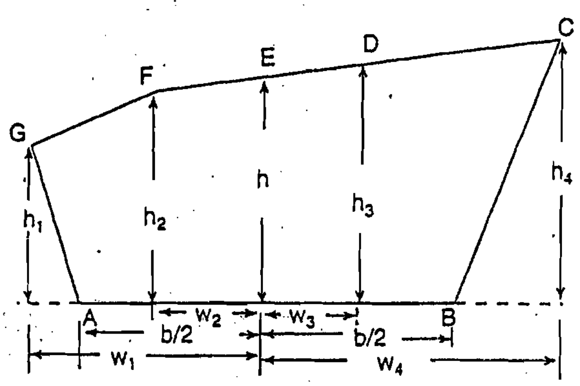


Fig. 14.18(b) Multilevel section.

14.3.2 FORMULAE FOR COMPUTATION OF VOLUMES

Volume by average end areas

The volume by this method is given by

$$V = \frac{1}{2} (A_0 + A_1) L \tag{14.14}$$

= Average end areas × Length between the sections.

Prismoidal Formula

The volume is,

$$V = \frac{L}{6} (A_0 + 4M + A_1) \tag{14.15}$$

Here A_0, A_1 are the end areas where as M is the area of the middle section.

The prismoidal formula very nearly gives the correct volume of the solid. The error in the use of the end area formula arises chiefly from the fact that in its application the volume of a pyramid is considered to be one-half the product of the base and the altitude whereas the actual volume is one-third the product of these quantities. The area of the middle section is obtained by taking dimension of the middle section and computing the area or by computing the area of a section whose dimensions are intermediate of the end dimensions.

Volume by mean area method

In this method the mean cross sectional area of the various sections is first computed as:

$$A_m = \frac{A_1 + A_2 + \ldots + A_n}{n}.$$

Volume $V = A_m \times L$ where L is the length between the first and last section.

Trapezoidal rule for computing volumes

If we have a series of sectional areas $A_1, A_2, \ldots, A_m$ at an equal interval of D, by end area methods:

$$\text{Volume between } A_1 \text{ and } A_2 = \frac{A_1 + A_2}{2} D$$

$$\text{between } A_2 \text{ and } A_3 = \frac{A_2 + A_3}{2} D$$

$$\text{between } A_{n-1} \text{ and } A_n = \frac{A_{n-1} + A_n}{2} D$$

Total volume then

$$V = D\left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \ldots + A_{n-1}\right] \quad (14.16)$$

This is known as trapezoidal rule.

Prismoidal rules for computing volumes

The volume of a prismoid

$$V = \frac{L}{6}(A_0 + 4M + A_1)$$

If three consecutive sections A_1, A_2 and A_3 are taken at interval of D, $L = 2D$ and the formula becomes

$$V_1 = \frac{2D}{6}(A_1 + 4A_2 + A_3)$$

The next prismoid

$$V_2 = \frac{2D}{6}(A_3 + 4A_4 + A_5)$$

$$\vdots$$

$$V_n = \frac{2D}{6}(A_{n+1} + 4A_{n+2} + A_{n+3})$$

Adding

$$V = \frac{D}{3}[A_1 + A_{n+3} + 4(A_2 + A_4 + \ldots + A_{n+2}) + 2(A_3 + A_5 + \ldots + A_{n+1})] \quad (14.17)$$

In other words, the prismoidal formula states that the total volume

$$V = \frac{\text{Interval between the sections}}{3} \times [\text{Area of 1st section}$$

$$+ \text{Last section} + 4 \text{ times the even sections} + 2 \text{ times the odd sections}]$$

To use the prismoidal rule the number of sections must be odd. If the number of sections is even the prismoidal formula cannot be applied to the last two sections which should be treated separately. Either the trapezoidal rule or the end area rule should be applied to compute the volume of the segment. If prismoidal rule is to be applied the area of the mid section should be computed separately and then the formula should be applied. The volume obtained by the trapezoidal rule is always greater than the volume obtained by the prismoidal formula.

14.3.3 PRISMOIDAL CORRECTION OR PRISMOIDAL EXCESS

It is the difference between the volumes computed by the trapezoidal rule and the prismoidal rule. As the volume calculated by trapezoidal rule is greater than that calculated by the prismoidal formula, the correction is negative and should be subtracted from the prismoidal formula to get the result of the prismoidal rule.

$$\text{Correction } C_P = \text{Volume by the trapezoidal rule} - \text{volume by the prismoidal rule}$$

$$= \frac{D}{2}(A_1 + A_2) - \frac{D}{6}(A_1 + 4A_m + A_2)$$

$$= \frac{D}{3}(A_1 - 2A_m + A_2)$$

14.3.4 CURVATURE CORRECTION

In computing volumes it is normally assumed that sections are parallel to one another. However, in a curved path the sections are radial and not parallel to one another. Hence the above formulae do not give correct results and suitable corrections are to be applied. This is based on Pappus' Theorem which states that the volume swept out by an area revolving about an axis is equal to the product of the area and the length of the path traced out by the centroid of the area. Therefore, if the area is constant

$$\text{Volume} = \text{Area} \times \text{distance travelled by centroid}$$

From the Fig. 14.19, distance travelled by centroid,

$$D_c = (R - e) \cdot \theta$$

where θ is in radian and is given by D/R, where D is the distance along the centre line.

Hence $$D_c = (R - e) \cdot \frac{D}{R}$$

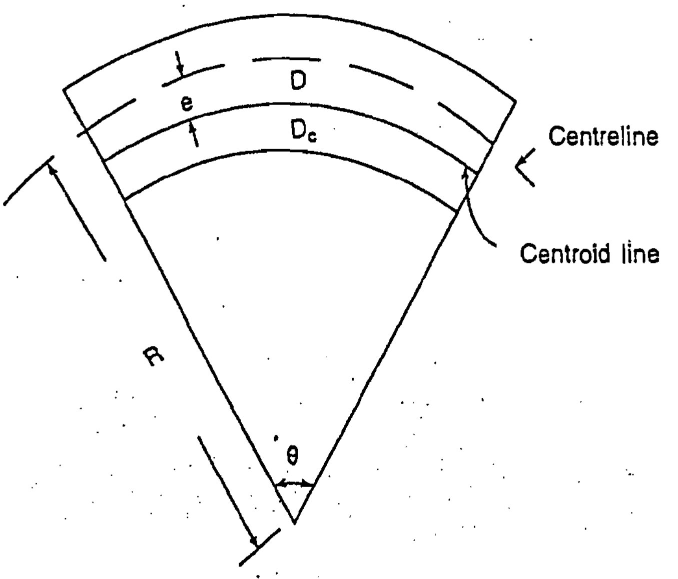


Fig. 14.19 Curvature correction.

When the centroid is away from the centreline

$$D_c = (R + e) \cdot \frac{D}{R}$$

Hence the volume by Pappus Theorem

$$V = \text{Area} \times \frac{D}{R}\,(R + e)$$

If the area is not constant but A_1 and A_2 with eccentricities e_1 and e_2

$$\therefore \text{Average area} = \frac{A_1 + A_2}{2}$$

and

$$\text{Average eccentricity } \bar{e} = \frac{e_1 + e_2}{2}$$

and

$$V = \left(\frac{A_1 + A_2}{2}\right) \times \frac{D}{R}\,(R \pm \bar{e}) \qquad (14.18)$$

The above formula can be split as:

$$V = \left(\frac{A_1 + A_2}{2}\right) \times D \pm \left(\frac{A_1 + A_2}{2}\right) \times D \times \frac{\bar{e}}{R} \qquad (14.18a)$$

$$= \text{Volume by trapezoidal rule} \pm \text{Correction.}$$

Plus sign is to be used when $\bar{e}$ is on the inside of the curve and minus sign when outside. The eccentricity of a particular cross-section can be determined by locating its centroid. The centroid is determined by dividing the cross section into small triangles and by taking moments about any line. The correction is normally small but when R is small the correction becomes large.

14.4 VOLUME THROUGH TRANSITION

In the case of railroad or highway construction there is both cut and fill. However, it is transitional. For example, there may be a number of full fill-sections and a number of full cut-sections. In between there may be a section of partial fill and cut. This is shown by a number of sections as in Fig. 14.20.

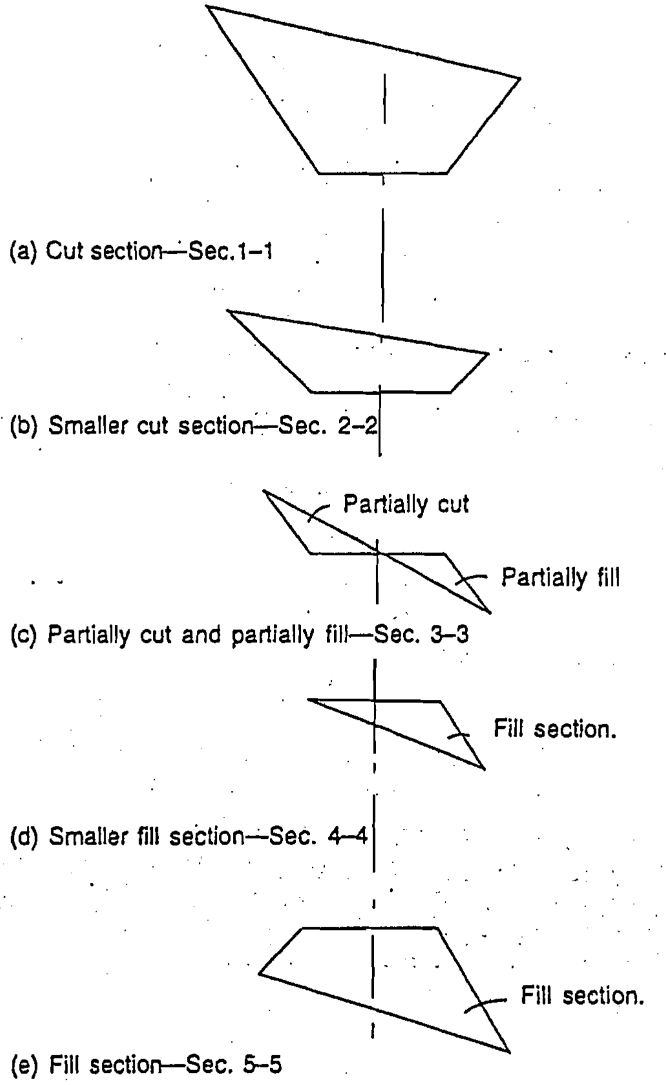


Fig. 14.20 Cross sections in a highway.

The volume between section 2-2 and section 4-4 consists of both cut and fill. The volume of the cut is computed usually taking cut areas at section 2-2 and section 3-3. The volume of fill between section 2-2 and 3-3 should be computed as a pyramid with the formula volume = base × altitude/3. Here area of the fill section at 3-3 is the base and the distance between sections 2-2 and 3-3 is the altitude. Similarly the volume of cut between sections 3-3 and 4-4 is obtained as a pyramid with partially cut area as the base and the distance between the sections as altitude.

Easa[4] has however suggested that the volume should be computed as frustum of pyramid.

Figure 14.21 shows the geometry of a pyramid frustum. For a frustum of pyramid the ratio of the corresponding dimensions of the end areas is constant. This ratio is given by

$$\frac{c_2}{c_1} = \frac{b_2}{b_1} = \frac{h-d}{h}$$

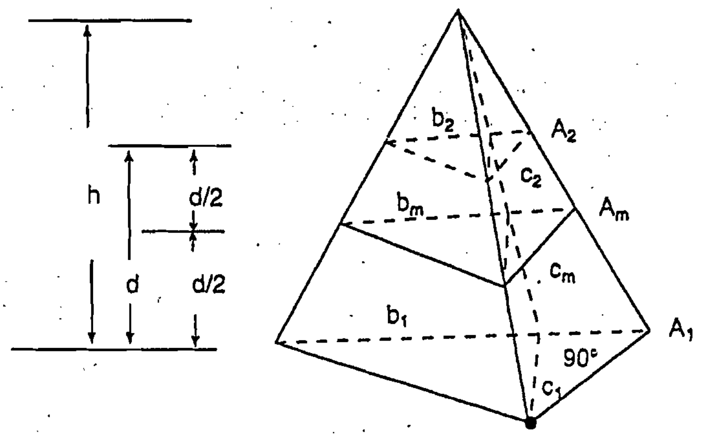


Fig. 14.21 Geometry of pyramid frustum.

Also, the areas of the end cross sections

$$A_1 = \frac{1}{2} b_1 c_1$$

$$A_2 = \frac{1}{2} b_2 c_2$$

The area of the middle cross section is computed as:

$$A_m = \frac{1}{2} b_m c_m$$

where b_m and c_m are given by

$$b_m = \left(\frac{b_1 + b_2}{2}\right)$$

$$c_m = \left(\frac{c_1 + c_2}{2}\right)$$

Substituting for b_m and c_m gives

$$A_m = \frac{1}{8} (b_1 c_1 + b_2 c_2 + b_1 c_2 + b_2 c_1)$$

But $\qquad b_1c_1 = 2A_1 \qquad b_2c_2 = 2A_2$

and $\qquad b_1c_2 = b_2c_1 = 2\sqrt{A_1A_2}$

Thus $\qquad A_m = \frac{1}{4}(A_1 + A_2 + 2\sqrt{A_1A_2})$

Finally substituting for A_m.

$$V = \frac{d}{3}(A_1 + \sqrt{A_1A_2} + A_2) \qquad (14.19)$$

which is the required pyramid frustum formula. It is interesting to note that this formula requires only the areas of the end cross sections.

14.5 VOLUME FROM SPOT LEVELS

When the area excavation is square, rectangular or consists of a number of vertical sides, as in the case of foundation of a water tank, underground reservoir, etc. The volume can be computed by taking levels of number of points along a grid. The difference between the formation level and the existing level of the ground will give the height of fill or cut at the corresponding points.

The volume of any square say a, b, c, d (Fig. 14.22)

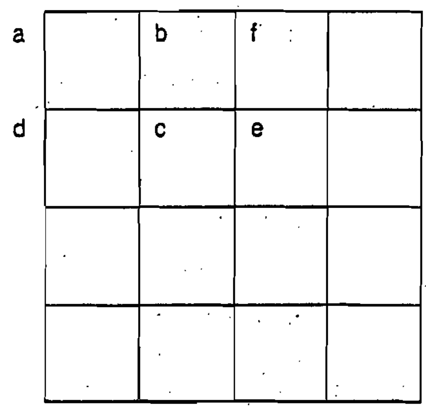


Fig. 14.22 Volume from spot levels.

$$\Delta V = \text{average height} \times \text{area of the square } abcd.$$

$$= \frac{h_a + h_b + h_c + h_d}{4} \times \text{area of the square } abcd$$

Similarly volume of $bcef$

$$= \frac{h_b + h_c + h_e + h_f}{4} \times \text{area of the square } bcef$$

If the areas of the squares are all equal and is given by A, summation of volumes can be expressed as

$$V = \frac{A}{4}\left(\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4\right) \qquad (14.20)$$

where Σh_1 = sum of depth used once
Σh_2 = sum of depth used twice
Σh_3 = sum of depth used thrice
Σh_4 = sum of depth used four times

which is the maximum number of times a vertical height can occur.

If the plan area is divided into a number of triangles, the volume ΔV will be that of a triangular prism, i.e.

$$\Delta V = \frac{h_a + h_b + h_c}{3} \times A$$

If all the triangular areas are equal

$$V = \frac{A}{3}(\Sigma h)$$

where h is the vertical height.

$$V = \frac{A}{3}\left(\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + \ldots + 8\,\Sigma h_8\right) \qquad (14.21)$$

where h_1 = sum of depth used once
h_2 = sum of depth used twice and so on.

14.6 VOLUME BY SIMPSON'S CUBATURE FORMULA

Volume can also be computed by a non linear profile formula known as Simpson's cubature formula which is applicable only when the grid has an even number of intervals in each direction. This is derived as follows:

Take the rectangular grid of Fig. 14.23 whose sides are divided into m and n intervals where m and n are even. The excavation depths $f(x_i, y_i)$ at the intersection points (x_i, y_i) with $i = 0, 1, \ldots, m$ and $j = 0, 1, \ldots, n$ are known. To calculate the excavation volume of the grid based on Simpson's 1/3 formula, first we consider the unit grid (2 × 2 intervals) shaded in Fig. 14.23.

The volume of the unit is given by

$$v = \int_{x_0}^{x_2}\int_{y_0}^{y_2} f(x, y)\, dy \cdot dx$$

First the inner integral is calculated using Simpson's 1/3 formula, giving

$$v = \int_{x_0}^{x_2} \frac{k}{3}\left[f(x, y_0) + 4f(x, y_1) + f(x, y_2)\right] dx$$

$$= \frac{k}{3}\left[\int_{x_0}^{x_2} f(x, y_0)\, dx + 4\int_{x_0}^{x_2} f(x, y_1)\, dx + \int_{x_0}^{x_2} f(x, y_2)\, dx\right].$$

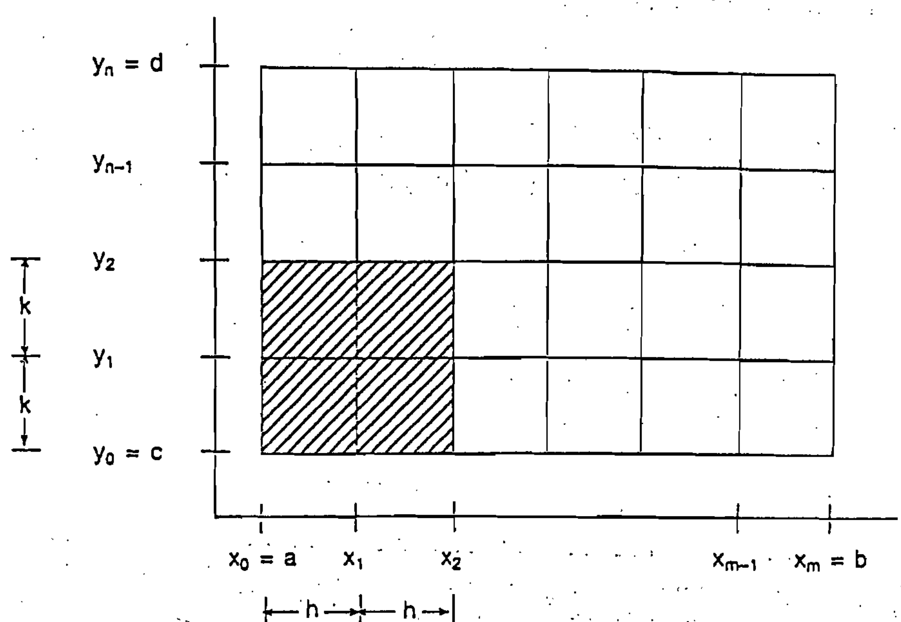


Fig. 14.23 Rectangular grids with equal intervals (m and n are even).

Simpson's 1/3 formula is then applied again to each integral to obtain

$$v = \frac{hk}{9}[f(x_0, y_0) + 4f(x_1, y_0) + f(x_2, y_0) + 4f(x_0, y_1) + 16f(x_1, y_1) + 4f(x_2, y_1) + f(x_0, y_2) + 4f(x_1, y_2) + f(x_2, y_2)]$$

Denoting $f(x_i, y_j)$ by f_{ij} the equation can be reduced to

$$v = \frac{hk}{9}[(f_{00} + f_{20} + f_{02} + f_{22}) + 4(f_{10} + f_{01} + f_{21} + f_{12}) + 16f_{11}] \tag{14.22}$$

Equation (14.22) is the Simpson's cubature (SC) formula which gives the excavation volume of the unit grid in terms of depth of intersection points. Note that the first term in parentheses is the sum of the depths of the corner points, the second term, sum of the depths of the boundary mid-points and the last term f_{11} is the depth of the centre point.

The volume of the total grid is simply the sum of the volumes of the unit grids. To calculate this volume, let

$$x_i = x_0 + ih \qquad i = 0, 1, \ldots, m$$

$$y_j = y_0 + jk \qquad j = 0, 1, \ldots, n$$

Then the composite formula for calculating the volume of the total grid V is

$$V = \int_{x_0}^{x_m} \int_{y_0}^{y_n} f(x, y)\, dy\, dx$$

For the whole grid

$$V = \frac{hk}{9} \sum_{i=0}^{m} \sum_{j=0}^{n} \lambda_{ij} f_{ij}$$

(Composite Simpson's Cubature Formula) (14.23)

in which λ_{ij} = the corresponding elements of the B matrix

$$B = \begin{bmatrix} 1 & 4 & 2 & \cdots & 4 & 2 & 4 & 1 \\ 4 & 16 & 8 & \cdots & 16 & 8 & 16 & 4 \\ 2 & 8 & 4 & \cdots & 8 & 4 & 8 & 2 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\ 4 & 16 & 8 & \cdots & 16 & 8 & 16 & 4 \\ 2 & 8 & 4 & \cdots & 8 & 4 & 8 & 2 \\ 4 & 16 & 8 & \cdots & 16 & 8 & 16 & 4 \\ 1 & 4 & 2 & \cdots & 4 & 2 & 4 & 1 \end{bmatrix}$$

The matrix corresponds to the grid points shown in Fig. 14.23. Note that the second and third columns are to be repeated as well as second and third rows. Thus to calculate V, one need only multiply each of the depths f_{ij} by the corresponding element and sum the results for all points. The method can be computerized.

14.7 VOLUME FROM CONTOUR PLAN

The contour plan can be utilized for computing the volume of earth work between different contour lines. There are four different methods:

1. By cross sections.
2. By equal depth contours.
3. By horizontal planes.
4. When the finished surface is a level surface (reservoir problem).

By cross sections

From the contour plan cross section of the existing ground surface can be plotted as shown in Fig. 14.24. When the formation level of the proposed road is known this can be superimposed on the cross section. This will give us the required cut and fill at various sections at the centre line of the proposed road. The area of the cross section is determined from the depth at the centre and side slopes. The volume of earth work is computed using the trapezoidal rule or the prismoidal formula.

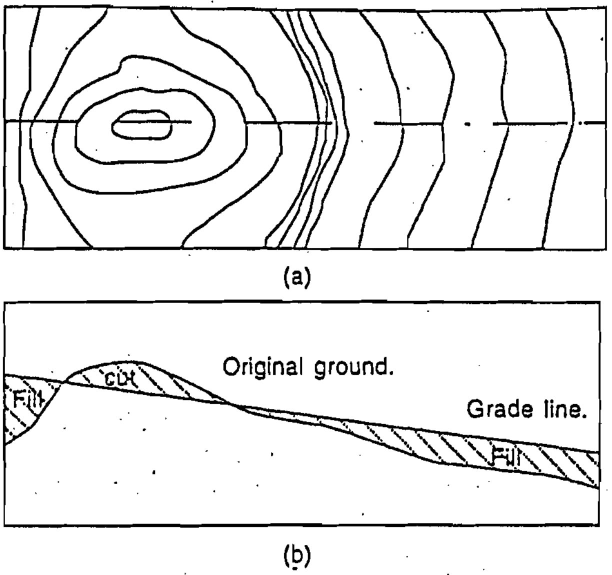


Fig. 14.24 Volume from cross section.

By equal depth contours

In this method the contours of the finished or graded surface are drawn over the contour map at the same interval as that of the contours. If the original contour is plotted with firm lines, the contour of the proposed surface is plotted with dotted lines. At the intersection of full line contour with a dash line contour, depths of cutting or filling can be determined. By joining the points of equal cut or fill a set of lines are obtained. These lines are the horizontal projection of lines cut from the existing surface by planes parallel to the finished surface. The irregular areas bounded by these lines are obtained by planimeter. The volume between any two successive areas is determined by multiplying the average of two areas by the depth between them (Fig. 14.25).

By horizontal planes

In this method, the volume of earth work is computed by taking horizontal sections on the contour plan. The existing contours are plotted as firm lines. The proposed formation surfaces are shown by dotted lines. The volume can be computed by the following steps:

(a) The dotted lines and the firm lines intersect at some points. They represent points of no cut or fill. All these points are joined to give a curve.

(b) Within these curves the original ground surface is at a higher level than the proposed grade surface and as such excavation of the ground is necessary.

(c) Similarly, outside this line fill is necessary.

(d) The amount of cut or fill at each section is plotted and from the hatched figure areas of cut A_1, A_3, A_5 and fill A_2, A_4, A_6 can be computed.

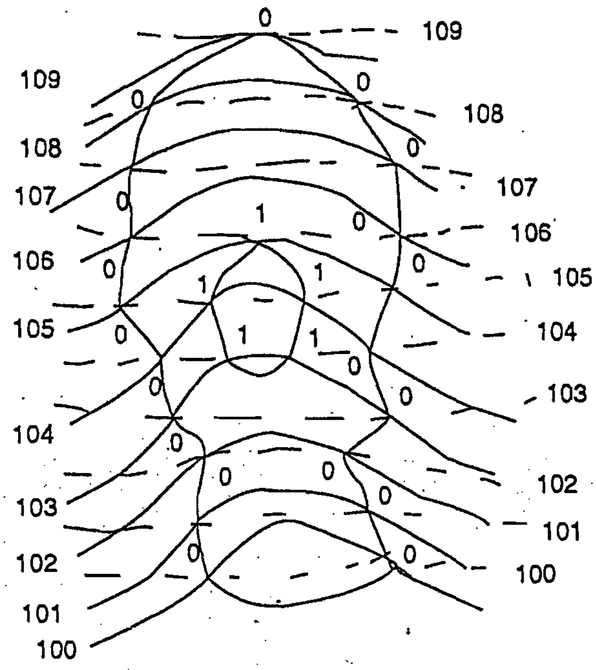


Fig. 14.25 Volume from equal depth contour.

(e) Compute the volume of the earthwork between two successive contours using the average end area method. The pyramidal formula should be applied for the end sections (Fig. 14.26).

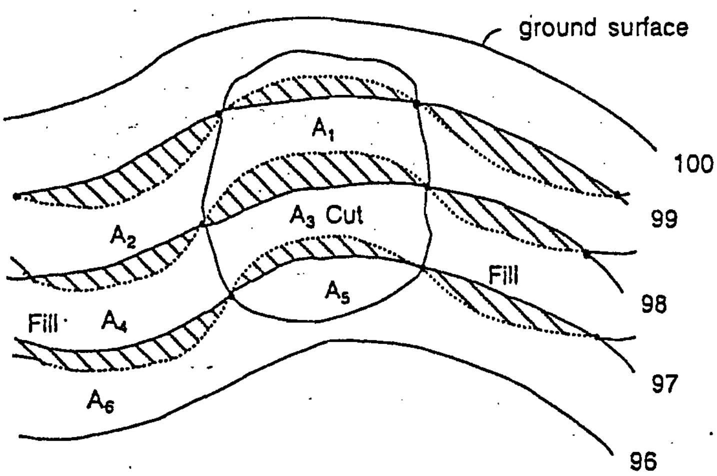


Fig. 14.26 Volume from horizontal sections.

Capacity of a reservoir

Two general methods are used for computing the reservoir volume—(i) By taking horizontal sections; (ii) By taking vertical sections.

In the first method the whole area under each contour line in computed. Let them be A_1, A_2, ..., A_n. If the trapezoidal rule is applied the volume

$$V = h\left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \ldots + A_{n-1}\right]$$

where h is equal to contour interval. If necessary prismoidal formula can also be applied (Fig. 14.27).

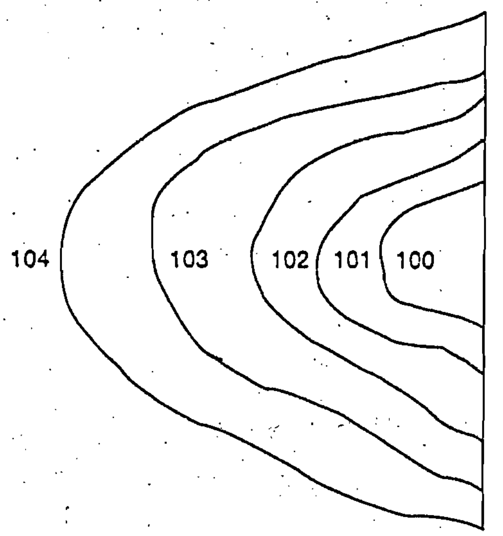


Fig. 14.27 Contour lines of a reservoir.

The second method is applied when the reservoir is regular in shape. From the contour map dimensions of the vertical cross sections are obtained. The volume is then calculated from the cross sectional areas.

14.8 MASS HAUL CURVE

In highway and railroad construction it is necessary to compute volumes of earthwork to be cut or to be filled in and distance through which cut earth is to be transported for filling purposes. This will determine the cost of the earthwork portion of the project. Mass haul curve where the cumulative volume of earthwork is plotted against distance helps in computation of the same. The curve is usually plotted below the longitudinal level section and is shown in Fig. 14.28.

The following information may be obtained from the mass-haul diagram.

1. An inspection of the mass diagram shows that a rising curve indicates cut and a falling curve indicates fill.

2. Maximum and minimum points on the mass curve occur at grade points on the profile.

3. The algebraic difference of ordinates between any two points indicates the volume of earth work between the two points.

4. If a horizontal line is drawn to intersect the diagram at two points excavation and embankment will be equal between the two stations represented by the point

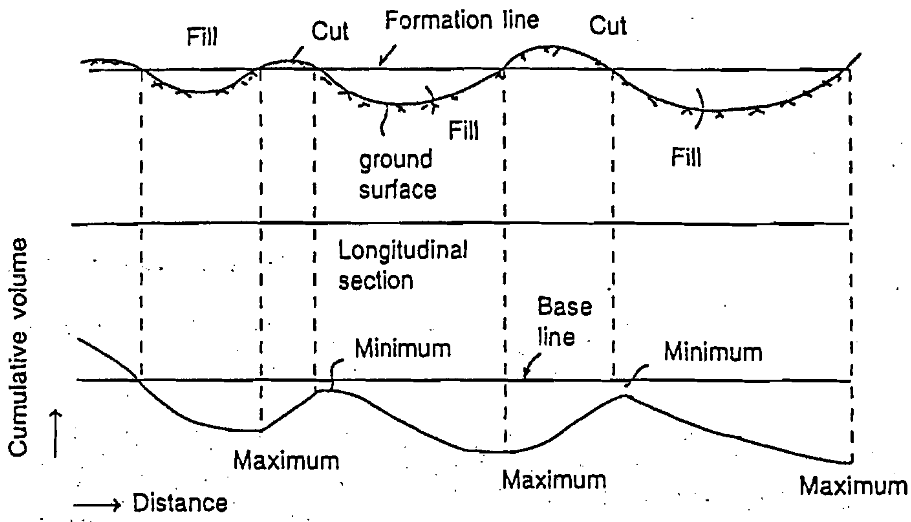


Fig. 14.28 Mass haul curve.

of intersection. Such a horizontal line is called a balancing line because the excavation balances the embankment between the two points at its ends.

5. Since the ordinates to the diagram represent algebraic sums of the volumes of excavation and embankment referred to the initial ordinate, the total volume of cuts and fills will be equal from one zero ordinate to another zero ordinate or if the initial positive ordinate is equal to the final negative ordinate.

When a material is cut and deposited in a fill, it does not come to its original volume. Initially when cut it expands in volume but finally when compressed by overburden at the top, it may show a lesser volume or shrink. The shrinkage is usually 5 to 15% depending on the character of the material handled and the condition of the ground on which the embankment is placed.

The following terms are used in computing the cost of earthwork utilizing mass haul curve.

Haul, haul distance, average haul distance

They are all connected with movement of earth from one point to another usually from cut to fill. Haul distance is the actual distance from the point of cut to the point of fill. Average haul distance is the distance between the centre of gravity of cut to the centre of gravity of fill. Haul, however, means product of volume of cut and average haul distance. It is also equal to the area between the mass haul curve and the balancing line. Haul is volume multiplied by distance. Hence its unit is $m^3 \times m$. However, it is usually expressed as station meter which means 1 m^3 of earth work moving through 1 station.

Free haul, overhaul

Contractor is to be paid for carrying the excavated material. Usually two rates are used for payment to the contractor. One is based on free haul limit (distance) for which the contractor is not paid any extra. Another is overhaul which is more than the free haul limit and for which contractor is to be paid extra.

Borrow and waste

Normally the balancing line is so adjusted that the amount of cut and fill are equal. However, this may require long overhaul. Sometimes, therefore, it is economical to fill an embankment by borrowing earth from outside. This is known as "borrow". Similarly sometimes it is wise to leave the cut material in spoil banks when the transportation distance is very large and will involve large overhaul. This is known as "waste".

Limit of economic haul (LEH)

It is the maximum limit of haul distance beyond which it is not economical to use the material obtained from cuts. Beyond the limit of economic haul, it is more economical to waste the material or to take the materials from the borrow pits than to haul it.

Lead and lift

Lead is the horizontal distance through which the excavated material is moved from the cut to the required embankment. Lift is the vertical distance through which an excavated material from a cut is moved to the required embankment. Lead and lift are general terms and can be used for any construction material, e.g. sand or cement.

Balancing line

If a horizontal line is drawn to intersect the mass haul curve at two points, excavation and embankment (with proper shrinkage correction) will be equal between the two stations represented by the point of intersection. Such a horizontal line is called a balancing line because the excavation balances the embankment between the two points at its ends.

Shrinkage

Earthwork when cut occupies a greater volume than its original position. Solid rock when broken up occupies a much greater volume than its original value. However when placed on embankment and compacted the volume will be less. Except rock, the final result is usually shrinkage and a shrinkage factor has to be applied to the excavated volume to compute the volume of embankment that would be filled up. The shrinkage is usually 5 to 15%.

Swelling

As already explained, rocky soil when excavated usually expands in volume and the ratio of the expanded volume and the volume in *in-situ* condition is the swelling factor. Table 14.1 gives an idea of the expansion and contraction in volume when excavated and subsequently compacted.

Table 14.1 Expansion and Contraction in Volume (Volume before excavation 1 m^3)

Material	Volume immediately after excavation m^3	Volume after compaction m^3
Rock (large pieces)	1.50	1.40
Rock (small pieces)	1.70	1.35
Chalk	1.80	1.40
Clay	1.20	0.90
Light sandy soil	0.95	0.89
Gravel	1.00	0.92

14.8.1 USE OF THE MASS DIAGRAM

The following points may be noted when using the mass diagram.

1. Points beyond which it is not feasible to haul material define the limits of a mass diagram. A limit point may be the beginning of a project, the end of a project, the bank of a river or an edge of a deep ravine where a bridge will be constructed.
2. Since the ordinates to the diagram represent the algebraic sums of the volumes of excavation and embankment referred to the initial ordinate, the total volumes of the excavation and embankment will be equal where the final ordinate equals the initial ordinate. If the final ordinate is greater than the initial ordinate, there is an excess of excavation, if it is less than the initial ordinate, the volume of embankment is greater and additional material must be obtained to complete the embankments.
3. Grade line is usually fixed keeping in mind that it should not exceed the permissible limit. Balancing lines should be drawn over moderate distances. Long balancing line though ensures balancing of earthwork may mean long overhaul distances and more cost. In such a case it may be economical to waste material at one place and obtain the volumes necessary for filling from borrow pits located along the right of way.
4. Costing of earthwork may be computed by using a mass diagram. The limit of economic haul (LEH) is the distance beyond which it is cheaper to borrow or waste material. It is determined from the following:

$$\text{LEH} = \text{Free haul distance} + \frac{\text{Cost of excavation}}{\text{Cost of overhaul}}$$

For example, if the free haul distance is 300 m, the cost of excavation is Rs. 3/m^3, the overhaul is Rs. 2 per station meter (1 station meter is movement of 1 m^3 of material through 1 station say, 30 m)

$$\therefore \text{LEH} = 300 \text{ m} + 3/2 \times 30 = 345 \text{ m.}$$

Suppose the distance is 400 m, i.e. 100 m beyond free haul distance.

$$\text{Cost of overhaul} = \frac{100 \times 2}{30} = \text{Rs. } 6.67.$$

Cost of excavation = Rs. 3.00

Hence it will be more economical to take material from the borrow pit than to haul it from the cut.

Figure 14.29 shows the earth profile, gradeline and the mass haul curve. *CD*, *FG* and *IJ* are the balancing lines. To minimize cost they are not necessarily equal. The following information may be obtained from a study of the mass-haul-curve.

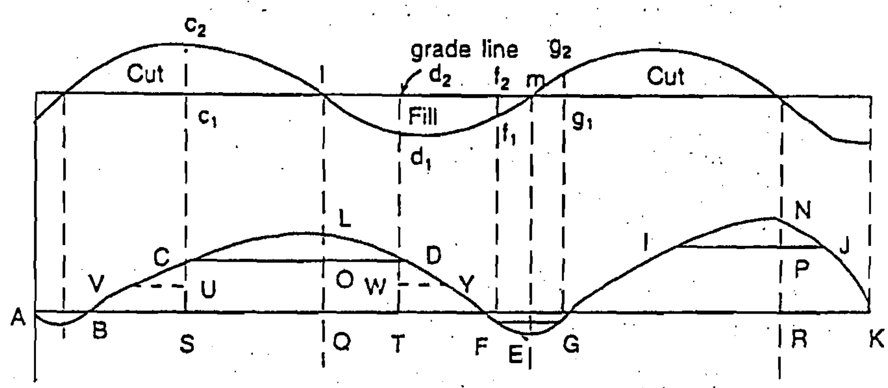


Fig. 14.29 Earth profile, gradeline and mass haul curve.

(a) When the loop of a mass haul curve cut off by a balancing line is above that line, the excavated material must be moved forward to the right in the direction of the increasing abscissa. However, when the loop is below the balancing line, the material must be moved backwards to the left in the opposite direction. For the balancing line *CD*, the movement is from *C* to *D* as mass c_1c_2l fills the void ld_1d_2. For the balancing line *FG*, the movement is from *G* to *F* as the cut g_1g_2m fills the void f_1f_2m.

(b) Haul is product of volume of earthwork and distance of travel. Hence area of the mass haul curve above balancing line is the corresponding haul. For example, the area *BCLDFTQSB* in Fig. 14.29 indicates the total haul between *B* and *F*. Similarly the total haul between *C* and *D* is the area *CLD*. If *CD* is the free haul distance, the volume of earth *CS* which will balance the fill *DT* is beyond free haul limit and must be paid at the overhaul rate.

(c) The c.g. of the volume of earth and the distance through which it should be moved can be obtained graphically as follows. Bisect the line *CS* at *U* and draw the line *UV* parallel to the base cutting the mass haul curve at *V*. Then *V* is the c.g. of the portion *BVCUS* of the curve. Similarly *Y* is the c.g. of the R.H. Portion. Hence the overhaul distance is *VY* and the over hauled volume of earth work is *CS*. The overhaul distance is determined by (implicitly) assuming that centroid is the location at which it has equal volumes on both sides. Despite the simplicity of determining the centroid by this procedure, it does not give the correct location of the centroid. The centroid, by definition, is the location at which the moment of the total volume of the section about any point is equal to the sum of the moments of incremental volume of that section about the same point. This does not necessarily mean that the volumes on both sides of the centroid are equal.

(d) The haul over any length is a minimum when the location of the balancing line is such that the arithmetic sum of areas cut off by it, ignoring the sign, is a minimum.

(e) Whenever there is a vertical interval between successive balancing lines, there is a waste if the succeeding balancing line is above the preceding balancing line. If we consider *AK* as a balancing line, there is no waste or borrowing but the length *BF* being large there will be overhaul if the length *CD* is the freehaul limit. But if we want to avoid overhaul and keep the balancing lines within freehaul limit as *CD*, *FG* and *IJ*, there will be waste or borrowing. As *IJ* is above *FG*, there is a waste of material between *G* and *I*. Similarly there is borrow of material between *D* and *F*.

(f) Minimum haul will not always ensure minimum cost as in that case there may be large waste and borrow. Minimum cost will depend upon freehaul distance, overhaul limit of economic haul, cost of excavation and borrowing.

Example 14.1 The following offsets were taken from a chain line to hedge:

Distance	0	20	40	60	80	120	160	220	280
Offset	9.4	10.8	13.6	11.2	9.6	8.4	7.5	6.3	4.6

Compute the area included between the chain line, the hedge and the offset by (i) Mid ordinate rule, (ii) Average ordinate rule, (iii) Simpson's rule, (iv) Trapezoidal rule.

Solution

(i) *Mid ordinate rule* Here the intervals are not all equal. Hence mid ordinate has to be computed for each interval.

$$d_1 = 20 \qquad M_1 = \frac{9.4 + 10.8}{2} = 10.10$$

$$d_2 = 20 \qquad M_2 = \frac{10.8 + 13.6}{2} = 12.20$$

$$d_3 = 20 \qquad M_3 = \frac{13.6 + 11.2}{2} = 12.40$$

$$d_4 = 20 \qquad M_4 = \frac{11.2 + 9.6}{2} = 10.40$$

$$d_5 = 40 \qquad M_5 = \frac{9.6 + 8.4}{2} = 9.00$$

$$d_6 = 40 \qquad M_6 = \frac{8.4 + 7.5}{2} = 7.95$$

$$d_7 = 60 \qquad M_7 = \frac{7.5 + 6.3}{2} = 6.90$$

$$d_8 = 60 \qquad M_8 = \frac{6.3 + 4.6}{2} = 5.45$$

$$\text{Total Area} = M_1d_1 + M_2d_2 + M_3d_3 + M_4d_4 + M_5d_5 + M_6d_6 + M_7d_7 + M_8d_8$$

$$= (20 \times 10.10) + (20 \times 12.20) + (20 \times 12.40) + (20 \times 10.40) + (40 \times 9.00) + (40 \times 7.95) + (60 \times 6.90) + (60 \times 5.45)$$

$$= 2321 \text{ sq units.}$$

(ii) Average ordinate rule:

$$\text{Average ordinate} = \frac{9.4 + 10.8 + 13.6 + 11.2 + 9.6 + 8.4 + 7.5 + 6.3 + 4.6}{9}$$

$$= 9.044.$$

$$\text{Area} = 9.044 \times 280$$

$$= 2532.32 \text{ sq units}$$

(iii) As the intervals are not all equal, Simpson's rule should be applied in parts. By Simpson's rule, taking three at a time

$$\text{Area} = \frac{d}{3}\,[O_1 + 4O_2 + O_3]$$

Area from 1st to 3rd Ordinates

$$= \frac{20}{3}\,[9.4 + 4(10.8) + 13.6]$$

$$= 441.333$$

Area from 3rd to 5th ordinates

$$= \frac{20}{3}\,[13.6 + 4(11.2) + 9.6]$$

$$= 453.333$$

Area from 5th to 7th ordinates

$$= \frac{40}{3}\,[9.6 + 4(8.4) + 7.5]$$

$$= 676.000$$

Area from 7th to 9th ordinates:

$$= \frac{60}{3}\,[7.5 + 4(6.3) + 4.6]$$

$$= 746.00$$

Hence total area $= 441.333 + 453.333 + 676.000 + 746.00$

$$= 2316.67 \text{ sq. units}$$

(iv) Trapezoidal rule

$$A = d\left(\frac{O_1 + O_n}{2} + O_2 + \ldots + O_{n-1}\right)$$

Considering 1st five ordinates

$$A_1 = 20\left(\frac{9.4 + 9.6}{2} + 10.8 + 13.6 + 11.2\right)$$

$$= 902.00$$

Considering 5th to 7th ordinates

$$A_2 = 40\left(\frac{9.6 + 7.5}{2} + 8.4\right)$$

$$= 678.00$$

Considering 7th to 9th ordinates

$$A_3 = 60\left(\frac{7.5 + 4.6}{2} + 6.3\right)$$

$$= 741.00$$

Total area $= 902.00 + 678.00 + 741.00$

$= 2321.00$ sq. units.

It can be observed from the above results that the mid ordinate rule and the trapezoidal rule give the same results.

Example 14.2 The following perpendicular offsets in *m* are measured from a straight line to an irregular boundary at regular intervals of 10 m.

$h_1 = 8.25$	$h_6 = 13.60$	$h_{11} = 20.05$
$h_2 = 13.85$	$h_7 = 15.25$	$h_{12} = 15.90$
$h_3 = 12.25$	$h_8 = 16.85$	$h_{13} = 12.25$
$h_4 = 10.85$	$h_9 = 14.95$	$h_{14} = 12.00$
$h_5 = 12.25$	$h_{10} = 17.35$	

Compute the area lying between the straight line and the irregular boundary by

(i) Trapezoidal rule.

(ii) Simpson's one-third rule (a) using h_1 as the first offset, (b) using h_{14} as the first offset.

Solution

(i) Trapezoidal rule

$$\text{Area} = d\left[\frac{O_1 + O_n}{2} + O_2 + O_3 + \ldots + O_{n-1}\right]$$

$$= 10\left[\frac{8.25 + 12.00}{2} + (13.85 + 12.25 + 10.85 + 12.25 + 13.60\right.$$

$$\left. + 15.25 + 16.85 + 14.95 + 17.35 + 20.05 + 15.90 + 12.25)\right]$$

$$= 1855.25.$$

(ii) Simpson's 1/3rd rule: No. of ordinates must be odd.

(a) Using h_1 as first offset and applying upto h_{13}.

$$\frac{10}{3}\ [8.25 + 12.25 + 4(13.85 + 10.85 + 13.60 + 16.85 + 17.35$$

$$+ 15.90) + 2(12.25 + 12.25 + 15.25 + 14.95 + 20.05)]$$

$$= 1745.33 \text{ sq. m.}$$

Area of last two offsets by trapezoidal rule

$$= \frac{12.25 + 12.0}{2} \times 10$$

$$= 121.25$$

Hence : Total area = 1866.58 m^2

(b) Taking h_{14} as first offset and h_2 as last offset for applying Simpson's rule;

$$A = \frac{10}{3}\ [h_{14} + h_2 + 4(h_{13} + h_{11} + h_9 + h_7 + h_5 + h_3)$$

$$+ 2(h_{12} + h_{10} + h_{08} + h_{06} + h_{04})]$$

$$= \frac{10}{3}\ [12 + 13.85 + 4(12.25 + 20.05 + 14.95 + 15.25 + 12.25 + 12.25)$$

$$+ 2(15.90 + 17.35 + 16.85 + 13.60 + 10.85)]$$

$$= 1743.1667 \text{ m}^2.$$

Area of last two offsets by trapezoidal rule

$$= \frac{8.25 + 13.85}{2} \times 10 = 110.5$$

Total area = 1853.67 m^2.

Example 14.3 A closed traverse *ABCDA* is run along the boundaries of a built-up area with the following results:

Side	W.C.B.	Length
AB	69°55′	262.0
BC	166°57′	155.0
CD	244°20′	268.0
DA	347°17′	181.0

Coordinate the stations B, C and D on A as origin and calculate the area of the traverse in hectares by

(i) Meridian distance method.
(ii) Double meridian distance method.
(iii) Double parallel distance method.
(iv) Departure and total latitude method.
(v) Coordinate method.

Solution The computation of independent coordinates of the points B, C and D with A as origin is shown in Table 14.2.

(i) Meridian distance method gives

$$A = \Sigma L \times M_d$$

Line	Latitude	Meridian distance	$L \times M_d$
AB	90.29	$\frac{1}{2}$ [246. 39] = 123.20	11123.72
BC	− 151.00	$\frac{1}{2}$ [246.39 + 246.39 + 35] = 263.89	− 39847.39
CD	− 116.10	263.89 + $\frac{1}{2}$ [35 − 241.55] = 160.62.	− 18647.98
DA	+ 176.81	160.62 + $\frac{1}{2}$ [− 241.55 − 39.84] = 19.93	+ 3523.82
			− 43847.83m²

1 hectare = 10^4 m²

Therefore, 43847.83m² = 4.38 hectare.

(ii) Double meridian distance method.

Line	Latitude	Double meridian distance	$L \times DMD$
AB	90.29	246.39	22246.553
BC	− 151.00	246.39 + 246.39 + 35.00 = 527.78	− 79694.780
CD	− 116.10	527.78 + 35 − 241.55 = 321.23	− 37294.803
DA	+ 176.81	+ 39.84	+ 7044.110
			− 87698.92

Area = 43849.46 m²

= 4.38 hectare.

Table 14.2 Example 14.3

Station	Line	Length in m	W.C.B.	Quadrantal bearing	Latitude N	Latitude S	Departure E	Departure W	Independent Coordinates N	Independent Coordinates S	Independent Coordinates E	Independent Coordinates W
A									500		500.00	
	AB	262.0	69°55′	N 69°55′E	90.04 (+ 0.25)		246.04 (+ 0.35)					
B									590.29		746.39	
	BC	155.0	166°57′	S 13°03′E	—	151.00	35.00					
C									439.29		781.39	
	CD	268.0	244°20′	S 64°20′W	—	116.10		241.55				
D									323.19		539.84	
	DA	181.0	347°17′	N 12°43′W	176.56 (+ 0.25)			39.84				
A									500		500.00	
					Σ 266.60 + (0.50)	Σ 267.10	Σ 281.04 (+ 0.35)	281.39				

Adjustment of computational error is shown in parentheses.

(iii) Double parallel distance method:

Line	Latitude	Double parallel Distance	Departure	D_{PD} × Dep.
DA	+ 176.81	+ 176.81	− 39.84	− 7044.11
AB	+ 90.29	+ 176.81 + 176.81 + 90.29 = 443.91	+ 246.39	+ 109374.98
BC	− 151.00	443.91 + 90.29 − 151.00 = 383.20	+ 35.00	+ 13412.00
CD	− 116.10	383.20 − 151.00 − 116.10 = 116.10	− 241.55	− 28043.95
				Σ − 87698.915

$$\text{Hence area} = \frac{87698.915}{2} = 43849.46 \text{ m}^2$$

$$= 4.38 \text{ hectare.}$$

(iv) Departure and total latitude method: The formula is

$2A = \Sigma$ total latitude of a point × (algebraic sum of two adjacent departures).

Total latitudes of points B, C, D with reference to the reference point A is

$$B = 0 + 90.29 = 90.29$$

$$C = 90.29 - 151.00 = -60.71$$

$$D = -60.71 - 116.10 = -176.81$$

Algebraic sum of the departures of the two lines meeting at B, C, D are

point $B = 246.39 + 35.00 = 281.39$

$C = 35 - 241.55 = -206.55$

$D = -241.55 - 39.84 = 281.39$

$$2 \text{ Area} = 90.29 \times 281.39 + (-60.71) \times (-206.55) + (-176.81) \times (-281.39)$$

$$= 87698.32 \text{ m}^2$$

$$\text{Area} = 43849.16 \text{ m}^2 = 4.38 \text{ hectare.}$$

(v) Coordinate method.

Coordinates of	X	Y
A	500.00	500.00
B	590.29	746.39
C	439.29	781.39
D	323.19	539.84

$$2 \text{ Area} = (X_A Y_B - X_B Y_A) + (X_B Y_C - X_C Y_B) + (X_C Y_D - X_D Y_C) + (X_D Y_A - X_A Y_D)$$

$$= (500 \times 746.39 - 590.29 \times 500) + (590.29 \times 781.39 - 439.29 \times 746.39) + (439.29 \times 539.84 - 323.19 \times 781.39) + (323.19 \times 500 - 500 \times 539.84)$$

$= 78050 + 133370 - 15391 - 108325.$

$= 87704 \text{ m}^2$

Area $= 43852 \text{ m}^2 = 4.38$ hectares.

Example 14.4 Calculate the area of a plan from the following readings of a planimeter.

Initial reading = 7.456.
Final reading = 1.218.

The zero of the disc passed the fixed index mark thrice in the clockwise direction. The anchor point was placed outside the plan and the tracing point was moved in the clockwise direction. Take $M = 100 \text{ cm}^2$

Solution

$$A = M(\text{F.R.} - \text{I.R.} \pm 10\,N + C)$$

As the anchor point was placed outside the plan $C = 0$. Therefore,

$$A = M(\text{F.R.} - \text{I.R.} \pm 10\,N)$$

Here $N = 3$ and the sign is plus as the zero mark passed in the clockwise direction.

Therefore $A = 100(1.218 - 7.456 + 30)$

$= 23.762 \text{ m}^2$

Example 14.5 Determine the area of a figure from the following readings of a planimeter.

Initial reading = 7.462
Final reading = 2.141
$M = 100 \text{ cm}^2$
$C = 20.5$

The zero mark of the disc passed once in the anticlockwise direction. The anchor point was placed inside the figure and the tracing point was moved in the clockwise direction along the outline.

Solution In this case $N = 1$ and the sign for N is negative as the zero mark passed in the anticlockwise direction.

$$A = M(\text{F.R.} - \text{I.R.} \pm 10\,N + C)$$

$$= 100(2.141 - 7.462 - 10 + 20.5)$$

$$= 517.90 \text{ cm}^2$$

Example 14.6 Find the area of the zero circle from the following observations. Take $M = 100 \text{ cm}^2$

(i) anchor point outside the figure:

Initial reading 7.452
Final reading 3.412

The zero of the disc passed the fixed index mark once in the clockwise direction.

(ii) Anchor point inside the figure

Initial reading 3.722
Final reading 5.432

The zero of the disc passed the fixed index mark twice in the anticlockwise direction.

Solution

(i) As anchor point is outside the figure

$$C = 0$$

$$A = M(\text{F.R.} - \text{I.R.} \pm 10N)$$

$$= 100(3.412 - 7.452 + 10)$$

$$= 596 \text{ cm}^2$$

(ii) With anchor point inside

$$A = M(\text{F.R.} - \text{I.R.} \pm 10N + C)$$

$$596 = 100(5.432 - 3.722 - 20 + C)$$

or

$$C = 24.25$$

Area of zero circle = $24.25 \times 100 = 2425 \text{ cm}^2$

Example 14.7 Calculate the area of a piece of property bounded by a traverse and circular arc described as follows: *AB* S 40°00′W 122 m, *BC* S 80°E 122 m, *CD* N35°00′ W 61 m and *DA* a circular arc tangent to *CD* at point *D* (Fig. 14.30(a)).

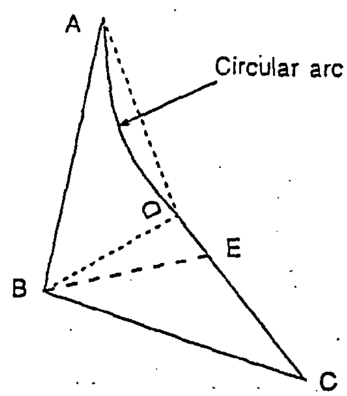


Fig. 14.30(a) Example 14.7.

Solution Gale's Traverse Table

Line	Length	Bearing	N	S	E	W
AB	122	S 40°00′W		93.45		78.42
BC	122	S 80°00′E		21.18	120.14	
CD	61	N 35°00′ W	49.96			34.98
DA	—	—	—	—	—	—
			49.96	114.63	120.14	113.40

$$\text{Latitude of } DA = 114.63 - 49.96$$
$$= 64.67\text{N}$$
$$\text{Departure of } DA = 120.14 - 113.40$$
$$= 6.74\text{W}$$
$$\text{Length of } DA = \sqrt{64.67^2 + 6.74^2}$$
$$= 65.02$$
$$\text{Bearing of } DA = \text{N} \tan^{-1} \frac{6.74}{65.02} \text{W}$$
$$= \text{N } 5°55'\text{W}$$
$$\text{Angle between } CD \text{ and } DA = 35°00' - 5°55'$$
$$= 29°05'$$

Taking coordinates of $A(0, 0)$

Coordinates of	x	y
B	− 78.42	− 93.45
C	+ 41.72	− 114.63
D	+ 6.74	− 64.67

Equation of the line *AD*

$$\frac{y-0}{x-0} = \frac{0-(-64.67)}{0-6.74}$$
$$y = -\frac{64.67}{6.74}x$$
$$= -9.59x$$

Equation of a line perpendicular to *AD*

$$y = \frac{1}{9.59}x + C$$

when the line passes through midpoint of *AD* whose coordinates are

$$(+3.37, -32.34)$$
$$-32.34 = \frac{1 \times 3.37}{9.59} + C$$

or $$C = -32.69$$

Equation of the line then becomes

$$y = \frac{x}{9.59} - 32.69$$

Equation of line *CD*

Coordinates of *C* 41.72, (– 114.63), *D* 6.74, (– 64.67)

Equation of the line *CD* is

$$\frac{y - (-114.63)}{x - 41.72} = \frac{-114.63 - (-(64.67)}{41.72 - 6.74}$$

or $$y = -1.428x - 55.05$$

Equation of a line perpendicular to *CD*

$$y = +\frac{1}{1.428}x + c'$$

when passing through the point 6.74, (– 64.67)

$$c' = -69.39$$

Equation of the line then becomes

$$y = +\frac{x}{1.428} - 69.39$$

Point of intersection of this line with the perpendicular bisector of *AD* can be obtained by solving simultaneously the two equations:

$$y = \frac{x}{9.59} - 32.69$$

$$y = \frac{x}{1.428} - 69.39$$

Solution gives the coordinates of the point of intersection

$$x = +61.58$$

$$y = -26.26$$

Length of radius of the curve $= \sqrt{61.58^2 + 26.26^2}$

$= 66.95$

Area of sector

$$A = R^2\left(\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right)$$

where θ = angle subtended at centre in degrees.
R = radius of circle.

Here $\theta/2 = 29°05'$

Hence $\theta = 58°10' = 58.17°$

$$A = 66.95^2 \left(\frac{\pi \times 2 \times 29.09}{360} - \frac{\sin 58.17}{2}\right)$$

$$= 371.618$$

Area in terms of coordinates

	X	Y
A	0	0
B	– 78.42	– 93.45
C	+ 41.72	– 114.63
D	+ 6.74	– 64.67

$$2 \text{ Area} = (X_AY_B - X_BY_A) + (X_BY_C - X_CY_B) + (X_CY_D - X_DY_C) + (X_DY_A - X_AY_D)$$

$$= 0 \times (-93.45) - (-78.42) \times 0 + (-78.42)(-114.63)$$

$$- (+41.72)(-93.45) + (+41.72)(-64.67) - (+6.74)(-114.63)$$

$$+ (+6.74)(0) - (0)(-64.67)$$

$$= 0 - 0 + 8989.28 + 3898.73 - 2698.03 + 772.60$$

$$= + 10962.50$$

$$\text{Area} = \frac{1}{2} \times (10962.50)$$

$$= 5481.25$$

Area of circular portion = 371.618

Hence net area = 5109.632 m^2

= 0.510963 hectare

Example 14.8 Divide the area of the plot in two equal parts by a line through point B. List in order the length and bearings of all sides for each parcel. Refer to the plot of Example 14.7.

Solution Join BD.

Coordinates of B, C and D are

	X	Y
B	0	0
C	+ 120.14	– 21.18
D	+ 85.16	+ 28.78

$$2 \text{ Area} = (X_BY_C - X_CY_B) + (X_CY_D - X_DY_C) + (X_DY_B - X_BY_D)$$

$$= (+28.78 \times 120.14) + (85.16 \times 21.18)$$

$$= 5261.318 \text{ m}^2$$

Area = 2630.659, is greater than 1/2 the area, i.e. 5109/2 = 2554.5 m^2

Difference = 2630.66 – 2554.50

= 76.16 m^2

Let area of triangle *BED* = 76.16 m²

Length and bearing of line *BD*,

$$\text{Length of } BD = \sqrt{28.78^2 + 85.16^2}$$
$$= 89.89 \text{ m.}$$

$$\text{Bearing of } DB = \tan^{-1}\frac{85.16}{28.78}$$
$$= \text{S } 71.32°\text{W}$$

$$\text{Bearing of } DE = \text{Bearing of } DC = \text{S } 35°00'\text{E}$$

$$\text{Angle between } DB \text{ and } DE = 71.32° + 35° = 106.32°$$

$$\text{Area of triangle } BDE = \frac{1}{2}\, BD \cdot DE \cdot \sin BDE$$

or

$$DE = \frac{2 \times \text{area}}{BD \sin BDE}$$

$$= \frac{2 \times 76.16}{89.89 \sin 106.32} = 1.76 \text{ m.}$$

$$BE = \sqrt{BD^2 + DE^2 - 2BD \cdot DE \cdot \cos BDE}$$

$$= \sqrt{89.89^2 + 1.76^2 - 2 \times 89.89 \times 1.76 \cos 106.32°} = 90.40 \text{ m.}$$

Again

$$\frac{BE}{\sin 106.32} = \frac{DE}{\sin DBE}$$

or

$$\sin DBE = \frac{DE \cdot \sin 106.32}{BE}$$

$$= \frac{1.76 \times \sin 106.32}{90.40} = .0186$$

or

$$DBE = 1.07° = 1°4.2'$$

Length and bearing of lines.

AB	122.00	S 40°00′W
BE	90.40	N 72°23′E
ED	1.76	N 35°00′W
BC	122	S 80°00′E
CE	59.24	N 35°00′W
EB	90.40	S 72°23′W.

Example 14.9 Partition the area given in Example 14.7 into two equal areas by a line parallel to *BC*. Tabulate in clockwise consecutive order the lengths and bearings of all sides.

Solution Draw *DE* parallel to *BC* (Fig. 14.30b). The traverse table for the quadrilateral *BCDE* is given below:

Line	Length	Quadrantal Bearings	Latitude N	Latitude S	Departure E	Departure W
BC	122	S 80°00′E		21.18	120.14	
CD	61	N 35°00′W	49.96			39.98
DE	l_1	N 80°00′W	$0.17l_1$			$0.98l_1$
EB	l_2	S 40°00′W	—	$0.766l_2$		$0.642l_2$

Since *BCDE* is a closed traverse

$$49.96 + 0.17l_1 - 0.766l_2 - 21.18 = 0$$
$$120.14 - 39.98 - 0.98l_1 - 0.642l_2 = 0$$

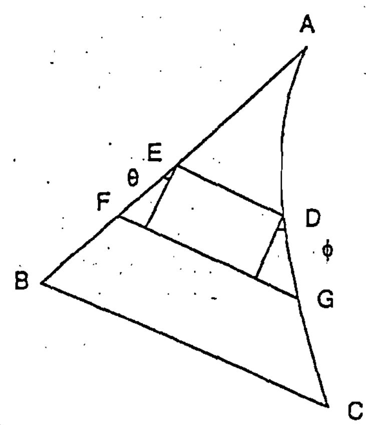


Fig. 14.30(b) Example 14.9.

Solving $\quad l_1 = 49.87 \quad l_2 = 48.64$

Independent coordinates of points

	x	*y*
B	0	0
C	120.14	– 21.18
D	80.16	28.78
E	31.29	37.26
B	00.00	0.00

$$2\text{ Area} = (120.14 \times 0) - (0 \times 21.18) + (28.78 \times 120.14) - (-21.18 \times 80.16) + (37.26 \times 80.16) - (28.78 \times 31.29) + (31.29 \times 0) - (37.26 \times 0.00)$$
$$= 7241.65$$
$$\text{Area} = 3620.82 \text{ m}^2$$

This is more than $\frac{1}{2}$ (total area) = $\frac{1}{2}$ (5109.84) = 2554.9 m^2.

Difference in area = 3620.82 – 2554.90 = 1065.92.

Draw *FG* parallel to *DE* so that area *DEFG*

= 1065.92 m^2

$$= DE \times h + \frac{1}{2} h^2 \tan \theta + \frac{1}{2} h^2 \tan \phi$$

$$= DE \times h + \frac{h^2}{2} [\tan \theta + \tan \phi]$$

Here $DE = 49.87$

$\theta = 80°00' + 40°00' - 90°00' = 30°$

$\phi = (360° - 80°) - (180° - 35°) - 90° = 45°$

Hence $1065.92 = 49.87 \times h + \frac{h^2}{2} [\tan 30° + \tan 45°]$

$= 49.87h + 0.788h^2$

or $h = 16.87$

$$GC = DC - DG = 61 - \frac{16.87}{\cos 45°} = 49.07$$

$$FG = 49.87 + 16.87 \times (1.577) = 76.48$$

$$BF = BE - FE = 48.64 - \frac{16.87}{\cos 30°} = 48.64 - 19.48 = 29.16$$

Length and Bearing of Sides

Line	Length	Bearing
CB	122	N 80°00′W
BF	29.16	N 40°00′E
FG	76.48	N 80°00′W
GC	49.07	S 35°00′E

Example 14.10 Prepare a table of end areas versus depth of fill from 0 to 10 m by increments of 1 m for level sections 20 m wide level road bed and side slopes 2 to 1 (Fig. 14.31).

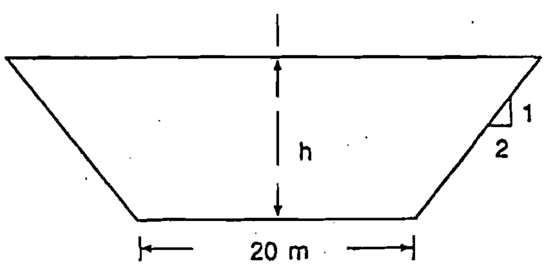


Fig. 14.31 Example 14.10.

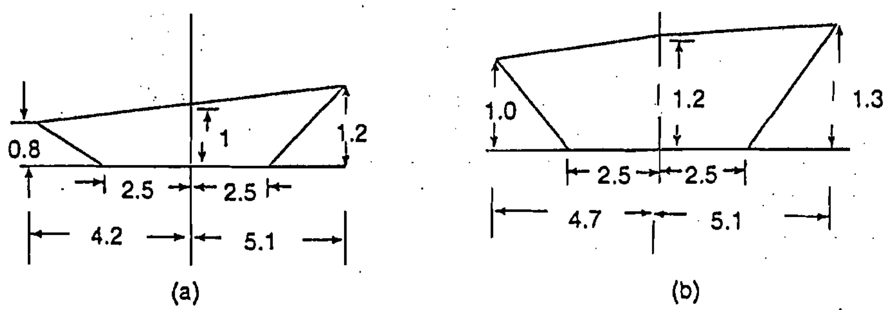
Solution We have area = $h(b + nh)$. Here $b = 20$ m and $n = 2$. Hence

$$\text{area} = h[20 + 2h]$$

The results are given in tabular form below:

h(m)	Area (m^2)	h(m)	Area (m^2)
0	0	6	192
1	22	7	238
2	48	8	288
3	78	9	342
4	112	10	400
5	150		

Example 14.11. An irrigation ditch is with $b = 5$ m and side slopes 2 to 1. Notes giving distance from centreline and cut ordinates for stations 52 + 00 and 53 + 00 are c 0.8/4.2, c 1.0, c 1.2/5.1 and c 1/4.7, c 1.2, c 1.3/5.1. Draw the cross sections and compute volumes by average end area method (Fig. 14.32).



(a) Section at 52 + 00. (b) Section at 53 + 00.

Fig. 14.32 Example 14.11.

$$\text{Area} = \frac{1}{2}\left[\frac{b}{2}(h_1 + h_2) + h(w_1 + w_2)\right]$$

$$\text{Area at section } 52 + 00 = \frac{1}{2}\left[\frac{5}{2}(0.8 + 1.2) + 1.0(4.2 + 5.1)\right] = 7.15 \text{ m}^2$$

$$\text{Area at section } 53 + 00 = \frac{1}{2}\left[\frac{5}{2}(1.0 + 1.3) + 1.2(4.7 + 5.1)\right] = 8.755 \text{ m}^2$$

Assuming station distance 30 m.

$$\text{Volume} = \frac{1}{2}(7.15 + 8.755) \times 30 = 238.575 \text{ m}^3$$

Example 14.12 For the data listed, tabulate cut, fill and cumulative volumes in cut between stations 10 + 00 and 20 + 00. Use a shrinkage factor of 1.30 for cuts. Take 30 m stations.

Station	End area (m²) Cut	Fill	Station	End area (m²) Cut	Fill
10 + 00	0		15 + 00		12.40
11 + 00	19.6		16 + 00		23.80
12 + 00	34.8		17 + 00		30.00
13 + 00	31.7		18 + 00		26.50
14 + 00	14.6		19 + 00		18.30
14 + 13.6	00.0	00.0	20 + 00		11.60

Solution

Table 14.3 Example 14.12

Station	End area m² Cut	Fill	Volume Cut	Fill	Cut volume / 1.3	Cumulative Volume
10 + 00	0					0.00
			+196.0		+ 150.77	
11 + 00	19.6					+ 150.77
			+ 816.0		+ 627.69	
12 + 00	34.8					+ 778.4
			+ 997.5		+ 767.30	
13 + 00	31.7					+ 1545.76
			+ 694.5		+ 534.23	
14 + 00	14.6					+ 2079.99
			+146.0		+ 112.30	
14 + 13.6	00.0	00.00				+ 2192.29
				– 124.0		
15 + 00		12.40				+ 2068.29
				– 543.0		
16 + 00		23.80				+ 1525.29
				– 807.0		
17 + 00		30.00				+ 718.29
				–847.5		
18 + 00		26.50				– 129.21
				– 672.0		
19 + 00		18.30				– 801.21
				– 448.5		
20 + 00		11.60				– 1249.71

The volumes between stations 10 + 00 and 11 + 00, 14 + 00 and 14 + 13.6 and 14 + 13.6 and 15 + 00 are computed as 1/3 base × altitude. Others are computed by end area methods.

Example 14.13 From the following excerpts from field notes (i) Plot the cross section on graph paper and superimpose upon it a design template for a 10 m wide road bed with fill slopes of 3:1 and a subgrade elevation at centre line of 324.90 m. Determine the end area graphically by counting squares.

$$H \cdot I. = 324.12 \text{ m}$$

$$20 + 00\ L_t + \frac{1.6}{17}\ \frac{1.7}{7.0}\ \frac{2.0}{C \cdot L}\ \frac{2.2}{4.0}\ \frac{2.7}{10.0}\ \frac{2.3}{17}$$

(ii) Also calculate slope intercepts and determine the end area by coordinate method. Check by computing areas of triangles and trapezoids.

Solution

(i) Elevations of the points are obtained by subtracting corresponding rod reading from the $H \cdot I.$ of the levelling instrument.

The new data can be written as follows:

$$\begin{array}{lcccccc} & 322.52 & 322.42 & 322.12 & 321.92 & 321.42 & 321.82 \\ 20 + 00\ L_t + & \frac{1.6}{17} & \frac{1.7}{7.0} & \frac{2.0}{C \cdot L} & \frac{2.2}{4.0} & \frac{2.7}{10.0} & \frac{2.3}{17} \end{array}$$

The cross section is plotted in Fig. 14.33.

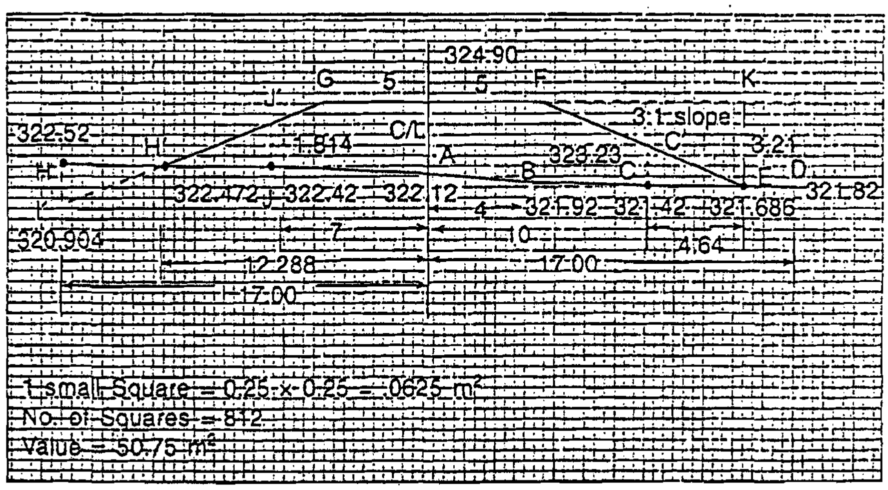


Fig. 14.33

$$\text{slope of line } CD = \frac{321.82 - 321.42}{7} = \frac{0.40}{7} = 0.057.$$

$$\text{slope of line } FE = \frac{1}{3} = 0.333.$$

$$R.L. \text{ of point } C' = 324.90 - 5 \times \frac{1}{3} = 323.23 \text{ m.}$$

$$R.L. \text{ of point } C = 321.42$$

$$CC' = 1.81 \text{ m}$$

Change of slope between $C'E$ and CE

$$0.057 + 0.333 = 0.390$$

The vertical distance CC' is covered in a horizontal distance of $\frac{1.81}{0.39} = 4.64$ m

Hence horizontal coordinate of $E = 10 + 4.64 = 14.64$ m.

$$\text{Vertical distance } KE = (14.64 - 5)(0.333) = 3.21 \text{ m.}$$

$$\text{slope of } JH = \frac{322.52 - 322.42}{10} = 0.01.$$

$$\text{slope of } GH' = 0.333.$$

$$\text{change of slope} = 0.01 + 0.333 = 0.343.$$

$$R.L. \text{ of } I' = 324.90 - (17 - 5)(0.333) = 320.904.$$

This is lower than the level of H which means H' lies between H and J.

$$R.L. \text{ of } J' = 324.90 - (7 - 5)(0.333) = 324.234$$

$$R.L. \text{ of } J = 322.420$$

$$JJ' = 1.814 \text{ m.}$$

With change of slope of 0.343, this vertical distance is covered in a horizontal distance of $1.814/0.343 = 5.288$ m.

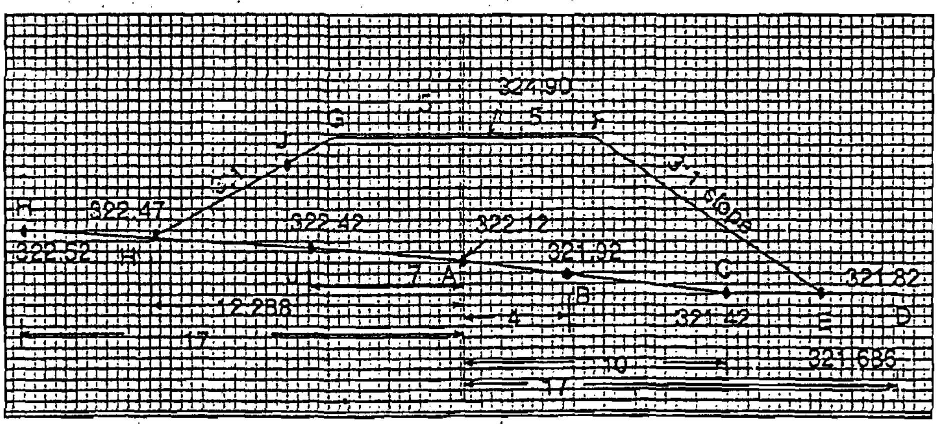


Fig. 14.34 Cross Section Example 14.13 (large scale).

Check

$$R.L. \text{ of } E \text{ (from } F) = 324.90 - (14.64 - 5) \times \frac{1}{3} = 321.686$$

$$R.L. \text{ of } E \text{ (from } C) = 321.42 + \frac{(321.82 - 321.42)}{7} \times 4.64 = 321.685$$

Hence O.K.

$$R.L. \text{ of } H' = 322.42 + \frac{322.52 - 322.42}{(17 - 7)} \times 5.288$$

$$= 322.42 + 0.052 = 322.472 \text{ m}$$

$$R.L. \text{ of } H' \text{ (from } G) = 324.90 - (7.288) \times \frac{1}{3} = 322.47.$$

Hence O.K.

Area by trapezoids and triangles. (Fig. 14.35)

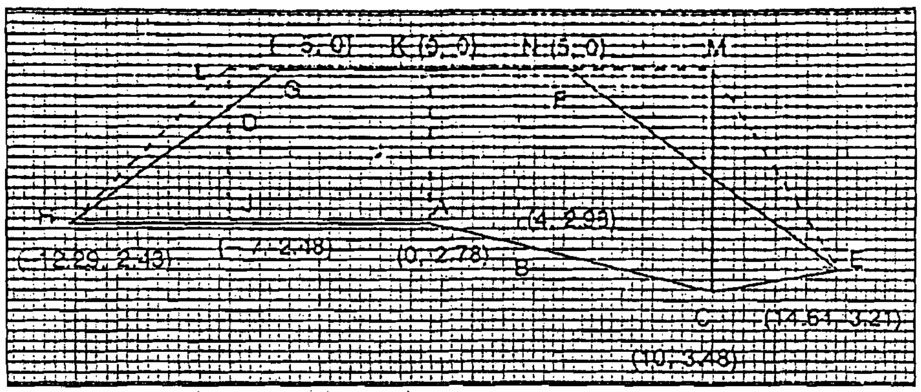


Fig. 14.35 Coordinates of controlling points, Example 14.13.

Figure	Computations	Area
HLJ	$\frac{1}{2} \times 2.48 \times 5.29$	6.56
LKAJ	$\frac{1}{2}(2.78 + 2.48) \times 7.00$	18.41
KNBA	$\frac{1}{2}(2.78 + 2.98) \times 4.00$	11.52
NMCB	$\frac{1}{2}(2.98 + 3.48) \times 6.00$	19.38
MEC	$\frac{1}{2} \times 3.48 \times 4.64$	8.07
– *HLG*	$-1/2 \times 2 \times 2.43$	– 2.43
– *EFM*	$-1/2 \times 5 \times 3.21$	– 8.03
		53.48 m^2

Area by coordinates

Point	*X*	*Y*	Plus +	Minus –
K	0.00	0.00		
F	5.00	0.00	0.00	0.00
E	14.64	3.21	0.00	–16.05
C	10.00	3.48	32.10	– 50.95
B	4.00	2.98	13.92	– 29.80
A	0.00	2.78	00.00	– 11.12
J	– 7.00	2.48	– 19.46	– 00.00
H	– 12.29	2.43	– 30.48	+ 17.01
G	– 5.00	0.00	– 12.15	+ 00.00
K	0.00	0.00	00.00	+ 00.00
			– 16.07	– 90.91

$$\text{Twice total area} = -16.07 - 90.91$$
$$= 106.98$$
$$\text{Area} = \frac{106.98}{2} = 53.49 \text{ m}^2$$

Example 14.14 From the accompanying final cross section notes compute the total volume of cut and the total volume of fill between station 43 + 00 and station 48 + 00 by the average end area method. The road bed is 8 m in cut and 7 m in fill and the side slopes are 1:1 in cut and $1^1/_2$:1 in fill.

Station	Ground Elevation	Grade Elevation	Cross section *L*	*C*	*R*
48 + 00	187.1	189.1	$\frac{F3.1}{8.0}$	$\frac{F2.0}{0}$	$\frac{F1.6}{5.7}$
47 + 00	188.6	189.40	$\frac{F1.6}{5.7}$	$\frac{F0.8}{0}$	$\frac{F0.7}{4.3}$
46 + 23	189.2	189.50	$\frac{F1.0}{4.8}$	$\frac{F0.3}{0}$	$\frac{F0.0}{3.3}$
46 + 00	189.8	189.74	$\frac{F0.6}{4.2}$	$\frac{C0.0}{0}$	$\frac{C0.9}{4.9}$
45 + 27	190.4	189.80	$\frac{C0.0}{4.0}$	$\frac{C0.6}{0}$	$\frac{C1.4}{5.4}$
45 + 00	191.6	190.40	$\frac{C0.7}{4.7}$	$\frac{C1.2}{0}$	$\frac{C2.1}{6.1}$
44 + 15	192.0	190.24	$\frac{C1.3}{5.3}$	$\frac{C1.8}{0}$	$\frac{C2.6}{6.6}$
44 + 00	192.2	190.44	$\frac{C1.2}{5.2}$	$\frac{C1.8}{0}$	$\frac{C2.6}{6.6}$
43 + 13	193.20	190.64	$\frac{C1.9}{5.9}$	$\frac{C2.6}{0}$	$\frac{C3.0}{7.0}$
43 + 00	193.50	190.77	$\frac{C1.9}{5.9}$	$\frac{C2.7}{0}$	$\frac{C3.9}{7.9}$

Solution For a three level section, the area is given by

$$A = b/4(f_l + f_r) + \frac{f}{2}(d_l + d_r)$$

Station	Area
43 + 00	$8/4(1.9 + 3.9) + \frac{2.7}{2}(5.9 + 7.9) = 30.23C.$
43 + 13	$8/4(1.9 + 3.0) + \frac{2.6}{2}(5.9 + 7.0) = 26.57C.$
44 + 00	$8/4(1.2 + 2.6) + \frac{1.8}{2}(5.2 + 6.6) = 18.22C.$

44 + 15	$8/4(1.3 + 2.6) + \frac{1.8}{2}(5.3 + 6.6) = 18.51C.$
45 + 00	$8/4(0.7 + 2.1) + \frac{1.2}{2}(4.7 + 6.1) = 12.68C.$
45 + 27	$8/4(0.0 + 1.4) + \frac{0.6}{2}(4.0 + 5.4) = 5.62C.$
46 + 00	$8/4(0.6) + 8/4(0.9) + \frac{0.0}{0}(4.2 + 4.9) = 1.2F + 1.9\ C.$
46 + 23	$8/4(1.0 + 0.0) + \frac{0.3}{2}(4.8 + 3.3) = 3.22F.$
47 + 00	$8/4(1.6 + 0.7) + \frac{0.8}{2}(5.7 + 4.3) = 8.60F.$
48 + 00	$8/4(3.1 + 1.6) + \frac{2.0}{2}(8.0 + 5.7) = 23.10F.$

Volume of cut

$$\frac{30.23 + 26.57}{2} \times 13 + \frac{26.57 + 18.22}{2} \times 17 + \frac{18.22 + 18.51}{2} \times 15$$

$$+ \frac{18.51 + 12.68}{2} \times 15 + \frac{12.68 + 5.62}{2} \times 27 + \frac{5.62 + 1.90}{2} \times 3$$

$$+ \frac{1}{3} \times 1.9 \times 23 = 1532.36 \text{ m}^3$$

Volume of fill

$$\frac{1}{3} \times 1.2 \times 3 + \frac{1.2 + 3.22}{2} \times 23 + \frac{3.22 \times 8.60}{2} \times 7$$

$$+ \frac{8.6 + 23.1}{2} \times 30 = 568.9 \text{ m}^3$$

Example 14.15 Given the following five level sections compute the volume of earthwork lying between them by (a) the average end area method, (b) the prismoidal formula. Compute the side slopes

Station 44 + 00	$\frac{c3.1}{14.2}$	$\frac{c3.4}{8.00}$	$\frac{c2.8}{0}$	$\frac{c2.6}{8.0}$	$\frac{c2.0}{11.9}$
Station 45 + 00	$\frac{c1.5}{10.3}$	$\frac{c1.0}{8.0}$	$\frac{c0.7}{0}$	$\frac{c0.5}{8.0}$	$\frac{c0.55}{9.1}$

Solution The area of Fig. 14.36 = $\frac{1}{2}(f_l' \cdot d_l + f.b. + f_r' \cdot d_r)$ in which f_l' and f_r' are the fill at the left and right edge of the road bed respectively, and d_l and d_r are the distances to the left and right.

As the section is cut the symbol c should be substituted for f.

$$\text{Area } A = \frac{1}{2}(c_l' \cdot d_l + c \cdot b + c_r' \cdot dr)$$

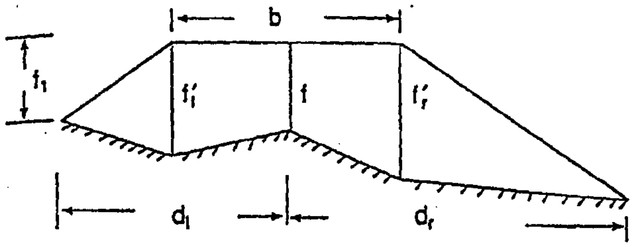


Fig. 14.36 Example 14.15.

$$\text{Station } 44 + 00 \; A = \frac{1}{2}(3.4 \times 14.2 + 2.8 \times 16 + 2.6 \times 11.9) = 62.01 \text{ m}^2$$

$$\text{Station } 45 + 00 \; A = \frac{1}{2}(1 \times 10.3 + 0.7 \times 16 + 0.5 \times 9.1) = 13.02 \text{ m}^2$$

$$\text{Volume by average end area} = \frac{1}{2} \times 30(62.01 + 13.02) = 1125.45$$

$$\text{Left hand side slope} = \frac{c_l}{d_l - b/2} = \frac{3.1}{14.2 - 8} = \frac{1}{2}$$

$$\text{Right hand side slope} = \frac{c_r}{d_r - b/2} = \frac{0.55}{9.1 - 8} = \frac{1}{2}$$

Volume by prismoidal rule Since the area of the middle section is not given, dimensions of the middle section are taken as mean of the corresponding dimensions of the end sections. Hence,

$$M \qquad \frac{c2.3}{12.25} \; \frac{c2.2}{8.0} \; \frac{c1.75}{0.00} \; \frac{c1.5}{8.0} \; \frac{c1.28}{10.5}$$

$$\text{Area} = \frac{1}{2}(2.2 \times 12.25 + 1.75 \times 16 + 1.5 \times 10.5) = 35.35$$

$$V = \frac{15}{3}(62.01 + 13.02 + 4 \times 35.35) = 1082.15$$

Example 14.16 Given the following notes for three irregular sections, compute the volume of earthwork lying between station 15 and station 16 by (a) the average end area method; (b) by the prismoidal formula. The road bed width is 20 m and the side slopes $1\frac{1}{2}$:1.

	A	B	C	D	E	F	G	H
Station 15 + 00	$\frac{F1.5}{12.3}$	$\frac{F2.3}{9.7}$	$\frac{F2.3}{6.3}$	$\frac{F3.1}{2.5}$	$\frac{F2.7}{0}$	$\frac{F2.6}{4.5}$	$\frac{F4.4}{14.1}$	$\frac{F6.6}{20.0}$

	A	B	C	D	E	F	G
Station 15 + 15	$\frac{F1.9}{12.8}$	$\frac{F2.0}{10.3}$	$\frac{F4.7}{7.7}$	$\frac{F3.1}{0}$	$\frac{F3.1}{3.7}$	$\frac{F3.8}{9.9}$	$\frac{F1.8}{12.7}$

$$\begin{array}{lcccccccc} & A & B & C & D & E & F & G & H \\ \text{Station } 16+00 & \frac{F0.6}{10.9} & \frac{F1.3}{9.6} & \frac{F0.8}{6.7} & \frac{F1.7}{2.0} & \frac{F1.7}{0} & \frac{F3.4}{3.9} & \frac{F2.5}{10.4} & \frac{F3.4}{15.1} \end{array}$$

Solution Taking the origin at *E*, the data with proper sign can be plotted as in Fig. 14.37

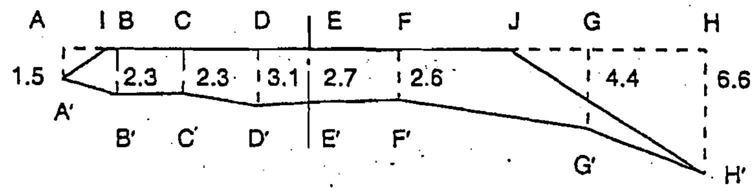


Fig. 14.37 Example 14.16.

$$\text{Slope of line } IA' = \frac{1.5}{12.3-10} = \frac{1.5}{2.3} = \frac{1}{1.5} = 1:1\frac{1}{2}$$

$$\text{Slope of line } JH' = \frac{6.6}{20-10} = \frac{6.6}{10} = \frac{1}{1.5} = 1:1\frac{1}{2}$$

$$\begin{aligned}\text{Area} &= \frac{1.5+2.3}{2} \times (12.3-9.7) + 2.3(9.7-6.3) \\ &+ \frac{2.3+3.1}{2}(6.3-2.5) + \frac{3.1+2.7}{2}(2.5-0) \\ &+ \frac{2.7+2.6}{2}(4.5-0) + \frac{2.6+4.4}{2}(14.1-4.5) \\ &+ \frac{4.4+6.6}{2}(20.0-14.1) - \frac{1}{2} \times 1.5 \times (12.3-10) \\ &- \frac{1}{2} \times 6.6 \times (20.0-10.0) \\ &= 4.94 + 7.82 + 11.93 + 33.6 + 32.45 - 1.73 - 33.0 = 56.01 \text{ m}^2\end{aligned}$$

Station 15 + 15 The data can be plotted as in Fig. 14.38

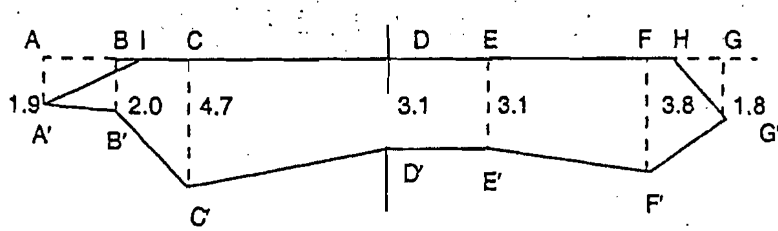


Fig. 14.38 Example 14.16.

$$\text{Slope of } IA' = \frac{1.9}{12.8-10} = \frac{1.9}{2.8} = 1:1.5$$

$$\text{Slope of } HG' = \frac{1.8}{12.7-10} = \frac{1.8}{2.7} = 1:1.5$$

$$\text{Area} = \frac{1.9 + 2.0}{2}(12.8 - 10.3) + \frac{2.0 + 4.7}{2}(10.3 - 7.7)$$

$$+ \frac{4.7 + 3.1}{2}(7.7 - 0) + 3.1(3.7 - 0)$$

$$+ \frac{3.1 + 3.8}{2}(9.9 - 3.7) + \frac{3.8 + 1.8}{2}(12.7 - 9.9)$$

$$- \frac{1}{2}(1.9)(12.8 - 10) - \frac{1}{2} \times 1.8(12.7 - 10)$$

$$= 4.87 + 8.71 + 30.03 + 11.47 + 21.39 + 7.84 - 2.66 - 2.43$$

$$= 79.22 \text{ m}^2.$$

Station 16 + 00. The data can be plotted as in Fig. 14.39.

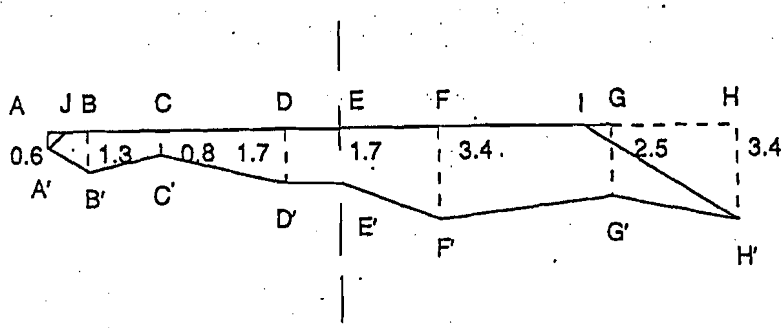


Fig. 14.39 Example 14.16.

$$\text{Slope } JA' = \frac{0.6}{10.9 - 10} = \frac{0.6}{0.9} = 1:1.5$$

$$IH' = \frac{3.4}{15.1 - 10} = \frac{3.4}{5.1} = 1:1.5$$

$$\text{Area} = \frac{1}{2}(0.6 + 1.3)(10.9 - 9.6) + \frac{1}{2}(1.3 + 0.8)(9.6 - 6.7)$$

$$+ \frac{1}{2}(0.8 + 1.7)(6.7 - 2.0) + \frac{1}{2}(1.7 + 1.7)(2.0 - 0)$$

$$+ \frac{1}{2}(1.7 + 3.4)(3.9 - 0.00) + \frac{1}{2}(3.4 + 2.5)(10.4 - 3.9)$$

$$+ \frac{1}{2}(2.5 + 3.4)(15.1 - 10.4) - \frac{1}{2}(0.6)(10.9 - 10.0)$$

$$- \frac{1}{2}(3.4)(15.1 - 10.0)$$

$$= 47.6 \text{ m}^2.$$

(a) Volume by average end area method

$$V_a = \frac{1}{2}(56.01 + 79.22)(15) + \frac{1}{2}(79.22 + 47.60)(15)$$

$$= 1965.375 \text{ m}^3.$$

(b) Volume by prismoidal rule

$$V = \frac{15}{3}(56.01 + 4(79.22) + 47.60)$$

$$= 2102.50 \text{ m}^3.$$

Example 14.17 A straight level road is to be constructed on a plane hillside with a lateral slope uniformly 9 horizontally to 1 vertically, the side slopes being like wise 1:1 and 2:1 in cut and fill respectively and the formation width is 10 m. Calculate the total volume of earthwork in a length of 200 m.

(i) when the areas of cut and fill in each cross section are equal; and

(ii) when the total earthwork in each cross section is a minimum stating the volume of cut in excess of the fill in the latter case.

Solution From cut,

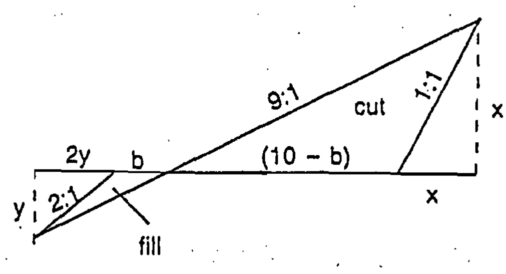


Fig. 14.40 Example 14.17.

$$\frac{x}{10 - b + x} = \frac{1}{9}$$

$$\text{or } x = \frac{(10 - b)}{8}$$

$$\therefore \quad \text{Area of cut} = \frac{1}{2}(10 - b) \cdot x$$

$$= \frac{(10 - b)^2}{16}$$

From fill, $$\frac{y}{2y + b} = \frac{1}{9}$$

$$\text{or } y = b/7$$

$$\text{Area} = \frac{1}{2} \cdot b \cdot y = \frac{1}{2} \cdot b \cdot \frac{b}{7}$$

$$= \frac{b^2}{14}$$

(i) Since areas of cut and fill are equal.

$$\frac{(10-b)^2}{16} = \frac{b^2}{14}$$

Solving

$$b = 4.83$$

$$10 - b = 5.17$$

$$\text{Area of cut} = \frac{5.17^2}{16} = 1.67$$

$$\text{Area of fill} = \frac{4.83^2}{14} = 1.67$$

(ii) Total earthwork, $E = \dfrac{(10-b)^2}{16} + \dfrac{b^2}{14}$

This will be minimum when

$$\frac{dE}{db} = 0$$

or

$$\frac{2(10-b)(-1)}{16} + \frac{2b}{14} = 0$$

or

$$b = 4.67 \text{ m.}$$

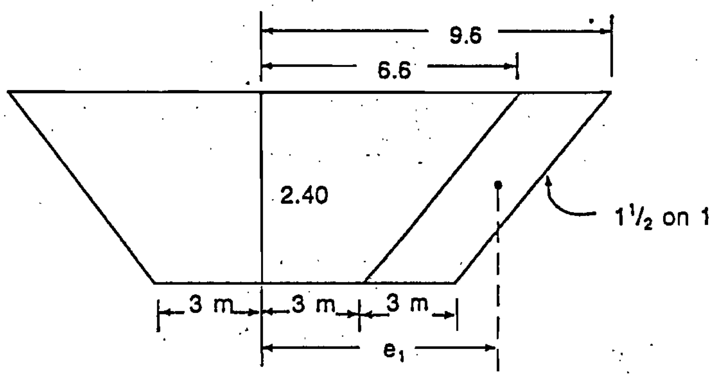
$$\text{Volume of cut} = \frac{(10-4.67)^2}{16} \times 200 = 355.00 \text{ m}^3.$$

$$\text{Volume of fill} = \frac{4.67^2}{14} \times 200 = 311.55 \text{ m}^3.$$

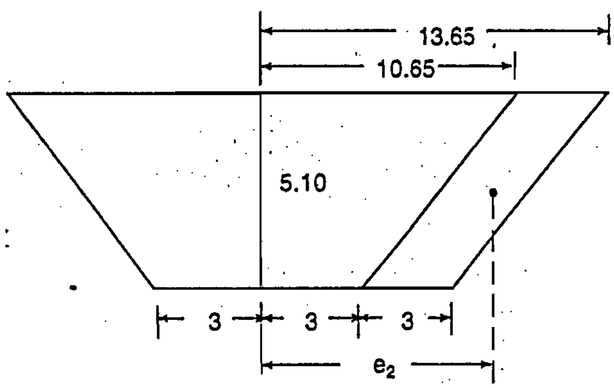
Total volume of earthwork $= 666.55$ m^3.

Excess of cut over fill = 43.45 m^3.

Example 14.18 The centre line of a highway cutting is on a curve of 120 m radius, the original surface of the ground being approximately level. The cutting is to be widened by increasing the formation width from 6 m to 9 m, the excavation to be entirely on the inside of the curve and to retain the existing side slopes of $1^1/_2$ horizontal to 1 vertical. If the depth of formation increases uniformly from 2.40 m to 5.10 m over a length of 90 m (Fig. 14.41), calculate the volume of earth to be removed in this length. [L.U.]



(a) Cross section at the beginning—$A_1 = 2.4 \times 3 = 7.2$ m^2.



(b) Cross section at the end—$A_2 = 3 \times 5.1 = 15.3$ m^2.

Fig. 14.41 Example 14.18.

Solution

$$e_1 = \frac{6 + 6.6}{2} = 6.3 \text{ m} \quad \text{or} \quad \frac{3 + 9.6}{2} = 6.3 \text{ m.}$$

$$e_2 = \frac{6 + 10.65}{2} = 8.33 \text{ m} \quad \text{or} \quad \frac{3 + 13.65}{2} = 8.33 \text{ m.}$$

Mean radius of the path of centroid of new excavation $= 120 - \left(\frac{6.30 + 8.33}{2}\right) =$ 112.69 m.

$$\text{Length of the path of centroid} = 90 \times \frac{112.69}{120} = 84.52 \text{ m.}$$

From Fig. 14.41, Volume swept $= L \times \frac{1}{2}(A_1 + A_2)$

$$= 84.52 \times \frac{1}{2}(7.2 + 15.3)$$

$$= 950.85 \text{ m}^3$$

Example 14.19 The areas within the contour lines at the site of a reservoir are as follows:

	Contour (m)	Area (m^2)
at	158	476,000
	156	431,000
	154	377,000
	152	296,000
	150	219,000
	148	164,000
	146	84,000
	144	10,000
	142	1,000

The level of the bottom of the reservoir is 142 m. Calculate (a) the volume of water in the reservoir when the water level is 158 m using the end area method. (b) the volume of water in the reservoir when the water level is 158 m using the prismoidal formula (every second area may be taken as mid area) and the water level when the reservoir contains 1,800,000 m^3.

Solution Since the contours are at regular interval of 2 m, the trapezoidal formula can be straightaway used.

$$\text{Volume} = \frac{2}{2}\,[(476000 + 1000) + 2(431000 + 377000 + 296000$$
$$+ 219000 + 164000 + 84000 + 10000)]$$
$$= 3.639 \times (10^6)\ m^3.$$

By prismoidal formula,

$$\text{Volume} = \frac{2}{3}\,[(476000 + 1000) + 4(10{,}000 + 164000 + 296{,}000$$
$$+ 431{,}000) + 2(84{,}000 + 219{,}000 + 377{,}000)]$$
$$= 3.627 \times (10^6)\ m^3.$$

Up to 150 m.

$$\text{Volume} = \frac{2}{3}\,[1000 + 219{,}000) + 2(10{,}000 + 84{,}000 + 164{,}000)]$$
$$= 736 \times (10^3)\ m^3.$$

Up to 154 m

$$\text{Volume} = 736 \times (10^3) + \frac{219 + 296}{2} \times 2 \times (10^3)$$
$$+ \frac{296 + 377}{2} \times 2 \times (10^3) = 1924 \times (10^3)\ m^3.$$

Hence the level of water lies between 152 m and 154 m.

Volume upto 152 m = $1251 \times (10^3)\ m^3$

Extra volume required = $549 \times (10^3)\ m^3$;

Let x be the distance beyond 152 m;

$$\text{extra volume} = \frac{x}{2}\left[\frac{(377000 - 296{,}000)}{2}x + 296{,}000 + 296{,}000\right]$$

This must be equal to 549×10^3. Hence

$$\frac{x}{2}\left(\frac{81}{2}x + 592\right) = 549$$

or $$20.25x^2 + 296x - 549 = 0$$

or $$x = 1.665$$

Hence the level of water = 152 + 1.665 = 153.665 m.

Example 14.20 The centre line of a highway cutting is on a circular curve of radius R. This cutting is to be widened by increasing the formation width from 10 m to 15 m, the excavation to be entirely on the outside of the curve and to retain the existing side slopes of 1 vertical to 2 horizontal. The ground surface and formation are each horizontal and the depth of formation increases uniformly at the centre line from 5 m at chainage 2700 to 8 m at chainage 2800. If neglecting the influence of curvature causes the volume of excavation over this length to be underestimated by 5 per cent, determine the radius of the curve. [C.E.I.].

Solution

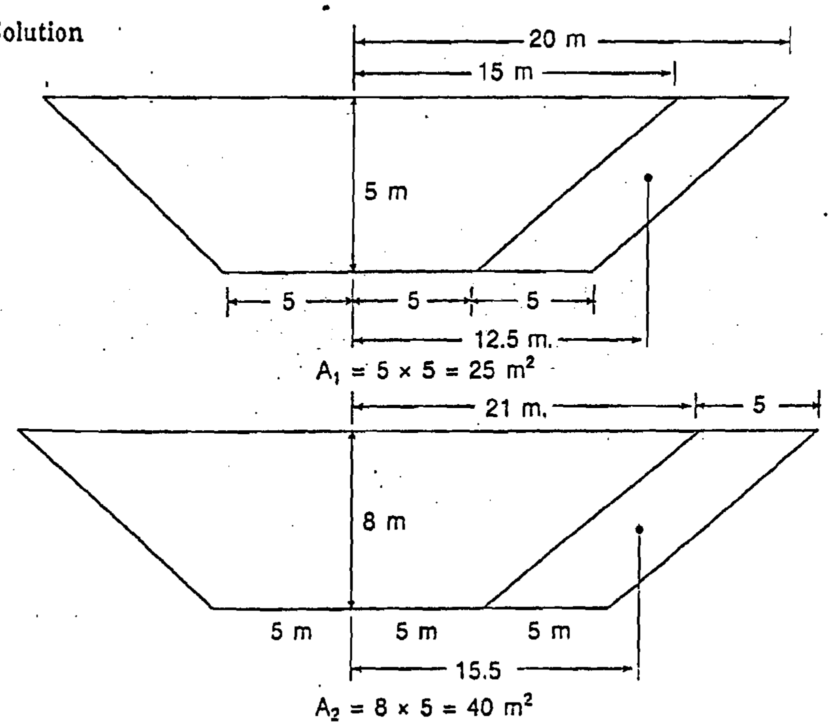


Fig. 14.42 Example 14.20.

$$e_1 = \frac{20+5}{2} = 12.5 \text{ m} \quad \text{or} \quad \frac{15+10}{2} = 12.5 \text{ m}.$$

$$e_2 = \frac{26+5}{2} = 15.5 \text{ m} \quad \text{or} \quad \frac{21+10}{2} = 15.5 \text{ m}.$$

If R be the original radius, mean radius

$$\text{of the excavation} = R + \frac{(12.5+15.5)}{2} = R + 14.$$

when radius is R length is 100 m.

$$\text{when radius is } R + 14 \text{ length is } \frac{100}{R} \times (R+14)$$

$$\text{Initial volume} = 100 \times \frac{(25+40)}{2} = 3250 \text{ m}^3$$

$$\text{Final volume with curvature} = \frac{100}{R} \times (R+14)(65) \cdot \left(\frac{1}{2}\right)$$

$$\text{Underestimation} = \frac{3250(R+14)/R - (3250)}{3250(R+14)/R} = \frac{5}{100}$$

$$\text{or } R = 266 \text{ m}.$$

Example 14.21 The centre line of a certain section of highway cutting lies on a circular curve in plan. This cutting is to be widened by increasing the formation width of 20 m to 26 m, the excavation being on the inside of the curve and retaining the original side slopes of 2 horizontal and 1 vertical. The ground surface and the formation are each horizontal and the depth to formation over a length of 400 m increases uniformly from 3 m to 5 m. Determine the radius of the centreline if the volume of excavation is over estimated by 5 percent when the influence of curvature is neglected.

[Salford]

Solution

$$e_1 = \frac{16+16}{2} = 16 \text{ m} \quad \text{or} \quad \frac{22+10}{2} = 16 \text{ m}.$$

$$e_2 = \frac{20+16}{2} = 18 \text{ m}, \quad \text{or} \quad \frac{26+10}{2} = 18 \text{ m}.$$

If R be the original radius, mean radius of the path of new excavation

$$= R - \frac{16+18}{2} = R - 17$$

when radius is R, length is 400 m.

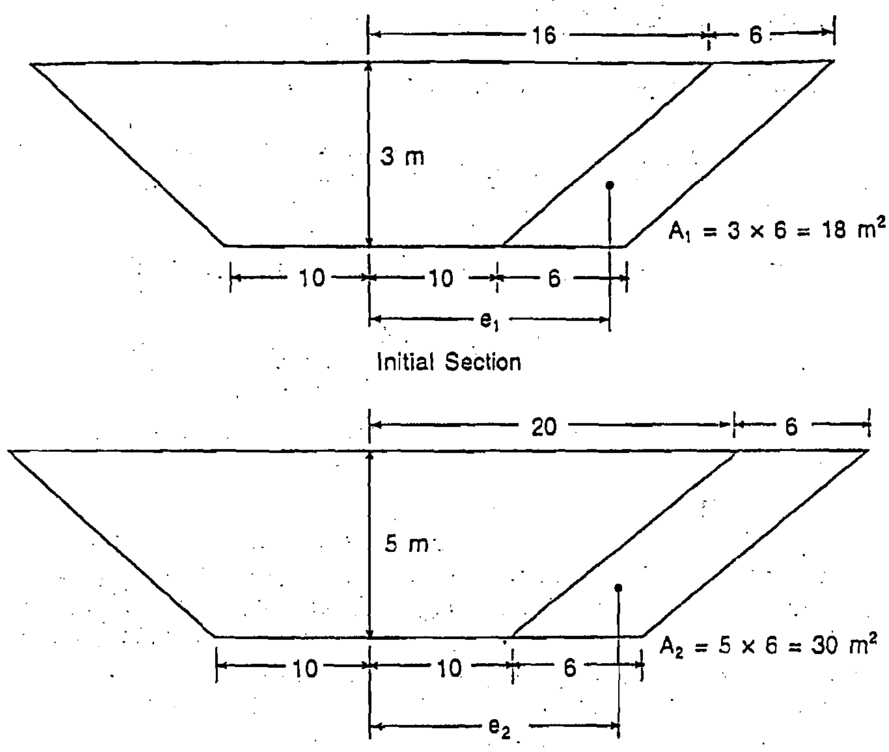


Fig. 14.43 Example 14.21.

when radius is $R - 17$, length $l = \frac{400}{R} \times (R - 17)$

$$\text{Initial volume} = \frac{400 \times (18 + 30)}{2} = 9600\ \text{m}^3$$

$$\text{Final volumes with reduced curvature} = \frac{400}{R} \times (R - 17) \times 24$$

$$= \frac{9600 \times (R - 17)}{R}$$

$$\text{Overestimation} = \frac{9600 - [9600 \times (R - 17)/R]}{9600\,(R - 17)/R} = \frac{5}{100}$$

or $R = 357$ m.

Accurate method Area of new excavation = 6 h.
For any given depth at the centreline of h.

$$x = \frac{h}{2} \times 2 = h \text{ m.}$$

Therefore eccentricity of centroid of excavation will be 13 + h from the former centreline.

Distance (m)	h (m)	$A = 6 \times h$ (m^2)	$\xi = 13 + h$ (m)	Mean distance of centroid from c/L
0	3	18	16.00	
				16.125
50	3.25	19.5	16.25	
				16.375
100	3.50	21.0	16.50	
				16.625
150	3.75	22.5	16.75	
				16.875
200	4.00	24.0	17.00	
				17.125
250	4.25	25.5	17.25	
				17.375
300	4.50	27.0	17.50	
				17.625
350	4.75	28.5	17.75	
				17.875
400	5.00	30.0	18.00	

Total Volume of Excavation.

Volume of excavation

0 – 50 $$\left(\frac{18 + 19.5}{2}\right) \cdot \frac{50(R - 16.125)}{R} = 50\left(18.75 - \frac{302.34}{R}\right)$$

50–100 $$\left(\frac{19.5 + 21.0}{2}\right) \cdot 50\left(\frac{R - 16.375}{R}\right) = 50\left(20.25 - \frac{331.59}{R}\right)$$

100–150 $$\left(\frac{21.0 + 22.5}{2}\right) \cdot \frac{50(R - 16.625)}{R} = 50\left(21.75 - \frac{361.59}{R}\right)$$

150–200 $$\left(\frac{22.5 + 24.0}{2}\right) \cdot \frac{50(R - 16.875)}{R} = 50\left(23.25 - \frac{392.34}{R}\right)$$

200–250 $$\left(\frac{24.0 + 25.5}{2}\right) \cdot \frac{50(R - 17.125)}{R} = 50\left(24.75 - \frac{423.84}{R}\right)$$

250–300 $$\left(\frac{25.5 + 27.0}{2}\right) \cdot \frac{50(R - 17.375)}{R} = 50\left(26.25 - \frac{456.09}{R}\right)$$

300–350 $$\left(\frac{27.0 + 28.5}{2}\right) \cdot \frac{50(R - 17.625)}{R} = 50\left(27.75 - \frac{489.09}{R}\right)$$

350–400 $$\left(\frac{28.5 + 30.0}{2}\right) \cdot \frac{50(R - 17.875)}{R} = 50\left(29.25 - \frac{522.84}{R}\right)$$

$$\Sigma = 50\left(192.00 - \frac{3279.72}{R}\right)$$

$$\text{Uncorrected volume} = \frac{50}{2}\,[(18 + 30) + 2(19.5 + 21.0 + 22.5 + 24.5 + 25.5 + 27 + 28.5)]$$

$$= 9600 \text{ m}^3$$

$$\text{Overestimation} = \frac{9600 - 50(192.00 - 3279.72/R)}{50(192.00 - 3279.72/R)} = .05$$

or $$R = 358.7 \text{ m}.$$

Using equivalent areas and the prismoidal formula.

Distance	A	ξ	$A\left(1 - \frac{\xi}{R}\right)$
0	18.0	16	$18\left(1 - \frac{16}{R}\right) = 18.0 - \frac{288.00}{R}$
50	19.5	16.25	$19.5\left(1 - \frac{16.25}{R}\right) = 19.5 - \frac{316.88}{R}$
100	21.0	16.50	$21.0\left(1 - \frac{16.50}{R}\right) = 21.0 - \frac{346.50}{R}$
150	22.5	16.75	$22.5\left(1 - \frac{16.75}{R}\right) = 22.5 - \frac{376.88}{R}$
200	24.0	17.00	$24.0\left(1 - \frac{17}{R}\right) = 24.0 - \frac{408.0}{R}$
250	25.5	17.25	$25.5\left(1 - \frac{17.25}{R}\right) = 25.5 - \frac{439.88}{R}$
300	27.0	17.50	$27.0\left(1 - \frac{17.50}{R}\right) = 27.0 - \frac{472.5}{R}$
350	28.5	17.75	$28.5\left(1 - \frac{17.75}{R}\right) = 28.5 - \frac{505.88}{R}$
400	30.0	18.00	$30.0\left(1 - \frac{18.00}{R}\right) = 30.0 - \frac{540.0}{R}$

Volume by prismoidal formula,

$$= \frac{50}{3}\Bigg[18.0 + 30.0 + 4(19.5 + 22.5 + 25.5 + 28.5) + 2(21.0 + 24.0 + 27.0) - \left(\frac{288}{R} + \frac{540.0}{R}\right) - 4(316.88 + 376.88 + 439.88 + 505.88)\,\frac{1}{R} - 2\,(346.5 + 408 + 472.5)\,\frac{1}{R}\Bigg].$$

$$\frac{50}{3}\left[48 + 384 + 144 - \frac{1}{R}(828 + 6558.08 + 2454)\right]$$

$$= \frac{50}{3}\left[576 - \frac{1}{R}(9840.08)\right]$$

Volume without considering curvature $= \frac{50}{3} \times 576 = 9600$

% overestimation.

$$\frac{9600 - \frac{50}{3}[576 - 9840.08/R]}{\frac{50}{3}[576 - 9840.08/R]} = 0.05$$

or $$R = 358.75 \text{ m}.$$

Example 14.22 The following notes refer to a 400 m section of a proposed railway and the earthwork distribution in this section is to be planned without regard to adjoining sections. The table shows the stations in 30 m units and the surface levels along the centre line, the formation being at an elevation above datum of 14.5 m at station 20 and thence rising uniformly on a gradient of 1.2 per cent. The corresponding earthwork volumes are recorded in m^3, the cut and fill volumes being prefixed respectively with the plus and minus signs.

(i) Plot the longitudinal section using a horizontal scale of 1 cm = 24 m and a vertical scale of 1 cm = 10 m.

(ii) Assuming a balancing factor of 0.8 applicable to fill volumes, plot the mass haul curve on a horizontal scale of 1 cm = 10 m and a vertical scale of 1 cm = 1000 m^3.

(iii) Calculate the total haul in station meter and indicate the haul limits on the curve and longitudinal section.

(iv) State which of the following estimates you would recommend: (a) No free haul at Rs. 20.00 per m^3 for excavating, hauling and filling. (b) Free haul limit of 100 m at Rs. 15.00 per m^3 plus Rs. 2.00 per station meter for "overhaul" or haul distance exceeding 100 m.

One station meter = 1 cubic meter hauled through one station, i.e. 30 m.

Table 14.4 Example 14.22

Station	Surface level (m)	Volume (m^3)	Station	Surface level (m)	Volume m^3
20	17.6		23	15.7	
		+ 1400			– 100.00
21	19.1		24	14.9	
		+ 1160			– 820.00
22	17.8		25	13.2	
		+ 416			– 1540.00

Station	Surface level (m)	Volume (m^3)	Station	Surface level (m)	Volume m^3
26	12.5		29	18.1	
		− 1600.00			+ 275.00
27	13.8		30	20.3	
		− 850.00			+ 550.00
28	16.5		31	20.7	
		− 180.00			+ 325.00
			32	26.2	

Solution

Table 14.5 Example 14.22

Station	Surface level (m)	Formation level	Cut/Fill	Volume	Reduced vol	Cumulative vol
20	17.60	14.50	+ 3.10			
				+ 1400.00	+ 1400.00	+ 1400.00
21	19.10	14.86	+ 4.24			
				+ 1160.00	+ 1160.00	+ 2560.00
22	17.80	15.22	+ 2.58			
				+ 416.00	+ 416.00	+ 2976.00
23	15.70	15.58	+ 0.12			
				− 100.00	− 80.00	+ 2896.00
24	14.90	15.94	− 1.04			
				− 820.00	− 656.00	+ 2240.00
25	13.20	16.30	− 3.10			
				− 1540.00	− 1232.00	+ 1008.00
26	12.50	16.66	− 4.16			
				− 1600.00	− 1280.00	− 272.00
27	13.80	17.02	− 3.22			
				− 850.00	− 680.00	− 952.00
28	16.50	17.38	− 0.88			
				− 180.00	− 144.00	− 1096.00
29	18.10	17.74	+ 0.36			
				+ 275.00	+ 275.00	− 821.00
30	20.30	18.10	+ 2.20			
				+ 550.00	+ 550.00	− 271.00
31	20.70	18.46	+ 2.24			
				+ 325.00	+ 325.00	+ 54.00
32	26.20	18.82	+ 7.38			

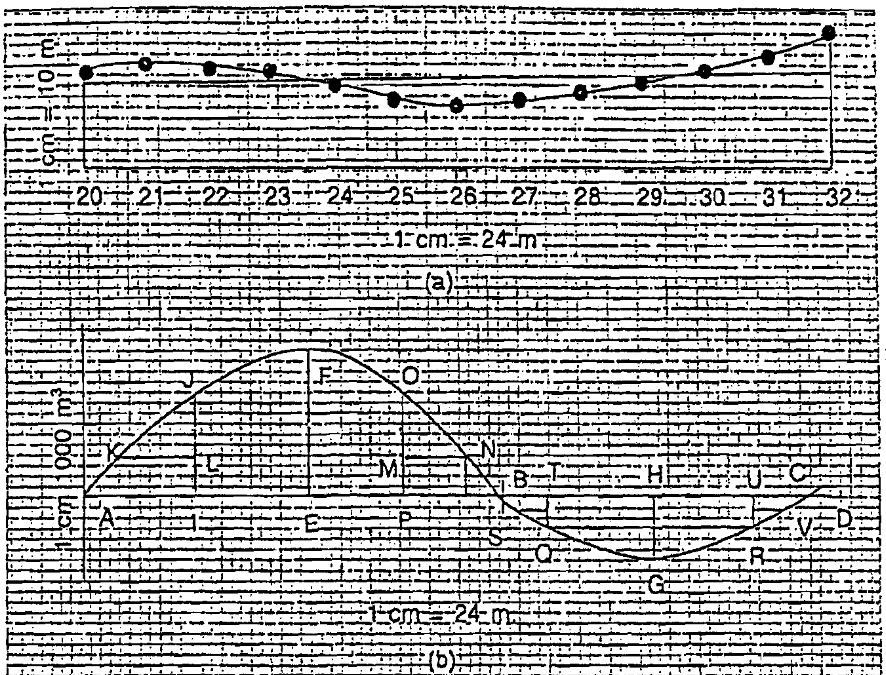


Fig. 14.44: Mass haul curve.

$$AB = 8.5 \text{ cm} = 8.5 \times 24 = 204 \text{ m.}$$

$$EF = 2.976 \text{ cm} = 2976 \text{ m}^3$$

$$\text{Haul} = \frac{2}{3} \times \frac{204 \times 2976}{30} = 13491.2 \text{ station meter}$$

$$BC = 6.2 \text{ cm} = 6.2 \times 24 = 148.8 \text{ m.}$$

$$GH = 1.2 \text{ cm} = 1200 \text{ m}^3$$

$$\text{Haul} = \frac{2}{3} \times \frac{148.8 \times 1200}{30} = 3968 \text{ station meter}$$

Total Haul = 17459.2 station meter

upto 31 + 19.2 m beyond carried forward

Cost Estimates

(i) Total volume of earthwork to be

hauled = 2976 + 1200 = 4176 m^3

At Rs. 20.00 per m^3 total haul = 20 × 4176 = Rs. 83520.00

(ii) Total volume of earthwork @ Rs. 15.00 per m^3 = 15 × 4176 = 62640.00

Cost of overhaul

In *AB*, Volume of earthwork, $IJ = 2.25 \times 1000 = 2250$ m^3.

Distance of movement, $KN = 6.4 \times 24$

$= 153.60$

$$\text{Station meter} = \frac{2250 \times 153.60}{30} = 11520.$$

In *BC*, Volume of earthwork, $QT = 0.55 \times 1000 = 550$ m^3

Distance of movement, $SV = 5.35 \times 24 = 128.4$ m

$$\text{Station meter} = \frac{550 \times 128.4}{30} = 2354$$

Total station meter = 11520 + 2354 = 13874

Cost @ Rs. 2.00 per station meter = Rs. 27748.00

Total cost = 62640 + 27748 = Rs. 90388.00

Hence 1st scheme is preferable.

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PROBLEMS

14.1 A plot of land *ABCDA* has four sides. The sides *AB* and *BC* are straight and the sides *CD* and *DA* are irregular. The plot was surveyed by chain and tape by the method of chain surveying, fixing four stations at *A*, *B*, *C*, and *D* and was moved and measured in the clockwise direction. The straight distances, measured from one station to the other are as follows:

AB = 150 m, *BC* = 165 m, *CD* = 155 m, *DA* = 162 m and *AC* = 230 m

The offsets measured from chain lines *CD* and *DA* to the irregular boundaries are as follows:

Distance from *C* (m)	Offsets (m)
0	0
30	1.50 left
60	2.00 left
90	2.25 left
120	1.75 left
155	0.00 left

Distance from *D* (m)	Offsets (m)
0	0
30	1.62 right
60	2.45 right
90	2.30 right
120	1.22 right
162	0.00 right

Calculate the area of the plot *ABCDA*. [AMIE Sec. B. Civil Winter 1979].

14.2 An abstract from a traverse sheet for a closed traverse is given below:

Line	Length	Latitude	Departure
AB	200 m	– 173.20	+ 100.00
BC	130 m	0.00	+ 130.00
CD	100 m	+ 86.00	+ 50.00
DE	250 m	+ 250.00	0.00
EA	320 m	– 154.90	– 280.00

(a) Balance the traverse by Bowditch's method
(b) Given the coordinates of A 200 N, 00 E determine the coordinates of other points
(c) Calculate the enclosed area in hectares by coordinates method.
[AMIE Sec. B Civil Winter 1980]

14.3 In order to layout a pond in a public park two perpendiculars *AD* and *BC* of 40 m and 80 m length respectively were erected at either end of a line *AB* of length 240 m. If the pond is to have straight sides lying along *AB* and *DC*, the ends being formed of circular arcs to which *AB*, *DC* and the end perpendiculars are tangential, calculate: (i) the radii of the two arcs, (ii) the perimeter of the pond. [AMIE Sec. B Civil Summer 1981]

14.4 The formation width of a certain road embankment was 20 m. Bank slopes are 2 horizontal to 1 vertical. Formation level at the starting point was 161.00. The ground levels along the centre line were as follows:

Distance (m)	0	100	200	300	400
Ground level	155.0	154.0	154.0	152.5	152.5

Assuming the ground to be level transversely, calculate the volume of earthwork in cubic meter by (a) Prismoidal formula (b) Trapezoidal formula.
[AMIE Sec. B Civil Summer 1981]

14.5 (a) Explain the terms lift and lead in earthwork.

(b) Write explanatory note on prismoidal correction in computation of earthwork quantities.

(c) Following data refer to a site of a reservoir.

Contour (m)	Area enclosed (hectare)
610	22
615	110
620	410
625	890
630	1158

The areas given are the ones which will be contained by the proposed dam and contour lines as given above. Calculate the volume of water in the reservoir. [AMIE Winter 1982]

14.6 (a) The following perpendicular offsets were taken from a chain line to a barbed wire fence

chainage (m)	0	20	40	60	80	95	110	140	170
offsets (m)	6.7	5.8	10.3	12.8	9.7	8.8	6.9	8.2	6.5

Calculate the area between the chainline, the barbed fence, and end offsets by (i) trapezoidal rule, (ii) Simpson's rule.

(b) A road embankment 500 m long is 15 m wide at formation level and has side slopes 2:1. The ground levels at every 100 m along the centre line are as follows:

Distance (m)	0	100	200	300	400	500
R.Ls (m)	105.2	106.2	107.6	107.2	108.3	108.8

The formation level at zero chainage is 107.0 m and the embankment has a rising gradient of 1:100 while the ground is level across the centre line. Calculate the volume of earthwork by the prismoidal rule.

[AMIE Summer 1986]

14.7 The following level readings were taken on the centreline of a highway alignment. The designed crest level and the crest width of the proposed embankment at reduced distance (R.D.) 1500 m is 61.80 m and 10 m respectively with a falling gradient of 1 in 100 and side slopes 1:1.

R.D.	B.S.	I.S.	F.S.	R.L.	Remarks B.M.
—	1.425			60.00	
1500		1.400			
1530		1.425			
1560	2.000		1.925		
1590		2.200			
1620	2.550		2.700		
1650		3.050			
1680			2.750		

Assuming the ground to be level across the alignment, calculate the earthwork in filling from R.D. 1530 m to R.D. 1650 m. [AMIE Winter 1985]

14.8 The following notes refer to three level cross section at two stations 50 m apart

Station	Cross section		
1	$\frac{1.9}{7.7}$	$\frac{3.0}{0}$	$\frac{4.8}{10.8}$
2	$\frac{3.1}{9.1}$	$\frac{3.9}{0}$	$\frac{7.1}{13.1}$

The width of cutting at the formation level is 12 m. Calculate the volume of cutting between the two stations using trapezoidal formula.
[AMIE Summer 1987]

14.9 (a) Derive an expression for the area of a three level section with the help of a neat sketch.
(b) A railway embankment 600 m long has a formation level width of 11.5 m with side slope 2 to 1. If the ground level and formation levels are as follows, calculate the volume of earthwork. The ground is level across the centre line:

Distance (m)	0	100	200	300	400	500	600
Ground level (m)	105.2	106.8	107	103.4	105.6	104.7	105
Formation level (m)	107.5	108.6	108.5	104.5	106.9	105.6	106

[AMIE Summer 1988]

14.10 A railway embankment, 500 m long, has a width at formation level of 9 m with side slopes 2 to 1. The ground levels at every 100 m along the centre line are:

Distance (m)	0	100	200	300	400	500
Ground level (m)	107.8	106.3	110.5	111.0	110.7	112.2

The embankment has a rising gradient of 1.2 m per 100 m and the formation level is 110.5 at zero chainage. Assuming the ground to be level across the centre line, compute the volume of earthwork. [AMIE Summer 1989]

14.11 In a proposed hydroelectric project a storage reservoir was required to provide a storage of 4.50 million cubic meters between lowest draw down (L.D.D) and top water level (T.W.L). The areas contained within the stated contours and upstream face of the dam were as follows:

Contour (m)	100	95	90	85	80	75	70	65
Area (hectares)	30	25	23	17	15	13	7	02

If L.D.D. was to be 68, calculate T.W.L. for full storage capacity. (*Note*: Calculate volumes using end area method.) [AMIE Summer 1990]

14.12 (a) Why must cut and fill volumes be totalled separately?
(b) Discuss comparative side slopes in cut and fill.
(c) Why a roadway in cut is normally wider than the same roadway in fill?

(d) List the information which can be extracted from a mass diagram.
(e) State two situations where prismoidal corrections must be applied.

14.13 Describe with neat sketches an "earthwork mass curve", stating the relationship it bears to the corresponding longitudinal section. Show concisely how the "haul" may be ascertained from the curve, also the "overhaul" for an assumed free haul limit. [U.L.]

14.14 With reference to civil engineering practice explain what is meant by the following:

(a) Trapezoidal and prismoidal rules.
(b) Prismoidal correction.
(c) Haul, free haul and overhaul.
(d) Mass diagram.

14.15 A cutting is to be through ground where the cross slope varies considerably. At *A* the depth of cut was 3 m at the centre line and the cross slope was 10 to 1. At *B* the corresponding figures were 5 m and 12 to 1 and at *C* 4 m and 8 to 1. *AB* and *BC* are each 30 m. The formation width is 10 m and the side slopes $1^1/_2$ to 1. Calculate the volume of excavation between *A* and *C*. [I.C.E]

14.16 A road embankment 35 m wide at formation level with side slopes 1:1 and with an average height of 12 m is constructed with an average gradient 1 in 30 from contour 140 m to 580 m. The ground has an average slope of 12 to 1 in direction transverse to centre line. Calculate (i) the length of the road, (ii) volume of the embankment in cm^3. [AMIE Winter 1993]

14.17 (a) Define a prismoid. State and prove the prismoidal formula. Discuss the prismoidal correction.
(b) The width of formation level of a certain cutting is 10.0 m and the side slopes are 1:1. The surface of the ground has a uniform side slope of 1 in 6. If the depths of cutting at the centreline of three sections 50.0 m apart are 3.0 m, 4.0 m and 5.0 m respectively determine the volume of earthwork involved in this length of cutting. Explain your answer with neat sketches. [AMIE Winter 1994]

HINTS TO SELECTED QUESTIONS

14.12 (a) Earthwork in cutting and filling involve different types of work and hence payment at different rates. Moreover, if there is insufficient material from cuts to make the required fills, the difference must be borrowed.
(b) Side slopes in fill usually are flatter than those in cuts because in fill soil is in artificial state whereas in cuts soil remains in natural state.
(c) Roadway cut is normally wider than fills to provide for drainage ditches.
(e) (i) When the middle area is not the average of the end areas.
(ii) Centre height is great and width narrow at one station and the centre height is small but width large at the adjacent station.

15

Tacheometry

15.1 INTRODUCTION

Tacheometric surveying (also called *stadia* surveying) is a rapid and economical surveying method by which the horizontal distances and the differences in elevation are determined indirectly using intercepts on a graduated scale and angles observed with a transit or the theodolite. The stadia method has many applications in surveying practice including traverse and levelling for topographic surveys. The value of stadia surveys can be appreciated in rugged terrain and in inaccessible areas where conventional methods are difficult and time consuming. The accuracy attained is such that under favourable conditions the error will not exceed 1/1000.

15.2 INSTRUMENTS

The following are the two main instruments:

(a) Tacheometer, and
(b) Stadia rod.

Tacheometer

It is a vernier theodolite fitted with stadia diaphragm. It has three horizontal hairs, one central and other two equidistant from central hair at top and bottom. In modern instruments these three lines are etched as also the vertical hair. Figure 15.1 shows different types of stadia hairs used.

A tacheometer differs from an ordinary theodolite in (i) High magnifying power; (ii) Large aperture of the objective—35-45 mm diameter.

Stadia rod

For short distances an ordinary levelling staff with 5 mm graduation can be used. For long distances, a special large staff called a stadia rod is used. It is usually 3 to 5 m long and in one piece. The width is between 50 to 150 mm. The graduations are very prominent so that they can be read from long distances. The graduations are generally in m, dm and cm. Figure 15.2 gives a typical graduated stadia rod (part view).

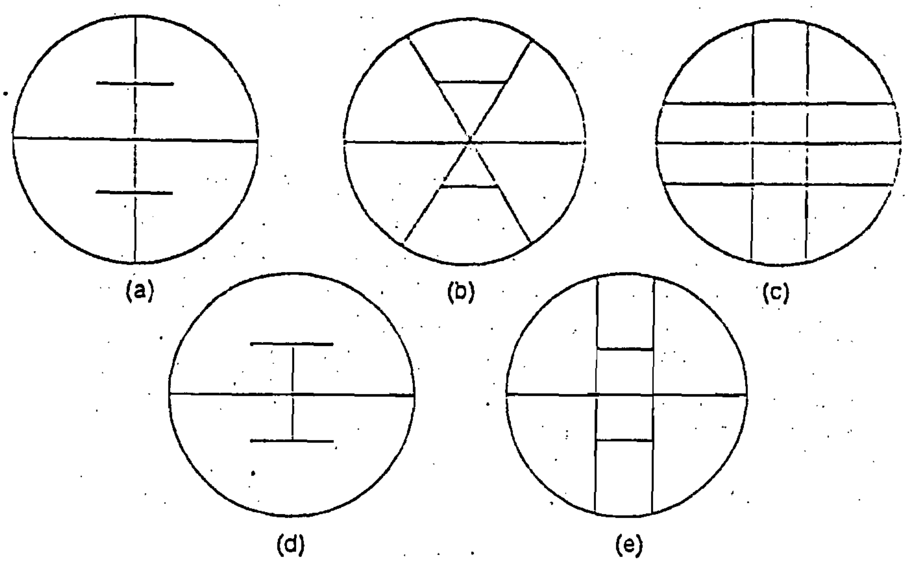


Fig. 15.1 Stadia hairs.

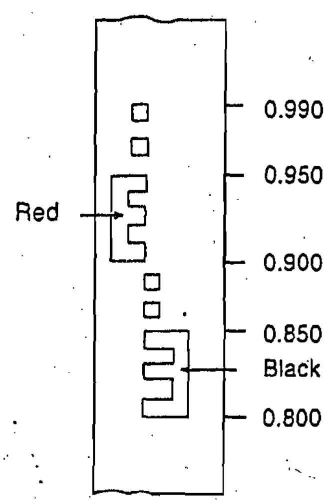


Fig. 15.2 Stadia rod.

15.3 DIFFERENT TYPES OF TACHEOMETRIC MEASUREMENTS

There are basically three types of tacheometric measurements.

1. Stadia system which again can be divided into—(i) Fixed hair method, and (ii) Movable hair method.
2. Tangential system.
3. Subtense bar system.

In the fixed hair method, as the name suggests, the hairs are fixed in position, i.e. the distance between them remains constant. The staff intercepts, i.e. the readings at which the hairs intersect the staff varies as the distance of the staff from the instrument station varies. This is shown in Fig. 15.3.

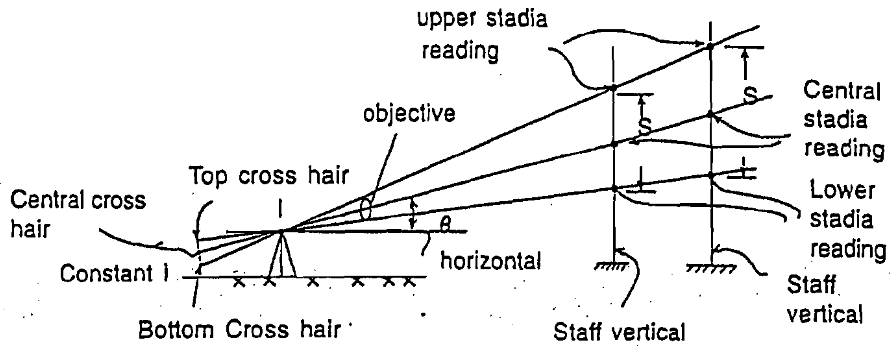


Fig. 15.3 Fixed hair method.

In the movable hair method, as the name suggests, the stadia hairs, i.e. the top and bottom hairs are movable and this is done by means of micrometer screws. The staff intercept '*S*', however, is kept fixed at a constant spacing of usually 3 m. While taking readings at different distances micrometer screws are adjusted such that the top and bottom hairs intersect the fixed targets. As it is difficult to measure the stadia interval accurately and adjust the stadia hair every time an observation is to be taken, this method is rarely used. On the other hand, the fixed hair method is frequently used.

In the tangential systems, the stadia hairs are not essential. However, two readings are to be taken at the two targets at a fixed distance '*S*' apart in the staff. This is shown in Fig. 15.4. This requires two settings of the instrument at θ_1 and θ_2 and only one hair—top, central or bottom should be used for the intersection.

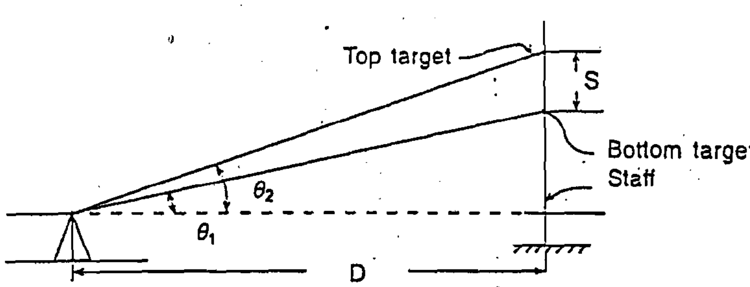


Fig. 15.4 Tangential system.

In the subtense bar system a special staff with two targets at two ends at a fixed distance apart known as subtense bar is placed horizontally and the angle between the targets from the instrument station is measured. This is shown in Fig. 15.5 where plan view of the subtense bar is shown.

15.4 PRINCIPLES OF STADIA METHOD

Figure 15.6 shows how the rays from the stadia hairs in an external focussing

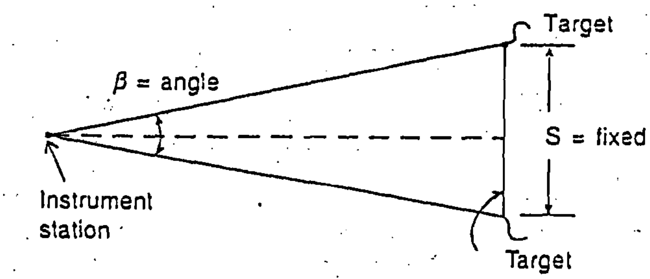


Fig. 15.5 Subtense bar.

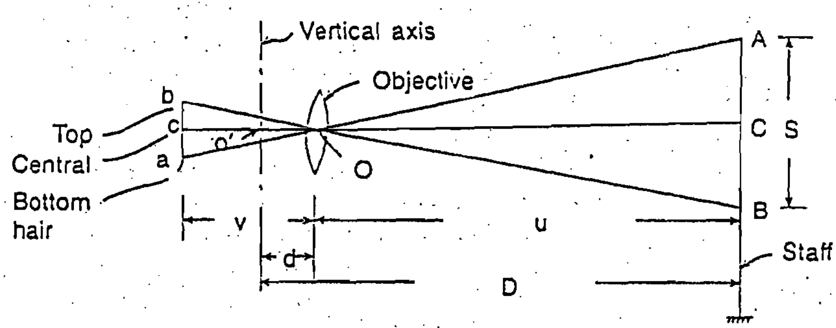


Fig. 15.6 Principle of Stadia method.

telescope intersects the staff held at a distance D from the axis of the instrument. From the principle of optics AB will be the staff intercept S corresponding to the stadia interval 'i'. From similar triangles aob and AoB

$$\frac{oc}{oC} = \frac{ab}{AB} = \frac{i}{S}$$

or

$$\frac{v}{u} = \frac{i}{S} \qquad (15.1)$$

where u = distance of the staff from the objective

v = distance of the stadia hairs from the objective

As u and v are the conjugate focal distances they are related by the lens formula.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

where f is the focal length of the objective, or

$$u = f + \frac{uf}{v}$$

But

$$\frac{u}{v} = \frac{S}{i}$$

Therefore

$$u = f + \left(\frac{S}{i}\right) f$$

Rearranging terms, we can write

$$u = \left(\frac{f}{i}\right) S + f \qquad (15.2)$$

From the Fig. 15.6, the distance of the staff from the axis of the instrument

$$D = u + d$$

$$= \left(\frac{f}{i}\right) S + (f + d) \qquad (15.3)$$

This is usually written as

$$D = KS + C$$

where $K = \frac{f}{i}$ = multiplying factor

$C = f + d$ = additive constant (15.4)

In an external focussing instrument when focussing is done by moving the objective lens d slightly changes with focussing and it usually lies between 0.3 to 0.6 m.

To avoid the additive constant Poro in 1840 devised the external focussing anallactic telescope. Here an additional convex lens, called an anallactic lens is placed between the diaphragm and the objective at a fixed distance from the latter. As a result the triangle AoB converges at the point o' from which D is measured.

Figure 15.7 shows the optical diagram of an external focussing anallactic telescope. The rays coming from A and B (corresponding to stadia hair a and b) along Ao' and Bo' are refracted by the object glass and meet at F'. The anallactic lens is placed at a distance f' from the point F' where f' is the focal length of the anallactic lens. Hence the rays passing through F' will become parallel after being refracted by the anallactic lens. Thus ab is the inverted image of the length AB of the staff. Without the intervening anallactic lens, the image would have been virtual and at b_2a_2 which is at a distance f_2 from the object glass.

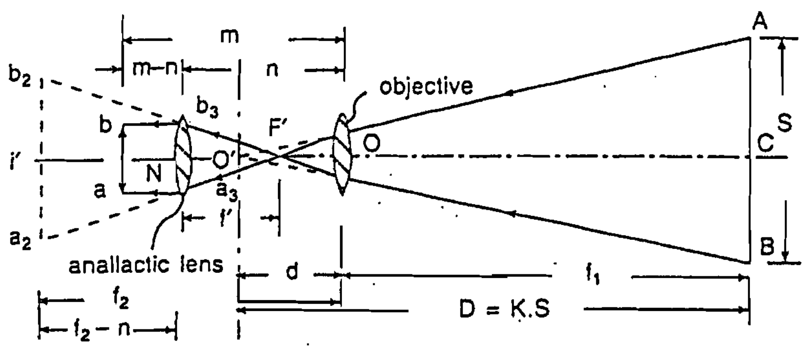


Fig. 15.7 Anallactic lens.

From the conjugate relationships of principle of optics,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (15.5)$$

As the length ab and a_2b_2 are proportional to their distance from the objective O

$$\frac{S}{i'} = \frac{f_1}{f_2} \qquad (15.6)$$

For the anallactic lens, a_2b_2 is the object and ab is the image and applying the conjugate relationship:

$$\frac{1}{f'} = -\frac{1}{(f_2 - n)} + \frac{1}{(m - n)} \qquad (15.7)$$

The minus sign before $(f_2 - n)$ has been used as both ab and a_2b_2 are on the same side of the anallactic lens.

Since the length ab and a_2b_2 are proportional to their distances from the centre N of the anallactic lens, we get,

$$\frac{i'}{i} = \frac{f_2 - n}{m - n} \qquad (15.8)$$

Multiplying Eq. (15.6) and Eq. (15.8), we get

$$\frac{S}{i} = \frac{f_1}{f_2}\,\frac{f_2 - n}{m - n}$$

From Eq. (15.5)

$$\frac{f_1}{f_2} = f_1/f - 1$$

$$= \frac{f_1 - f}{f}$$

or

$$f_2 = \frac{f f_1}{f_1 - f}$$

Similarly from Eq. (15.6)

$$\frac{f_2 - n}{f'} = \frac{f_2 - n}{m - n} - 1$$

or

$$\frac{f_2 - n}{m - n} = \frac{f_2 - n}{f'} + 1$$

$$= \frac{f_2 - n + f'}{f'}$$

Hence

$$\frac{S}{i} = \frac{f_1 - f}{f}\,\frac{f_2 - n + f'}{f'}$$

$$= \frac{f_1 - f}{f}\left\{\frac{\left(\frac{f f_1}{f_1 - f}\right) - n + f'}{f'}\right\}$$

$$= \frac{ff_1 + (f_1 - f)(f' - n)}{ff'}$$

$$= \frac{f_1(f + f' - n)}{ff'} + \frac{f(n - f')}{ff'}$$

or
$$f_1 = \frac{S}{i} \cdot \frac{ff'}{f + f' - n} + \frac{f(n - f')}{ff'}$$

$$D = (f_1 + d)$$

$$= \frac{ff'}{(f + f' - n)} \frac{S}{i} - \frac{f(n - f')}{f + f' - n} + d$$

$$= KS + C \tag{15.9}$$

where $K = \frac{ff'}{(f + f' - n)i}$ and $C = d - \frac{f(n - f')}{f + f' - n}$

In order that $D = KS$ additive constant C should be zero,

or
$$\frac{f(n - f')}{f + f' - n} = d$$

or
$$fn - ff' = fd + f'd - nd$$

or
$$n(f + d) = f'(d + f) + df$$

or
$$n = f' + \frac{df}{f + d} \tag{15.10}$$

In such a case the apex of the vertical triangle $AF'B$ will become $Ao'B$. K will be equal to 100 when the multiplier

$$\frac{ff'}{(f + f' - n)i} = 100$$

f and n being already fixed f' and i can be suitably adjusted to make the multiplying constant 100.

15.5 INTERNAL FOCUSSING TELESCOPE

In an internal focussing telescope, there is a concave lens between the eyepiece and the objective. The purpose of the concave lens is to focus the telescope, that is, to bring the image of the object in the plane of the crosshairs. The equation can be derived as follows:

The focal length of the object lens is f. The focal length of the internal focussing lens is f_1. If there was no internal focussing lens, the image of the point P will be at P' (Fig. 15.8).

The lens formula is

$$\frac{1}{u} + \frac{1}{v'} = \frac{1}{f} \tag{15.11}$$

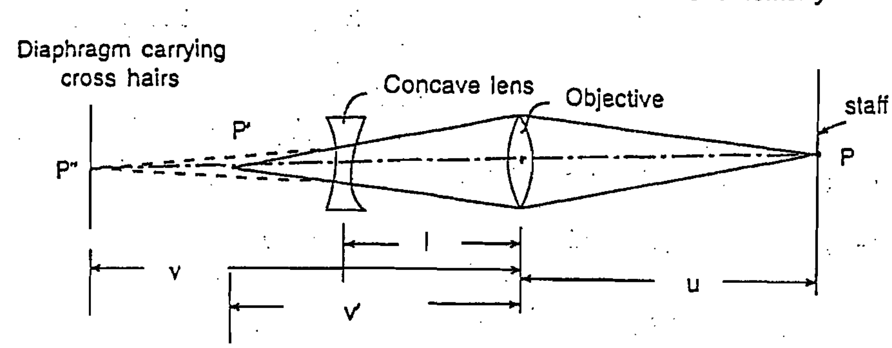


Fig. 15.8 Internal focussing telescope.

or
$$v' = \frac{fu}{u-f} \tag{15.12}$$

Then treating P' as an object before the focussing lens giving an image at P'' gives,

$$\frac{1}{v-l} - \frac{1}{v'-l} = \frac{1}{f_1} \tag{15.13}$$

(Negative sign is used as both the object and the image are on the same side).

or
$$f_1 = \frac{(v-l)(v'-l)}{v-v'} \tag{15.14}$$

Cross multiplying,

$$f_1(v-v') = (v-l)(v'-l)$$

Substituting,

$$f_1\left(v - \frac{fu}{u-f}\right) = \left(\frac{fu}{u-f} - l\right)(v-l)$$

or
$$(u-f)\,l^2 - \{v(u-f) + uf\}\,l - vuf_1 + vuf + uff_1 + vff_1 = 0 \tag{15.15}$$

This is a quadratic equation in l and can be solved to give l in terms of f, f_1, v and u. Here f, f_1 and v are constants, only u is a variable on which l depends. When the formula of the type $D = KS + C$ is applied to observations through internal focussing instruments, it is found that the value of K changes slightly when focussing the internal lens. This change is almost compensated for throughout the range of focus by a corresponding change in the value of C. This fact allows the constant C to be neglected, i.e. $C = 0$. The value of the stadia interval factor is then assumed to be constant throughout the focussing range.

15.6 DETERMINATION OF TACHEOMETER CONSTANTS

There are two constants in an external focussing tacheometer: (i) Multiplying constant K; (ii) Additive constant C.

The additive constant $C = f + d$.

By focussing a distant object say 300 to 400 m away and measuring the distance between the objective and the crosshairs, the focal length of the objective can be determined. The telescope is then focussed at an object between 100 to 150 m. The distance between the objective lens and the centre of the instrument is then measured. This is the distance d and the constant C can then be obtained.

The multiplying constant f/i is obtained by taking a number of readings on a fairly level straight line 100 m to 200 m. length at an interval of 15 m. For internal focussing instrument the distances are measured from a point which is directly under the instrument. For external focussing instrument the distances are measured from a point which is $(f + d)$ cm in front of the axis of the instrument. Then distances $D_1, D_2, D_3, \ldots$ and corresponding staff intercepts $S_1, S_2, S_3, \ldots$ are noted. Then $D = KS$ gives the formula $D_1 = K_1S_1, D_2 = K_2S_2, \ldots$ and so on.

The mean values of K_1, K_2 and K_3 gives the multiplying constant K.

Alternately, the formula $D = KS + C$ can be applied to a number of pairs of distances D_1 and D_2 and getting corresponding stadia intercepts C_1 and C_2. Solving a number of such simultaneous equations and taking their mean values will give the value of K.

15.7 DISTANCE AND ELEVATION FORMULAE

In tacheometry the most general approach will be to derive expressions when the telescope is inclined to the horizontal and the stadia rod inclined to the vertical at an arbitrary angle. However, usually the stadia rod is kept either vertical or normal to the line of sight. Hence formulae are derived for the above cases only.

Inclined sight and staff vertical
(θ, an angle of elevation)

The three points on the staff which are cut by the three stadia hairs are A, C and B, C being the point of intersection of the central hair. Here AB is the staff intercept. Draw $A'B'$ perpendicular to IC, the axis of the telescope which makes an angle θ with the horizontal (Fig. 15.9).

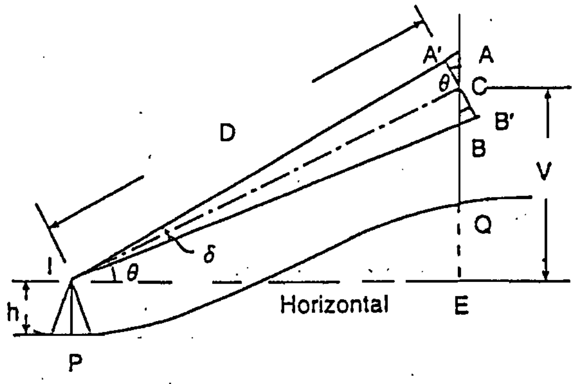


Fig. 15.9 Inclined sight (elevation).

As $A'CA = \theta$, $A'B' = AB \cos\theta$ with the small approximation of $\angle CA'A$ and $\angle CB'B$ being considered right angles. Actually $CA'A$ is $90° + \delta/2$ and $CB'B$ is $90° - \delta/2$.

But AB is the staff intercept S and

$$A'B' = S', \text{ hence } S' = S\cos\theta$$

Hence

$$D = KS' + C$$

$$= KS\cos\theta + C \qquad (15.16)$$

Horizontal distance

$$H = D\cos\theta = KS\cos^2\theta + C\cos\theta \qquad (15.17)$$

Similarly

$$V = IC\sin\theta$$

$$= (KS\cos\theta + C)\sin\theta$$

$$= KS\cos\theta\sin\theta + C\sin\theta$$

$$= \frac{1}{2}KS\sin 2\theta + C\sin\theta \qquad (15.18)$$

As the additive constant is zero for an external focussing telescope fitted with an anallactic lens and it is very small for an internal focussing telescope, the terms containing C can be neglected and we can write

$$H = KS\cos^2\theta \qquad (15.19)$$

$$V = \frac{1}{2}KS\sin 2\theta \qquad (15.20)$$

The R.L. of P is known. Hence R.L. of Q can be obtained as

$$\text{R.L. of } Q = \text{R.L. of } P + h + V - CQ$$

where CQ is the central hair reading.

When the angle θ is an angle of depression (Fig. 15.10)

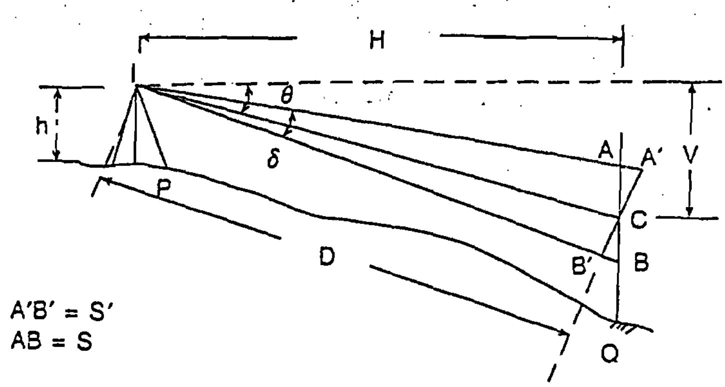


Fig. 15.10 Inclined sight (depression).

As before

$$D = KS' + C$$
$$= KS \cos \theta + C$$
$$H = D \cos \theta = KS \cos^2 \theta + C \cos \theta$$
$$V = D \sin \theta = KS \sin \theta \cos \theta + C \sin \theta$$
$$\text{R.L. of } Q = \text{R.L. of } P + h - V - CQ$$

where CQ is the central hair reading.

Inclined sight with staff normal to the line of sight (θ an angle of elevation), Fig. 15.11.

In this method, the staff is held normal to the line of sight, that is, IC is perpendicular to $AB = S$

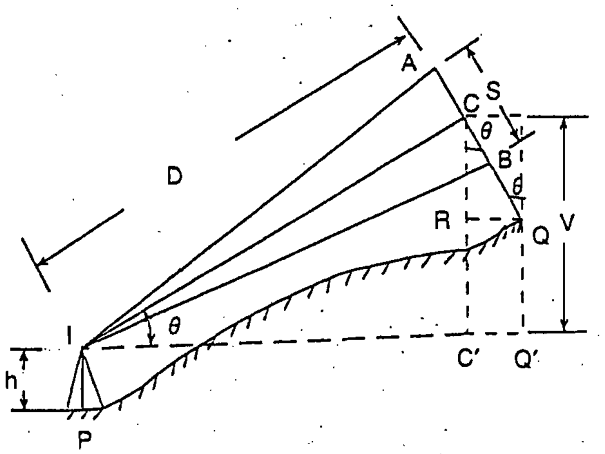


Fig. 15.11 Sight normal (when θ is angle of elevation).

Hence $$D = KS + C$$

If the distance of the middle hair reading from Q, i.e. CQ is taken as r, then

$$RQ = CQ \sin \theta = r \sin \theta$$

and $$CR = CQ \cos \theta = r \cos \theta$$

Hence horizontal distance between P and Q

$$IQ' = IC' + C'Q'$$
$$= D \cos \theta + r \sin \theta$$
$$= (KS + C) \cos \theta + r \sin \theta$$
$$\text{R.L. of } Q = \text{R.L. of } P + h + V - CR$$
$$= \text{R.L. of } P + h + D \sin \theta - r \cos \theta$$

When the line of sight is depressed downward (Fig. 15.12)

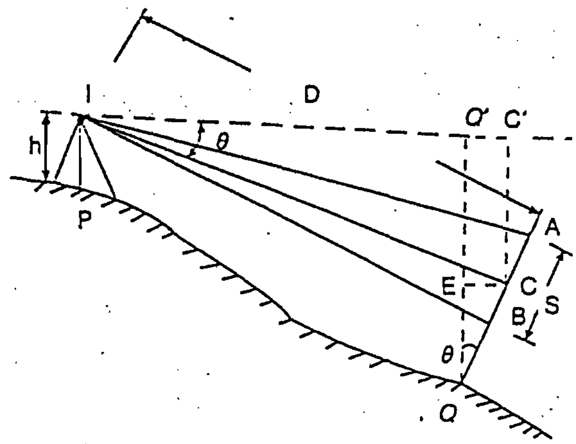


Fig. 15.12 Sight normal (When θ is angle of depression).

As before

$$IC = D = KS + C$$

$$IC' = IC \cos\theta = (KS + C).\cos\theta$$

If the distance of the middle hair $CQ = r$

$$CE = Q'C' = CQ \sin\theta = r \sin\theta$$

$$QE = CQ \cos\theta = r \cos\theta$$

Hence horizontal distance between P and Q

$$\begin{aligned} IQ' &= IC' - Q'C' \\ &= IC \cos\theta - r \sin\theta \\ &= D \cos\theta - r \sin\theta \\ &= (KS + C)\cos\theta - r \sin\theta \end{aligned}$$

$$\begin{aligned} \text{R.L. of } Q &= \text{R.L. of } P + h - CC' - EQ \\ &= \text{R.L. of } P + h - D \sin\theta - r \cos\theta \end{aligned}$$

15.8 MOVABLE HAIR METHOD

As already explained in fixed hair method, the stadia interval is fixed while the staff intercept changes with distance of the staff from the instrument station. This is shown in Fig. 15.13.

In the movable hair method, stadia interval varies with the movement of the hairs and as such the tacheometric angle β is not constant but changes to β_1, β_2 and β_3. Corresponding to stadia intercepts i_1, i_2 and i_3. However, the staff intercept S is kept fixed by having targets at a fixed interval say 3 m apart. This is shown in Fig. 15.14.

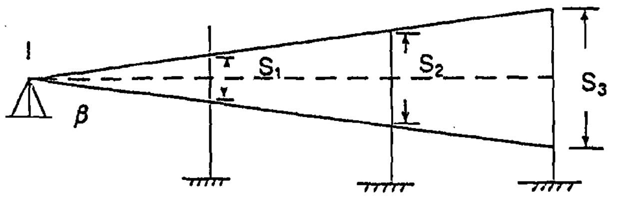


Fig. 15.13 Fixed hair method.

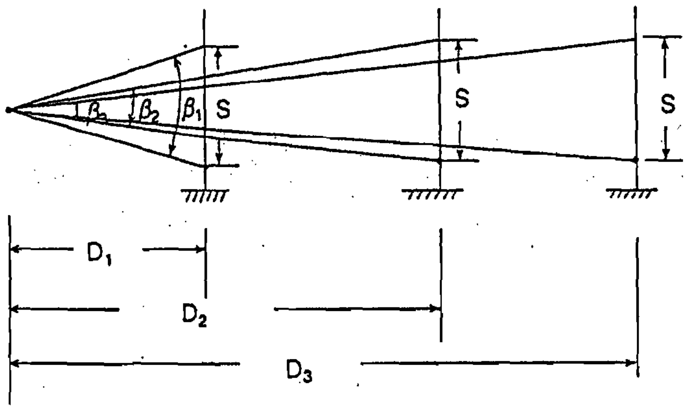


Fig. 15.14 Movable hair method.

Figure 15.15 shows the optical diagram of a subtense theodolite with movable hairs.

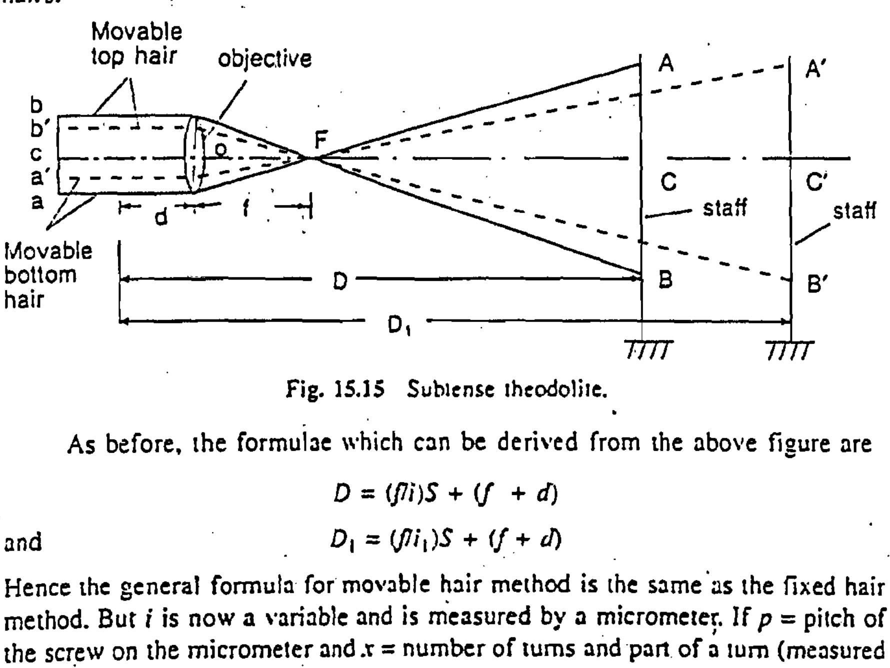


Fig. 15.15 Subtense theodolite.

As before, the formulae which can be derived from the above figure are

$$D = (f/i)S + (f + d)$$

and
$$D_1 = (f/i_1)S + (f + d)$$

Hence the general formula for movable hair method is the same as the fixed hair method. But i is now a variable and is measured by a micrometer. If p = pitch of the screw on the micrometer and x = number of turns and part of a turn (measured on the micrometer drum) to bring hairs to the targets, then,

$$i = px$$

and therefore $$D = \frac{fS}{px} + (f + d)$$

$$= \frac{KS}{x} + C \qquad (15.21)$$

where $K = f/p$

15.9 TANGENTIAL SYSTEM OF MEASUREMENT

In this method two readings are taken with the central hair at two targets spaced at a known fixed distance apart say 2 m or 3 m. Figures 15.16 (i) to 15.16 (iii) show how the observations are made when: (i) both angles are angles of elevation, (ii) both angles are angles of depression, and (iii) one angle, angle of elevation and the other angle of depression.

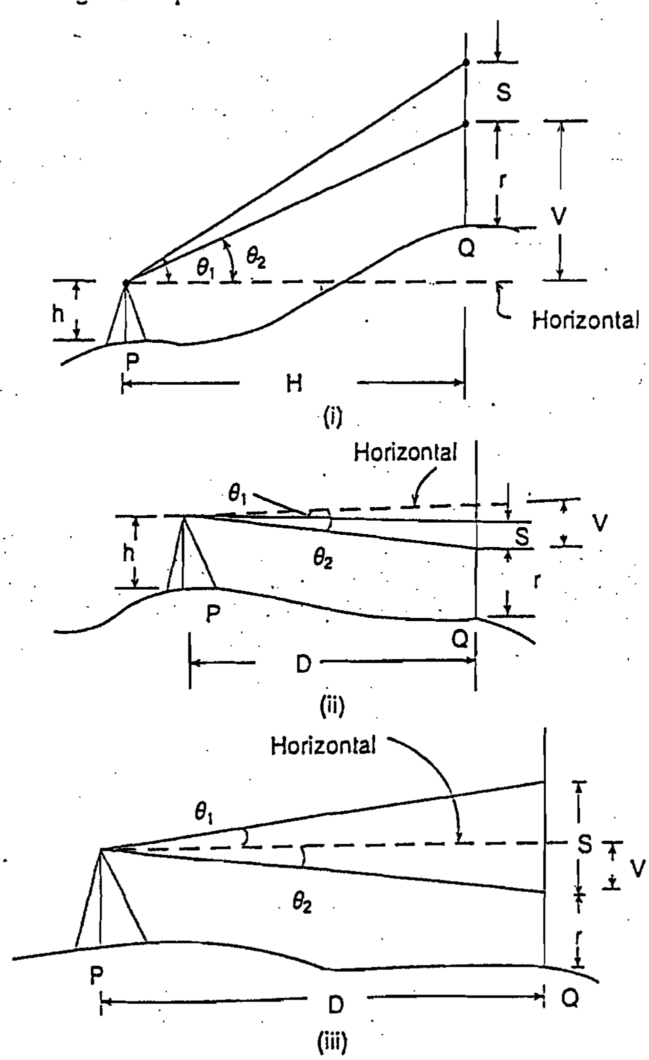


Fig. 15.16 Tangential system of measurement.

Case i When both angles are angles of elevation.

From Fig. 15.16(i) $V = H \tan \theta_2$

$$V + S = H \tan \theta_1$$

or $S = H(\tan \theta_1 - \tan \theta_2)$

or
$$H = \frac{S}{\tan \theta_1 - \tan \theta_2} \qquad (15.22)$$

$$V = H \tan \theta_2 = \frac{S}{\tan \theta_1 - \tan \theta_2} \tan \theta_2$$

$$= \frac{S \cos \theta_1 \cos \theta_2}{\sin \theta_1 \cos \theta_2 - \cos \theta_1 \sin \theta_2} \frac{\sin \theta_2}{\cos \theta_2}$$

$$= S \frac{\cos \theta_1 \sin \theta_2}{\sin (\theta_1 - \theta_2)} \qquad (15.23)$$

$$\text{R.L. of } Q = \text{R.L. of } P + h + V - r$$

Similar equations can be derived for case (ii) and case (iii).

15.10 SUBTENSE BAR

It is a bar of accurate length mounted horizontally over a tripod with a levelling head. The outer steel casing is hinged at the middle and contains invar wires anchored there and tensioned by springs at the target ends. The target holders themselves are of brass. Invar is used so that error in length due to variation of temperature is minimum. Fig. 15.17 shows the plan view of the subtense measurement.

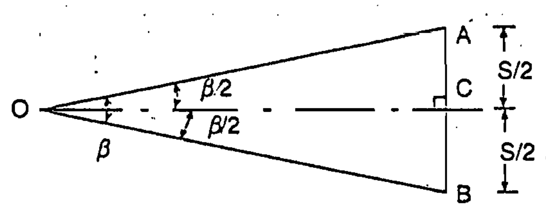


Fig. 15.17 Subtense bar (Plan view).

The theodolite is located at O. The two targets are at A and B and the midpoint of the targets is at C. From Fig. 15.17

$$\tan \beta/2 = \frac{AC}{OC}$$

or

$$OC \text{ (the perpendicular length)} = \frac{AC}{\tan \beta/2}$$

$$= \frac{S/2}{\tan \beta/2} = \frac{S}{2 \tan \beta/2}$$

If β is small $\tan \beta/2 = \beta/2$ where β is in radian.

$$= \frac{1}{2}\,\frac{\beta}{206265}$$

if β is in seconds.
As 1 radian = 206265 seconds.

$$D = \frac{S \times 206265}{\beta} \tag{15.24}$$

where β is in second

Unit of D will be unit of S. S is usually 2 m. A table is supplied with the subtense bar which gives distances in meters corresponding to values of β. The instrument for subtense measurement should be accurate to 1″ of angle or less. This accuracy can be obtained by measuring the angle with a 1″ theodolite in several positions of the circle. There is no need to reverse the telescope in these measurements because both targets are at the same vertical angle and at the same distance from the theodolite. Also the angle is obtained by the difference between the two directions to the targets. Thus an instrumental error for one pointing equals that for the other pointing. These errors are eliminated by the subtraction of one direction from the other.

15.11 COMPUTATIONS WITH INCOMPLETE INTERCEPTS

For long sights and large vertical angles, the stadia intercept may exceed the rod length. Therefore reading both the lower and upper crosshairs is not possible. In practice this problem is overcome by observing the half-intercept between the centre and lower (or upper) crosshairs and doubling the observed value for use with the standard stadia formulas. Doubling the half-intercept or quadrupling the quarter intercept provides the precise full intercept only for horizontal sights. For inclined sights, the computed value would be different from the actual full intercept because the half or quarter intercepts are not equal. The resulting discrepancies in the computed horizontal distance and difference in elevation may be substantial. To eliminate these discrepancies, it is necessary to determine the precise full intercept that corresponds to the observed half or quarter intercept.

When it is necessary to observe the half (or quarter) intercept, the corresponding stadia intercept S is not generally equal to twice (or four times) the observed value.

Establishing S for half-intercepts

The stadia intercept S is the sum of the lower half intercept l and the upper half intercept u. Thus

$$S = l + u$$

l and u are equal only when $\alpha = 0$. For positive α, l is always smaller than u and vice versa for negative α. On the other hand half intercepts of S' are always equal.

From the triangle PEO, $EO = u \cos \alpha$, $EP = u \sin \alpha$ and from triangle PEP'

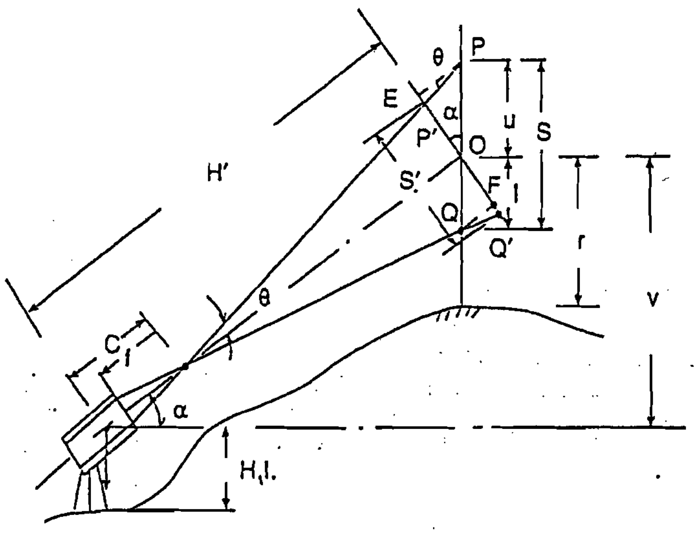


Fig. 15.18 Computations with incomplete intercept.

$$EP' = EP \tan \theta \doteq u \sin \alpha \tan \theta$$

$$OP' = S'/2 = OE - EP' = u \cos \alpha - u \sin \alpha \tan \theta$$

where θ is half the angle between the upper and lower cross hairs.
Rearranging,

$$u = \frac{S'}{2(\cos \alpha - \sin \alpha \tan \theta)} \tag{15.25}$$

Similarly, from the triangles $QQ'O$ and $QQ'F$, the following relationship can be obtained

$$l = \frac{S'}{2(\cos \alpha + \sin \alpha \tan \theta)} \tag{15.26}$$

Substituting u and l in the equation

$$S = u + l$$

$$S' = S(\cos \alpha - \tan \alpha \sin \alpha \tan^2 \theta) \tag{15.27}$$

As $$\tan \theta = \frac{S'}{2H'}$$

$$\tan \theta = \frac{S'}{2(KS' + C)} = \frac{1}{2K}$$

since C is very small compared to KS'.

The second term in parentheses on the right hand side of Eq. (15.27) is quite negligible compared to cos α (For $\alpha = 50°$, $K = 100$, the second term equals 2×10^{-5}). Eq. (15.27) then reduces to

$$S' = S \cos \alpha$$

Substituting for S' and $\tan\theta$
S can be expressed in terms of u and l.

$$S = 2u\left(1 - \frac{\tan\alpha}{2K}\right) \text{ (for the half intercept } u) \tag{15.28}$$

$$= 2l\left(1 + \frac{\tan\alpha}{2K}\right) \text{ (for the half intercept } l) \tag{15.29}$$

As expected, the two half intercepts are equal for $\alpha = 0$. For $\alpha > 0$, $l < u$ and for $\alpha < 0$, $l > u$.

For the lower half intercepts, the stadia formulae changes to

$$H = 2Kl\left(1 + \frac{\tan\alpha}{2K}\right)\cos^2\alpha + C\cos\alpha \tag{15.30}$$

$$V = Kl\left(1 + \frac{\tan\alpha}{2K}\right)\sin 2\alpha + C\sin\alpha \tag{15.31}$$

Example 15.1 A theodolite is fitted with an ordinary telescope in which the eyepiece end moves in focussing, the general description being as follows:

Focal length of objective $f = 22.5$ cm

Fixed distance c between objective and vertical axis = 11.25 cm

Diaphragm lines are on glass in cell which may be withdrawn.

It is desired to convert the instrument into an anallactic lens tacheometer by inserting an additional positive lens in tube and ruling a new diaphragm so as to give a multiplier of 100 for intercepts on a vertical staff and in this connection it is found that 18.75 cm will be a convenient value for the *fixed* distance between the objective and the anallactic lens. Determine (a) a suitable value for the focal length f' of the anallatic lens (b) the exact spacing of the lines on the diaphragm.

Solution By simple conjugate relations

$$c = \frac{f(d - f')}{(f + f' - d)} \tag{i}$$

and by the general theorem

$$100 = \frac{ff'}{i(f + f' - d)}$$

or
$$d = (f + f') - \frac{ff'}{100\,i}$$

By (i)
$$11.25 = \frac{22.5\,(18.75 - f')}{3.75 + f'}$$

or $f' = 11.25$ cm

$$i = \frac{ff'}{100(f + f' - d)} = \frac{22.5 \times 11.25}{100(22.5 + 11.25 - 18.75)}$$

$$= 0.16875 \text{ cm}$$

Example 15.2 In a telescope the focal lengths of the objective and anallactic lens are 22.5 and 11.25 respectively and the constant distance between these is 20 cm for a multiplier of 100. Determine the error that would occur in horizontal distance D when reading intercepts S, also in m, with an error of 1/500 of a cm in the 0.175 cm interval between the subtense lines.

Solution We have

$$D = \frac{ff'S}{(f+f'-d)i}$$

Differentiating,

$$\Delta D = -\frac{ff'S\,\Delta i}{(f+f'-d)i^2}$$

$$= -\frac{22.5 \times 11.25}{(22.5 + 11.25 - 20)} \frac{S \times 100}{500} \frac{1}{(0.175)^2}$$

$$= -120.22\ S \text{ cm where } S \text{ is in m}$$

Example 15.3 An internal focussing telescope has a negative lens of 15 cm focal length, and the fixed distances from the objective to the diaphragm and vertical axis are 22.5 cm and 10 cm respectively, the focal length of the objective being 20 cm. A subtense interval is to be scribed on the diaphragm so as to give an anallactic multiplier of 100 for a horizontal sight on a staff held vertically 100 m horizontally from the axis of the theodolite. Determine the exact spacing of the subtense lines.

Solution

$$f_1 = 10000 - 10 = 9990 \text{ cm}$$

$$f_2 = \frac{ff_1}{f_1 - f} = \frac{20 \times 9990}{9990 - 20} = 20.04 \text{ cm}$$

$$f_2 = \frac{ld + lf' - d^2}{l - d + f'} = 20.04$$

or

$$\frac{22.5\,d + 22.5 \times 15 - d^2}{22.5 - d + 15} = 20.04$$

or

$$d^2 - 42.54d + 414 = 0$$

or

$$d = 15.07 \text{ cm or } 27.48 \text{ cm}$$

Admissible

$$d = 15.07 \text{ cm}$$

$$f_1 - f = \frac{f}{i}\left(\frac{l - d + f'}{f'}\right)S$$

or

$$9970 = \frac{20}{i}\left(\frac{22.5 - 15.07 + 15}{15}\right)100$$

or

$$i = 0.3 \text{ cm}$$

Example 15.4 The readings given below were made with a tacheometric theodolite having a multiplying constant of 100 and no additive constant. The reduced level at station *A* was 100.0 m and the height of the instrument axis 1.35 m above the ground. Calculate the gradient expressed as the horizontal distance one meter rise or fall vertically between the stations *B* and *C* (Fig. 15.19).

Station	To	Whole circle bearing from N	Vertical angle	Stadia readings
A	*B*	48°00′	+ 11°30′	2.048/1.524/1.000
	C	138°00′	− 17°00′	2.112/1.356/0.600

Solution From station *A* to *B*

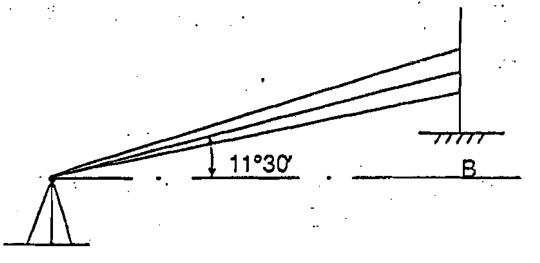


Fig. 15.19 Example 15.4.

$$H = KS \cos^2 \theta$$

Here $K = 100, \quad S = 2.048 - 1.000 = 1.048$

$$\theta = 11°30'$$

$$H = 100 \times 1.048 \times \cos^2 11°30'$$

$$= 100.634 \text{ m}$$

$$V = \frac{1}{2} KS \sin 2\theta$$

$$= \frac{1}{2} \times 100 \times 1.048 \times \sin 23°$$

$$= 20.47 \text{ m}$$

$$\text{R.L. of station } B = \text{R.L. of } A + 1.35 + 20.47 - 1.524$$

$$= \text{R.L. of } A + 20.296 \text{ m}$$

From station *A* to *C* (Fig. 15.20)

$$H = KS \cos^2 \theta$$

$$= 100\,(2.112 - 0.600) \times \cos^2 17°$$

$$= 138.28 \text{ m}$$

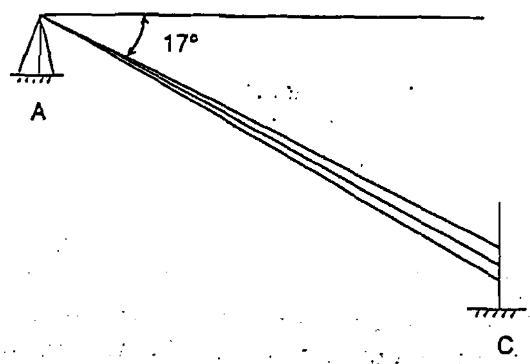


Fig. 15.20 Example 15.4.

$$V = \frac{1}{2} KS \sin 2\theta$$

$$= \frac{1}{2} \times 100 \times 1.512 \times \sin 34°$$

$$= 42.27 \text{ m}$$

R.L. of station C = R.L. of A + 1.35 − 42.27 − 1.356

= R.L. of A − 42.276

Difference of level between B and C (Fig. 15.21)

= 62.572 m

$$BC = (100.634^2 + 138.28^2)^{1/2}$$

$$= 171.022$$

$$\text{slope } BC = \frac{62.572}{171.022}$$

$$= \frac{1}{2.73}$$

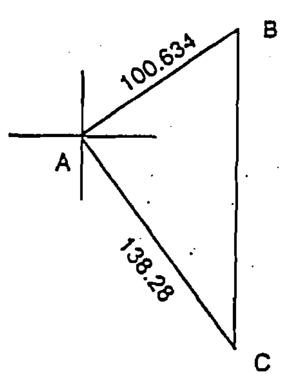


Fig. 15.21 Example 15.4.

Example 15.5 Two sets of tacheometric readings were taken from an instrument station A the reduced level of which was 15.05 m to a staff station B.

(a) Instrument P—multiplying constant 100, additive constant 360 mm, staff held vertical.

(b) Instrument Q—multiplying constant 95, additive constant 380 mm. Staff held normal to line of sight.

Instrument	At	To	Height of instrument (m)	Vertical angle	Stadia readings (m)
P	A	B	1.38	+ 30°	0.714/1.007/ 1.300
Q	A	B	1.36	+ 30°	—

What should be the stadia readings with instrument Q?

Solution

Instrument P

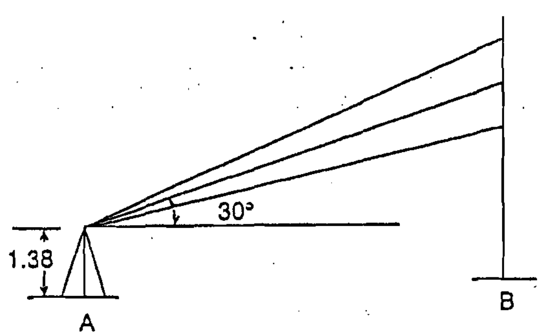


Fig. 15.22 Example 15.5.

$$H = KS\cos^2\theta + C\cos\theta$$

Here, $K = 100$, $S = (1.300 - 0.714) = 0.586$

$$\theta = 30°$$

$$\therefore \quad H = 100 \times 0.586 \times \cos^2 30 + 0.36 \times \cos 30°$$

$$= 44.26 \text{ m}$$

$$V = \frac{KS}{2}\sin 2\theta + C\sin\theta$$

$$= \frac{100 \times 0.586}{2} \cdot \sin 60° + 0.36 \times \sin 30°$$

$$= 25.55 \text{ m}$$

$$\text{R.L. of } B = 15.05 + 1.38 + 25.55 - 1.007$$

$$= 40.973 \text{ m}$$

Instrument Q

$$H = D\cos\theta + r\sin\theta$$

$$D = KS + C$$

$$= 95\,S + 0.38$$

$$44.26 = (95\,S + 0.38)\cos 30° + r\sin 30°$$

$$= (95\,S + 0.38)(0.866) + r\,(0.5)$$

$$= 82.27\,S + 0.33 + 0.5\,r$$

or $$82.27\,S + 0.5\,r = 43.93 \quad (1)$$

$$V = D\sin\theta = (95\,S + 0.38)\,0.5$$

$$= 47.5\,S + 0.19$$

$$\text{R.L. of } B = \text{R.L. of } A + 1.36 + (47.5\,S + 0.19) - 0.86\,r$$

or $$40.973 = 15.05 + 1.36 + (47.5\,S + 0.19) - 0.86\,r$$

or $$47.57\,S - 0.86\,r = 24.37 \quad (2)$$

Solving simultaneously (1) and (2)

$$r = 0.896$$

$$S = 0.528$$

$$S/2 = 0.264$$

Hence readings are: 0.632/0.896/1.160

Example 15.6 You are given a theodolite fitted with stadia hairs, the object glass of telescope being known to have a focal length of 230 mm and to be at a distance of 138 mm from the trunnion axis. You are told that the multiplying constant for the instrument is belived to be 180. The following tacheometric readings are then taken from an instrument station *A*, the reduced level of which is 15.05 m.

Instrument at	Height of instrument (m)	To	Vertical angle	Stadia readings	Remarks
A	1.38	*B*	+ 30°	1.225/1.422/1.620	Staff held vertical R.L. of B = 40.94 m
A	1.38	*C*	+ 45°	1.032/1.181/1.330	Staff held normal to line of sight

What is the error in the calculated reduced level of *C* if the multiplying constant of the instrument is taken as 180? Calculate the horizontal distance *AC* using the correct multiplying constant.

Solution

$$V = \frac{KS}{2} \sin 2\theta + C \sin \theta$$

Here $S = (1.620 - 1.225)$

$= 0.395$ m

$\theta = 30°$

$C = 230 + 138 = 368$ mm

$= 0.368$ m

$$V = \frac{K}{2} 0.395 \sin 60° + 0.368 \sin 30°$$

$= 0.171\ K + 0.184$

R.L. of B = R.L. of $A + h + V -$ central hair reading

$= 15.05 + 1.38 + 0.1710\ K + 0.184 - 1.422$

or $40.94 = 15.05 + 1.38 + 0.1710\ K + 0.184 - 1.422$

or $K = \dfrac{25.748}{0.171} = 150.57$

Error $= KS \sin \theta$

$= (180 - 150.57)\ (1.330 - 1.032) \sin 45°$

$= 29.43 \times .298 \times \dfrac{1}{\sqrt{2}}$

$= 6.202$ m

$H = (KS + C) \cos \theta + r \sin \theta$

$= (150.57 \times .298 + 0.368) \cos 45° + 1.181 \sin 45°$

$= 32.83$ m

15.12 RELATIVE MERITS OF HOLDING THE STAFF VERTICAL OR NORMAL

Vertical holding

This is usually done by fitting a small circular spirit level or a single level tube with its axis perpendicular to the face of the staff. In some case a plumb bob is attached to ensure that the staff is held vertical while holding. Effect of non verticality on distance can be studied. Let the vertical staff $ABCD$ tilt backward or forward through a small angle β. Assuming that $\angle BXX_1$, $\angle BYC$, B_1XA_1, and B_1Y_1Y are all right angles, then from Fig. 15.23(i) with θ an angle of elevation.

$$XY = AC \cos \theta = S \cos \theta$$

$$X_1Y_1 = A_1C_1 \cos (\theta + \beta) = S_1 \cos (\theta + \beta)$$

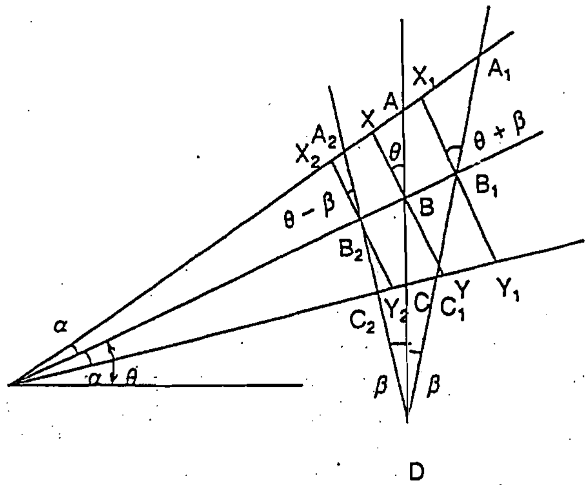


Fig. 15.23(i) Staff deviated from vertical.

Assuming $$XY \approx X_1Y_1$$

$$S \cos\theta = S_1 \cos(\theta + \beta)$$

or $$S = \frac{S_1 \cos(\theta + \beta)}{\cos\theta}$$

This gives the corrected reading S with staff vertical compared with the actual reading S_1 taken on the inclined staff.

The error e in horizontal length

$$H_T - H_A = KS\cos^2\theta - KS_1\cos^2\theta$$

$$= KS_1 \frac{\cos(\theta \pm \beta)}{\cos\theta}\cos^2\theta - KS_1\cos^2\theta$$

$$= KS_1\cos^2\theta\left[\frac{\cos(\theta \pm \beta)}{\cos\theta} - 1\right]$$

Expressed as a ratio,

$$\frac{H_T - H_A}{H_A} = \frac{\cos(\theta \pm \beta)}{\cos\theta} - 1$$

$$= \frac{\cos\theta\cos\beta \mp \sin\theta\sin\beta - \cos\theta}{\cos\theta}$$

$$= \cos\beta \mp \tan\theta\sin\beta - 1$$

If β is small and is less than 5°

$$\cos\beta \approx 1$$

$$\sin\beta \approx \beta$$

$$\frac{H_T - H_A}{H_A} = \beta\tan\theta$$

It can be observed that error is dependent on θ and it increases very rapidly with increase of θ as $\tan\theta$ increases very rapidly with increase of θ.

Normal holding

Figure 15.23 (ii) shows what happens when the staff deviates from normal by a small angle β. Assuming

$$A_1C_1 = AC = S$$

$$A_1C_1 = A_2C_2 \cos\beta$$

i.e.

$$S = S_1 \cos\beta$$

Then the error ratio $= \dfrac{KS_1 - KS}{KS_1} = 1 - \dfrac{S}{S_1} = 1 - \cos\beta$

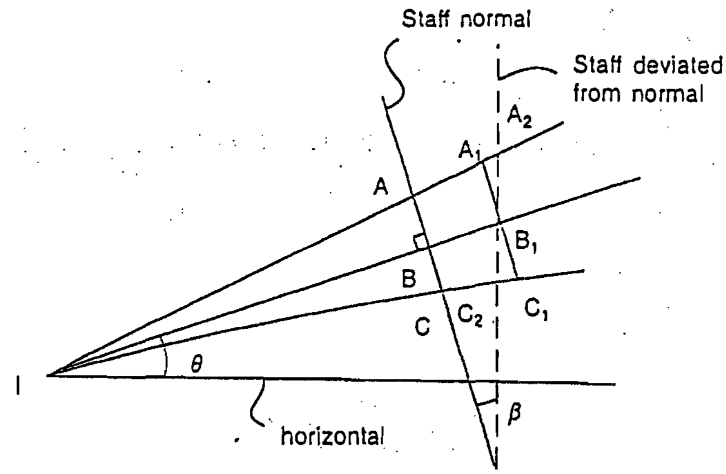


Fig. 15.23 (ii) Staff deviated from normal.

Thus the ratio is independent of the angle θ

Now $$H = D\cos\theta + r\sin\theta$$

$\therefore$ $$\frac{\partial H}{\partial \theta} = -D\sin\theta + r\cos\theta$$

or $$\partial H = (-D\sin\theta + r\cos\theta)\,\partial\theta$$

To keep the staff normal to the line of sight, a small collimator, provided with a lens and a reflecting mirror is attached to the staff and the staff man looks through the reflector inclining the staff at the same time till the light flash is obtained which happens when the line of sight strikes the object glass of the tacheometer.

Example 15.7 In a tacheometric survey, an interval of 0.825 m was recorded on a staff which was believed to be vertical and the vertical angle measured on the theodolite was 15°. Actually the staff which was 4 m long was 150 mm out of plumb and leaning backwards away from the instrument position. If the multiplying constant of the instrument is 100 what is the error in horizontal distance? In what conditions will the effect of not holding the staff vertical be most serious? What alternative procedure can be adopted in such condition?

Solution

$$\beta = \tan^{-1}\frac{0.150}{4.00} = 2.1475°$$

$$S = \frac{S_1 \cos(\theta + \beta)}{\cos\theta} = \frac{0.825 \cos(15 + 2.1475)}{\cos 15°}$$

$$\delta H = KS_1 \cos^2\theta \left[\frac{\cos(\theta+\beta)}{\cos\theta} - 1\right]$$

$$= 100\ (0.825) \cos^2 15° \left[\frac{\cos 17.1475}{\cos 15°} - 1\right]$$

$$= 100\ (0.825)\ (0.933) \left[\frac{0.9555}{0.9659} - 1\right] = -0.831 \text{ m}$$

The error will be very serious when the angle θ, vertical angle measured on the theodolite is very large. In such a case staff should be held normal to the line of sight.

15.13 PROBLEMS IN PRACTICAL APPLICATION OF TANGENTIAL METHOD

In tangential method θ_1, θ_2, and S are to be measured. There are two options:

(a) Constant base, i.e. S is kept fixed while θ_1 and θ_2 are observed for each position of the staff.

(b) Variable base i.e. S is variable for each staff station but θ_1 and θ_2 are pre-selected.

It is already derived

$$H = \frac{S}{\tan\theta_1 - \tan\theta_2}$$

It is possible to select θ_1 and θ_2 such that $\tan\theta_1$ and $\tan\theta_2$ become simple fraction of 100, such as, 0.03 or 3%, 0.02 or 2%. Then

$$H = \frac{S}{.03 - .02} = \frac{S}{.01} = 100\,S$$

This enables quicker and simpler computation of S. From trigonometrical tables it is found that

$$\tan 0°\ 34'\ 24'' = 0.01$$

$$\tan 1^\circ\ 08'\ 45'' = 0.02$$
$$\tan 1^\circ\ 43'\ 06'' = 0.03$$

Similarly angles for other percentages can be obtained from tables. Though computation becomes easier it is difficult to set angles at such odd values. Furgusson has solved this problem by dividing a circle inscribed in a circle in eight octants. This is shown in Fig. 15.24. Each line such as *OB* is divided into 100 parts. Lines are then drawn from the centre to these points, thus dividing each octant into 100 unequal parts. The points of division on the circle are marked 0 to 100. The divisions are further read to 0.01 of a unit by means of special drum micrometer.

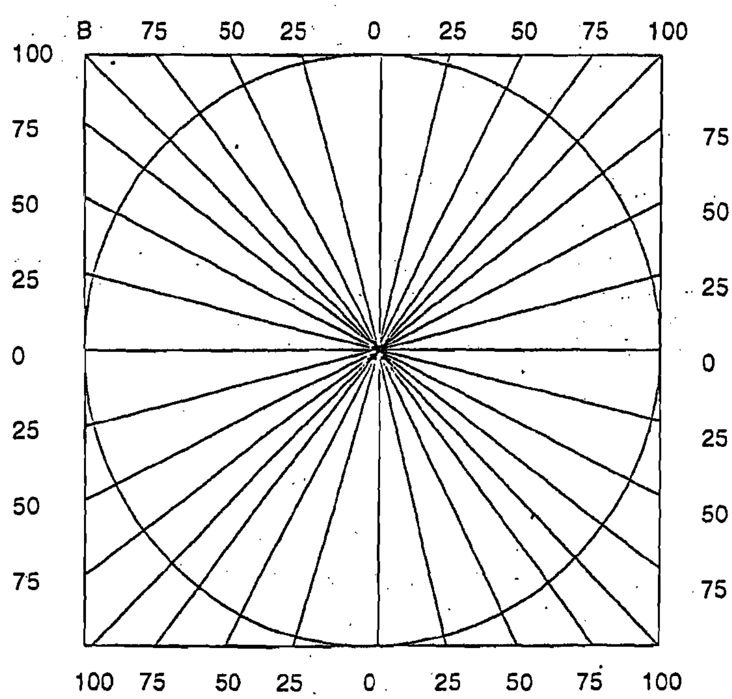


Fig. 15.24 Fergusson's chart. (Tangential method, smaller divisions not shown.)

Effect of Angular Error in tangential measurement can be mathematically studied. It is known

$$H = \frac{S}{\tan\theta_1 - \tan\theta_2}$$

when θ_1 and θ_2 are both angles of elevation. If the probable error of measuring each of the angles is 20″, then $\delta\theta_1 = +20''$ and $\delta\theta_2 = -20''$.

Then
$$H_1 = \frac{S}{\tan(\theta_1 + 20'') - \tan(\theta_2 - 20'')}$$

Let
$$\tan(\theta_1 + 20'') = \tan\theta_1 + a_1$$
$$\tan(\theta_2 - 20'') = \tan\theta_2 - a_2$$

Then
$$H_1 = \frac{S}{\tan\theta_1 + a_1 - \tan\theta_2 + a_2}$$
$$= \frac{S}{(\tan\theta_1 - \tan\theta_2) + (a_1 + a_2)}$$

For small differences in angles θ_1 and θ_2 the tangent differences are approximately equal. Hence $a_1 \approx a_2 = a$.

If $\tan\theta_1 - \tan\theta_2$ is taken equal to q

$$S = Hq$$
$$S = H_1 (q + 2a)$$

or
$$\frac{H}{H_1} = \frac{q + 2a}{q}$$

or
$$\frac{H - H_1}{H_1} = \frac{2a}{q}$$

or
$$\frac{e}{H_1} = \frac{2a}{q}$$

where e is the error in horizontal length $H - H_1$

Then, Ratio of error
$$r = \frac{e}{H_1} \approx \frac{e}{H}$$

or
$$r = \frac{2a}{q} = \frac{2a}{S/H} = \frac{2aH}{S}$$

As an example let H = 100 m, $\theta_1 = 5°$ and S = 3 m

$$\tan 5° = 0.0874887$$
$$\delta\theta_1 = \pm 5''$$
$$a = 0.0000978$$
$$r = \frac{2aH}{S} = \frac{2 \times 0.0000978 \times 100}{3}$$
$$= \frac{1}{153374}$$

Under such condition if r is not to exceed 1/500, we get

$$\frac{1}{500} = \frac{2 \times a \times 100}{3}$$

or
$$a = \frac{3}{500 \times 2 \times 100}$$
$$= 3 \times 10^{-5}.$$
$$\tan 5° = 0.0874887$$
$$\tan (5 + x) = 0.0875187$$

$$\text{or} \qquad 5 + x = 5.0017058°$$

$$\text{or} \qquad x = .0017058°$$

$$= 6.14''$$

Thus the permissible angular error is ± 6.14″.

Example 15.8 The vertical angles to vanes fixed at 1 m and 3 m above the foot of staff held vertically at a station 'A' were 3°10′ and 5°24′ respectively (Fig. 15.25). Find the horizontal distance and the reduced level of A if the height of the instrument axis is 138.556 meters above datum.

[AMIE Surveying Summer 1987]

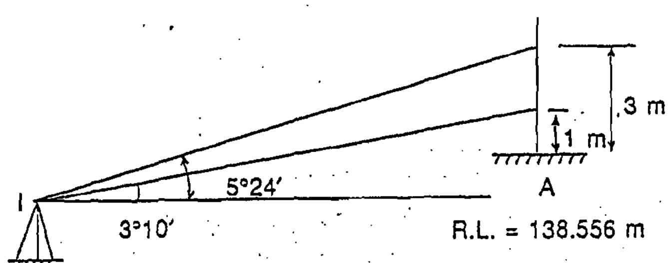


Fig. 15.25 Example 15.8.

Solution

Here $S = 3 - 1 = 2$ m, $\theta_1 = 5°24'$, $\theta_2 = 3°10'$

$$H = \frac{S}{\tan\theta_1 - \tan\theta_2}$$

$$= \frac{2}{\tan 5°24' - \tan 3°10'}$$

$$= \frac{2}{0.0945278 - .0553251}$$

$$= 51.0168 \text{ m}$$

$$\text{R.L. of } A = \text{R.L. of } I + 51.0168 \tan 5°24' - 3.00$$

$$= 138.556 + 4.822 - 3.00$$

$$= 140.378 \text{ m}$$

Example 15.9 An observation with a percentage theodolite gave staff readings as 1.155 and 2.655 for angles of elevation 4.5% and 5.5% respectively. On sighting the graduation corresponding to the height of the instrument axis above the ground, the vertical angle was 5%. Compute the horizontal distance and the elevation of the staff station if the instrument has an elevation of 500.512 m.

Solution

$$\tan\theta_1 = .055$$

$$\tan\theta_2 = .045$$

$$\therefore \quad H = \frac{S}{\tan\theta_1 - \tan\theta_2} = \frac{2.655 - 1.155}{.055 - .045}$$

$$= 150.00 \text{ m.}$$

$$V = H \tan\theta_2 = 150.00 \times .045$$

$$= 6.75 \text{ m}$$

Let the angle to the graduation corresponding to the height of the instrument be α_3 so that $\tan \alpha_3 = 0.05$

If S' is the corresponding staff intercept

$$H = \frac{S'}{\tan\theta_1 - \tan\theta_3} = \frac{S'}{.055 - .05}$$

or $$S' = .005 \times 150 = 0.75 \text{ m}$$

If r be the staff reading corresponding to the height of the instrument

$$r = 2.655 - 0.75 = 1.905 \text{ m}$$

$$\therefore \quad \text{R.L. of staff} = \text{R.L. of } IA + V - 1.155$$

$$= (500.512 + 1.905) + 6.750 - 1.155$$

$$= 508.012 \text{ m}$$

15.14 TACHEOMETRIC CALCULATIONS AND REDUCTIONS

The great disadvantage of tacheometry is that it requires elaborate calculations to find out the horizontal distances and elevations of points. To help in computations (i) stadia tables, (ii) stadia diagrams, (iii) stadia slide rules can be used. Special instruments like Beaman Stadia Arc and Jeffcot Direct reading tacheometer enable immediate reductions of horizontal distances and elevations of points.

Beaman stadia arc

This is a mechanical device fitted to the vertical circle of a tacheometer (Fig. 15.26). There are two scales: one for calculating horizontal distances and the other for vertical distances. In tacheometry for inclined sights $H = KS \cos^2 \theta$.
If the line of sight is assumed horizontal

$$H' = KS$$

Hence $$H' - H = KS\,(1 - \cos^2 \theta)$$

$$= KS \sin^2 \theta$$

The horizontal scale, therefore, gives the values of the percentage correction 100 $\sin^2 \theta$ that should be subtracted from 100 S. The graduations on the scale meant for vertical distances are in terms of $100 \times \sin 2\theta/2$ $\left[\text{as } V = KS\frac{1}{2}\sin 2\theta\right]$ and are read against an index mark. The central graduation of the V scale is marked 50 and a reading of less than 50 indicates that the telescope is inclined downward

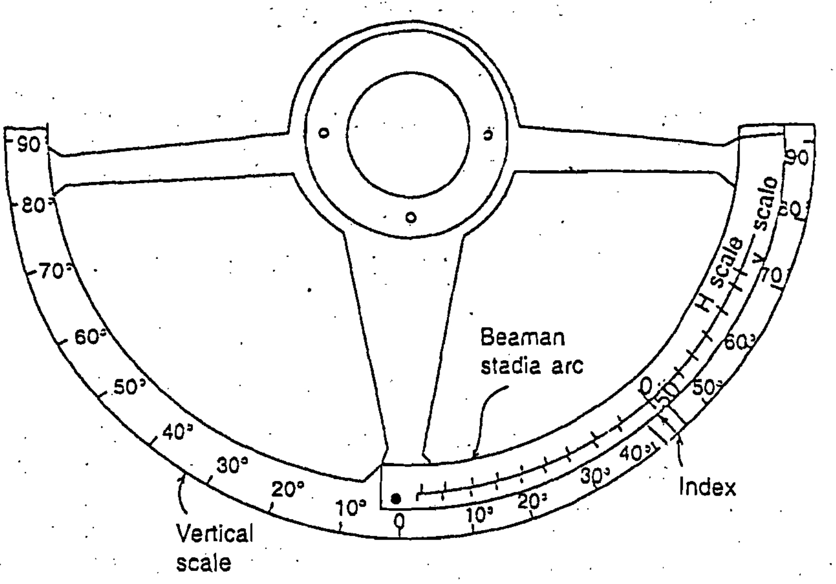


Fig. 15.26 Beaman stadia arc (schematic diagram).

while a reading greater than 50 shows it is inclined upwards. The value of V is then given by $V = S \times$ (Reading on V scale – 50).

The graduations on the vertical scale is based on the computation that $\frac{1}{2}$ sin 2θ for each graduation is a magnitude of 0.01. Hence the first division is 0.01, second division 0.02 and so on. The corresponding angles of θ which gives

$$\frac{1}{2} \sin 2\theta = 0.01 \text{ or } \theta = 0°34'23''$$

$$\frac{1}{2} \sin 2\theta = 0.02 \text{ or } \theta = 1°08'46''$$

and so on.

If the index is not against the whole number reading of the V scale, it is brought to a whole number by the tangent screw. This does not appreciably change the value of S and hence the result.

Example: Central hair reading — 1.315 m

Reading on V scale — 55

Reading on H scale — 3

Staff intercept = 1.145

Elevation of I.A. = 140.50

Assume K = 100 and C = 0

$$V = 1.145 \times (55 - 50) = +\ 5.725$$

$$\text{Elevation of staff} = 140.50 + 5.725 - 1.315 = 144.910 \text{ m}$$

Horizontal Correction = 1.145 × 3
= 3.435 m
Horizontal distance = 1.145 × 100 − 3.435
= 114.500 − 3.435
= 111.065 m

Jeffcot direct reading tacheometer

This is the first direct reading tacheometer which was invented by H.H. Jeffcot to directly obtain the horizontal distance and vertical intercept and thus avoiding tedious tacheometer computations. The diaphragm consists of three platinum-iridium pointers. The central one is fixed while the other two are movable and actuated by cams. The two cams are fixed in position but so shaped that the interval between the pointers is adjusted automatically with the variation of the vertical angle of the telescope. The intercept between the central pointer and the bottom pointer on the right side multiplied by 100 gives the horizontal distance *D* and the intercept between the central pointer and the top left hand pointer multiplied by 10 gives the vertical component. Figures 15.27(a), (b), (c) and (d) explain

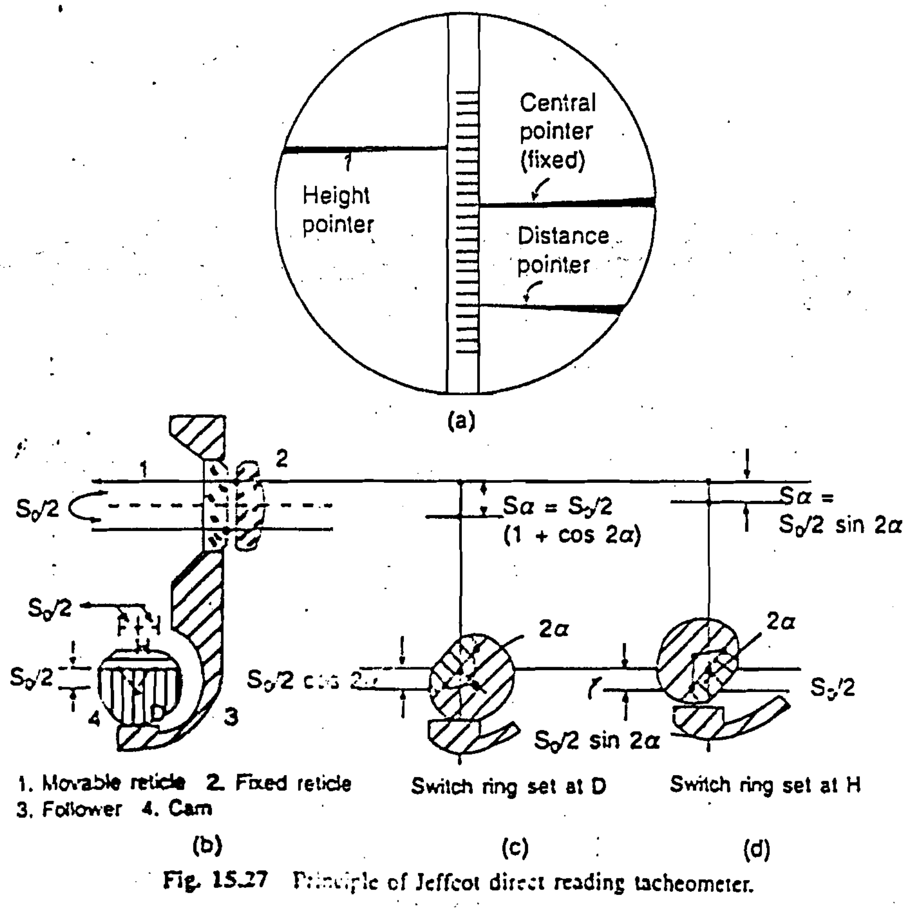


Fig. 15.27 Principle of Jeffcot direct reading tacheometer.

the principle. The stadia rod readings are taken by first setting the fixed pointer at a whole decimeter mark and then reading the other two pointers. If the reading of the fixed pointer is greater than that of height pointer, the vertical intercept (V) is positive and vice versa.

Example Let the readings of the

$$
\begin{aligned}
\text{distance pointer} &= 2.850\\
\text{height pointer} &= 0.350\\
\text{fixed pointer} &= 2.000\\
\text{Horizontal distance} &= 100 \times (2.850 - 2.000)\\
&= 85 \text{ m}\\
\text{Vertical intercept} &= 10(2.0000 - 0.350)\\
&= 10(1.650) = +16.50 \text{ m}
\end{aligned}
$$

This instrument is not much used these days because: (i) Pointers cannot be read easily, (ii) It is difficult to measure half intercepts, and (iii) Parallax error is difficult to avoid.

Szepessy direct reading tacheometer

It is a very popular direct reading tacheometer of the tangent group. It uses percentage angles. A scale of tangents of vertical angles is engraved on a glass arc which is fixed to the vertical circle cover of the instrument. The scale is divided to 0.005 but marked at every 0.01. As the graduation is in percentage 0.005 means an angle whose tangent is 0.00005 or 0.005%. A number of prisms reflect the scale in the view of the eye piece and when the staff is sighted the image of the staff is seen along with the scale as shown in Fig. 15.28. The following are the steps for using the tacheometer:

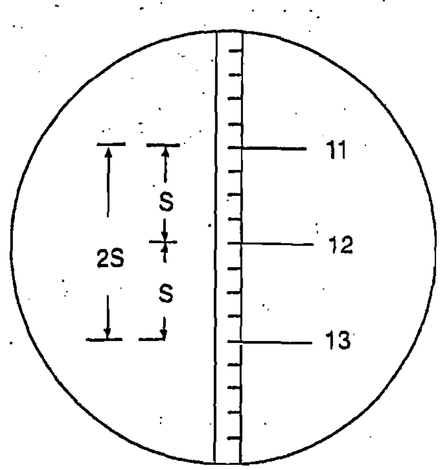


Fig. 15.28 Szepessy tacheometer.

(a) Sight the stadia and clamp the instrument at some convenient position.

(b) By the vertical circle tangent screw bring the axial hair to a whole number division.

(c) Read the stadia between two consecutive whole numbers. The staff intercept multiplied by 100 gives the horizontal distance D.

(d) The vertical intercept is obtained by multiplying the staff intercept with the axial hair reading.

For example, if the staff intercept $S = 1.45$ and the axial hair reading is 12, then

$$H = 1.45 \times 100 = 145 \text{ m}$$

$$V = 1.45 \times 12 = 17.40 \text{ m}$$

Self reducing tacheometers

They differ from conventional tacheometers in the fact that the interval between the stadia lines varies automatically with the telescope inclination. This is made possible by having stadia lines etched on a special glass circle called a diagram through which the line of sight is directed. This replaces the diaphragm in the telescope. The diagram rotates about the trunnion axis as it is connected to the telescope through a system of gears such that different parts of the diagram and hence different stadia lines are seen in the field of view as the telescope moves. Figure 15.29 shows the field of view of a typical diagram tacheometer. Instead of conventional stadia lines, there are three continuous curved lines. The lower curve is called the zero curve and it is placed on a convenient full graduation on the stadia rod. The upper curve is the horizontal distance curve. The stadia interval between the upper and lower curve multiplied by 100 gives the horizontal distance D. The middle curve determines the vertical distance interval together with the factor for that part of the curve which is being used.

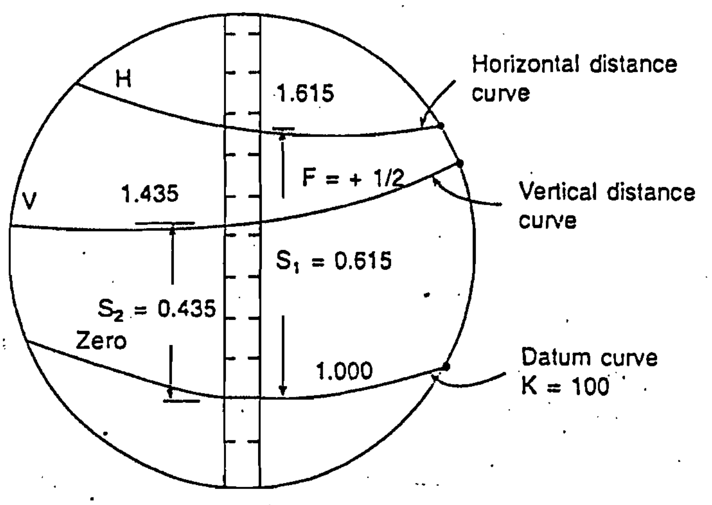


Fig. 15.29 Self-reducing tacheometer.

Example Let the lower curve reading = 1.000

Upper curve reading = 1.615 m

The difference = 0.615 m

and the horizontal distance = 0.615 × 100

= 61.5 m

If the middle curve reading = 1.435

with F = 1/2

$$V = \frac{1}{2}\ (1.435 - 1.000) \times 100 = 21.75 \text{ m}$$

Theory

The basic stadia formula $D = KS \cos^2 \theta$.

$$= (f/i)\ S \cos^2 \theta$$

In ordinary theodolite i is fixed but in self reducing tacheometer the interval i varies with θ as $i \cos^2 \theta$. Hence

$$D = \left(\frac{f}{i \cos^2 \theta}\right) S_1 \cos^2 \theta = \frac{f}{i} S_1 = KS_1$$

S_1 is the interval between the upper or horizontal distance line and the datum line.

In ordinary tacheometer

$$V = \frac{1}{2}(KS \sin 2\theta)$$

$$= (f/i)\left(\frac{S}{2}\right) \sin 2\theta$$

In diagram tacheometer the stadia interval (S_2) used for obtaining height differences varies as $\frac{1}{2}$ (i sin 2θ) so that

$$V = \frac{f}{\frac{1}{2}(i \sin 2\theta)} \frac{S_2}{2} \sin 2\theta$$

$$= \left(\frac{f}{i}\right) S_2 = KS_2$$

S_2 is the stadia reading between the middle curve line and the lower curve reading. i.e. the datum line. As S_2 varies with $\frac{1}{2}$ (i sin 2θ) it becomes very small for angles less than about 25°. So curve of i sin 2θ, (2.5) i sin 2θ and (5) i sin 2θ are introduced. This magnifies the value of S_2 which must be reduced accordingly. Hence multiplying factor or diagram constant 1/2, 0.2 or 0.1 are marked on the middle curve with positive or negative sign indicating elevation or depression of the telescope.

15.15 ERRORS IN TACHEOMETRIC SURVEYING

The errors can be classified in three groups (a) Instrumental errors, (b) Errors due to manipulation and sighting, (c) Errors due to natural causes.

Instrumental errors

The accuracy of stadia measurements is largely dependent on the instrument and the rod used. For longer sights error depends on the magnifying power of the telescope, the coarseness of the stadia hairs and the type of rod used. Error due to imperfections in the graduation of the rod can be kept to minimum by standardizing the rod. Errors occur due to imperfect adjustments of the tacheometer which are dependent upon (i) adjustment of altitude level, (ii) index error and (iii) accuracy of reading of the vertical circle.

It should always be checked that altitude bubble is at the centre of its run when readings are taken.

Errors may also occur if the multiplying and additive constants are not checked from time to time.

Errors due to manipulation and sighting

These are due to (i) inaccurate centring, levelling and bisection, (ii) incorrect estimation of the staff intercept, (iii) inaccurate reading of the vertical angle, and (iv) error due to non-verticality of staff.

Errors due to natural causes

These errors are due to—

Refraction This occurs due to varying density of different air strata due to temperature change. To minimize this error, readings with line of sight passing within 1 m of the ground surface should be avoided.

Sun Working directly under hot sun should be avoided.

Wind Working under strong wind should be avoided as it is difficult to keep the staff vertical or normal under such a condition.

Poor visibility Working under poor visibility should be avoided as under such conditions readings will be incorrect.

15.16 USES OF TACHEOMETRY

(a) It is a rapid method of surveying though not highly accurate. So where low accuracy is acceptable, this is recommended.

(b) It is useful for topographic survey where distance and elevations of points are both required.

(c) It is used in plane table survey in the form of telescopic allidade.

(d) Tacheometer is used to complete field survey required for photographic mapping.

(e) It can also be profitably used in differential levelling, profile levelling and in indirect trigonometrical levelling.

15.17 MISCELLANEOUS EXAMPLES

Example 15.10 Upto what vertical angle may sloping distances be taken as horizontal distances without the error exceeding 1 in 200, the staff being held vertically and the instrument having an anallactic lens? (U.L.)

Solution

True horizontal distance $D = KS\cos^2\theta$

Sloping distance $L = KS$

$$\frac{\text{Sloping distance}}{\text{Horizontal distance}} = \frac{L}{D} = \frac{KS}{KS\cos^2\theta} = \sec^2\theta$$

Permissible error 1 in 200.

$$\therefore \quad \frac{L}{D} = \frac{200+1}{200} = \frac{201}{200}$$

or

$$\sec^2\theta = \frac{201}{200}$$

$$\theta = \sec^{-1}\sqrt{\frac{201}{200}}$$

$$= \cos^{-1}\sqrt{\frac{200}{201}} = 4.04° = 4°2'24''$$

Example 15.11 A tacheometer was set up at station 'A' and the following readings were obtained on a vertically held staff.

Station	Staff station	Vertical Angle	Hair readings	Remarks
A	B.M.	– 2°18′	3.225, 3.550, 3.875	R.L. of B.M.
	B	+ 8°36′	1.650, 2.515, 3.380	= 425.515 m

Calculate the horizontal distance from *A* to *B* and the R.L. of *B* if the constants of the instruments are 100 and 0.4.

Solution

$$H = KS\cos^2\theta + C\cos\theta$$

$$V = \frac{KS\sin 2\theta}{2} + C\sin\theta$$

when $\theta = 2°18'$

$$\cos\theta = 0.999$$
$$\sin\theta = 0.040$$
$$\sin 2\theta = 0.080$$

$$V = 100(3.875 - 3.225)\cdot\frac{0.080}{2} + 0.4(0.04)$$
$$= 2.6 + 0.016$$
$$= 2.616 \text{ m}$$

For angle of depression

Elevation of staff station

= Elevation of Instrument station + $h - V - r$ (axial hair reading)

425.515 = [Elevation of instrument station + h] − 2.616 − 3.550

or [Elevation of instrument station + h] = 425.515 + 2.616 + 3.550 = 431.681

For angle of elevation

Elevation of staff station:

= Elevation of instrument station + $h + V - r$

$$V = 100(3.380 - 1.65)\frac{\sin 2(8°36')}{2} + 0.4\sin 8°36'$$
$$= 100(1.73)\frac{(0.2957)}{2} + 0.4\,(0.1495)$$
$$= 25.578 + 0.0598 = 25.6378 \text{ m}$$

Elevation of staff station = 431.681 + 25.6378 − 2.5150 = 454.8038 m

$$H = KS\cos^2\theta + C\cos\theta$$
$$= 100\,(1.73)\,(\cos 8°36')^2\ \ 0.4\cos 8°36'$$
$$= 100\,(1.73)\,(0.988)^2 + (0.4)\,(0.988)$$
$$= 169.26811 \text{ m}$$

Example 15.12 An internal focussing telescope has a length l from the objective to the diaphragm. The focal length of the objective and the internal focussing lens are f_1 and f_2 respectively. Find the distance d of the focussing lens from the objective when the object focussed is at a distance u_1 from the objective.

Solution For the objective (convex lens)

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1}$$
$$= \frac{u_1 - f_1}{f_1 u_1}$$

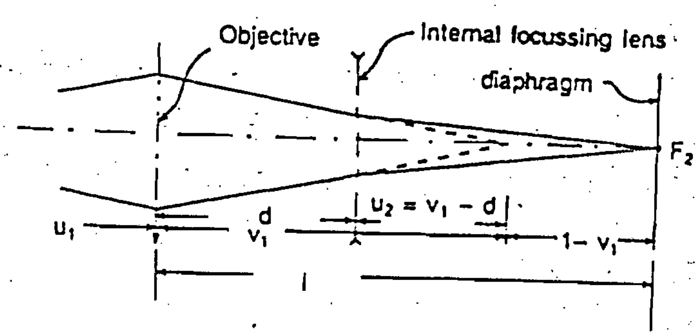


Fig. 15.30 Example 15.12.

For the focussing lens (concave)

$$-\frac{1}{f_2} = -\frac{1}{u_2} + \frac{1}{v_2}$$

or
$$\frac{1}{f_2} = \frac{1}{u_2} - \frac{1}{v_2}$$

$$= \frac{1}{v_1 - d} - \frac{1}{l - d}$$

or
$$(v_1 - d)(l - d) = f_2(l - d) - f_2(v_1 - d)$$

or
$$d^2 - d(l + v_1) + \{lv_1 - f_2(l - v_1)\} = 0$$

or
$$d^2 - d(l + v_1) + \{v_1(l + f_2) - f_2 l\} = 0$$

Solving

$$d = \frac{1}{2}\left[(l + v_1) \pm \sqrt{\{(l + v_1)^2 - 4(v_1 l + v_1 f_2 - l f_2)\}}\right]$$

$$= \frac{1}{2}\left[(l + v_1) \pm \sqrt{\{(l - v_1)^2 + 4f_2(l - v_1)\}}\right]$$

$$= \frac{1}{2}\left[(l + v_1) \pm \sqrt{\{(l - v_1)(l + 4f_2 - v_1)\}}\right]$$

Example 15.13 In a telescope, the object glass of focal length 180 mm is located 220 mm away from the diaphragm. The focussing lens is midway between these when a staff 18 m away is focussed. Determine the focal length of the focussing lens.

Solution

$$f_1 = 0.18 \text{ m}$$
$$u_1 = 18.000 \text{ m}$$

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{0.180} - \frac{1}{18.00}$$

$$= 5.500 \text{ or } v_1 = 0.182$$

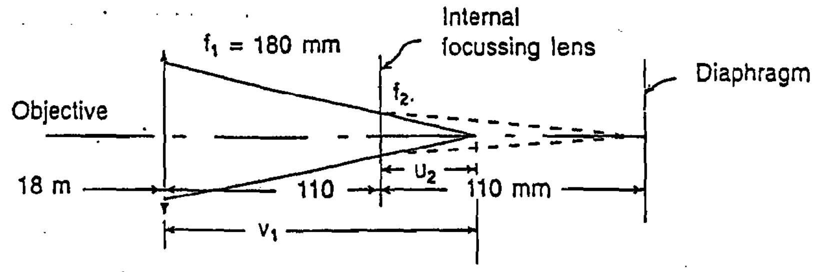


Fig. 15.31 Example 15.13.

For the focussing lens

$$u_2 = v_1 - 0.110 = 0.182 - 0.110 = 0.072$$

$$v_2 = 0.110$$

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$= -\frac{1}{0.072} + \frac{1}{0.110}$$

$$= -4.7979 \text{ m}$$

$$f_2 = -0.208 \text{ m} = -208 \text{ mm}$$

i.e. The lens is concave.

Example 15.14 In an internally focussing telescope, the objective of the focal length 125 mm is 200 mm from the diaphragm. If the internal focussing lens is of focal length 250 mm, find its distance from the diaphragm when focussed at infinity.

Solution For the objective $f_1 = 125$ mm and thus the position of F_1 will be 125 mm from C_1. Therefore,

$$C_2F_1 = 125 - d$$

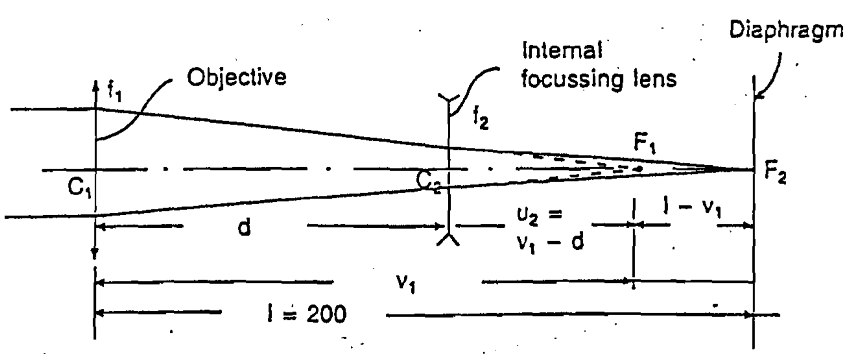


Fig. 15.32 Example 15.14.

For the internal focussing lens

$$f_2 = -250$$

$$u_2 = -(125 - d)$$

$$v_2 = 200 - d$$

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$-\frac{1}{250} = -\frac{1}{(125-d)} + \frac{1}{(200-d)}$$

or $$-(125-d)(200-d) = -250(200-d) + 250(125-d)$$

or $$-(25000 - 325d + d^2) = -50{,}000 + 250d - 250d + 31250$$

or $$d^2 - 325d + 6250 = 0$$

$$d = 304.47 \text{ or } 20.52$$

$\therefore$ $$v_2 = 200 - 20.52$$

$$= 179.48 \text{ mm}$$

Thus the internal focussing lens will be 179.48 mm away from the diaphragm when focussed to infinity.

Example 15.15 Derive expression for the spacing of the stadia lines to give a multiplying factor K for a given sight distance D if i = stadia interval and S = stadia intercept.

Solution

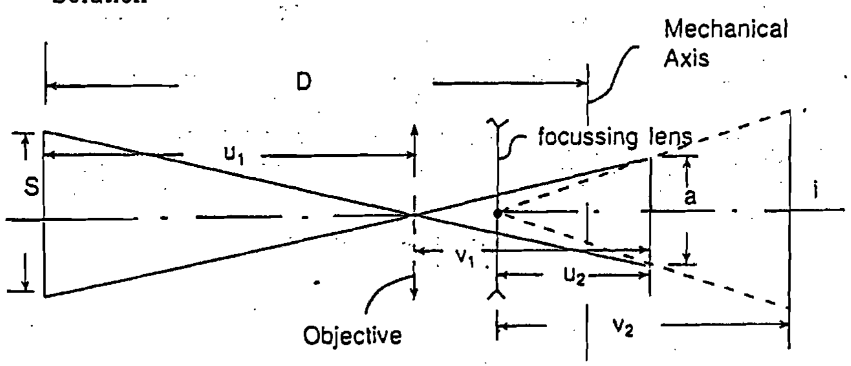


Fig. 15.33 Example 15.15.

For the convex lens, i.e. objective

$$\frac{a}{S} = \frac{v_1}{u_1} = m_1, \text{ i.e. } a = S\frac{v_1}{u_1} = m_1 S$$

where m_1 = magnifying power

For the concave lens (internal focussing)

$$\frac{i}{a} = \frac{v_2}{u_2} = m_2, \text{ i.e. } i = a\,\frac{v_2}{u_2} = m_2 \cdot a$$

or $$i = m_1 m_2\, S$$

$$D = KS$$

$\therefore$ $$S = \frac{D}{K} \qquad i = \frac{D m_1 m_2}{K}$$

Example 15.16 An internal focussing telescope has an objective 140 mm from the diaphragm. The focal lengths of the objective and the concave lens are 120 mm and 240 mm respectively. Find the distance apart of the stadia lines to produce a multiplying factor of 100 when the staff is 150 m away.

Solution Here $l = 140$ mm, $f_1 = 120$, $f_2 = 240$, $K = 100$, $D = 150$ m

At 150 m, $$u_1 = 150.00 - \frac{0.140}{2} = 149.93 \text{ m}$$

as the axis lies midway between the objective and diaphragm. For the objective,

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{u_1 - f_1}{u_1 f_1}$$

or $$v_1 = \frac{u_1 f_1}{u_1 - f_1}$$

$$= \frac{149.93 \times 0.120}{149.93 - 0.120} \times 1000$$

$$= 120.096 \text{ mm}$$

$$l + v_1 = 140.0 + 120.096 = 260.096 \text{ mm}$$

$$l - v_1 = 140.0 + 120.096 = 19.904 \text{ mm}$$

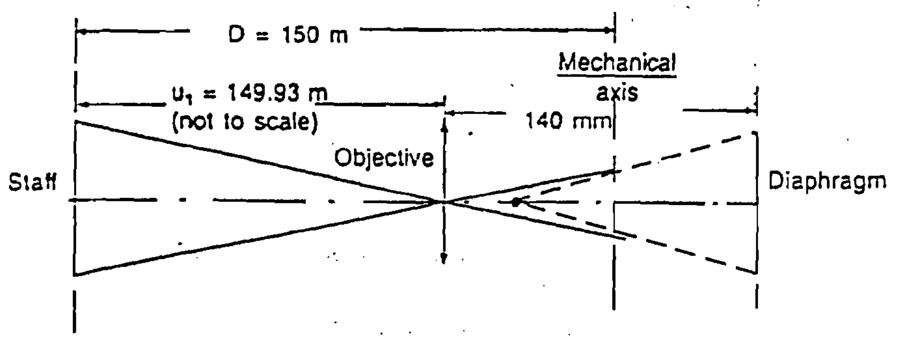


Fig. 15.34 Example 15.16.

$$d = \frac{1}{2}\,[(l + v_1) \pm \sqrt{(l - v_1)\,(l + 4f_2 - v_1)}]$$

$$= \frac{1}{2}\,[260.096 \pm \sqrt{19.904 \times 979.904}]$$

$$= \frac{1}{2}\,[260.096 \pm 139.656]$$

$$= 199.876 \text{ or } 60.22$$

However 199.876 is inadmissible as it does not fit physical conditions.

$$v_2 = l - d = 140 - 60.22 = 79.78 \text{ mm}$$

$$u_2 = v_1 - d = 120.096 - 60.220 = 59.876 \text{ mm}$$

$$i = \frac{Dv_1 v_2}{Ku_1 u_2}$$

$$= \frac{150 \times 1000\,(120.096)\,(79.78)}{(100)\,(149.93)\,(59.876\,(1000)}$$

$$= 1.60 \text{ mm}$$

Example 15.17 What errors will be introduced if the above instrument is used for distances 30, 100 and 150 m?

Solution At 30 m

$$u_1 = 30{,}000 - 70 = 29{,}930 \text{ mm}$$

$$v_1 = \frac{29{,}930 \times 120}{29{,}930 - 120} = 120.48$$

$$l + v_1 = 140 + 120.48 = 260.48$$

$$l - v_1 = 140 - 120.48 = 19.52$$

$$d = \frac{1}{2}\,[(l + v_1) \pm \sqrt{(l - v_1)(l + 4f_2 - v_1)}]$$

$$= \frac{1}{2}\,[260.48 \pm \sqrt{(19.52)(979.52)}]$$

$$= \frac{1}{2}\,[260.48 \pm 138.28]$$

$$= 199.38 \text{ or } 61.10$$

[199.38 does not fit physical conditions hence inadmissible]

$$v_2 = l - d = 140 - 61.10 = 78.90$$

$$u_2 = v_1 - d = 120.48 - 61.10 = 59.38$$

$$S = i \times u_1 u_2 / v_1 v_2$$

$$= \frac{1.60 \times 29{,}930 \times 59.38}{120.48 \times 78.90}$$

$$= 299.1403 \text{ mm}$$

The value should be 300 mm

$$\text{Error} = 300 - 299.1403$$
$$= 0.8597 \text{ mm}$$

Therefore D = 29.914 m and should be 30.000 m.
Hence error = 0.086 m/30 m

At 100 m

$$u_1 = 100{,}000 - 70$$
$$= 999{,}30 \text{ mm}$$
$$v_1 = \frac{999{,}30 \times 120}{999{,}30 - 120}$$
$$= 120.144 \text{ mm}$$
$$l + v_1 = 140.00 + 120.144 = 260.144 \text{ mm}$$
$$l - v_1 = 140.00 - 120.144 = 19.856 \text{ mm}$$
$$d = \frac{1}{2}[(l + u_1) \pm \sqrt{(l - v_1)(l + 4f_2 - v_1)}$$
$$d = \frac{1}{2}[260.144 \pm \sqrt{(19.856)(979.856)}]$$
$$= \frac{1}{2}[260.144 \pm \sqrt{(19.856)(979.856)}]$$
$$= \frac{1}{2}[260.144 \pm 139.484]$$
$$= 199.814 \text{ or } 60.33$$

As before,

$$d = 60.33$$
$$v_2 = 140 - 60.33 = 79.67$$
$$u_2 = 120.144 - 60.330 = 59.814$$
$$S = i \times \frac{u_1 u_2}{v_1 v_2}$$
$$= \frac{1.60 \times (999.30)(59.814)}{120.144 \times 79.67}$$
$$= 999.129 \text{ mm}$$

The value should be 1000.00 mm.

$$\text{Error} = 1000 - 999.129$$
$$= 0.881 \text{ mm}$$

Therefore D = 99.9129 m

and should be 100.00 m

$$\text{Error} = 0.0871 \text{ m/100 m}$$

At 150 m error = 0 as the diaphragm spacing has been set accordingly.

Example 15.18 The constant for an instrument is 800 and the value of constant $C = 0.45$ m. The intercept $S = 3$ m. Calculate the distance from the instrument to the staff when the micrometer readings are 4.265 and 4.267 and the line of sight is inclined at $+10°36'$. The staff was held vertical

Solution Sum of micrometer readings = 4.265 + 4.267 = 8.532

$$D = K\frac{S}{n} + (f + d)$$

For inclined sights

$$D = \frac{KS}{n}\cos^2\theta + (f + d)\cos\theta$$

$$= \frac{800 \times 3}{8.532}\cdot\cos^2 10°36' + 0.45\cos 10°36'$$

$$= 271.77 + 0.44$$

$$= 272.21 \text{ m}$$

Example 15.19 The following values were recorded during a theodolite tacheometric survey

Stadia readings	: 3.33 (top) 2.20 (middle) 1.07 (bottom)
Vertical angle	: 11°40′
Instrument height	: 1.48 m
Height of collimation	: 269.01 m

Find the horizontal distance between the staff and instrument station, and the reduced level of the staff station. Assume that the telescope is anallactic, the multiplying constant 100 and the staff vertical. Determine the error in the horizontal and vertical distances due to an error of ± 5 minutes of arc in the measurement of the vertical angle. [CEI]

Solution

$$H = KS\cos^2\theta + C\cos\theta$$

with $C = 0$ and $K = 100$

$$H = 100\, S\cos^2\theta$$

$$= 100\,(3.33 - 1.07)\cos^2 11°40'$$

$$= 100\,(2.26)\,(0.979)^2$$

$$= 216.75 \text{ m}$$

Reduced level of the staff station

= R.L. of instrument station + h + V − central hair reading

$$V = \frac{KS}{2}\sin 2\theta$$

$$= \frac{100 \times 2.26}{2} \sin 2\,(11°40')$$

$$= \frac{100 \times 2.26}{2}\,(.396)$$

$$= 44.75 \text{ m}$$

Hence reduced level of staff station

$$= 269.01 + 44.75 - 2.20 = 311.56$$

as height of collimation = R.L. of instrument station + height of instrument h = 269.01 m

$$H = KS \cos^2 \theta$$

$$\frac{dH}{d\theta} = -KS \sin 2\theta$$

$$dH = \pm KS \sin 2\theta \cdot d\theta$$

$$= \pm 100\,(2.26) \sin 2(11°40') \frac{5}{60} \cdot \frac{\pi}{180}$$

$$= \pm 100\,(2.26)\,(0.396) \cdot \frac{(5)\,(\pi)}{(60)\,(180)}$$

$$= \pm 0.130 \text{ m}$$

$$V = \frac{KS}{2} \sin 2\theta$$

$$\frac{dV}{d\theta} = \frac{KS}{2}\, 2 \cos 2\theta$$

$$dV = \pm KS \cos 2\theta\, d\theta$$

$$= \pm (100)\,(2.26) \cos 2\,(11°40') \frac{5}{60} \frac{\pi}{180}$$

$$= \pm (100)\,(2.26)\,(0.918) \left(\frac{5}{60}\right)\left(\frac{\pi}{180}\right)$$

$$= \pm 0.30 \text{ m}$$

REFERENCE

1. Easa, Said M., "Modelling of Stadia Surveying with Incomplete Intercepts", *ASCE Journal of Surveying Engineering*, Vol 116, No. 3, August 1990, pp 139–148.

PROBLEMS

15.1 A tacheometer is placed at a station A and readings on a staff held upon a B.M. of R.L. 100.00 and a station B are 0.640, 2.200, 3.760 and 0.010, 2.120 and 4.230 respectively. The angle of depression of the telescope in the first case is $-6°19'$ and in the second case $-7°42'$. Find the horizontal distance from *A* to *B* and the R.L. of station *B*. Assume f/i = 100 and $(f + d)$ = .3 m. [AMIE Surveying Winter, 1978]

15.2 (a) Draw a neat diagram and derive from first principle an expression for the horizontal distance between a tacheometer and a vertically held staff for a horizontal line of sight.

(b) Find the error that would occur in horizontal distance with an ordinary stadia telescope if an error of 0.0025 cm exists in the interval between stadia lines.

Focal length of object glass = 25 cm
Multiplying constant = 100
Additive constant = 35 cm

[AMIE Surveying Summer 1980]

15.3 Two sets of tacheometric readings were taken from an instrument station *A* (R.L. 100.00) to a staff station *B*.

Instrument	P	Q
Multiplying constant	100	95
Additive constant	30 cm	45 cm
Height of instrument	1.40 m	1.45 m
Staff held	Vertical	Normal to line of sight

Instrument	At	To	Vertical angle	Stadia readings
P	*A*	*B*	5°44′	1.090, 1.440, 1.795
Q	*A*	*B*	5°44′	?

Determine

(a) The distance between instrument station and staff station.

(b) R.L. of staff station *B*.

(c) The stadia readings with instrument *Q*.

[AMIE Surveying Winter 1980]

15.4 (a) Describe the procedure to determine the constants of a tacheometer in the field.

(b) The stadia readings with horizontal sight on a vertical staff held 50 m away from a tacheometer were 1.284 and 1.780. The focal length of object glass was 25 cm. The distance between the object glass and trunnion axis of the tacheometer was 15 cm. Calculate the stadia interval.

[AMIE Surveying Winter 1981]

15.5 (a) Derive the distance equations for the tangential system of tacheometry when both the sightings are angles of depression.

(b) Staff readings observed with a percentage theodolite corresponding to angles of elevation of 4% and 5% are 1.525 and 2.925 respectively. If the vertical angle on sighting the staff reading equal to the height of the trunnion axis above ground was 4.5%, calculate, (i) the horizontal distance between instrument and staff; (ii) the elevation of the staff station if that of he instrument station was 493.700.

[AMIE Surveying Summer 1981]

15.6 (a) Explain in brief any direct reading tacheometer you know.

(b) A tacheometer was set up at an intermediate station *C* on the line *AB* and following readings were obtained:

Staff station	Vert. angle	Staff readings
A	– 6°20′	0.445, 1.675, 2.905
B	+ 4°20′	0.950, 1.880, 2.810

The instrument was fitted with an anallactic lens and the constant was 100. Find the gradient of the line joining station *A* and station *B*.

[AMIE Surveying Summer 1982]

15.7 (a) Explain in brief the essential features, the merits and demerits, of the Jeffcot direct reading tacheometer.

(b) A line was levelled tacheometrically with a tacheometer fitted with an anallactic lens, the value of the constant being 100. The following observations were made with the staff held vertical on each station:

Instrument station	Ht. of axis	Staff station	Vertical angle	Staff reading	Remarks
P	1.60	B.M.	– 2°18′	1.650, 2.500, 3.350	
P	1.60	*Q*	+ 8°36′	1.720, 2.670, 3.620	
Q	1.50	*R*	+ 10°42′	1.055, 2.055, 3.055	

R.L. of B.M. = 250.250

Determine the gradient of the line *QR*. [AMIE Surveying Winter 1984]

15.8 (a) What is an anallactic lens? Explain the object of providing an anallactic lens in a tacheometer.

(b) Explain how you will obtain in the field the constants of a tacheometer.

(c) The top of a hill subtends an angle of 9°30′ at a point *A*. The same point on the top of the hill subtends an angle of 12°30′ at point *B* which is in direct line joining point *A* and top of the hill. Distance *AB* was measured and found to be 1600 m. Determine the elevation of the top of the hill and its horizontal distance from point *A*, given the elevation of point *A* is 430.650 m. [AMIE Surveying Summer 1984]

15.9 (a) Compare the fixed hair stadia and tangential method of tacheometric survey.

(b) Derive expression for determining the elevation of staff station and its distance from instrument station when one vertical angle is of elevation and the other, of depression. Sketch the illustrative diagram.

(c) A traverse *ABCD* was run by a tacheometer fitted with an anallactic lens and having a multiplying constant as 100. The following readings were taken with the staff held normal:

Line	Bearing	Vertical angle	Staff intercept
AB	27°38′	+ 7°04′	1.90
BC	300°24′	+ 4°32′	1.47
CD	236°45′	– 2°10′	1.75

Find the length and bearing of *DA*. [AMIE Surveying Winter 1989]

16

Plane Table Surveying

16.1 INTRODUCTION

The plane table surveying is a very quick method of surveying where field observations and plotting of the plan proceed simultaneously. The necessity of transferring the field data to office and preparation of map is completely avoided. The plane table—alidade combination is an extremely useful and a versatile instrument. It can perform all the usual survey functions with the exception of field astronomy. It is also very useful as a basic instrument for teaching fundamental concepts of surveying because the geometric principles are readily grasped and the objective of a survey operation is clearly indicated in the map sheet. While photogrammetry is being extensively used for topographical survey these days, there are features which can be mapped in more detail by ground survey techniques. In large scale mapping of built-up areas where a considerable volume of underground detail and paved surfaces is encountered, plane table method can adequately plot the details. Even in construction layout staking, plane table method is very useful.

16.2 EQUIPMENTS REQUIRED

The following equipments are required for plane table survey.

1. Plain table
2. Alidade for sighting
3. Plumbing fork and plumb bob
4. Spirit level
5. Compass
6. Plane table sheet of suitable drafting media.

The plane table

It consists of a small drawing board mounted on light tripod in such a way that the boards can be rotated about the vertical axis and can be clamped in any position. I.S. 2539–1963 specifies three types of boards with dimensions given below:

Designation	Dimensions (mm)		
	A	B	C
Large	750	600	15
Medium	600	500	15
Small	500	400	15

The details of the board, the brass circular disc which firmly secures the clamping assembly to the board and the clamping assembly along with the tripod are given in Figs. 16.1 and 16.2. The plane table stand is 1250 mm in height from the top of the clamping head to the shoe.

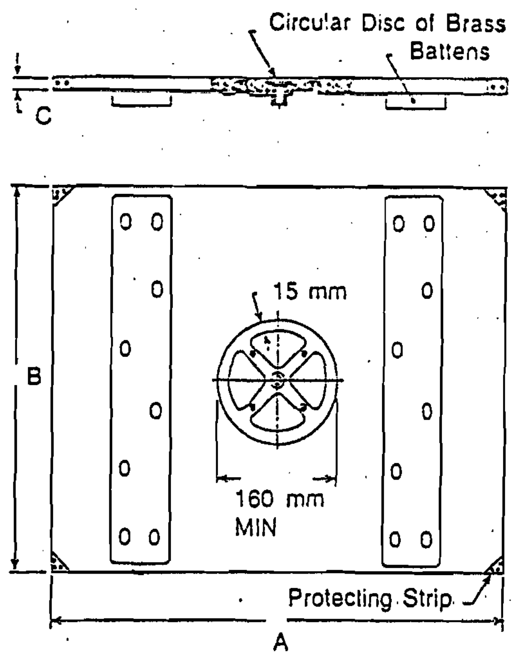


Fig. 16.1 Plane Table Board.

Alidade (sight rule)

This may be (i) Plain or (ii) Telescopic. The details of a *Plain alidade* are given in Fig. 16.3. The materials and dimensions of sight rules are given in Table 16.1. The sight rules are made of wood or metal but the sight vane is made of metal only. The front or object vane and the rear or sight vane are of folding type. It is possible to suitably clamp them in vertical position. The front vane has a thread across its length and the rear vane has a fine slit to view through it. The two vanes can be linked through a thread when required for sighting at high elevations or depressions. The stretched thread, object vane thread and sight vane vertical slit are coplanar and this plane is parallel to the edge of the sight rule and normal to the plane table board when levelled. The bottom surface of the sight rule is truly plane and has bevelled edges.

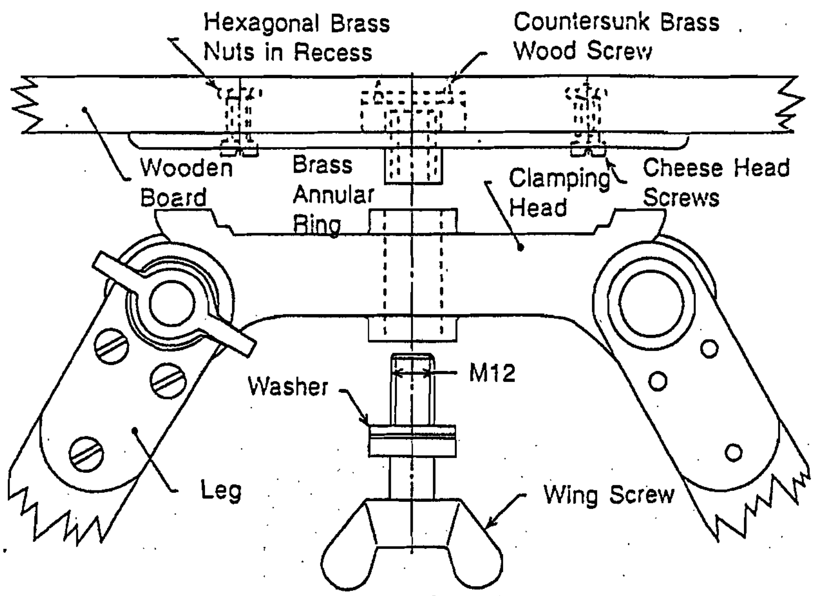


Fig. 16.2 Clamping Assembly.

Table 16.1 Material and Dimensions of Sight Rules

Designation	L	A	B	C	D	Material
Large	750	25	50	15	3	Brass or aluminium alloy
Medium	600	25	50	15	3.	Brass or aluminium alloy
Small	500	20	30	10	2	Wood, brass or aluminium alloy
		or	or	or	or	
		25	40	15	3	

A *telescopic alidade* consists of a base or blade composed of either a plain or articulated fiducial straight edge that rests directly on the plane table. On the base is: (i) a pylon or pedestal, (ii) a trunnion axis, and (iii) a telescope which can rotate against the trunnion axis.

The telescope eyepiece is generally swivel mounted so that the observer's eye need not be aligned with the telescope's longitudinal axis. All alidades are equipped with a circle for measuring vertical angles. The horizontal distance between the instrument and the point sighted can be computed by stadia readings on the staff kept at the point. The elevation of the point can also be computed by using usual tacheometric relations. Now a days EDMI is mounted on allidade and facilitates plane table operations. (Fig. 16.4).

Plumbing fork

This is a U-shaped piece of metal with parallel arms of equal length, a plumb bob being attached to the free end of the lower arm. The point of the upper arm and the plumbline are in the same vertical line. The plumbing fork is used for

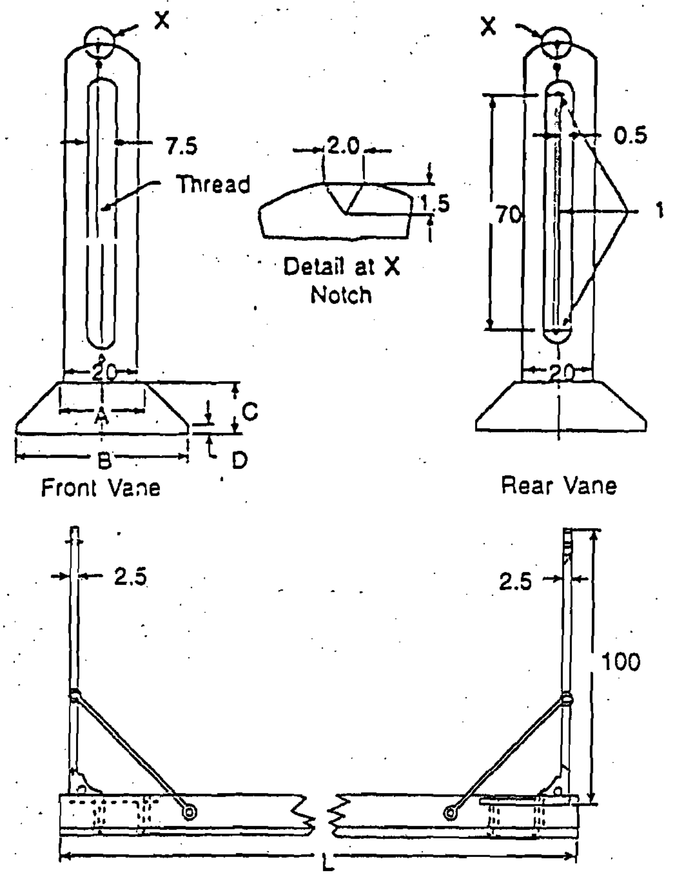


Fig. 16.3 Sight Rule (Alidade). (All dimensions in millimeters.)

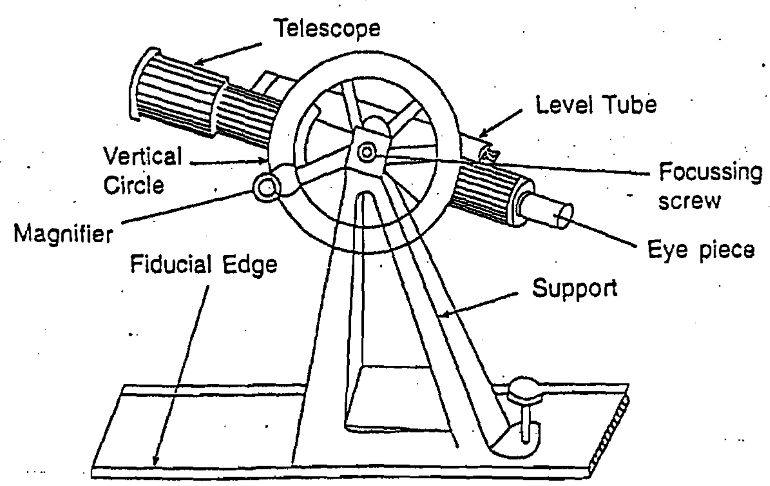


Fig. 16.4 Telescopic allidade.

Centring the table Here the upper end of the plumbing fork is placed over the plotted point and the plane table is so adjusted that the plumb bob is on the station point below.

Transferring of the ground point Here the plane table is centred over the underground point by means of the plumb bob while the upper arm of the fork gives the point to be plotted on the drawing sheet (Fig. 16.5)

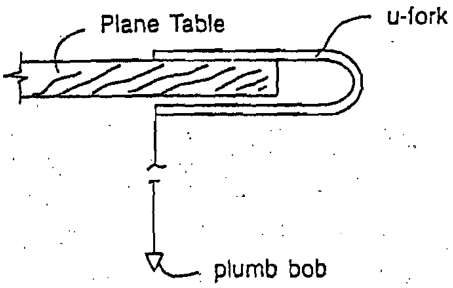


Fig. 16.5 U-fork and plumb bob.

Spirit level

A small spirit level either of the tubular variety or of the circular type is used to check that the plane table when in use is level. This can be ascertained by placing the level in two directions at right angles to each other and observing that the bubble is central in both cases.

Compass

Usually a trough compass is used. The longer sides of the trough are parallel and flat so that either side can be used as a ruler or laid down to coincide with a straight line drawn on the paper.

Plane table sheet

It should be of superior quality and should be moisture proof and non-hygroscopic. The dimensions of the sheet should remain stable under variable conditions of temperature and humidity. It should be capable of withstanding repeated erasures. It should also be stiff and tough and suitable for longtime archival, quality storage.

16.3 WORKING WITH PLANE TABLE

Before mapping work can start with plane table, the following steps are needed:

Fixing The plane table should be fixed to the tripod. The working sheet should be carefully mounted with spring clips, thumb screws or drafting tape.

Levelling The plane table should be levelled. For small work, it is through eye estimation. For more accurate work spirit level is to be used.

Centring The table should be so placed that the plotted point a is exactly over the ground point A. This is done through plumbing U-fork.

Orientation This is done by rotating the plane table such that plotted lines in the plane table sheet are all parallel to the corresponding lines on the ground. This is essential when more than one instrument station is to be used. Orientation is done by (i) Trough compass, (ii) Back sighting.

Orientation by trough compass

This is an approximate but quick method of orienting the plane table. The usual method is to place the trough compass on the plane table sheet and to rotate the plane table till the needle floats centrally. This is the direction of magnetic north and a fine pencil line is ruled against the long side of the box. At any other station, where the table is to be oriented, the compass is placed against this line and the table is turned till the needle freely floats in the middle. The table is then said to be oriented.

Orientation by back sighting

This is a more accurate method and two cases may arise depending on whether it is possible to set the plane table on a point already plotted on the sheet by way of observation from previous station or not. In the first case orientation is done by back sighting. Suppose the line *ab* has been plotted on the plane table corresponding to the ground line *AB*. After shifting the plane table from *A* to *B*, orientation will be done by (i) placing the point *b* exactly over the station *B* with the help of *U* fork. (ii) by rotating the plane table such that from station *B*, alidade which is placed on *ba* sights the pole at *A*. When this is achieved, the line *ba* coincides with the ground line *BA* and orientation is achieved. The table is then clamped in position.

When the plane table cannot be placed over a plotted point method of resection has to be applied. This has been explained in subsequent sections.

16.4 DIFFERENT METHODS OF PLANE TABLE WORK

There are four methods of plane table work: (i) Radiation, (ii) Intersection, (iii) Traversing, and (iv) Resection.

16.4.1 RADIATION

Here plane table is placed over station point *P* and alidades are pointed towards *A*, *B*, *C*, *D*, *E* and *F*. The lengths are measured and points *a*, *b*, *c*, *d*, *e* and *f* are plotted as shown in Fig. 16.6.

16.4.2 INTERSECTION

One of the great advantages of plane table is the ease with which a point can be located by intersection. Here *P* and *Q* are known stations and the plane table is first placed over station *P* and alidade points towards *PQ*. The line *PQ* is measured and *pq* is plotted on the sheet (Fig. 16.7). To locate points *A* and *B* rays are drawn from *p* towards *A* and *B*. The instrument is then shifted to *Q* and it is so placed

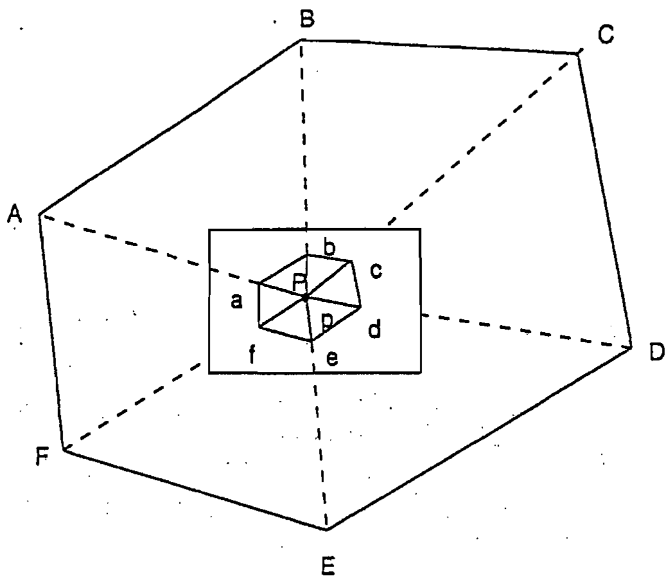


Fig. 16.6 Radiation method,

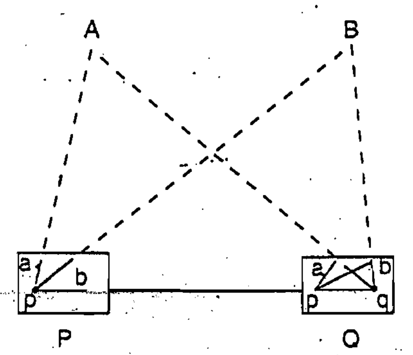


Fig 16.7 Method of intersection

that q is over Q and pq corresponds to the line PQ. In such a case the plane table is properly oriented at Q. With the alidade pivoted at q, rays are drawn toward A and B on the plane table sheet. The intersection of corresponding two rays defines the map position of A and B. The difference in elevation between A and P and that between A and Q can be obtained if the vertical angle to A has been measured from each set up. The product of the distance AP, scaled from the map and the tangent of the vertical angle at P is the difference in elevation between the alidade at P and A. The difference in elevation between the alidade at Q and the point A can be determined in the same manner.

16.4.3 TRAVERSING

A traverse consists of a series of straight lines connected together. In a plane table traverse, the angles are directly plotted without measuring them (Fig. 16.8). Here initial station A is occupied and then AB is sighted and measured. Then station B

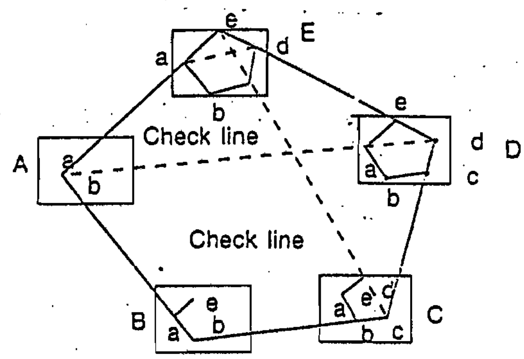


Fig. 16.8 Traversing.

is occupied and *BA* sighted. The distance *BA* is measured and the average of *AB* and *BA* used in laying out *ab*. The next point *C* is observed with the blade touching *b*, distance *BC* and *CB* measured and average value plotted as *bc*. In similar fashion, succeeding points can be occupied and traverse lines plotted. Whenever possible, check sights should be taken over previously occupied points. Small discrepancies are adjusted but if a plotted point is missed by an appreciable distance, some or all measurements must be repeated.

16.4.4 RESECTION

Resection is a method of orientation used when the table occupies a position not yet located on the map. There are two field conditions: (i) the three point problem, and (ii) the two point problem.

The three-point problem

Here three points in the field and their corresponding positions in the plane are known. The plane table is placed in an arbitrary position from where the three points are visible. It is necessary to locate the position of the observer. The solution enables the surveyor to place the plane table at any suitable position for taking details. The point can then be located by observing three known points such as church steeples, water towers, flag poles or any other prominent object. The three-point problem has long been employed in navigation to ascertain a ship's position by observing with a sextant on three recognizable features on the shore. There are many methods for solving the three-point problem. They are:

(a) Tracing paper method,
(b) Lehmann's method,
(c) Analytical method,
(d) Graphical solution.

Tracing paper method This method consists of the following steps.

(a) Here *A, B, C* are three known stations and *a, b, c* are their plotted points on a drawing sheet.

(b) Instrument is set up at point *P*. It is required to locate the corresponding point *p* on the sheet.

(c) Orient the table approximately with eye or compass so that AB is parallel to ab.

(d) Fix a tracing paper on the sheet and locate the point P approximately as p' by means of plumbing fork.

(e) Sight the stations A, B and C and draw $p'a'$, $p'b'$ and $p'c'$ on the tracing paper.

(f) The tracing paper is then moved above the drawing sheet until the three radiating lines $p'a'$, $p'b'$ and $p'c'$ pass through corresponding points a, b and c previously plotted on the map.

(g) This point is marked and the board turned to make the lines radiate to the signals A, B, and C, on the ground.

(h) The board is then clamped (Fig. 16.9).

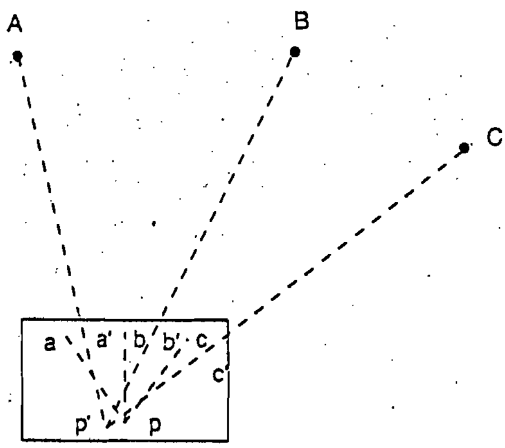


Fig. 16.9 Tracing paper method.

Lehmann's method This is the easiest and quickest solution. The principles of the method are as follows:

(a) When the board is properly oriented and the alidade sighted to each control signals A, B and C, rays drawn from their respective signals will intersect at a unique point.

(b) When rays are drawn from control signals, the angles at their intersections are true angles whether or not the board is properly oriented.

Procedure

1. Set the table over the new station P and approximately orient it.

2. With alidade on a sight A, similarly sight B and C. The three rays Aa, Bb and Cc will meet at a point if the orientation is correct. Usually, however, they will not meet but will form a small triangle known as the triangle of error.

3. To reduce the triangle of error to zero, another point p' is chosen as per Lehmann's rule.

4. Keep the alidade along $p'a$ and rotate the table to sight A. Clamp the

table. This will give next approximate orientation (but more accurate than the pervious one). Then sight B with alidade at b and C with alidade at c. The rays will again form a triangle of error but much smaller.

5. The method has to be repeated till the triangle of error reduces to zero.

Lehmann's rules There are three rules to help in proper choice of the point p'.

1. If the plane table is set up in the triangle formed by the three points (i.e. P lies within the triangle ABC) then the position of the instrument on the plan will be inside the triangle of error; if not it will be outside.

2. The point p' should be so chosen that its distance from the rays Aa, Bb and Cc is proportional to the distance of P from A, B and C respectively. Since the rotation of the table must have the same effect on each ray.

3. The point p' should be so chosen that it lies either to the right of all three rays or to the left of all three rays, since the table is rotated in one direction to locate P.

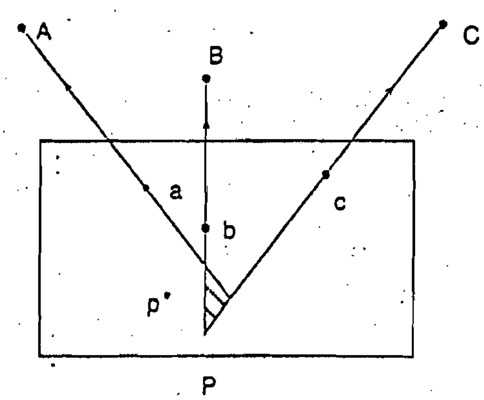


Fig. 16.10 Triangle of error.

Referring to the Fig. 16.10:

By rule 1 p is outside the small triangle as P is outside the triangle ABC.

By rule 2, using the proportions for the perpendiculars given by scaling the distances PA, PB and PC, it must be in the left hand sector as shown.

By rule 3, it cannot be in either of the sectors contained by the rays PA, PB and PC.

Failure of the fix When the three points A, B and C and the instrument position P are so chosen that they all lie on the circumference of a circle, there is failure of the fix and the solution becomes indeterminate. This is because no matter how the board is oriented, the rays will meet at a point, though not at the same point. Because the two angles subtended by the three points at the circumference of the cirucumscribing circle will always be the same. Hence the rays will always meet at a point. Hence the observer should choose the prominent points such that they do not lie on a circle.

Analytical and Graphical Solutions are given in Section 16.7.

The two point problem Here two points A and B are visible from the instrument station C and the corresponding points a and b are given in the plane table sheet. Two cases can arise either the points can be occupied by the plane table, or the points cannot be occupied.

Case I When the points can be occupied by the plane table. Let a and b be the corresponding points of the ground points A and B.

(a) The plane table is set up at B and oriented by sighting A. From B a line bx is drawn towards C.

(b) The table is then shifted to C and oriented by back sighting B along xb and clamped.

(c) To locate point c which is on the line bx, the alidade is placed over a and A is sighted. The line Aa when produced backward cuts the line bx at c to fix the point C (Fig. 16.11).

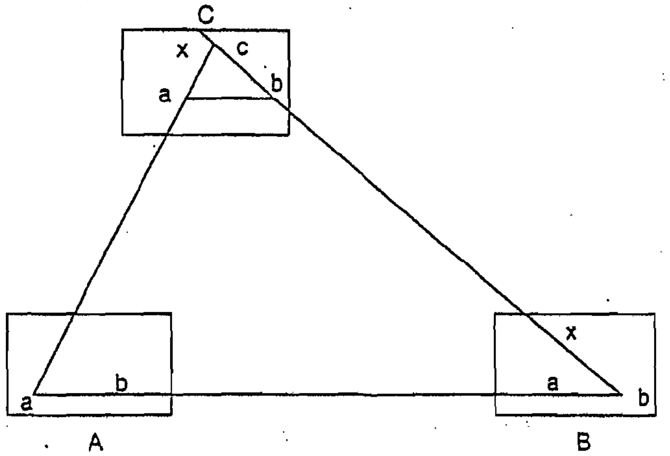


Fig. 16.11 Two-point problem, case I.

Case II When the plane table cannot be placed in the controlling stations. (Fig. 16.12):

(a) An auxilliary station point D is to be chosen near C. Set the table at D in such a way that ab is approximately parallel to AB. Clamp the table.

(b) With alidade at a, sight A and draw a line. Similarly with alidade at b sight B. The two rays intersect at point d.

(c) From station D and keeping the alidade at d sight C. Measure DC by estimation and mark c_1.

(d) Shift the table to C, take back sight to D with reference to c_1.

(e) With alidade at a, sight A. This ray intersects the previously drawn ray from D in c_2. Thus c_2 represents C with reference to the approximate orientation made at D.

(f) From c_2 sight B. Draw the ray to intersect the ray drawn from d to b in b'. Thus b' is the approximate location of B with respect to the orientation made at D.

(g) The angle between ab and ab' is the error in orientation. The board

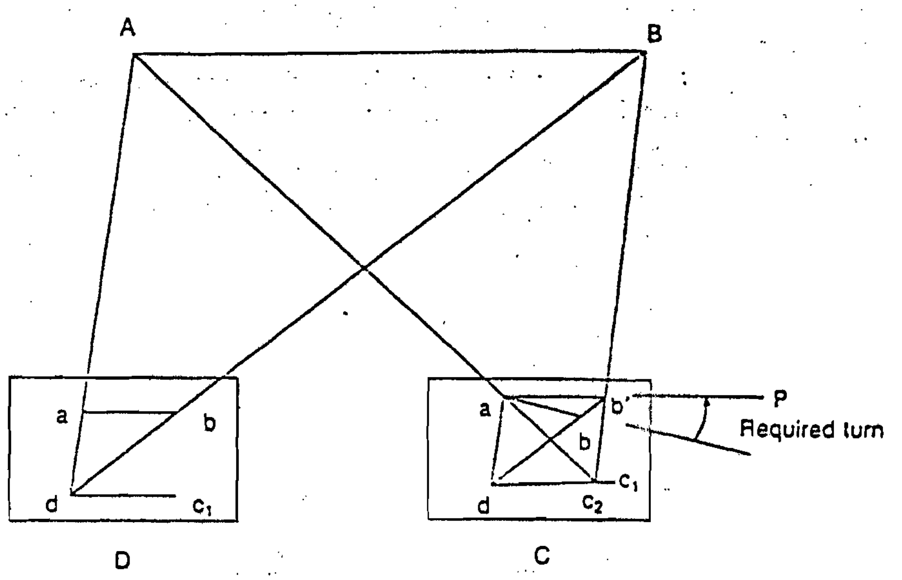


Fig. 16.12 Two-point problem, case II.

should be rotated through the angle *bab'*. This is done by fixing a pole at *P* such that it is in line with *ab'*. The plane table is then rotated till *ab* comes in line with *P*. The table is thus correctly oriented.

(h) From this new position draw rays *Aa* and *Bb*. They will intersect at *c* (not shown) which will be the point corresponding to *C*.

Two point problem does not give accurate result as with finite distance of the point *P* it is difficult to rotate and orient the table at *C*. Moreover, the setting up of an auxilliary station involves more work in a two point problem compared to the three point problem.

16.5 ERRORS IN PLANE TABLE

Three types of errors are involved: (i) Instrumental errors, (ii) Errors in plotting, (iii) Error due to manipulation and sighting.

Instrumental errors

These may consists of the following:

(a) The top surface of the plane table may not be perfectly plane.

(b) The fiducial edge of the alidade may not be straight.

(c) The plane table may not be stable due to loose fittings

(d) If the magnetic needle is sluggish accurate orientation may not be possible.

(e) If the sight vane is not perpendicular to the base of the alidade, there will be error in sighting.

(f) With defective level tube, plane table will not be horizontal even if the bubble is in the centre of its run.

Errors in plotting

(a) If the drawing paper is not of good quality, with temperature changes it will shrink or expand and there will be errors in plotting.

(b) Plotting error will occur if the alidade is not properly pivoted against the point or if thicker pencils are used. This is particularly so if the scale of drawing is small.

Errors in sighting and manipulation

(a) If the plane table is not exactly over the station point, centring error will occur.

(b) If the plane table is not properly oriented there will be angular error in location of points

(c) If the plane table is not properly clamped, between observations it will move and there will be error.

(d) Sighting error will occur if the object is not bisected at the middle.

16.6 ADVANTAGES AND DISADVANTAGES OF PLANE TABLE SURVEY

Advantages

(a) The map is made while looking at the area. Hence minutest detail can be plotted.

(b) No field book is necessary.

(c) Irregular lines such as stream banks and contours can be checked.

(d) The plane table can be used even in magnetically sensitive areas where compass survey is not possible.

(e) It is very rapid and less costly.

Disadvantages

(a) The method is not very accurate.

(b) It is not possible to work under rain or scorching sun.

(c) Without any field data, it is not possible to replot the plan in a different scale.

(d) More field time is required as the plotting has to be done in the field itself.

(e) The control points are usually fixed by triangulation and interior fillings only are done by plane table.

(f) The workers are to be very skilled as field work and plotting has to be done simultaneously and necessary computations have to be done in the field itself.

Example 16.1 The plane table operator sets over an unknown ground-point and measures a distance of 1.29 m from the ground to the alidade. The rod man holds the rod on a point whose elevation is 482.75 m. The plane table operator reads a stadia interval of 1.664 m, a V-scale reading of + 8, and a centre crosshair reading of 1.78 m on the rod. Compute the elevation of the unknown ground point.

Solution

Stadia interval	= 1.664 m
V-Scale	= + 8
Product	= + 8 × 1.664
	= + 13.312 m
Elevation of Known point	= 482.75 m
Central hair reading	= + 1.78 m
	484.530 m
– V.D	= 13.312
	= 471.218
Elevation of Instrument Axis	= 471.218 m
– Elevation of alidade	= – 1.290 m
Elevation of ground	= 469.928 m

Example 16.2 Derive an expression for inaccurate centring of the plane table. In setting up the plane table at a station *P* the corresponding point on the plan was not accurately centred above *P*. If the displacement of *P* was 30 cm in a direction at right angles to the ray, how much on the plan would be the consequent displacement of a point from its true position if

$$r = \frac{1}{2000}, \quad r = \frac{1}{200} \quad \text{and} \quad \frac{1}{20}?$$

Solution Let *P*, the original point and *p* the plotted position of point *P*. Let *A* and *B* be the two sighted stations. The plotted angle then is *APB*, and the correct

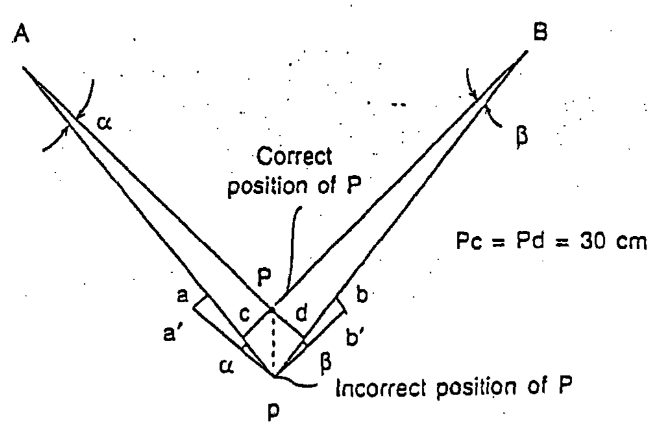


Fig. 16.13 Example 16.2.

angle APB and the error in angle is the difference between APB and ApB. The linear error in centring is Pp. Angular error in centring is $\alpha + \beta = \gamma$. a' and b' are the correct positions of A and B.

The error in the plotted positions are

$$aa' = pa \sin \alpha \approx pa \cdot \alpha = pa \times \frac{Pc}{AP}$$

$$bb' = pb \sin \beta \approx pb\ \beta = pb \times \frac{Pd}{BP}$$

If R.F. = r

$$pa = AP \times r$$

$$pb = BP \times r$$

$$\therefore \quad aa' = \frac{AP \times r \times Pc}{AP} = r \times Pc$$

$$bb' = \frac{BP \times r \times Pd}{BP} = r \times Pd$$

In this case $Pc = Pd = e = 30$ cm $= 300$ mm

(i) when $r = \frac{1}{2000}$, $aa' = \frac{1}{2000} \times 300$
$= .15$ mm. (small)

(ii) when $r = \frac{1}{200}$, $aa' = \frac{1}{200} \times 300$
$= 1.5$ mm. (large)

(iii) when $r = \frac{1}{20}$, $aa' = \frac{1}{20} \times 300$
$= 15$ mm. (very large)

16.7 ANALYTICAL AND GRAPHICAL SOLUTIONS

In three-point method it is necessary to fix a point by making observations to three known points. The solution of this problem means the computation of the position of a station from observations to three known points. It may be frequently found necessary to locate additional points that are subsequently used as instrument stations.

Analytical methods

From station P, A, B, C are observed. Hence (i) The observed angles θ_1, θ_2 and (ii) lengths AB, BC, i.e. L_1, L_2 (iii) angle β are known.

Method I (Fig. 16.14)

In $\Delta\ ABP$ $\quad \sin \alpha_1 = \frac{BP \sin \theta_1}{L_1}$

In $\Delta\ BCP$ $\quad \sin \alpha_2 = \frac{BP \sin \theta_2}{L_2}$

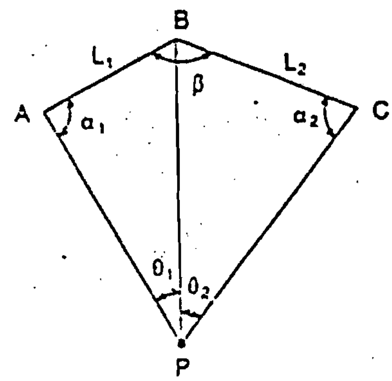


Fig. 16.14 Analytical solution of three point problem.

$$\therefore \qquad \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{L_2 \sin \theta_1}{L_1 \sin \theta_2} = K$$

In *ABCP*,

$$\alpha_1 + \alpha_2 = 360 - (\theta_1 + \theta_2 + \beta)$$

$$= \phi.$$

$$\alpha_1 = (\phi - \alpha_2) \text{ or } \sin \alpha_1 = \sin (\phi - \alpha_2) = K \sin \alpha_2$$

or

$$\sin \phi \cos \alpha_2 - \cos \phi \sin \alpha_2 = K \sin \alpha_2$$

Dividing by $\cos \alpha_2$

$$\sin \phi - \tan \alpha_2 \cos \phi = K \tan \alpha_2$$

or

$$\tan \alpha_2 = \frac{\sin \phi}{K + \cos \phi}$$

from which α_2 can be found out. Then *AP*, *BP* and *CP* can be found out applying the sine rule.

Method II From triangle *ABP* (Fig. 16.14)

$$PB = \frac{L_1 \sin \alpha_1}{\sin \theta_1}$$

From triangle *PBC*

$$PB = \frac{L_2 \sin \alpha_2}{\sin \theta_2}$$

$$\therefore \qquad \frac{L_1 \sin \alpha_1}{\sin \theta_1} = \frac{L_2 \sin \alpha_2}{\sin \theta_2}$$

or

$$\sin \alpha_2 = \frac{L_1 \sin \alpha_1 \sin \theta_2}{L_2 \sin \theta_1}$$

If

$$\psi = \frac{\alpha_1 - \alpha_2}{2}$$

$$\phi = \alpha_1 + \alpha_2$$

and $$\tan\theta = \frac{L_1 \sin\theta_2}{L_2 \sin\theta_1}$$

then $$\tan\psi = \cot(\theta + 45°) \tan\phi/2$$

This can be derived as follows:

$$\frac{\tan\frac{1}{2}(\alpha_1 - \alpha_2)}{\tan\frac{1}{2}(\alpha_1 + \alpha_2)} = \frac{\sin\frac{1}{2}(\alpha_1 - \alpha_2)\cos\frac{1}{2}(\alpha_1 + \alpha_2)}{\sin\frac{1}{2}(\alpha_1 + \alpha_2)\cos\frac{1}{2}(\alpha_1 - \alpha_2)}$$

$$= \frac{\sin\alpha_1 - \sin\alpha_2}{\sin\alpha_1 + \sin\alpha_2}$$

$$= \frac{1 - \dfrac{\sin\alpha_2}{\sin\alpha_1}}{1 + \dfrac{\sin\alpha_2}{\sin\alpha_1}}$$

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{L_1 \sin\theta_2}{L_2 \sin\theta_1} = \tan\theta$$

Therefore $$\frac{\tan\frac{1}{2}(\alpha_1 - \alpha_2)}{\tan\frac{1}{2}(\alpha_1 + \alpha_2)} = \frac{1 - \tan\theta}{1 + \tan\theta}$$

Now $$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

or $$\cot(A + B) = \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

If A is taken to be 45° and B is taken to be θ

$$\cot(45° + \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

$\therefore$ $$\frac{\tan\frac{1}{2}(\alpha_1 - \alpha_2)}{\tan\frac{1}{2}(\alpha_1 + \alpha_2)} = \cot(45° + \theta)$$

or $$\tan\psi = \cot(45° + \theta)\tan\phi/2$$

From the above relation $(\alpha_1 - \alpha_2)$ can be found out. As $(\alpha_1 + \alpha_2)$ is known, α_1 and α_2 can be computed.

Method III Tienstra's Method.

Let the coordinates of A, B, C be E_A, N_A : E_B, N_B and E_C, N_C. The coordinates of P can then be determined by Tienstra's formulae which state

$$E_P = \frac{K_1 E_A + K_2 E_B + K_3 E_C}{K_1 + K_2 + K_3}$$

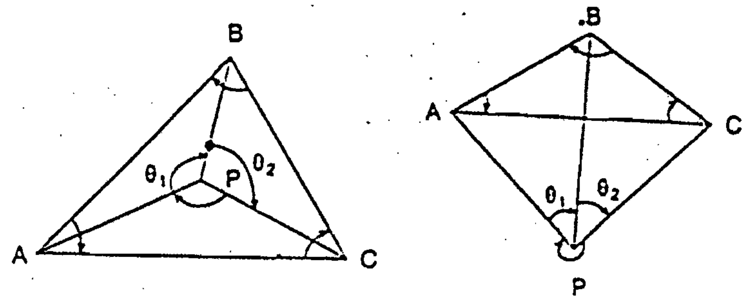


Fig. 16.15 Tienstra's method.

$$N_P = \frac{K_1 N_A + K_2 N_B + K_3 N_C}{K_1 + K_2 + K_3}$$

In the above formulae

$$K_1 = \frac{1}{(\cot B\hat{A}C - \cot B\hat{P}C)}$$

$$K_2 = \frac{1}{(\cot C\hat{B}A - \cot C\hat{P}A)}$$

$$K_3 = \frac{1}{\cot A\hat{C}B - \cot A\hat{P}B}$$

Angles are measured clockwise as shown in Fig. 16.15.

Graphical solution

Let A, B and C be the three known points. θ_1 and θ_2 the measured angles.

Method I Join AC. At A draw a line AD making an angle θ_2 and at C draw a line CD making an angle θ_1. These lines intersect at D. Draw a circle passing through A, D and C. Join DB and produce it to cut the circle in P which is the required point (Fig. 16.16).

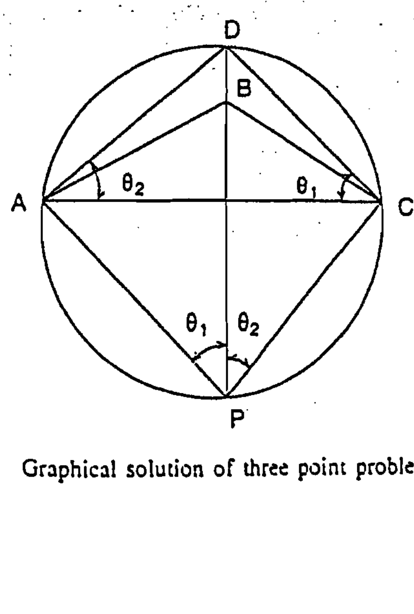


Fig. 16.16 Graphical solution of three point problem (Method I).

Proof $\angle DAC = \angle DPC = \theta_2$. $\angle DCA = \angle APD = \theta_1$ as A, D, C, P are concyclic.

Method II Join AB and BC. At A and B draw $90° - \theta_1$ with AO_1 and BO_1 respectively. They cut at O_1. Similarly draw $90° - \theta_2$ at B and C when they cut at O_2. With O_1 as centre draw a circle through A and B and similarly with O_2 as centre draw a circle through B and C. The circles intersect at the required point P (Fig. 16.17).

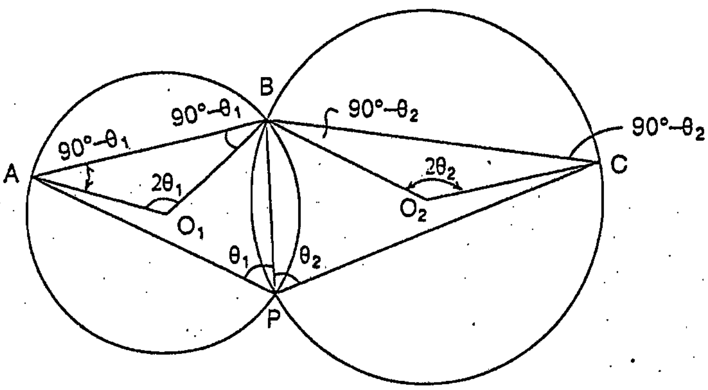


Fig. 16.17 Graphical solution of three point problem (Method II).

Proof $\angle APB = \frac{1}{2} \angle AO_1B = \frac{1}{2} \cdot 2\theta_1 = \theta_1$

as A, P, B are concyclic.

Similarly $$\angle BPC = \frac{1}{2} \angle BO_2C = \frac{1}{2} \cdot 2\theta_2 = \theta_2$$

Method III Join AB and BC. At B draw a line at an angle of $90° - \theta_1$ and at A draw a perpendicular. They intersect at E. Similarly draw a line at B at angle of $90° - \theta_2$ and a perpendicular at C. They interesect at D. Join ED. Drop a perpendicular from B on ED. This cuts DE at P which is the required point (Fig. 16.18)

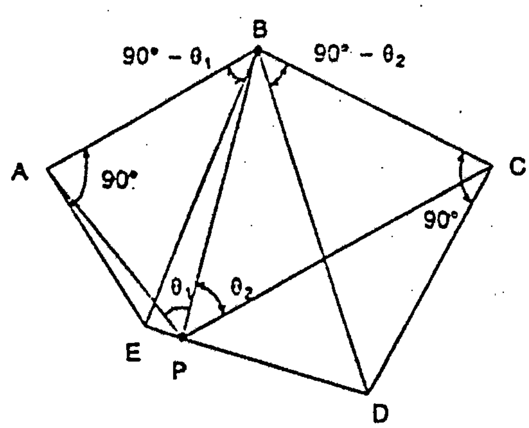


Fig. 16.18 Graphical solution of three point problem (Method III).

Proof As *A*, *B*, *P* and *E* are concyclic $\angle APB = \angle AEB = \theta_1$ and $\angle BPC = \angle BDC = \theta_2$.

Method IV Bessel's method—1. Set up and level the plane table over *P*.

2. With the alidade along *ba* sight *A*, *a* being towards *A*. *Clamp* and place the alidade over *b* to sight *C*. Draw the line *bdd'*. Fig. 16.19(i).

3. *Unclamp* and with the alidade along *ab* sight *B*, *b* being towards *B*. *Clamp* and sight *C* with the alidade passing through a cutting *d'bd* at *d*. Fig. 16.19(ii).

4. *Unclamp* and sight *C* with the alidade along *dc* and *clamp*. The table is now oriented with the help of alidade draw *Aa*. This will intersect *cd* produced at *p* to locate *P*. *Bb* should now pass through *p*. Fig. 16.19 (iii)

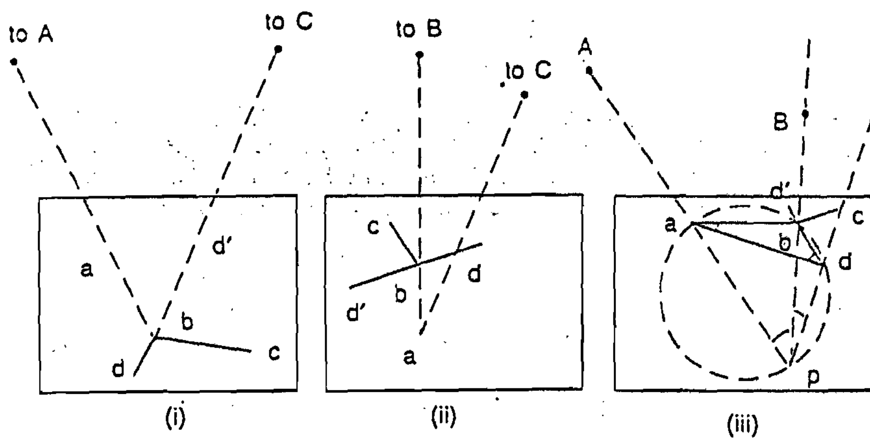


Fig. 16.19 Bessel's solution (Plane Table over station P).

Proof From field observations

$$abd' = APC \text{ and } bad = BPC$$

$\therefore$ $bda = APC - BPC = APB.$

But $bpc = BPC$

$\therefore$ $bpc = bad.$

$\therefore$ b, a, p, d are concyclic. Hence $bda = bpa$

$\therefore$ $apb = APB$

Hence *p* simultaneously subtends with *a*, *b*, *c* the required angles *APB* and *BPC*.

Example 16.3 The sides *AB* and *BC* of a triangle *ABC* with stations in clockwise order are 2001 m and 3114 m respectively and the angle *ABC* is 154°24′. Outside this triangle, a station *O* is established, the stations *B* and *O* being on the opposite sides of *AC*. The position of *O* is to be found by three point resection on *A*, *B* and *C*, the angles *AOB* and *BOC* being respectively 24°12′ and 36°06′. Determine the distances *OA* and *OC*. (U.P.)

Solution

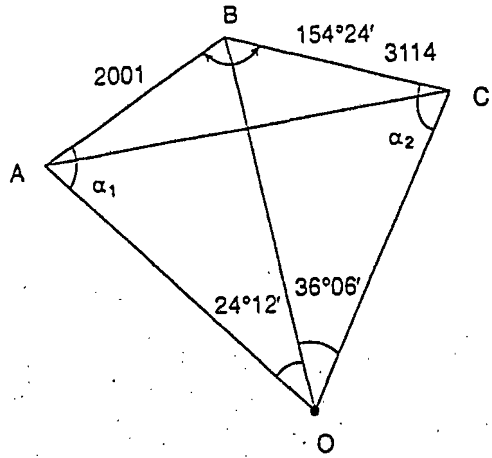


Fig. 16.20 Example 16.3.

Method I

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{L_2 \sin \theta_1}{L_1 \sin \theta_2} = \frac{3114 \cdot \sin 24°12'}{2001 \cdot \sin 36°06'}$$

$$= \frac{3114 \times 0.4099}{2001 \times 0.5892} = 1.0826 = K$$

$$\alpha_1 + \alpha_2 = 360 - (24°12' + 36°06' + 154°24')$$

$$= 145°18' = \phi.$$

$$\tan \alpha_2 = \frac{\sin \phi}{K + \cos \phi}$$

$$= \frac{0.5692}{1.0826 - (0.8221)}$$

$$= \frac{0.5692}{0.2605} = 2.1850$$

$$\alpha_2 = 65.40 = 65°24'$$

$$\alpha_1 = 145°18' - 65°24'$$

$$= 79°54'$$

$$\angle ABO = 180° - (79°54' + 24°12')$$

$$= 75°54'$$

$$\angle OBC = 78°30'$$

Applying sine rule

$$\frac{AB}{\sin 24°12'} = \frac{OB}{\sin 79°54'} = \frac{OA}{\sin 75°54'}$$

$$OB = \frac{2001 \sin 79°54'}{\sin 24°12'}$$

$$= \frac{2001 \times .9845}{0.4099} = 4805.74 \text{ m}$$

$$OA = \frac{2001 \times \sin 75°54'}{\sin 24°12'} = \frac{2001 \times 0.9698}{0.4099}$$

$$= 4734.60 \text{ m}$$

Similarly $$\frac{BC}{\sin 36°06'} = \frac{OC}{\sin 78°30'}$$

or $$OC = \frac{3114 \sin 78°30'}{\sin 36°06'}$$

$$= \frac{3114 \times 0.9799}{0.5892}$$

$$= 5178.93 \text{ m.}$$

Method II

$$\frac{\sin \alpha_2}{\sin \alpha_1} = \frac{L_1 \sin \theta_2}{L_2 \sin \theta_1} = \frac{2001 \sin 36°06'}{3114 \sin 24°12'}$$

$$= 0.9237 = \tan \theta$$

or $$\theta = 42.7287°$$

$$\tan \psi = \cot (45 + \theta) \tan \phi/2$$

$$= \cot 87.7287° \tan 72°39'$$

$$= 0.0396 \times 3.2 = 0.1267$$

$$\psi = 7.2238°$$

$$\alpha_1 - \alpha_2 = 14.4476°$$

$$\alpha_1 + \alpha_2 = 145.3000°$$

or $$2\alpha_1 = 159.74°$$

$$\alpha_1 = 79.87°$$

$$\alpha_2 = 65.43°$$

Rest can be-computed as before.

Example 16.4 The coordinates of three stations A, B, and C are given in Table 1. A point O is set up inside the triangle and the observations in Table 2 are taken. Calculate the coordinates of station O.

Table 1 Example 16.4

Station	Easting (m)	Northing (m)
A	24078.31	29236.48
B	26266.48	31493.20
C	28377.67	29661.04

Table 2 Example 16.4

Angle	Adjusted value
BOA	142°48′32″
COB	92°12′22″
AOC	124°59′06″

Solution From the coordinates

$$AB = [(26266.48 - 24078.31)^2 + (31493.20 - 29236.48)^2]^{1/2}$$
$$= 3143.53 \text{ m.}$$
$$BC = [(28377.67 - 26266.48)^2 + (31493.20 - 29661.04)^2]^{1/2}$$
$$= 2795.34 \text{ m.}$$
$$CA = [(28377.67 - 24078.31)^2 + (29661.04 - 29236.48)^2]^{1/2}$$
$$= 4320.48$$

$$\angle BAC = \cos^{-1}\left(\frac{AC^2 + AB^2 - BC^2}{2 \times AC \times AB}\right)$$
$$= \cos^{-1}\frac{4320.48^2 + 3143.53^2 - 2795.34^2}{2 \times 4320.48 \times 3143.53}$$
$$= \cos^{-1} 0.7633$$
$$= 40°14'24''$$

$$\angle CBA = \cos^{-1}\frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$
$$= \cos^{-1}\frac{3143.53^2 + 2795.34^2 - 4320.48^2}{2 \times 3143.53 \times 2795.34}$$
$$= 93°9'36''$$

$$\angle ACB = \cos^{-1}\frac{CA^2 + CB^2 - AB^2}{2\, CA \cdot CB}$$
$$= \cos^{-1}\frac{4320.48^2 + 2795.34^2 - 3143.53^2}{2 \times 4320.48 \times 2795.34}$$
$$= 46°35'24''$$

check: $40°14'24'' + 93°9'36'' + 46°35'24''$

$= 179°59'24'' \approx 180°$

Coordinates of O, as given by Tienstra's formulae are:

$$E_O = \frac{K_1 E_A + K_2 E_B + K_3 E_C}{K_1 + K_2 + K_3}$$

$$N_O = \frac{K_1 N_A + K_2 N_B + K_3 N_C}{K_1 + K_2 + K_3}$$

where

$$K_1 = \frac{1}{(\cot BAC - \cot BOC)}$$

$$= \frac{1}{\cot 40°14'24'' - \cot 92°12'22''}$$

$$= \frac{1}{1.18149 + .0385}$$

$$= 0.8197$$

$$K_2 = \frac{1}{\cot CBA - \cot COA}$$

$$= \frac{1}{\cot 93°9'36'' - \cot 124°59'06''}$$

$$= \frac{1}{-.0552 - (-0.6998)}$$

$$= \frac{1}{0.6446} = 1.5513$$

$$K_3 = \frac{1}{\cot ACB - \cot AOB}$$

$$= \frac{1}{\cot 46°35'24'' - \cot 142°48'32''}$$

$$= \frac{1}{0.94598 - (-1.31787)}$$

$$= \frac{1}{2.26385} = 0.4417$$

$$K_1 + K_2 + K_3 = 0.8197 + 1.5513 + 0.4417$$

$$= 2.8127$$

$$E_O = \frac{0.8197 \times 24078.31 + 1.5513 \times 26266.48 + 0.4417 \times 28377.67}{2.8127}$$

$$= 25960.322 \text{ m}$$

$$N_O = \frac{K_1 N_A + K_2 N_B + K_3 N_C}{K_1 + K_2 + K_3}$$

$$= \frac{0.8197 \times 29236.48 + 1.5513 \times 31493.20 + 0.4417 \times 29661.04}{2.8127}$$

$$= 30547.81 \text{ m}$$

PROBLEMS

16.1 What are the different methods of 'plane tabling'? Describe them fully with neat sketches. [AMIE Surveying Summer 1978].

16.2 (a) State the advantages and disadvantages of plane table surveying.
(b) Explain with neat sketches any one method of solving the 'three point problem' [AMIE Surveying Summer 1980].

16.3 (a) Enumerate the different methods of plane tabling and highlight the topographical conditions under which each one is preferred.
(b) Explain with neat sketches any one method of plane tabling for locating the details. [AMIE Surveying Winter 1980]

16.4 (a) What is meant by "two point problem" in plane table survey?
(b) Explain with neat sketches the solution of "two point problem" in the field. [AMIE Surveying Winter 1981].

16.5 (a) Describe the advantages and disadvantages of plane table survey.
(b) What do you understand by orientation in plane table survey? Explain different methods of orientation. [AMIE Surveying Summer 1983]

16.6 (a) What are the accessories required for a plane table surveying?
(b) State three point problem in plane tabling. Describe its solution by trial and error method. Briefly indicate the rules which may be followed in estimating the position of the point sought. [AMIE Surveying Winter 1984].

16.7 (a) State the advantages and disadvantages of plane table survey over other types of survey.
(b) Explain with sketches any one method of solving the three point problem. [AMIE Surveying Winter 1984]

16.8 (a) With the help of neat sketches describe the plane table survey operations of radiation and intersection.
(b) Explain what is understood by orientation of a plane table and how the method of resection is useful for this purpose. Define the three point problem and with the help of neat sketches. Describe stepwise the solution of the problem in field by the Lehmann's rules. [AMIE Summer 1986]

16.9 (a) Enumerate different methods of plane table survey. Under what field conditions each method is used?
(b) What do you understand by strength of fix? Explain with the help of neat sketches, the terms good fix, bad fix and failure of fix.

(c) Enumerate the various sources of error in plane table survey. What precautions will you take against each?

[AMIE Surveying Winter 1986]

16.10 (a) What is three point problem? How is it solved by Bessel's method?

(b) Compare the advantages and disadvantages of plane table surveying with those of chain surveying.

(c) In setting up the plane table at a station '*A*', the corresponding point on the plan was not accurately centred above *A*. If the displacement of *A* was 25 cm in a direction at right angles to the ray, how much on the plan would be the consequent displacement of the point from its true position, if (i) scale 1 cm = 100 m and (ii) 1 cm = 2 m.

[AMIE Surveying Summer 1987]

16.11 (a) Explain with sketches the methods of orienting plane table by back sighting.

(b) Describe, with neat sketches, the application of Lehmanns' Rules in solving three point problem. [AMIE Surveying Winter 1990]

16.12 (a) List the accessories used in plane tabling highlighting their purpose.

(b) Enumerate the methods of plane tabling and state the conditions under which each one is preferred.

(c) Describe the methods of orienting a plane table

[AMIE Surveying Summer 1991]

16.13 (a) List the instruments and accessories used in plane table survey.

(b) Describe the graphical method of adjustment of plane table traverse.

(c) State the methods used for plane tabling. Under what conditions is each of these preferred to?

(d) What is two point problem and how it can be solved in the field?

[AMIE Surveying Winter 1993]

16.14 (a) What is meant by plane tabling? When do you recommend it? State the advantages and disadvantages of plane tabling.

(b) Describe with neat sketch, the method of resection. For what purpose it is used?

(c) Explain clearly the two point problem and how it is solved.

[AMIE Surveying Winter 1994]

16.15 (a) Discuss the advantages and disadvantages of plane table surveying.

(b) Explain the three-point problem and show how it is solved by (i) tracing paper method (ii) trial and error method.

(c) In setting up the plane table at a station *A*, it was found that the point *a*, representing the station *A* on the plan was not exactly above the corresponding station *A* on the ground. If the displacement of *a* in a direction at right angles to a ray to *P* (*AP*) was 30 cm, find the consequent displacement of *p* from its true position, given that (i) scale of plan is 1 cm = 150 m, distance *AP* = 2000 m, (ii) scale of plan (RF) = 1/600, distance of *AP* = 40 m and (iii) scale of plan is 1 cm = 2 m, *AP* = 20 m. [AMIE Surveying Summer 1996]

17

Topographical Surveying

17.1 INTRODUCTION

The object of topographical surveying is to produce a topographic map showing elevations, natural and artificial features and forms of the earth's surface. It is drawn from field survey data or aerial photographs. Instruments required include transit, plane table and alidade, level, hand level, tape and levelling in various combinations. Total station EDM's are used to advantage in topographic surveying. Though aerial photographic methods are extensively used in preparing topographic maps, ground methods are still required for checking aerial photogrammetry and also for plotting details. For any engineering project topographic survey is a must. Whether it is laying a railway or highway or design of an irrigation or drainage system, the topographical features of the place must be known so that correct engineering decisions may be taken.

17.2 CONTROL FOR TOPOGRAPHIC SURVEYS

A topographic map should be drawn in three phases as given in the following.

1. Develop horizontal control producing a frame work for plotting details.

2. Plot all points of known elevation and locations of artificial or natural features for vertical control.

3. Construct contour lines from plotted points of elevation, drawing all features and symbols.

Horizontal control is provided by two or more points on the ground precisely fixed in position horizontally by distance and direction. It is the basis for map scale and locating topographical features. Usual methods are traversing, triangulation, trilateration or inertial or satellite methods.

Vertical control is provided by benchmarks in or near the tract to be surveyed. Elevations are found out at all traverse points.

Once the horizontal control is obtained, any other point can be obtained by using geometric principles as shown in Fig. 17.1. The following informations are required:

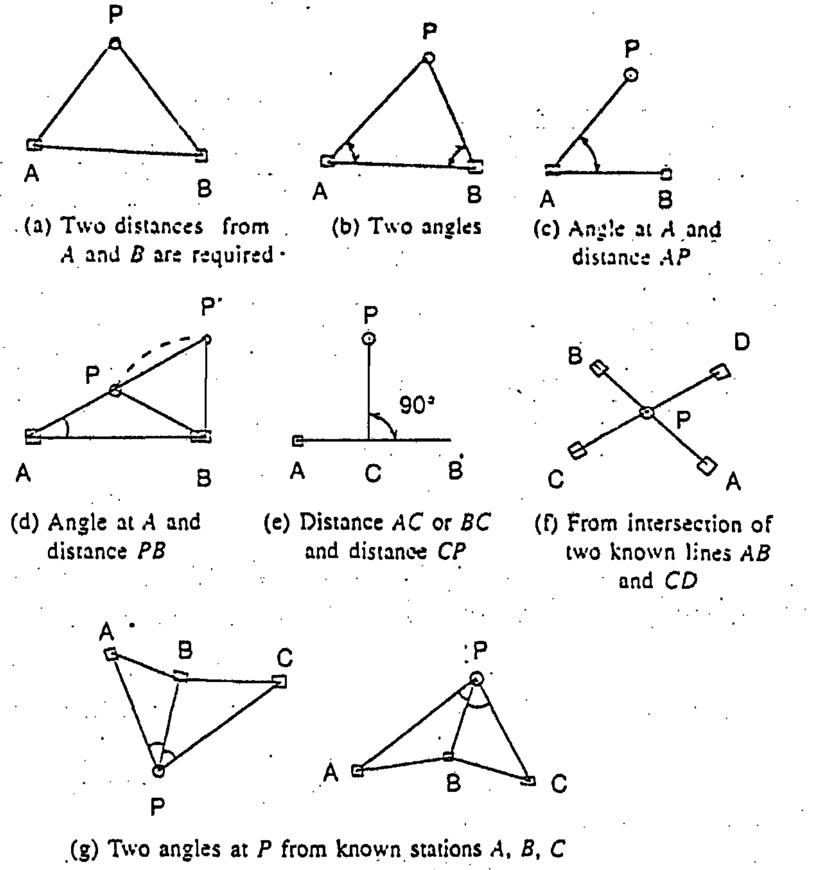


Fig. 17.1 Locating a point P.

(a) two distances;
(b) two angles;
(c) One angle and adjacent distance;
(d) One angle and the opposite distance. The solution is not unique as two points can be obtained;
(e) One distance and a right angle offset;
(f) The intersection of two known lines;
(g) Two angles at the point to be located.

17.3 PLOTTING OF CONTOURS

In a topographic map the elevations of different points are shown by means of contours. From the study of the contours the surface features such as hills, mountains, depressions or undulations of the earth can be easily understood (Fig. 17.2). A contour line is an imaginary line containing points of equal elevation and it is obtained when the surface of the ground is intersected by a level surface. This can be understood by studying the contours of a hill. Suppose a hill is cut by

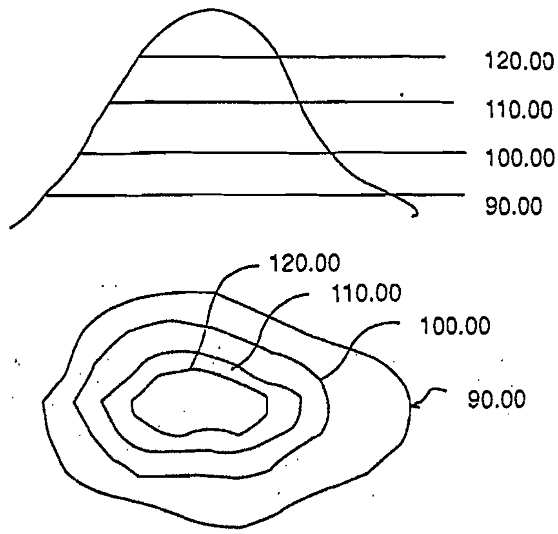


Fig. 17.2 Contour of a hill.

(Fig. 17.2) imaginery level surfaces at 90.00, 100.00, 110.00 and 120.00. Then the plan of the cut surfaces will give the contour line. The contour lines will be circular if the level surfaces cut a vertical cone, elliptical if they cut a sloping cone, straight lines if the surface is uniformly sloping.

The vertical distance between any two successive contours is known as contour interval. The contour interval is kept constant for a contour plan, otherwise interpretation of contour will be difficult. The contour interval depends on (i) nature of the ground, (ii) scale of the map, (iii) purpose and extent of survey, (iv) time and expense of field and office work.

The contour interval should be small when the ground is flat, the scale of the map is large, the survey is detailed survey for design work and longtime and large cost can be accepted.

The contour interval may be large when the ground is of steep slope, the scale of the map is small, the survey is preliminary and the survey is to be completed in a short time and cost should be small.

17.4 CHARACTERISTICS OF CONTOUR

1. Contours must close upon themselves though necessarily within the map.

2. Contours are perpendicular to the direction of maximum slope.

3. The slope between contours of equal intervals is assumed to be uniform and difference in contour divided by the distance gives the steepness of a slope. Hence if the contours are widely spaced the slope is gentle, if the contours are closely spaced the slope is steep. When the contours are evenly and parallel spaced, it indicates uniform slope.

4. Concentric closed contours that increase in elevation represent hills. If they decrease in elevation it is a pond. Depression contours will have inward facing radial marks to avoid confusion.

5. Contours of different elevation never meet except on a vertical surface such as overhanging cliff or cave (Fig. 17.3).

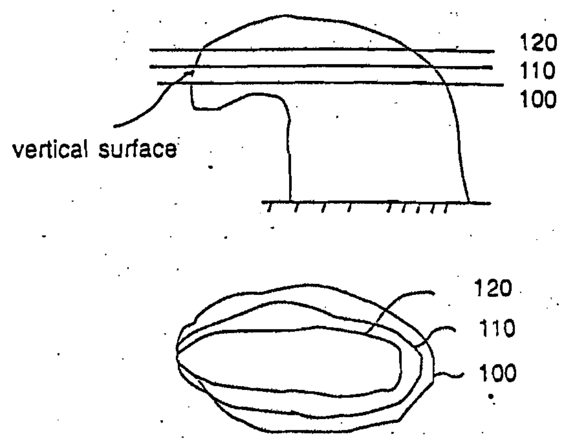


Fig. 17.3 Overhanging cliff with vertical surface.

6. Two contour lines having the same elevation cannot unite and continue as one line. Similarly a single contour cannot split into two lines. Two contours of same elevation meeting in a line indicates knife edge condition which is seldom found in nature.

7. Contour lines cross at right angles ridge crest in the form of U's. Similarly it crosses a valley also at right angles in the form of Vs. As shown in Fig. 17.4 contour lines go in pairs up valleys and sides of the ridges.

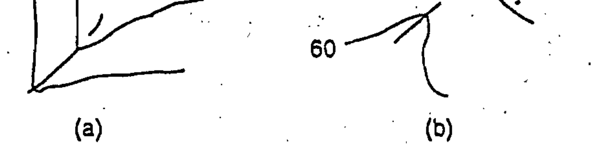


Fig. 17.4 Contours of (a) Ridge, (b) Valley.

17.5 METHODS OF LOCATING CONTOURS

There are two principal methods of locating contours (1) Direct method also known as trace contour method; (2) Indirect method also known as controlling point method.

In the direct method, the contour to be plotted is actually traced on the ground. Only those points are surveyed which happen to be plotted. Say R.L. of point *A* is 120.45 m. If elevation of the instrument is 1.05 m. H.I. = 121.50 (Fig. 17.5). If we want to plot contour lines of 119, 120, 121, etc. subsequent rod readings should be 2.5, 1.5 and 0.5 respectively. The rod person has to move to different points *X*, *Y*, *Z* to locate the different contour points. This method of plotting contours is accurate and is useful for an engineering study involving drainage or irrigation. However the method becomes impractical as too much time is required with ordinary methods of reducing stadia interval to get the required difference in elevation.

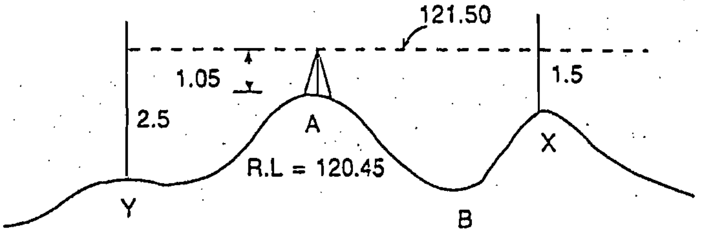


Fig. 17.5 Direct method of contouring.

In the indirect method, the contours are located by determining the elevations of well chosen points from which the positions of points on the contours are determined by interpolation. With the instrument set at *A* elevations of points *B*, *C*, *D*, *E* can be obtained. *B*, *C*, *D* and *E* are the controlling points from which contours will be interpolated by topographer from experience and by judgment.

17.6 FIELD METHODS OF OBTAINING TOPOGRAPHY

There are many methods for obtaining contours. Two data are obviously required. (i) Location of the points, (ii) Elevation of the points.

Instruments like transit or theodolite, plane table, hand level, EDMI are used in various combinations to plot the points and compute their levels. The different methods for obtaining topography are as follows.

Radiation method

In this method, the traverse stations (previously plotted) are occupied with a transit or theodolite and angles to desired contour points and features are measured. Levels are taken along these radial lines at measured distances from the centre. Interpolation is used to give the contour line. This method is particularly suitable for contouring small hills. The method is quick if sophisticated equipment like a combination theodolite—EDMI (total station instrument) with self reduction capacity is used.

Stadia method

In this method distances and elevations of points are obtained by stadia interval, azimuths and vertical angles. The method is rapid and sufficiently accurate for most topographic surveys.

Plane table survey

Here plane table procedures are employed to locate a point. A stadia distance and vertical angles are read. Sometimes, the observation of vertical angle is avoided by using the alidade as a level. Contour is plotted immediately at the site either by direct or indirect method. This ensures correct reproduction of the area.

Coordinate squares method

Here the area is divided into a number of squares, the side of he square depends on the terrain and accuracy of the survey. The instrument is then placed at a suitable position and readings are taken at the corners of the squares. Contours are then interpolated between the corner elevations by estimation or by proportionate distances assuming the slope between points to be uniform.

Cross section methods

This is usually done in connection with route survey. The longitudinal profile is drawn along the centre line of the route. Levels are then taken at right angles to the centre line at suitable intervals and at all break points so that a true profile of the area is obtained. These cross-section data can be used for compiling contours. They can also be used for earth work computation. On some surveys contour points are directly located along with any important change in ground slope. For example, if 2 m contours are required, the page of the field book will be as follows:

			L		C/L		R		
80	78	76	74	70	72.5	74	76	78	80
8.0	6.7	5.5	4.8	3.0	0.00	2.0	3.5	4.8	6.2

Contours by hand level

A hand level can be used for finding the height of a point when very high precision is not required. In this method from a known elevation and measuring the height of the eye of the observer, the elevation of the observer's eye is known. When levelling uphill, the point at which the observer's eye strikes the ground is noted. The observer then moves to the new station and adding again observer's height a new elevation is obtained.

17.7 SOURCES OF ERRORS IN TOPOGRAPHICAL SURVEYS

Instrumental errors

(i) Since while plotting contours different instruments like transit, theodolite, plane table, hand level, EDMI are used, errors will occur if there are maladjustments in the instruments. (ii) Errors may occur in reading the instruments.

Errors may also occur due to

(i) Control traverse not being properly established, checked and adjusted. (ii) Control points not properly selected for easy coverage of the area. (iii) Instrument points not properly selected for clear visibility of distant points or for contour delineation.

Errors may also occur in mapping due to

(i) inaccurate line work from blunt or too soft pencil. (ii) inaccurate angular plotting with a protractor. (iii) inaccurate plotting to scale. (iv) improper selection of scale.

Mistakes

Mistakes may occur due to (i) instrument (ii) misorientation, (iii) misinterpretation of field notes, (iv) inadequate number of contour points, (v) omission of some topographical details.

17.8 INTERPOLATION OF CONTOURS

Interpolation is a process of spacing the contours proportionately between the plotted ground points established by indirect method. It can be done by

1. *Estimation* Here the positions of contour points between the guide points are located by estimation based on experience and judgment.

2. *Direct calculation* The intervening horizontal distance between the guide points is measured and contour points are located by arithmetic calculations using theory of proportion.

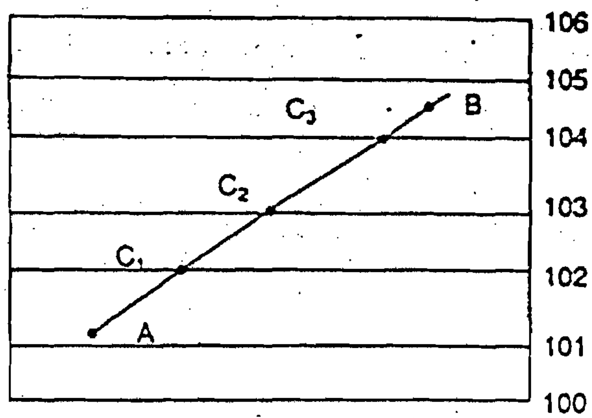
3. *Mechanical interpolation* A rubber band marked with uniform series of marks can be stretched to find the correct interval for each line.

4. *Graphical method* In this method, the interpolation is done with the help of a tracing paper or tracing cloth. In the first method parallel lines are drawn on the tracing cloth representing different lines of elevation. For interpolation of contours between two points *A* and *B* of known elevation, the tracing paper is placed over *A* and *B* in known elevation lines. The intermediate elevations are then pricked on the line. In the second method converging lines are drawn on the tracing paper. Moving the tracing cloth over the plan so that point *A* lies on the radial line representing elevation of *A* and point *B* on the radial line representing *B*, the points having other elevations can be pricked. These are explained in Figs. 17.6 and 17.7.

17.9 USES OF CONTOURS

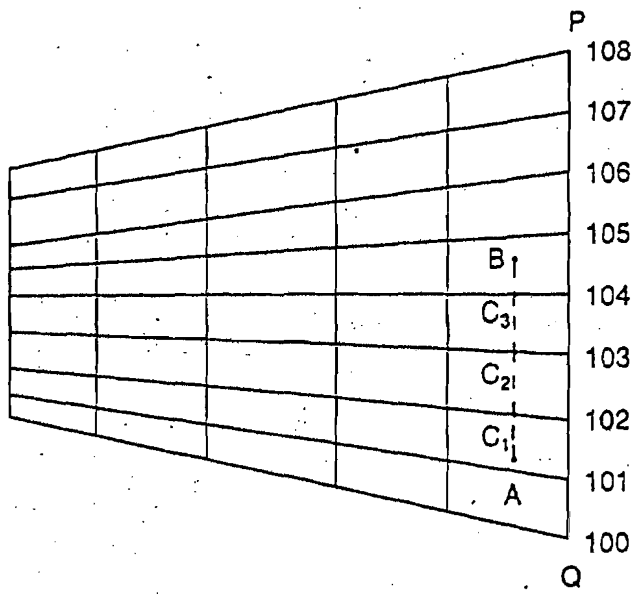
The following are the important uses of contours.

1. *Drawing of sections* From the contour lines the section along any given direction can be drawn. This is shown in Fig. 17.8. The levels of the points where



Interpolation of contours (A 101.2, B 104.6, C_1 102, C_2 103 and C_3 104)

Fig. 17.6 Tracing paper method (parallel lines).



Interpolation of contour A 101.2, B 104.6, C_1, C_2, C_3, – 102, 103, 104 respectively. Adjust the tracing paper so that AB is parallel to PQ.

Fig. 17.7 Tracing paper method (converging lines).

the section line cuts the contour lines are known and hence they can be plotted to a scale and the profile of the section obtained.

2. *Intervisibility between points* From the contour lines intervisibility between two points A and B can be determined. The points A and B are joined by a line and section of the ground is drawn. From the location of the points in the section and the ground profile, it is possible to find out whether A and B are intervisible (Fig. 17.9).

3. *Plotting contour gradient and location of route* Contour gradient is a line lying through out on the surface of the ground and preserving a constant

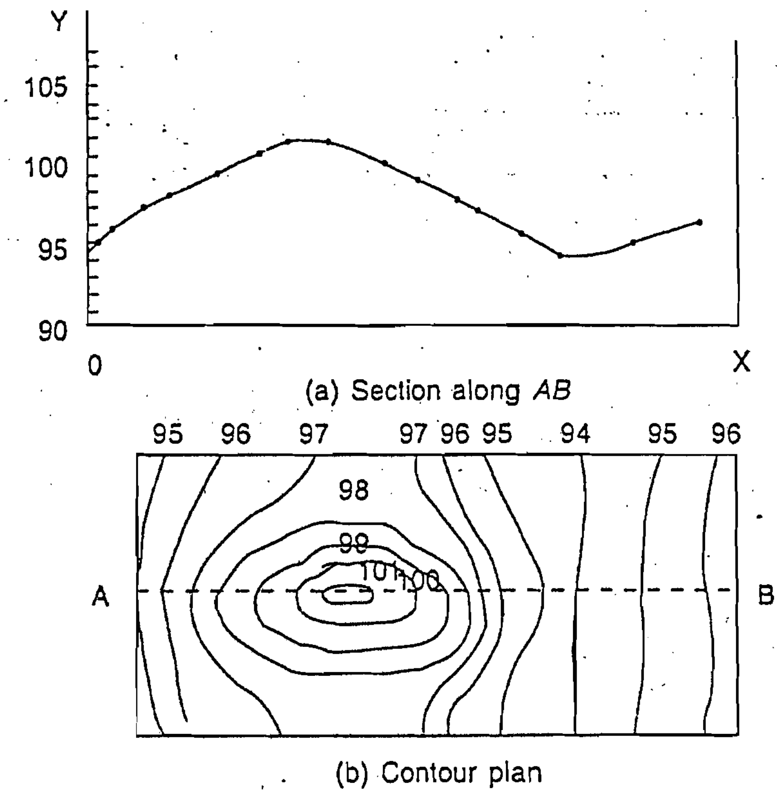


Fig. 17.8 Section from plan of a contoured area.

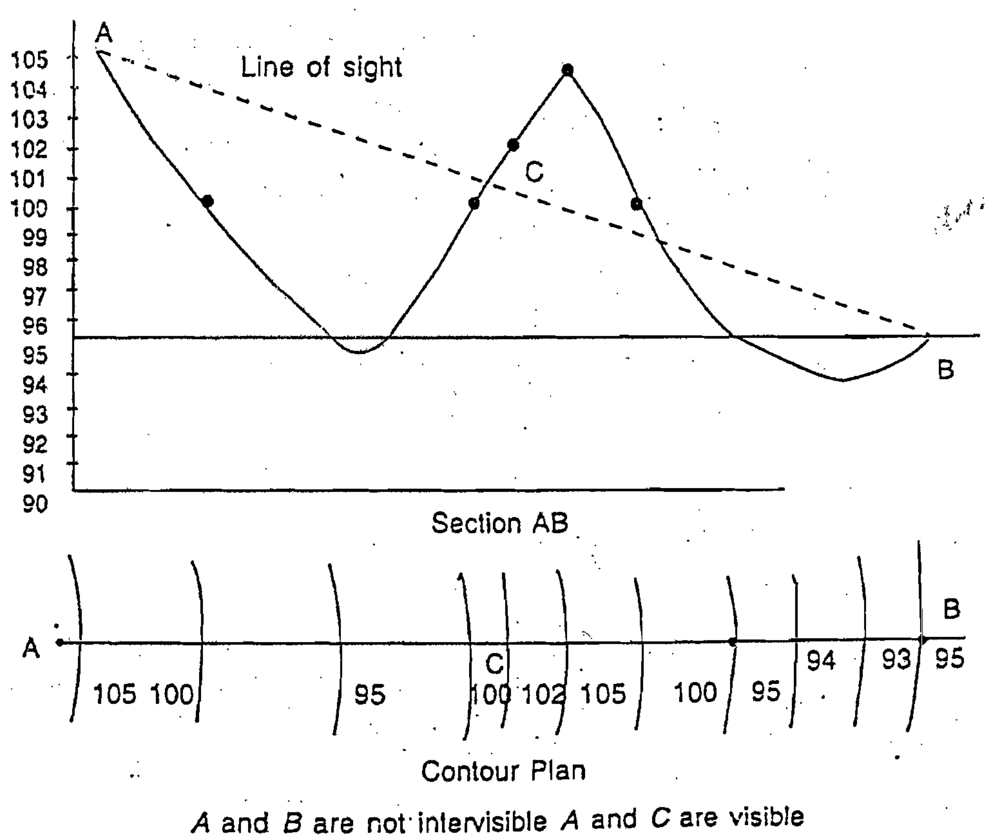


Fig. 17.9 Intervisibility of points.

inclination to the horizontal: If a highway, railway canal or any other communication line is to be laid at a constant gradient, the alignment can easily be plotted on the plan or map. If the contours are at an interval of m meters and if the gradient is 1 in n, the horizontal distance to cover m meters is m/n meter. From the initial point A, an arc should be drawn with distance m/n to locate the first point a. Similarly from a to b and so on. The distances should be in the same scale as contour map.

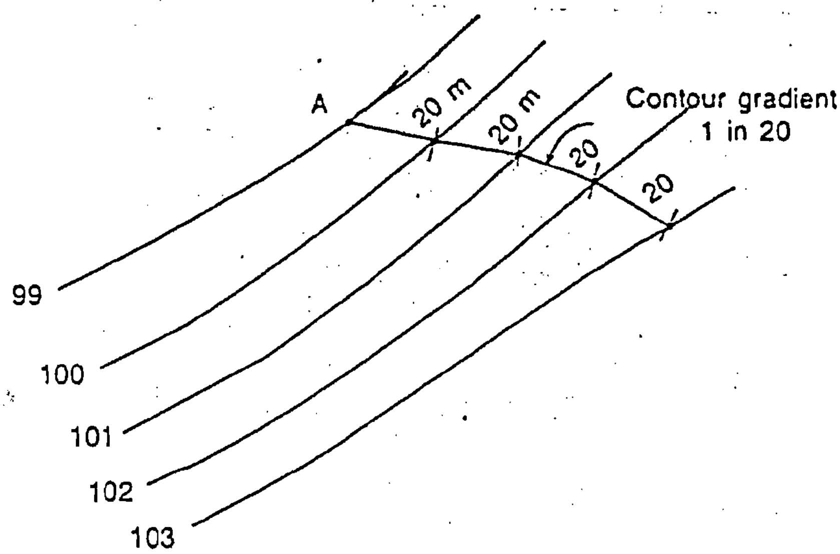


Fig. 17.10 Plotting contour gradient.

4. *Determination of catchment area* . A study of the contour enables us to calculate the catchment area of a river. The catchment area of a river has a typical pattern with ridges and saddles. Watershed line is defined as the line which separates the catchment basin of a river from the rest of the area. This line crosses the contour lines at ridges and saddles at right angles. This area can be calculated with the help of a planimeter (Fig. 17.11).

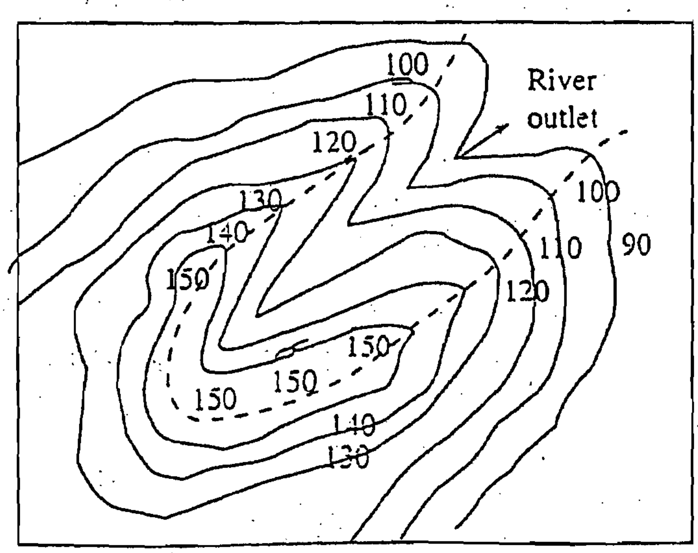


Fig. 17.11 Catchment area.

5. *Estimation of reservoir capacity* From a study of the contour lines, the reservoir capacity of a dam can be computed. The areas between different contour lines can be measured by planimeter. Average area into depth gives the volume between contour interval.

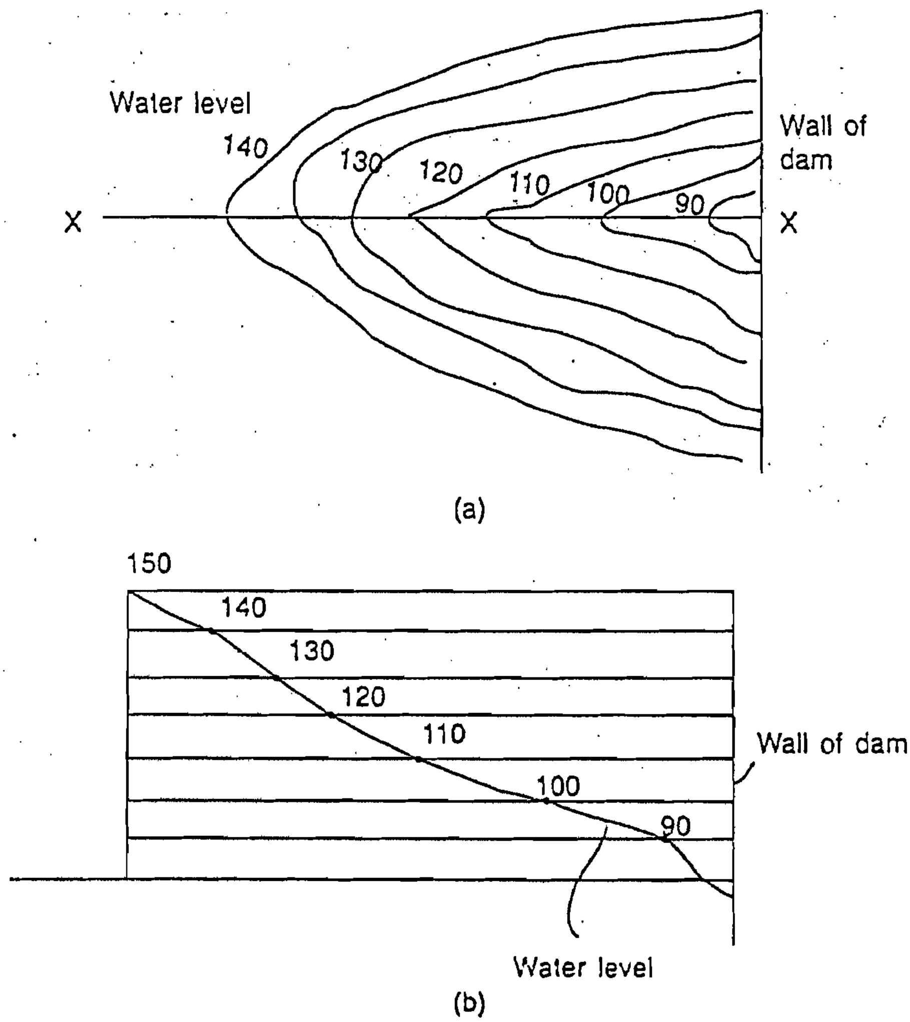


Fig. 17.12 Estimation of reservoir capacity from contour.

PROBLEMS

17.1 (a) What are the different methods of 'contouring'? Describe any one of them.

(b) What are the uses of a contour map? How will you determine the intervisibility of a point if the contour map is given to you? Explain by giving an example. [AMIE Surveying Summer 1978]

17.2 (a) What is a contour? Define and explain.

(b) Describe the method of squares for finding the contours in a map of a plot of land.

(c) What is meant by (i) contour interval, (ii) Contouring by direct method, (iii) contour gradient? [AMIE Surveying Winter 1979]

17.3 (a) Show with neat sketches the characteristic features of contour lines for the following:

(i) A pond, (ii) A hill, (iii) A ridge, (iv) A valley, (v) A vertical cliff.

(b) Enumerate the uses of contours and illustrate one such use with a sketch. [AMIE Summer 1980]

17.4 (a) List the uses of a contoured topographic map. Show with the help of neat sketches, the characteristic formation of contour lines for the following topographic features:

(i) Vertical cliff (ii) Over hanging cliff, (iii) Valley, and (iv) Ridge.

(b) The areas enclosed by contour lines at 5 m interval for a reservoir upto the face of the proposed dam are as shown below:

Value of Contour lines (m)	1005	1010	1015	1020	1025	1030	1035
Area (m^2)	400	1500	3000	8000	18,000	25,000	40,000

Taking 1005 m and 1035 m as the bottom most level and the highest water level achievable of the reservoir determine the capacity of the reservoir by (i) Trapezoidal formula and (ii) Prismoidal formula.

18

Construction Surveying

18.1 INTRODUCTION

In every country construction is a major activity and setting out, therefore, becomes an important work for the surveyor. Normally surveying involves preparation of a map or plan showing existing features of the ground. Setting out is the reverse process of fixing on the ground the details shown in a map or plan.

18.2 EQUIPMENTS FOR SETTING OUT

Normally ordinary equipments as described before, e.g. levels, theodolites, tapes and EDMI's are used. However, for vertical control Automatic laser levels are being frequently used these days. They provide a continuous sharp beam of visible light at a given grade (selectable by the operator) and maintain it at the same grade precisely at all times. The laser beam can be intercepted at any point by a special target. This way one knows one's own level without any one giving readings from the instrument end. An extended development of such laser level is to provide a continuously rotating beam with a given grade thereby giving a plane in the same grade. They can be applied for tunnel alignment, machine alignment, elevator shaft alignment, pipe laying, false roofing installation, etc. They expedite placement of grade stakes over large areas such as airports, parking lots, etc. Laser methods have the advantage of being (i) convenient, (ii) quick, and (iii) accurate. However, they are quite expensive. Theodolites combined with EDMIs that can automatically reduce measured slope distances to their horizontal and vertical components and "total-station" instruments are also very convenient for construction stakes.

18.3 HORIZONTAL AND VERTICAL CONTROL

The importance of a good frame work for horizontal and vertical control in a project area cannot be over-emphasized. It is important for a surveyor in charge of a project to describe and reference all major horizontal control monuments. Methods shown in Fig. 18.1 can be used with intersection angle as close to 90° as possible.

To preserve vertical control monuments (benchmarks) it is recommended

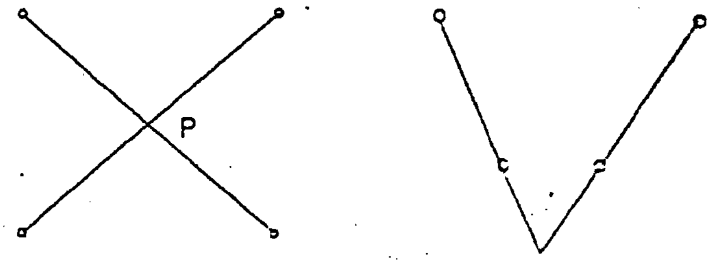


Fig. 18.1 Horizontal control.

that an adequate number of differential level circuits be run to establish supplementary benchmarks removed from areas of construction and possible displacements, yet close enough for efficient use by construction personnel.

For large projects, it is common engineering practice to establish a rectangular grid system. Usually such a system is based on a local coordinate system. The ends of major x and y grids are fixed by concrete monument supporting a metal disc. Intermediate points are fixed by wooden stakes 50 × 100 mm.

18.4 SETTING OUT A PIPE LINE

Pipelines are of two types (i) Gravity flow lines, (ii) Pressure flow lines. Slopes must be very carefully maintained in gravity flow lines because it utilizes only the force of gravity for maintaining flow. In contrast, pressure flow lines generally depend upon a pump to provide movement of liquids through the line. There are mainly two methods: (i) Conventional Methods of using sight rail, boning rods, etc. (ii) By means of laser.

Conventional methods

The steps in the *conventional methods* are as follows.

Principal points such as manhole locations and the beginnings and ends of curves are established on the ground along the designed pipeline centre line location. An offset line parallel to the pipeline centre line and far enough from it to prevent displacement during excavation and construction is established. Marks should be closer together on horizontal and vertical curves than on straight segments. A marker stake is placed behind the grade stake (The side of the grade stake opposite the pipe centre line). On the flat side of the marker stake is marked "C" to the invert or pipe flow line and the grade stake's station location. On the reverse side is shown the horizontal offset from the pipe centre (Fig. 18.2).

On hard surfaces where stakes cannot be driven, points are marked by paint, spikes or by other means.

The work is usually done in two steps: (i) excavation with trenching machine and setting of guide line (Fig. 18.3), (ii) transferring the invert grade from guideline to pipe invert. For transferring invert grade the following procedure needs to be followed.

(a) Grade boards are erected by driving a 5 × 10 cm upright on each side of trench and nailing a 2.5 × 15 board to them.

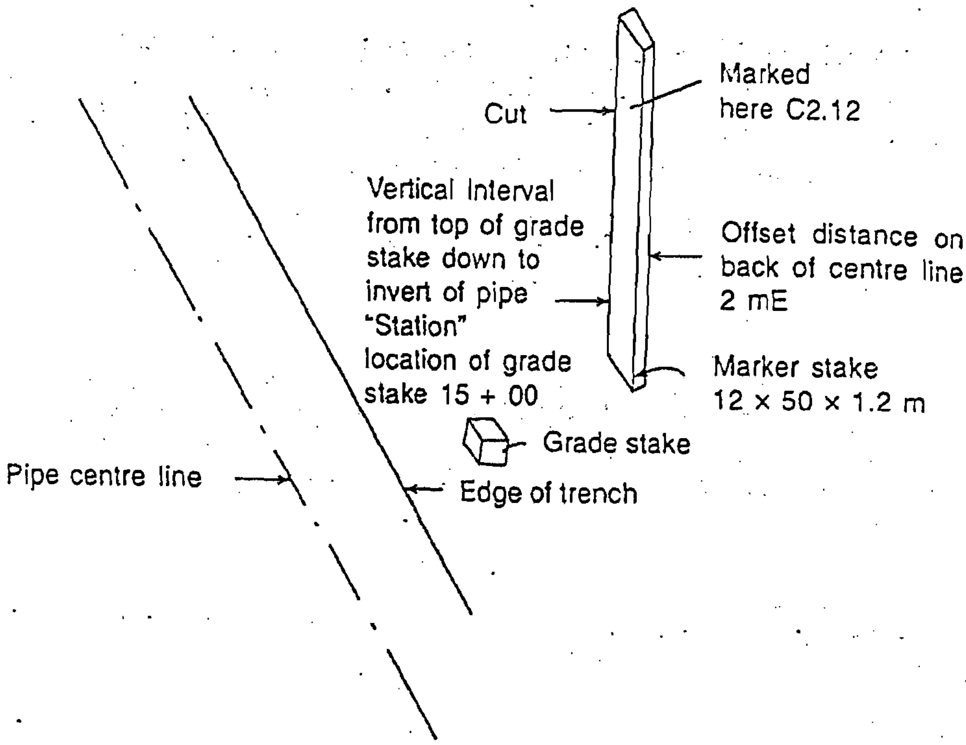


Fig. 18.2 Laying a pipe line (conventional method).

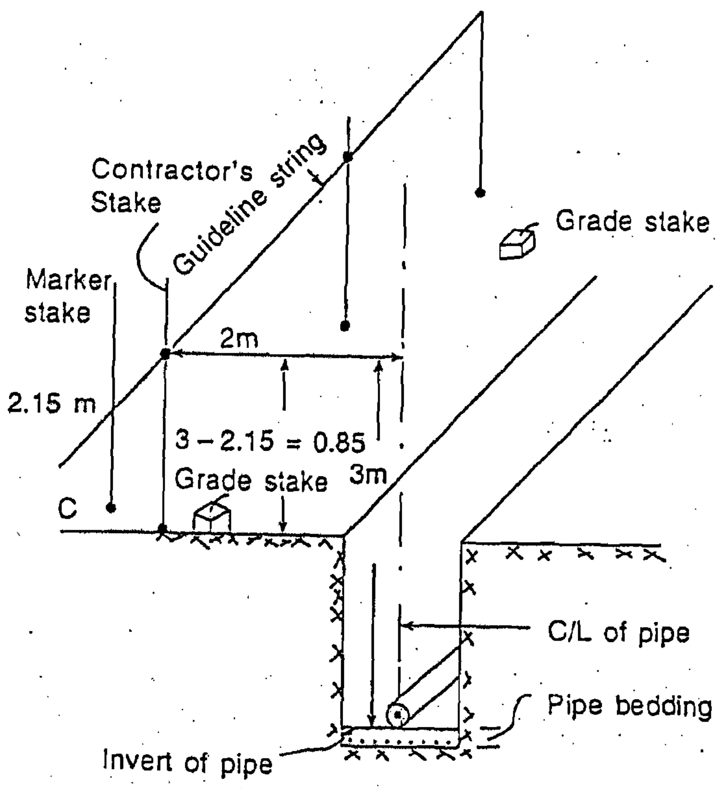


Fig. 18.3 Excavation with trenching machine.

(b) The top edge of the cross or grade board is set at a full no vertical interval (3 m in this case).

(c) A nail is driven into the top edge of the grade board to mark horizontal alignment and a string or wire stretched between each alignment nail to provide a checking line for pipe installation. A grade pole (long wooden pole with a right angle foot on its bottom end) transfers the invert grade from guide line to pipe invert.

Here level of C = 2.15 m. Guideline string is so fixed that it is 3 m (a full meter number) above invert of pipe. Hence guideline is to be set 0.85 m above grade stake (Fig. 18.4).

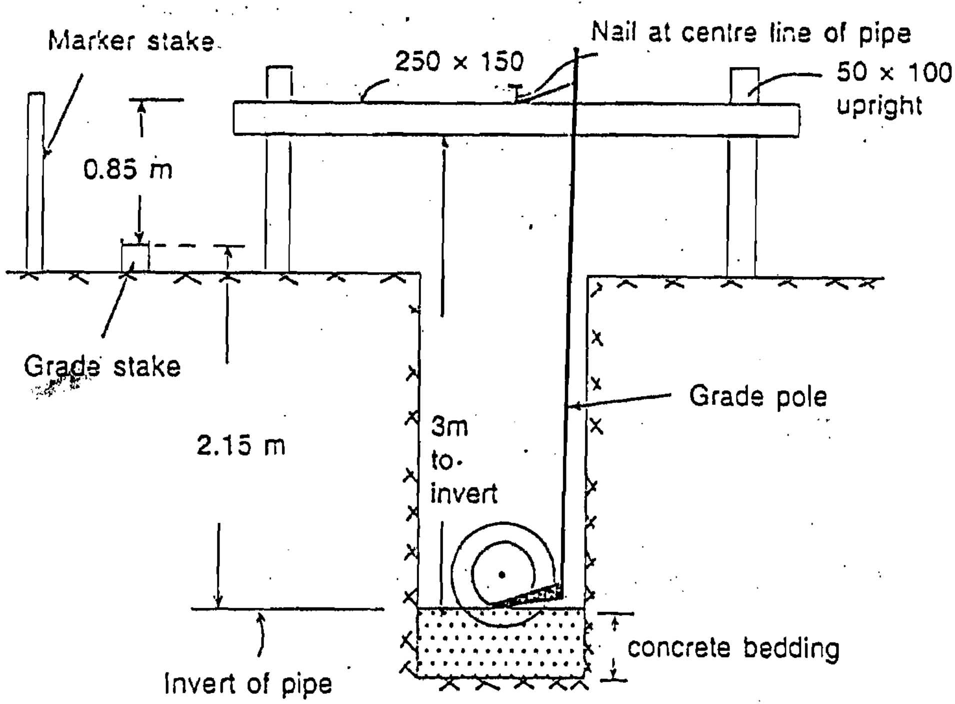


Fig. 18.4 Transferring grade to pipe invert.

Laying pipeline through laser

Laser equipped survey instruments can also be used in laying accurately pipeline. Though laser replaces guidelines and much grade setting and checking work, conventional survey methods are still required for correct positioning and alignment. As already stated laser methods have the following advantages: (i) Less Labour is required (ii) Line and grade can be accurately staked (iii) On going work can be easily checked (iv) Trench can be back filled as soon as the pipe is installed.

Figure 18.5 shows how the laser beam provides a horizontal and vertical alignment and replaces the guideline.

Figure 18.6 shows how the laser beam can be placed inside a pipeline and aligned along centre line and slope.

18.5 SETTING OUT OF BUILDINGS AND STRUCTURES

The first task in setting out a building or a structure is to locate the ownership line. This is required for (i) to provide a baseline for layout, (ii) to check that the proposed building does not encroach on adjoining properties.

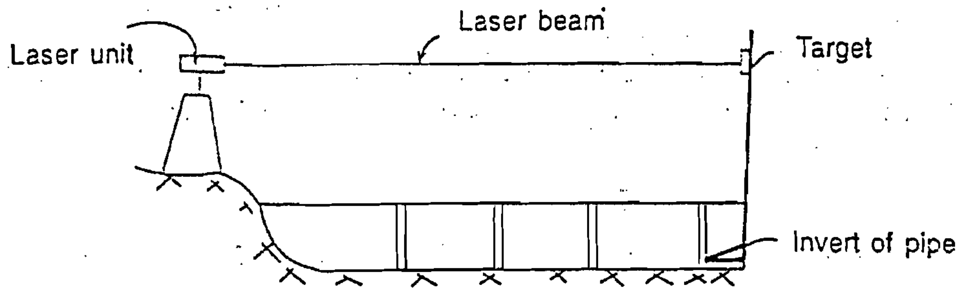


Fig. 18.5 Laying pipeline through laser.

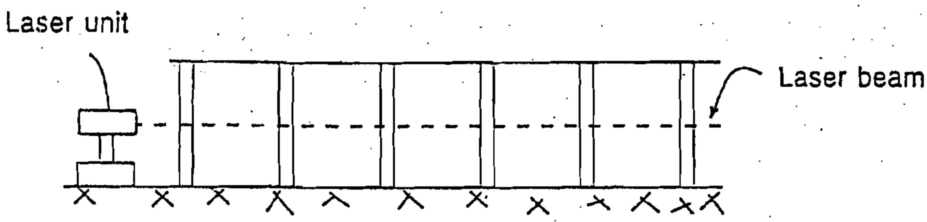


Fig. 18.6 Laser beam inside pipeline.

Stakes may be set initially at the exact building corners as a visual check on positioning of the structure but these points are lost as soon as excavation begins. Hence batter boards are necessary. Figure 18.7 shows a typical batter board.

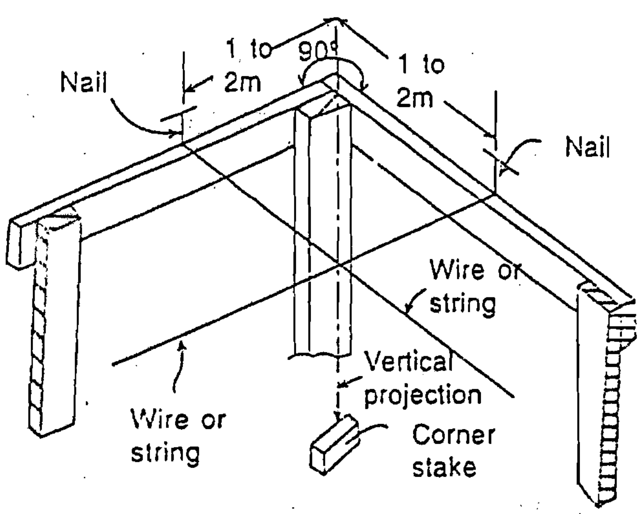


Fig. 18.7 Typical batter board.

It is usually set 1 to 2 m from each end of the intersecting building lines. Top of the cross pieces are nailed a full number of meter above the footing base or at first floor elevation. If possible all batter boards should be set at the same elevation so that a level line is created. Corner stakes and batter board points are checked by measuring diagonals for comparison with each other and their computed values. Benchmarks (beyond the construction area are required to control elevations. After erection of batter boards, excavation of the structures' footings or basement can begin.

Staking out a building can be quickened by taking minimum instrument setups and staking many points from a single setup of the instrument. Final building dimensions, however, should be checked by tapes, EDMI's etc.

18.6 STAKING OUT A HIGHWAY

The staking of a highway project is usually done in the following steps.

1. The first step is to provide the contractor with stakes showing points marking limits of the construction project. This will enable the contractor to clear the site and hence these stakes are known as *clearing stakes*. They are 1.5 cm × 2.5 cm × 1.25 m wooden lathe and are placed 1 chain apart.

2. Next, rough cut stakes are to be provided so that the contractor can undertake "rough-cut" grading operations.

These stakes are set (i) along the project centre line at 15 m interval, (ii) at the beginning and end of all horizontal curves, (iii) at any other grade or alignment transition. The stakes are 2.5 cm × 5.0 cm × 45 cm. On the stakes are marked *C* or *F* indicating cut or fill.

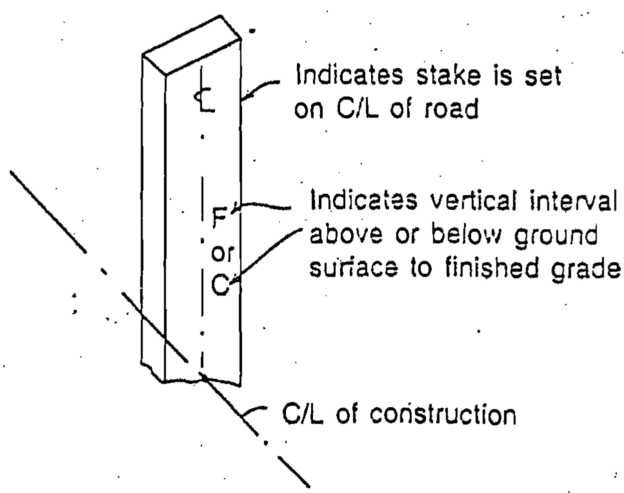


Fig. 18.8 Typical marking on stakes.

3. To guide a contractor in marking final excavations and embankments slope stakes are driven at the slope intercepts (intersections of the original ground and each side slope) or off set a short distance perhaps a meter. Figure 18.9 shows the principles of slope stakes.

Grade stakes are set at points that have the same ground and grade elevation. Three transition sections normally occur in passing from cut to fill and a grade stake is set at each one. Figure 18.10 shows slope staking and grade points at transition sections.

Example 18.1 Describe the procedure of setting out of a two storeyed building with 200 mm thick load bearing outer walls all round. The foundation width is 750 mm. The building is absolutely rectangular and outside dimensions above plinth level are 12,000 mm × 18,000 mm. Draw the 'foundation plan' of the outer

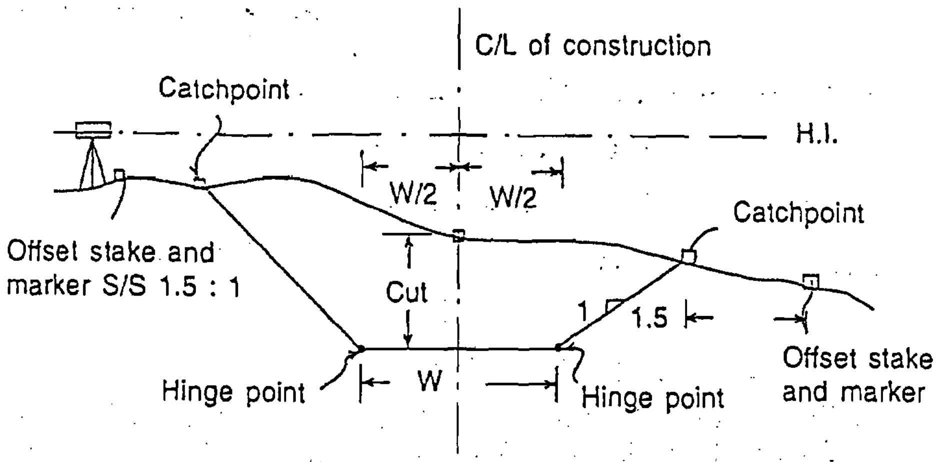


Fig. 18.9 Principles of slope stakes.

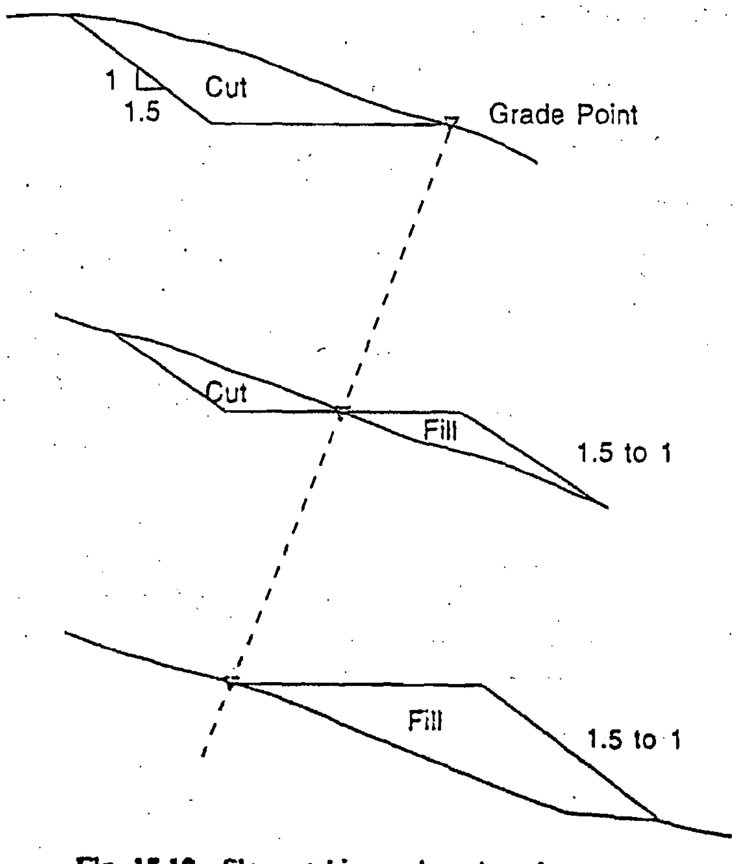


Fig. 18.10 Slope staking and grade points.

walls showing all the dimensions (not to scale). In the above plan show the positions of different pegs required for setting out of the above building.

(AMIE Surveying Winter 1978)

Solution The outside dimensions are

12,000 mm × 18,000 mm

With walls 200 mm thick centre line dimensions of building

$$(12{,}000 - 200) \times (18{,}000 - 200)$$

or

$$11{,}800 \times 17{,}800$$

With foundation width 750 mm, the outside dimensions of foundation wall are

$$12{,}550 \times 18{,}550$$

Inside dimensions are 11,050 × 17,050. These are shown in Fig. 18.11. Since this is a small building, batter boards may not be required. Instead offset pegs are used. The following are the steps:

(i) Mark the centre line of the longest wall on the ground by stretching a string between offset pegs 1-1.

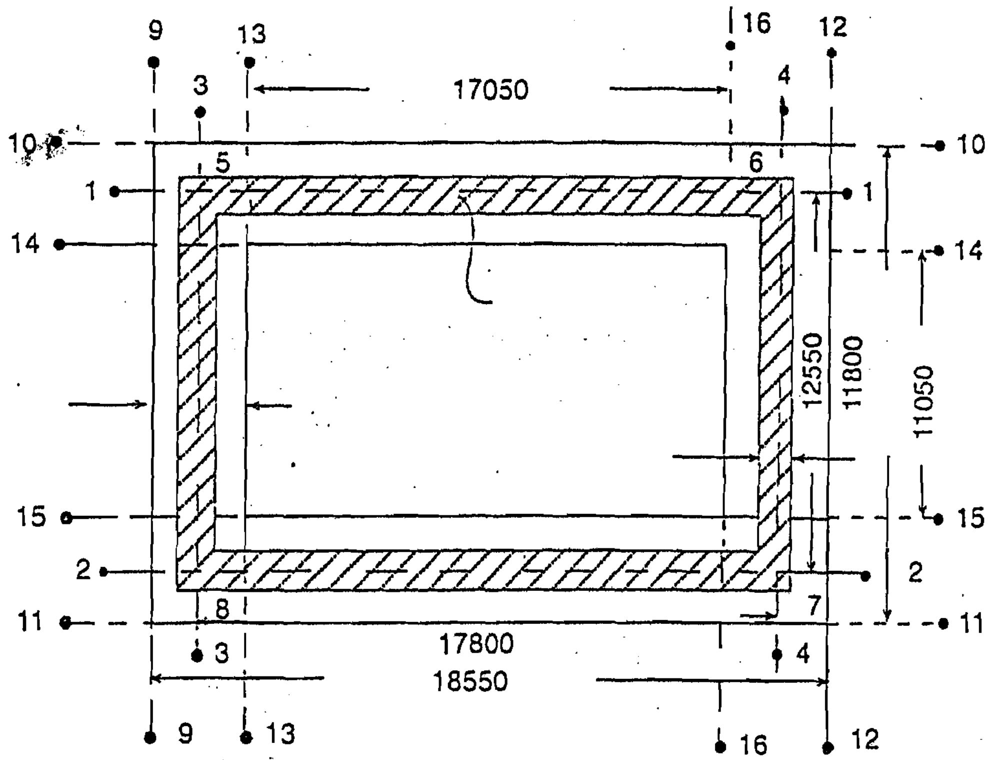


Fig. 18.11 Example 18.1 (not to scale).

(ii) Similarly stake strings along 22, 33 and 44.

(iii) Obviously 33 should be at right angles to 11, 22 at right angles to 33, 44 at right angles to 22. The peg should project 25 to 50 mm above the ground level.

(iv) The strings intersect at 5, 6, 7, 8 and obviously 57 and 68 should be equal. To ensure right angles at the corners optical square or theodolite may be used or right angled triangles should be formed by measuring sides in the ratio 3 : 4 : 5. If the distances 57 and 68 are not equal, the right angles should be checked.

(v) Similarly lines 9-9, 10-10, 11-11, 12-12, 13-13, 14-14, 15-15, and 16-16 should be staked and the equality of the diagonals of the points of inter section checked .

(vi) The pegs with nail at the top should be fixed at least 1–2 m from the outside excavation line so that they are not disturbed during excavation.

Example 18.2 During redevelopment in a city area a block of flats rectangular in plan is to be built with the long elevation oriented due east-west.

Line	Whole circle bearing	Horizontal distance (m)
AP	251°00′	80.69
PQ	283°00′	20.64

The building, which is to measure 86 m long by 21 m wide, is located initially by means of a peg at the south-west corner (Point *A*, 1,200 mE, 600 m N). The remainder of the building cannot be set out as existing properties have not yet been demolished and it is essential to check its clearance from a proposed new building line. The observations in the Table are therefore made. Point *Q* is located on the proposed building line which is straight with a WCB of 13°30′ (Fig. 18.12). Calculate the minimum distance from the proposed development to the new building line. [Salford]

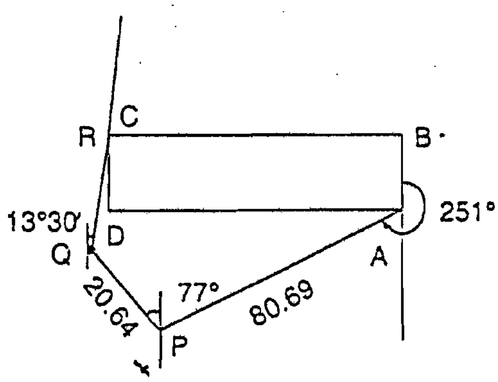


Fig. 18.12 Example 18.2.

Solution

$$\text{WCB of } AP = 251°00'$$

$$\text{Reduced bearing} = \text{S } 71°00' \text{ W}$$

$$\text{WCB of } PQ = 283°00'$$

$$\text{Reduced bearing} = \text{N } 77°00' \text{ W}$$

From the data the following chart can be obtained.

Equating Northing and Southing

$$4.64 + 0.97\, l_1 = 26.27 + 21.00$$

Table 18.1 Example 18.2

Line	Length (m)	R.B.	Latitude N	Latitude S	Departure E	Departure W
AP	80.69	S 71° W		26.27		76.29
PQ	20.64	N 77° W	4.64			20.11
QR	l_1	N 13°30′ E	$0.97l_1$		0.23 l_1	
RB	l_2	N 90° E	—		l_2	
BA	21.00	S 180° E		21.00	0	

or

$$0.97\ l_1 = 42.63$$

$$l_1 = 43.94 \text{ m}$$

Equating Easting and Westing

$$0.23\ l_1 + l_2 = 96.40$$

$$l_2 = 96.40 - 0.23 \times 43.94$$

$$= 86.29$$

$$RC = RB - CB = 86.29 - 86 = 0.29$$

Minimum perpendicular distance = 0.29 sin 76°30′ = 0.28 m.

19

Underground Surveys

19.1 INTRODUCTION

Underground surveying embraces the survey operations performed beneath the surface of the earth in connection with tunnelling, exploration of caves and construction in subterranean passageways.

Underground surveys are essentially similar to three-dimensional surveys on the surface in that the purpose of all angle and distance measurements is to obtain the horizontal and vertical coordinates of a point, the position of which is unknown with respect to a point of established location.

The following peculiarities of underground surveys indicate how they differ from surface surveys.

1. The lighting in underground passageways is generally poor and artificial illumination must be provided to view instrument crosshairs, to read verniers, to sight targets, and to permit normal movements of survey presonnel in executing their duties.

2. The working space in passageways is often cramped.

3. In certain types of operations, survey lines must be carried through locks in pressure chambers.

4. In many instances the underground workings are wet, with considerable water dripping from the roofs of passage ways and running along the floors.

5. Instrument stations and benchmarks for levelling must often be set into the roof of a passageway to minimize disturbance from the operations being carried on in the workings.

6. Instrument stations are set with some difficulty since plugs must be driven into drill holes in rock.

7. Lines of sight are frequently very short either because of crooked passage ways or because alignment must often be brought down from the surface through small shafts. Care must therefore be taken in all surveying operations involving the alignment of tunnels or the running of underground traverses.

8. The sights taken in shafts and sloping passageways are often sharply

inclined and it is frequently necessary to observe both horizontal and vertical angles through a prismatic telescope or eyepiece or through an auxilliary telescope mounted either above the main telescope or to one side of the instrument standards.

9. It is much more difficult to keep satisfactory survey notes when the workings are wet or dirty and the illumination is poor.

10. Plumbing down the shaft constitutes a special problem which is peculiar to underground surveying.

11. Two vertical dimensions, from a line of sight to both the floor and ceiling of the passage are involved in underground surveying whereas only one vertical dimension is normally encountered in surface surveys.

19.2 APPLICATION OF UNDERGROUND SURVEYS

The major application of underground surveys is in the construction of tunnels and other underground utilities. Tunnel is constructed when open excavation becomes uneconomical usually when it is more than 20 m. It (1) reduces the grade; (2) shortens the distance between given points separated by a dividing mountain or ridge; (3) meets the demand of modern rapid transit in a city. Tunnel is a very costly venture, hence it must follow the best line adopted to proposed traffic and it must be economical in construction and operation. Hence survey of tunnel is very important and explained in detail.

The following engineering operations are to be performed in any tunnel construction. (i) exact alignment, (ii) proper gradient, and (iii) establishment of permanent stations marking the line or the proposed route.

The survey work in connection with tunnelling can be divided into four types: (a) surface survey, (b) transferring the alignment underground, (c) levels in tunnels, (d) underground setting out.

19.2.1 SURFACE SURVEY

Surface survey connects points representing each portal of a tunnel. A traverse connecting the portal points determines the azimuth, distance and differences in elevation of each end of the proposed tunnel. The methods to be adopted for connecting survey work depends on local conditions and length of proposed tunnel. It is always advisable that the survey is based on suitable local coordinate system. The alignment is permanently referenced by a system of monuments within an area outside each tunnel portal. A working sketch of how tunnelling work proceeds is shown in Fig. 19.1.

Centre line and grade stakes within the tunnel are usually set in the roof to avoid displacement and destruction by the constant flow of people and machinery as construction proceeds. If stakes are set on the floor they should be offset into an area along the tunnel's edge.

19.2.2 VERTICAL SHAFTS

For long tunnels excavation are carried inward from both portals. But vertical

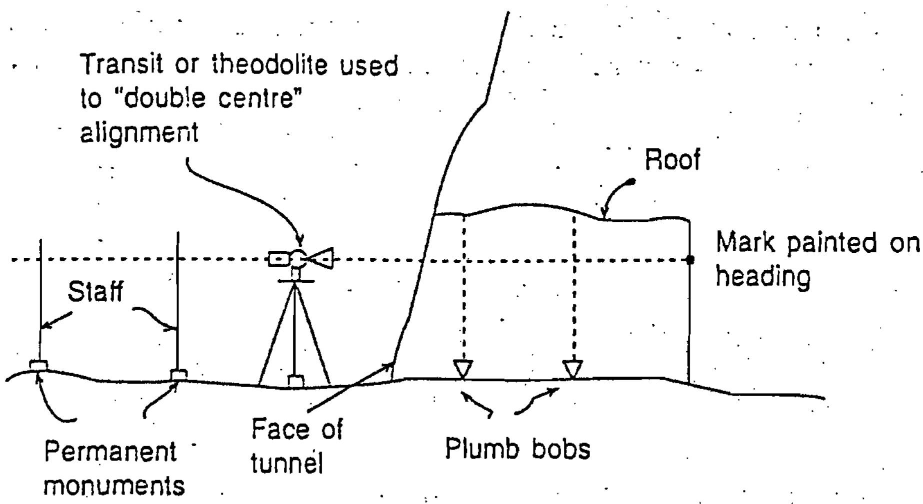


Fig. 19.1 Surface survey suspended from hooks fitted in roof.

shafts are also sunk upto required depth along the alignment of the tunnel at intermediate locations along the routes. The vertical alignment can be done by (i) Plumb bob, (ii) Optical collimator, (iii) Laser.

A heavy plumb bob (5 to 10 kg) is suspended on either a wire or heavy twine. Oscillations of the bob can be controlled by suspending it in a pot with high viscosity oil. The bob is suspended from a removable bracket attached to the surface side of the shaft.

Optical plumbing becomes important with the increase in depth of internal shaft. Various types of plummet are available for upwards and downward sighting to allow the establishment of a vertical line and these are normally manufactured so as to be interchangeable with theodolites on their tripods. As the line of sight of a theodolite in adjustment will transit in a vertical plane, it can also be used to check perpendicularity.

Pentagonal prisms attached to the objective end of the telescope of a theodolite can facilitate transfer of a given bearing to different levels as in the case of surface and underground lines as well as upward and downward plumbing.

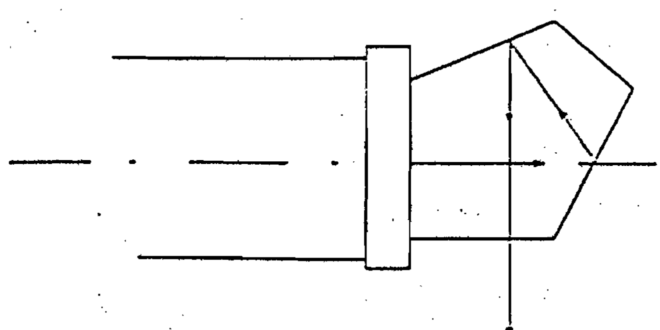


Fig. 19.2 Pentagonal prism.

The advantages of an optical collimator are: (i) More convenient than a plumb bob, (ii) Can be used to set marks directly on the floor of a completed shaft, (iii) No wires, as in case of plumb bob, is necessary.

A laser equipment can be used to provide a vertical line of sight. The laser generates a light beam of high intensity and of low angular divergence and can be projected over long distances since the spread of the beam is very small to provide a visible line for constant reference. Laser used in surveying equipment is helium neon gas laser which produces a bright red beam which clearly intersects a scale or rule. Here $\lambda = 0.6328 \times 10^{-6}$ m and $f = 4.74 \times 10^{14}$ Hz. It is characterized by an extremely long range of upto 100 km. Relatively good accuracy (± 5 mm to ± 1 mm/km) is possible with laser carriers since laser is a source of very coherent and stable radiation. The disadvantages of laser are (i) Bulky (ii) Requires more electrical power (iii) Can damage the eye.

19.2.3 LEVELS IN TUNNELS

In transferring levels underground, little difficulty is encountered at the ends of the tunnel but at the shaft use is made of (i) steel tape, (ii) chain, (iii) specially constructed rods, (iv) steel wires. In all cases the main idea is to deduct the height of the shaft measured from the top of a benchmark of known value. In modern days EDMI is also used. Figure 19.3 shows how depth is measured by a steel tape.

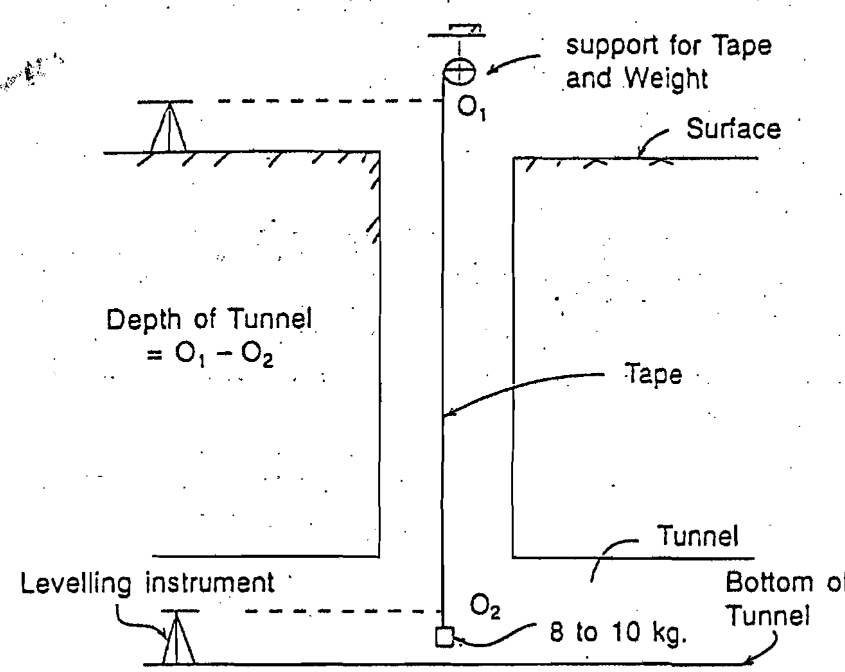


Fig. 19.3 Transferring level underground.

Figure 19.4 shows how depth is measured by EDMI. The important features of laser measurement are:

(a) EDMI unit and reflectors should be in the same vertical line.
(b) Both are mounted on stable support.
(c) Visibility should be good for EDMI to operate.

19.3 ALIGNING THE THEODOLITE

There is diversity in practice as to the position of the theodolite at both the surface and the bottom of the shaft. The above ground survey is linked with the working

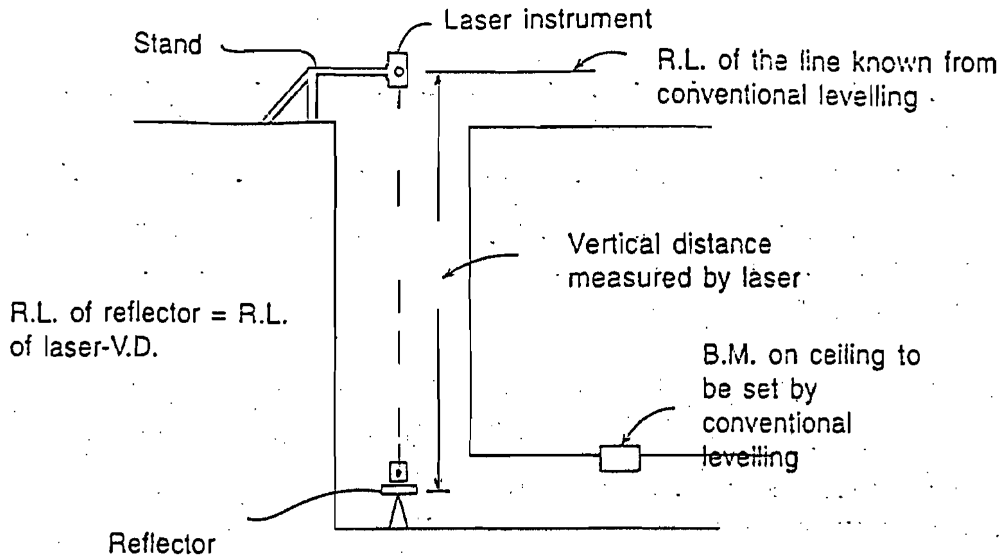


Fig. 19.4 Use of Laser.

surface of a tunnel through vertical shaft. Transfer of surface alignment to underground may be done through plumb bobs or by vertical collimator. Transfer of alignment down shaft by plumb bobs is shown in Fig. 19.5.

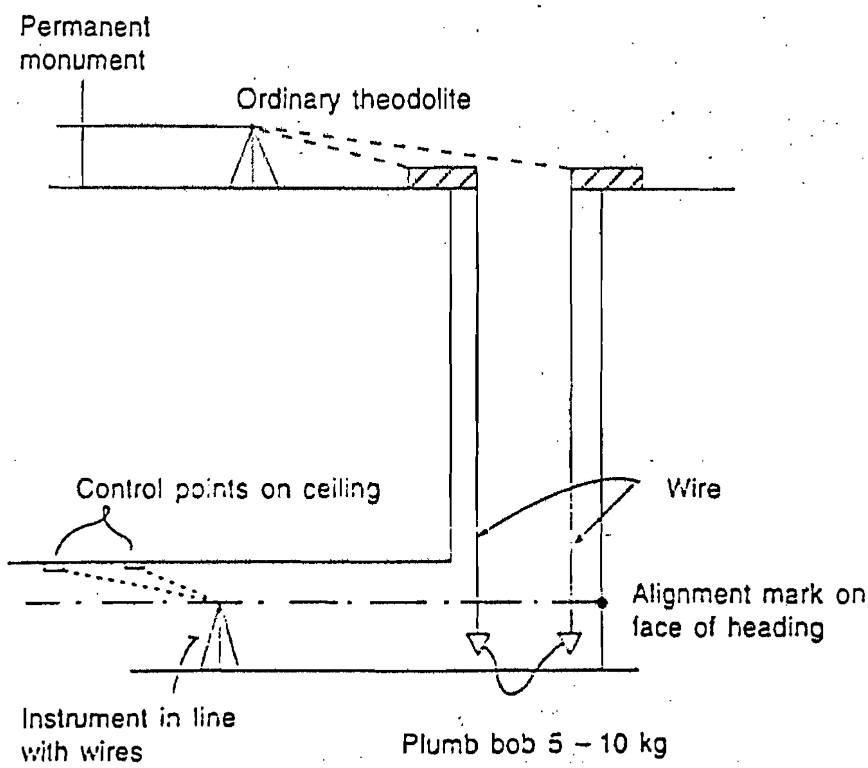


Fig. 19.5 Transfer of alignment.

The important features of this are:

1. The theodolite is placed on the correct alignment at the surface.

2. Two wires with plumb bobs are suspended from points along the alignment marked by theodolite pointings.

3. Theodolite at the tunnel floor is now aligned in line with the two plumb bobs.

4. With the aligned theodolite control points are set on the tunnel roof.

In the case of vertical collimator, two points on the top of shaft are transferred on the floor through vertical line of sight. These points are utilized in a precise double centring operation to prolong the tunnel alignment.

19.4 DETERMINATION OF AZIMUTH BY GYROSCOPE

Reliable azimuths are ordinarily determined in survey work by measuring angles from existing reference azimuth or by making observations on the sun or the stars. However, in many situations such as in underground mine surveys or where the area is perpetually overcast with clouds, these methods would not be possible. The gyroscope, on the other hand, permits the determination of reliable azimuths under these adverse conditions.

A gyroscope is usually attached to the top of a theodolite. Inside the gyrocompass a gyromotor is suspended on a thin tape similar to a plumb bob. The upper end of the tape is connected to the upper end of the tubular housing at the top. A schematic cross sectional view of the gyrocompass is shown in Fig. 19.6. The moving mark oscillates with the gyroscope. This mark is converted to a V-shaped index which is viewed through the eyepiece. The eyepiece is attached to the instrument and as such it is fixed with respect to the allidade of the theodolite.

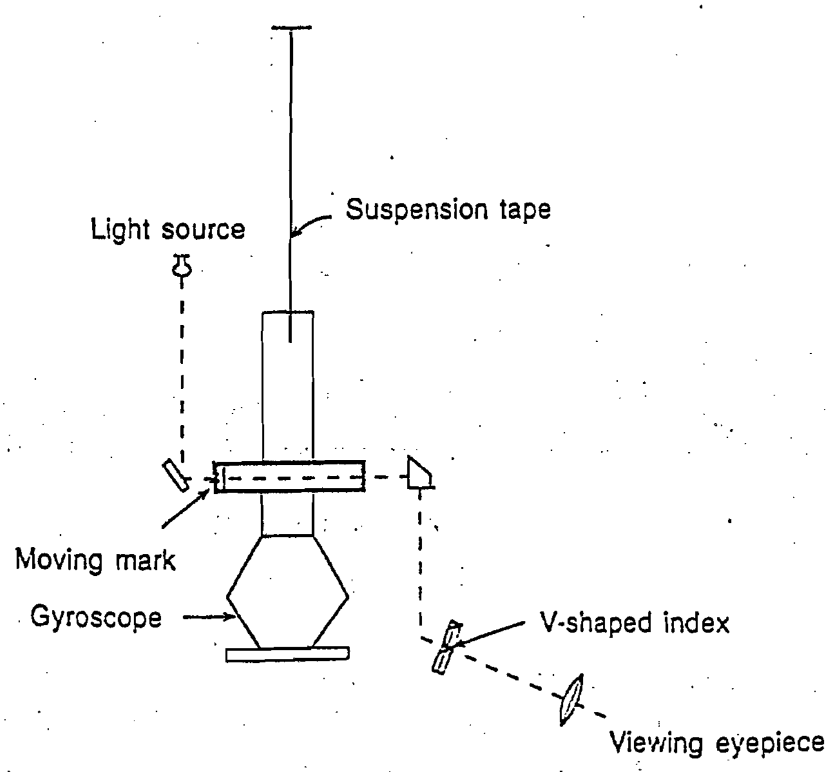


Fig. 19.6 Schematic diagram of gyrocompass.

In practice, the theodolite telescope is set up roughly pointing towards the North by means of prismatic compass or tubular compass (taking into account the declination of the compass needle). The gyrocompass is then fitted on the theodolite with the telescope in the face left position. The gyromotor is now turned on and allowed to run upto full operating speed. As the spinner oscillates about the meridian, the gyroindex mark can be observed through the eyepiece as moving ~~across the~~ graduations contained in the eyepiece scale. Thus if the mid-position of the oscillations is established, the telescope line of sight at the station is oriented towards the north and the purpose of all observations is to determine that position.

One method of doing the above is the *Turning Point Method*. Here the observer follows the oscillation of the moving mark and tries to keep it at the centre of the V-index by turning the horizontal circle tangent screw. When a turning point is reached, the moving mark will remain stationary on the V-shaped mark and the corresponding horizontal circle reading has to be obtained quickly. The observer then follows the movement of the moving mark following the V-index until the opposite turning point is reached. The horizontal circle is again read and recorded. If the three successive turning points are r_1, r_2 and r_3, the mean value r_0 is given by

$$r_0 = \frac{r_1 + 2r_2 + r_3}{4}$$

This approximate value is known as *Schuler Mean*. If the readings are more than three, mean of the Schuler means of three consecutive values is taken. This gives the reading for True North. The azimuth of a reference mark is then obtained. The azimuth of a reference line is established by sighting on a reference mark with both face left and face right conditions and taking the mean of both readings. If M is the mean circle reading of reference mark and N is the true North reading, the azimuth of the line is $M - N$ as shown in Fig. 19.7.

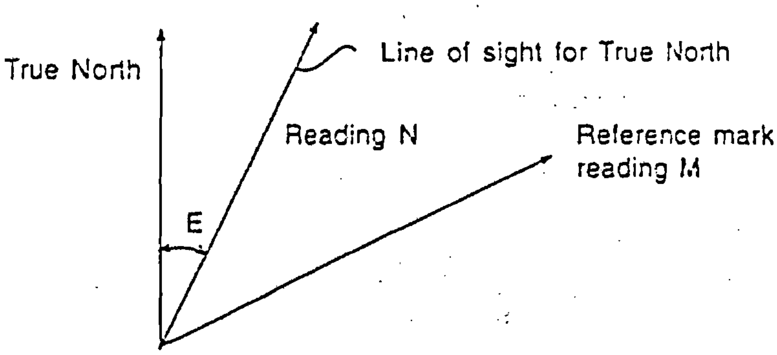


Fig. 19.7 Azimuth of line.

If, however, the direction of the line of sight of the telescope of the theodolite was not on the symmetrical line of the oscillatory mark and differed by angle E then azimuth of the lines becomes

$$A = M - N + E$$

as shown in Fig. 19.7. The procedure may take 20 to 30 min but yields an azimuth with a standard deviation of 20". The instrument is expensive and can be used profitably only in large projects.

The other method of determining azimuths using the gyrocompass is the *transit method*. The theodolite is first set up and oriented to approximate North, the clockwise horizontal circle is read. With the help of a stop watch the time t_L taken for the moving mark to move from the V-index to the left and back to the V-index is noted. This is known as *positive swing time*. Similarly, the negative swing time t_R is the time taken for the moving mark to travel from the V-index to the right and back to the V-index (negative swing time). If the average t_L is equal to the average t_R, then the line of sight points to the true north (except for E). If not, and the difference is Δt a correction ΔN must be applied to the initial circle reading N' to get the true north N. ΔN is given by the expression

$$\Delta N = ca\Delta t$$

where c is a proportionality factor relating scale readings to time, a is the mean of amplitudes left and right and Δt is equal to $t_L - t_R$. Then,

$$N = N' + \Delta N$$

To get the proportionality factor c two observations should be made with directions N_1' and N_2' about 15' to the west and east respectively of the middle oscillations of the moving mark. The following two equations are then obtained

$$N = N_1' + c\Delta t_1 a_1$$

$$N = N_2' + c\Delta t_2 a_2$$

Solving $$c = \frac{N_2' - N_1'}{\Delta t_1 a_1 - \Delta t_2 a_2}$$

19.5 WEISBACH TRIANGLE

This is a method of connecting the surface and underground surveys and avoids direct alignment. Here the theodolite is set up at C near and almost in line with AB so that the effect of error in BC or AC on θ is least when θ is least, $\theta = ACB$ being usually less than 30'. The angle θ is measured very accurately as also the distances AC, BC and as a check, AB (Fig. 19.8). In order to get a check on θ, the angles ACD, BCD which AC and BC make with any line CD may be measured, θ being found by subtraction. If desired, the offset CE from the vertical plane of AB can be calculated and a line parallel to the centre line set out. The weakest

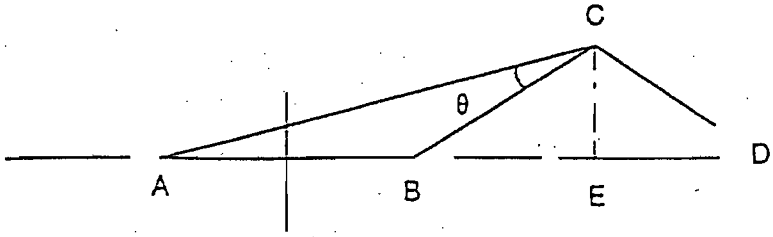


Fig. 19.8 Weisbach triangle.

point in the method is the fact that lines and angles are measured from unsteady points *A* and *B*, rendering it necessary to take the extreme positions of the plumb wire.

Example 19.1 The following notes refer to the alignment down a shaft by means of Weisbach triangle, *A* and *B* being the plumb wires, *C* and *D* the respective surface and underground theodolite stations, and *P* and *Q* the reference points accordingly. Determine the angle between the reference lines *CP* and *DQ*, given that zero readings were taken on the reference points on each case.

Station	Line	Length in m	Angle
C	*CA*	4.34	72°16′25″
	CB	8.83	72°16′21″
—	*AB*	4.48	—
D	*DA*	4.91	176°4′36″
	DB	9.40	176°4′27″

Solution

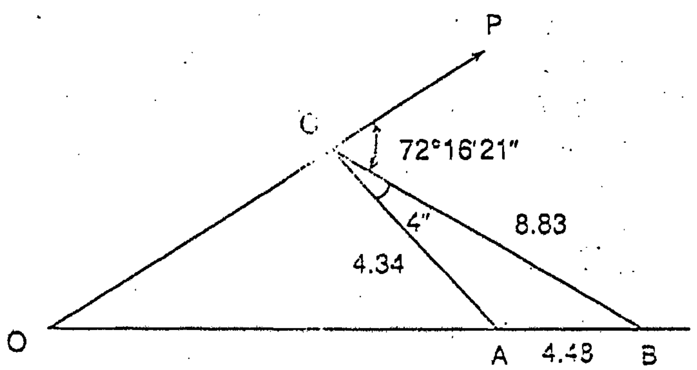


Fig. 19.9. Example 19.1.

On the surface For small angles (Fig. 19.9)

$$CBA = \frac{4''}{4.48} \times 4.34 = 3.875''$$

Therefore $\quad CAO = 4 + 3.875 = 7.875''$

and

$$COA = PCA - CAO$$
$$= 72°16'25'' - 7.875''$$
$$= 72°16'17.125''$$

Underground Similarly (Fig. 19.10)

$$DBA = \frac{9 \times 4.91}{4.48}$$
$$= 9.87''$$

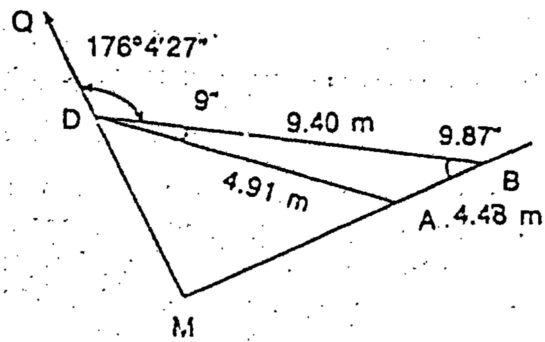


Fig. 19.10 Example 19.1.

Therefore $DAM = 9 + 9.87 = 18.87''$

and $DMA = 176°4'36'' - 18.87''$

$= 176°4'17.13''$

But $COA = 72°16'17.125''$

Hence angle between

CP and $DQ = 103°48'0.005''$

Example 19.2 Two plumb wires *A* and *B* in a shaft are 3.642 m apart. A theodolite was set up at *C* slightly off the line *AB* and at a distance of 6.165 m from the wire *B*. The angle *ACB* was found to be 121" (Fig. 19.11). Calculate the rectangular distance from *C* to the line *AB* produced. (log sin 1" = $\bar{6}.6855749$)

[I.C.E]

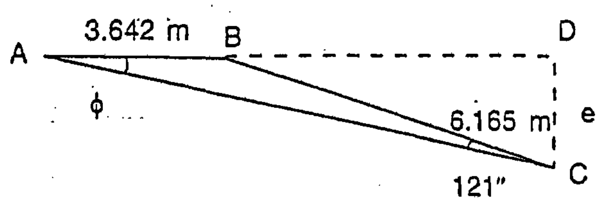


Fig. 19.11 Example 19.2.

Solution

$$\sin \phi = 6.165 \frac{\sin 2'1''}{3.642}$$

or

$$\phi = 240.82'' = 3'24.82''$$

$$e = AC \sin \phi$$
$$= (3.642 + 6.165) \sin \phi$$
$$= 9.807 \times 9.93 \times 10^{-4}$$
$$= 97.38 \times 10^{-4} \text{ m} = 9.738 \text{ mm}$$

Example 19.3 The centre line of a tunnel *AB* shown in Fig. 19.12 is to be set out to a given bearing. A short section of the main tunnel has been constructed along the approximate line and access is gained to it by means of adit connected to a shaft. Two wires *C* and *D* are plumbed down the shaft and readings are taken on to them by a theodolite set up at station *E* slighly off the line *CD* produced. A point *F* is located in the tunnel and sighting is taken on to this from station *E*. Finally a further point *G* is located in the tunnel and the angle *EFG* measured. From the surface survey initially carried out, the coordinates of *C* and *D* have been calculated and found to be N 1119.32 and E 375.78 and N 1115.70 and E 375.37 m respectively. Calculate the coordinates of station *F* and *G*. [I.C.E.]

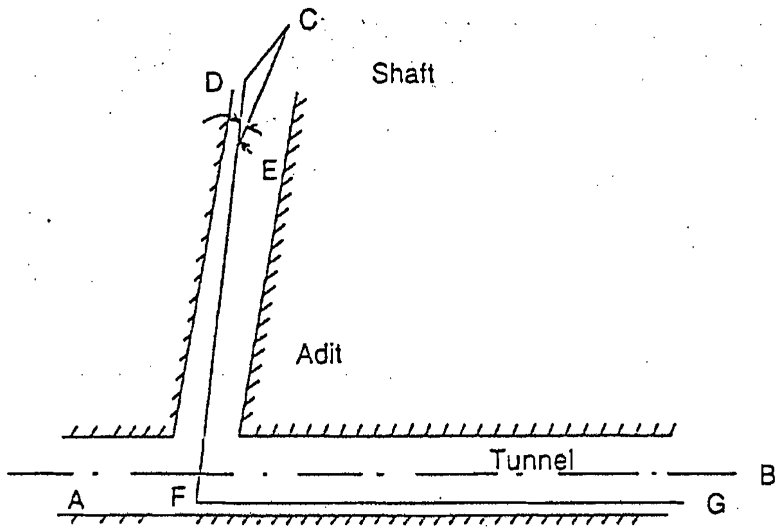


Fig. 19.12 Example 19.3. *CD* = 3.64 m, *DE* = 4.46 m, *EF* = 13.12 m, *FG* = 57.50 m, Angle *DEC* = 38″, Angle *CEF* = 167°10′20″, Angle *EFG* = 87°23′41″..

Solution

$$\angle DCE = \frac{4.46}{3.64} \times 38''$$

$$= 46.6''$$

$$CE \approx CD + DE$$

$$= 3.64 + 4.46$$

$$= 8.10 \text{ m}$$

$$\frac{CE}{\sin CDE} = \frac{3.64}{\sin 38''}$$

or

$$\sin CDE = \frac{CE \sin 38''}{3.64}$$

$$= \frac{8.10}{3.64} \sin 38''$$

or

$$\angle CDE = 84.56''$$

$$= 01'24.56''$$

This is external angle.

Bearing of DC

Coordinates of C = N 1119.32 m E 375.78 m

Coordinates of D = N 1115.70 m E 375.37 m

$$\text{Bearing of } DC = \text{N} \tan^{-1}\left(\frac{375.78 - 375.37}{1119.32 - 1115.70}\right) \text{E}$$

= N 6°27′42.4″ E

Bearing of ED = N (6°27′42.4″ − 01′24.56″) E

= N 6°26′17.84″ E

Bearing of EF = 6°26′17.84″ + 38″ + 167°10′20″

= 173°37′15.84″

Quadrantal bearing = S 06°22′44.16″ E

Bearing of FG = 173°37′15.84″ − (180° − 87°23′41″)

= 173°37′15.84″ − 92°36′19″

= N 81°00′54.84″ E

Coordinates of F = Coordinates of D ± Latitude/Departure of DE ± Latitude/Departure of EF

Bearing of ED = N 6°26′17.84″ E

Bearing of DE = S 6°26′17.84″ W

Latitude of DE = 4.46 cos 6°26′17.8″

= 4.432 m

Departure of DE = 4.46 sin 6°26′17.8″

= 0.500 m

Bearing of EF = S 06°22′44.16″ E

Latitude = 13.12 cos 06°22′44.16″

= 13.038 m

Departure = 13.12 sin 06°22′44.16″

= 1.457 m

Bearing of FG = N 81°00′54.84″ E

Latitude = 57.50 cos 81°54.84″

= 8.9798 m

Departure = 57.50 sin 81°00′54.84″

= 56.7944 m

Latitude of F = 1115.70 − 4.432 − 13.038

= 1098.23 N

$$\text{Departure of } F = 375.37 - 0.500 + 1.457$$
$$= 376.327 \text{ E}$$
$$\text{Latitude of } G = 1098.23 + 8.98$$
$$= 1107.21 \text{ N}$$
$$\text{Departure of } G = 376.327 + 56.794$$
$$= 433.121 \text{ E}$$

Example 19.4 Bore holes are sunk at points *A*, *B* and *C* to locate a coal seam. The coordinates of *B* and *C* relative to *A* are respectively in m (N 1334, E 33) and (N 167, E 867) (Fig. 19.13). The data levels and bore hole depths to the seam are: *A* = 200 M.S.L. and 506 m, *B* 187 m M.S.L. and 460 m; *C* = 213 M.S.L. and 587 m. Find the magnitude and direction of the dip of the seam.

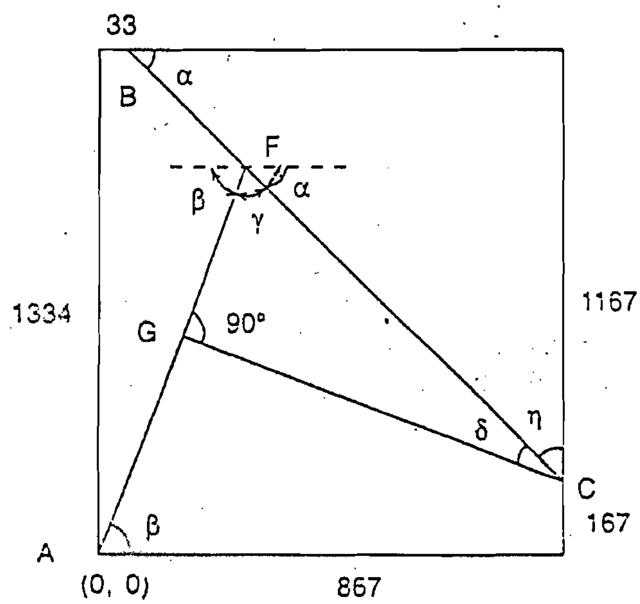


Fig. 19.13 Example 19.4.

Solution Data are given in the tabular form below:

Table 19.1 Example 19.4

Point	Coordinates N	Coordinates E	Ground level	Seam depth	R.L. of seam	Level relative to *C*
A	0	0	200	506	− 306	+ 68
B	1334	33	187	460	− 273	+ 101
C	167	867	213	587	− 374	0.00

$$\alpha = \tan^{-1} \frac{1167}{867} = \tan^{-1} 1.34 = 53.39°$$

$$BC = 1167 \text{ cosec } 53.39°$$
$$= 1453.8 \text{ m.}$$

Let F be the point in the seam below BC at the same level as the seam at A.

Then $CF = \frac{68}{101} \times BC = \frac{68}{101} \times 1453.8 = 978.79$ m

For CF

$$\text{Lat.} = 978.79 \sin \alpha$$
$$= 978.79 \times 0.802$$
$$= 785.78 \text{ m}$$
$$\text{Dep.} = -978.79 \cos \alpha = -583.72 \text{ m}$$

Hence F is 167 + 785.78 = 952.78 m. North of A

and 867 – 583.72 = 283.28 m East of A

$$\text{Angle } \beta = \tan^{-1} \frac{952.78}{283.28} = 73.44°$$
$$\angle AFC = 180° - (\alpha + \beta)$$
$$= 180° - (53.39 + 73.44)$$
$$= 53.17 = \gamma$$

Drop a perpendicular CG from C onto AF. Then CG is the line of the greatest slope of the seam (i.e. the dip of the seam).

$$\text{Length } CG = CF \sin \gamma$$
$$= 978.79 \sin 53.17$$
$$= 783.44 \text{ m}$$

Angle $\delta = 90 - \gamma = 36.83°$

$$\eta = 90 - \alpha = 90 - 53.39 = 36.61°$$

∴ Whole circle bearing of GC, the direction of the dip of the seam

$$= 180° - 73.44° = 106.56°$$

and magnitude of dip = 68 m in 783.44 m

= 1 in 11.52 m

Example 19.5 The following table gives the coordinates and reduced levels of two points P and Q on the centre line of a straight tunnel, together with those of three points A, B and C on the upper plane surface of a stratum of rock as determined by bore holes. Determine the coordinates and reduced level of the point at which the centre line of the tunnel meets the upper surface of the stratum.

Point	Coordinates (m)		Reduced Level (m)
	N	E	
P	67.00	0.00	265
Q	200.00	700.00	272
A	370.00	430.00	335
B	0	470.00	262
C	100.00	700.00	215

Solution Take the origin of three dimensional coordinates at the projection on to datum of the origin of the two dimensional N–E coordinates. Take the x-axis to run east, the y-axis towards north and the z-axis vertically upwards. The equation of the straight line joining points (x_1, y_1, z_1) and (x_2, y_2, z_2) are,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Therefore the equation of the line PQ joining points P (0, 67, 265) and Q (700, 200, 272)

$$\frac{x - 0}{700} = \frac{y - 67}{133} = \frac{z - 265}{7} = \lambda \text{ (say)} \tag{19.1}$$

The equation of the plane passing through the three points (430, 370, 335), (470, 0, 262) and (700, 100, 215) is

$$\begin{vmatrix} x & y & z & 1 \\ 430 & 370 & 335 & 1 \\ 470 & 000 & 262 & 1 \\ 700 & 100 & 215 & 1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} \frac{x}{100} & \frac{y}{100} & \frac{z}{100} & 1 \\ 4.3 & 3.70 & 3.35 & 1 \\ 4.7 & 0.00 & 2.62 & 1 \\ 7.0 & 1.00 & 2.15 & 1 \end{vmatrix} = 0$$

or

$$\frac{x}{100} \times \begin{vmatrix} 3.7 & 3.35 & 1.0 \\ 0 & 2.62 & 1.0 \\ 1.0 & 2.15 & 1.0 \end{vmatrix} - \frac{y}{100} \times \begin{vmatrix} 4.3 & 3.35 & 1.0 \\ 4.7 & 2.62 & 1.0 \\ 7.0 & 2.15 & 1.0 \end{vmatrix}$$

$$+ \frac{z}{100} \times \begin{vmatrix} 4.3 & 3.7 & 1.0 \\ 4.7 & 0.0 & 1.0 \\ 7.0 & 1.0 & 1.0 \end{vmatrix} - 1 \begin{vmatrix} 4.3 & 3.7 & 3.35 \\ 4.7 & 0.0 & 2.62 \\ 7.0 & 1.0 & 2.15 \end{vmatrix} = 0$$

Expanding the determinants

or $$2.469\,x - 1.836\,y + 8.91\,z - 3565 = 0 \tag{19.2}$$

From Eq. (19.1) $$x = 700\,\lambda \tag{19.3}$$

$$y = 67 + 133\,\lambda \tag{19.4}$$

$$z = 265 + 7\,\lambda \tag{19.5}$$

Substituting in Eq. (19.2) and simplifying

$$\lambda = 0.857$$

Substituting the values of λ in Eqs. (19.3) to (19.5) we get

$$x = 700 \times 0.857 = 600 \text{ m E}$$

$$y = 67 + 133 \times 0.857 = 180.98 \text{ m N}$$

$$z = 265 + 7 \times 0.857 = 271 \text{ m R.L.}$$

Example 19.6 Using the Schuler mean calculate the bearing of reference object *B* from the following observations taken at station *A* using a gyro theodolite.

Horizontal circle readings to *B*: face left 42°26′15″, face right 222°26′25″ Angular readings of successive gyro turning points were as follows:

left	276°20.1′	right	280°32.4′
	276°21.6′		280°30.8′
	276°23.3′		280°29.5′

The calibration constant of the instrument is + 2.6′. [Bradford]

Solution Schuler mean of three turning point observations is given by the expression

$$\frac{(a_1 + 2a_2 + a_3)}{4}$$

Hence

$$\text{1st Mean} = \frac{276°20.1' + 2(280°32.4') + 276°21.6'}{4}$$

$$= 278°26.625'$$

$$\text{2nd Mean} = \frac{280°32.4' + 2(276°21.6') + 280°30.8'}{4}$$

$$= 278°26.60'$$

$$\text{3rd Mean} = \frac{276°21.6' + 2(280°30.8') + 276°23.3'}{4}$$

$$= 278°26.625'$$

$$\text{4th Mean} = \frac{280°30.8' + 2(276°23.3') + 280°29.5'}{4}$$

$$= 278°26.725'$$

$\therefore$ Mean value = 278°26.6′

The azimuth of a line

$$A = M - N + E$$

where M is the mean circle reading of Reference mark.

N = True north reading

E = Calibration constant

Here M = Corrected reading from face left and face right observations

= 42°26′20″

N = 278°26.6′

But $M - N$ becomes negative hence 360° should be added.

Therefore A = 42°26′20″ + 360°00′00″ − 278°26′36″ + 2′36″

= 124°2′20″ = 124°02.3′

Example 19.7 The following 'transit' observations were recorded with a gyro theodolite attachment on a laboratory base line bearing 128°17′52″.

Observations east of true north
Horizontal circle reading during transit oscillations = 15°30.00′
Horizontal circle reading to reference object = 143°32.45′
Transit times: 0 min 0 s, 03 min 57.7 s, 07 min 20.5 s, 11 min 18.5 s, 14 min 41.1 s Amplitudes − 10.8, + 8.3, − 10.7, + 8.2.

Observations west of true north
Horizontal circle reading during Transit oscillations = 15°00.00′

Horizontal circle reading to reference object = 143°32.45′
Times of transit: 0 min 0 s, 04 min 05.7 s
07 min 20.4 s, 11 min 26.0 s, 14 min 41.2 s.
Amplitudes: + 7.9, − 5.6, + 7.9, − 5.5

Determine the additive constant and the proportionality factor for this particular attachment stating carefully the units of both. [CEI]

Solution

Table 19.2 Example 19.7

Time of transit	Time of swing left + (s)	Time diff Δt(s)	Amplitude reading	Mean amplitude reading
0 min 00.0 s				
03 min 57.7 s	− 237.7		− 10.8	
07 min 20.5 s	+ 202.8	− 34.90	+ 8.3	10.05
11 min 18.5 s	− 238.00	− 35.20	− 10.7	9.50
14 min 41.1 s	+ 202.6	− 35.40	+ 8.2	9.45

$$\text{Mean } \Delta t_E = -35.17 \text{ s}$$

$$\text{Mean amplitude } a_E = 9.67$$

Table 19.3 Example 19.7

Time of transit	Time of swing left + (s)	Time diff Δt(s)	Amplitude reading	Mean amplitude reading
0 min 00.0 s				
04 min 05.7 s	+ 245.7		+ 7.9	
07 min 20.4 s	− 194.7	51.0	− 5.6	6.75
11 min 26.0 s	+ 245.6	50.2	+ 7.9	6.75
14 min 41.2 s	− 195.2	50.4	− 5.5	6.70

$$\text{Mean } \Delta t_W = 50.77 \text{ s}$$

$$\text{Mean amplitude } a_W = 6.73$$

$$\Delta t_E \times a_E = -35.17 \times 9.67 = -340.09$$

$$\Delta t_W \times a_W = +50.77 \times 6.73 = +341.68$$

$$N_1' = 15°00.00'$$

$$N_2' = 15°30.00'$$

$$C = \frac{15°30.00' - 15°00.00'}{341.68 - (-340.09)}$$

$$= \frac{30}{681.77}$$

$$= .044'/\text{s}$$

$$C a_W \Delta t_W = 0.044 \times 341.68 = 15.03'$$

$$C a_E \Delta t_E = 0.044 \times 340.09 = 14.97'$$

For the eastern setting

$$\text{Corrected azimuth} = M - N + E$$

$$128°17'52'' = 143°32'27'' - (15°30'00'' - 14'58.2'') + E$$

$$E = 26.8''$$

For the western setting similarly

$$128°17'52'' = 143°32'27'' - (15°00'00'' + 15'18'') + E$$

$$E = 43''$$

$$\text{Average } E = 34.9'' = 0.58'$$

19.6 PROBLEMS IN TUNNEL SURVEY

As already explained tunnelling involves (i) Surface alignment, (ii) Transferring the alignment underground usually through vertical shafts. Hence four cases may arise.

1. Surface alignment is possible and depth is not great so that vertical shaft can easily be constructed. Here vertical shaft is to be constructed and surface alignment is transferred underground as explained in Section 19.3 and Fig. 19.5.

2. When the vertical shaft can be constructed but surface alignment is not possible. This occurs when tunnelling in town or other built up areas. Two problems may arise: (a) It is impossible to set out the line on the surface. (b) The shafts cannot be placed on the centre line of the tunnel. In the first case, the position, direction and chainage of the centre line at any specified point can be obtained by the use of a precise traverse and the corrected coordinates of the traverse. In the second case which occurs when the tunnel is under a major highway, it is necessary to use an eccentric shaft, the latter is then connected with the tunnel by means of an 'adit' or entry tunnel. This is explained in Example 19.3.

3. When the depth below the surface is too great for shafts to be sunk but surface alignment is possible. Here brick or concrete structures styled 'observatories' are erected at different points *A*, *B*, *C*, *D*, *E*, *F*, etc. with a separate pier for carrying the theodolite. Work progresses simultaneously from both ends and alignment is checked from both *B* and *C*. Shafts are constructed at the ends for alignment purpose.

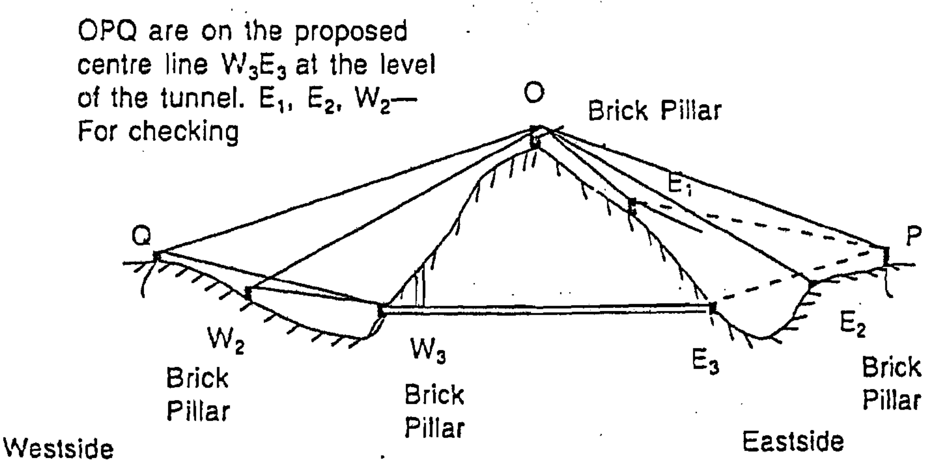


Fig. 19.14 Surface alignment.

4. When both vertical shaft and surface alignment are not feasible. Here surface alignment is done through triangulation and tunnelling operation through both ends as in case 3.

19.7 ANALYTICAL DERIVATIONS OF UNDERGROUND SURVEYS

Sloping plane surfaces

Let *AA* and *BB* be plane contours or horizontal lines in the plane *AB*, *a* the

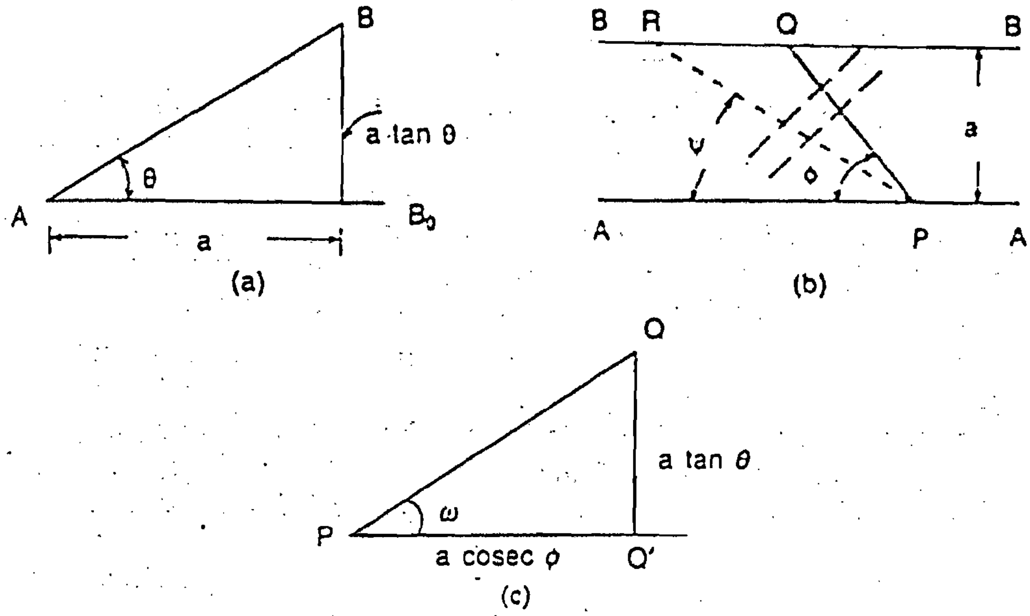


Fig. 19.15 Sloping plane surfaces.

horizontal distance between these lines and θ the angle of steepest slope in AB (Fig. 19.15 a, b).

1. If AA be the assumed direction of meridian and a line PQ of bearing ϕ and inclination ω to the horizontal lie in the plane (Fig. 19.15 b). Then since the height of B above A or P is $a \tan \theta$, it is equal to the height of Q above P.

Hence
$$\tan \omega = \frac{a \tan \theta}{a \operatorname{cosec} \phi}$$
$$= \tan \theta \sin \phi$$

or
$$\sin \phi = \frac{\tan \omega}{\tan \theta}$$

2. If, however, the direction ψ is prescribed for a given slope ω along PR, cutting will result if $\psi > \phi$ and filling if $\psi < \phi$ when ω rises from P to R and the difference in elevation between R and B will be

$$a (\tan \theta - \operatorname{cosec} \psi \tan \omega)$$

3. The lateral slope λ in the stratum at right angles to PQ, as shown dotted will be such that $a \sec (90° + \phi - 90°) \tan \lambda = a \tan \theta$

or
$$\tan \lambda = \frac{\tan \theta}{\sec \phi}$$

while at right angles to PR, but in the surface

$$\tan \lambda = \frac{\tan \theta}{\sec \psi}$$

Three point problem

Given the borings to three points on a stratum to determine dip and strike.

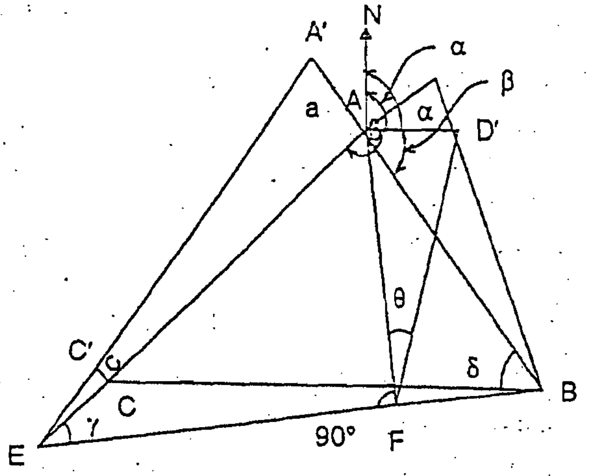


Fig. 19.16 Three point problem.

Let A, B and C the positions of the bore holes. A is the highest point and B is the lowest point. At A, AA' is drawn perpendicular to AC and CC' is drawn at C perpendicular to C. 'a' represents the level difference between A and B and c (small) the level difference between B and C. Join $A'C'$ and when produced it cuts AC produced at E. Join BE which is the direction of the strike. Drop perpendicular AF from A to line BE at F. Draw AD' perpendicular to AF and equal to a. Join $D'F$ (Fig. 19.16). $\angle AD'F$ = angle of dip θ.

Analytically

$$AE = \frac{a}{(a - c)} \times AC$$

From the triangle EAB

$$\frac{AE - AB}{AE + AB} = \frac{\tan \frac{1}{2} (\delta - \gamma)}{\tan \frac{1}{2} (\delta + \gamma)}$$

or

$$\tan \frac{1}{2} (\delta - \gamma) = \frac{AE - EB}{AE + EB} \cdot \tan \frac{1}{2} (\delta + \gamma)$$

$$= \frac{AE - EB}{AE + EB} \cdot \tan \frac{1}{2} \{180° - (\alpha - \beta)\}$$

Hence $\frac{1}{2} (\delta - \gamma)$ can be calculated.

Also $\frac{1}{2} (\delta + \gamma) = \frac{1}{2} \{180° - (\alpha - \beta)\}$ The bearing of the strike will be

$$180° + \beta - \delta \quad \text{or} \quad \alpha + \gamma$$

where α and β are the given bearings.

$$\tan \theta = \frac{AD'}{AF} = \frac{a}{AB \sin \delta}$$

Intersection of the centre line of a tunnel with a rock stratum

Let *A*, *B*, *C* be the plans of the borings and *P*, a given point of the centre line of the tunnel whose coordinates or bearings and distances are known. Let *B* be the datum point so that the heights *a*, *p* and *c* of points *A*, *P* and *C* are known.

Let the centre line *PQ* of the tunnel cut *AB* in *D* and *AC* in *E*. At *P*, *PP'* is drawn perpendicular to *PQ* and equal to *p*. At *E*, *EE'* is erected perpendicular to *PQ* and equal to *p* + *PE/n* where *n* is the gradient of the line. Join *P'E'*. At *A*, perpendiculars *AA'* and *AA"* equal to *a* are drawn. *A'B* is joined. At *D*, perpendicular *DD'* is drawn. *DF* is drawn perpendicular to *PQ* and made equal to *DD'*. At *C*, *CC'* is erected perpendicular to *CA* and equal to *c*. Here *C* is assumed to be below *B* and hence it is drawn downwards. *A"C'* is then joined. From *E*, perpendicular is drawn over *AC* which cuts *A"C'* at *M*. At *E*, *EN* is drawn perpendicular to the centre line *PQ* and is made equal to *EM*. *FN* is then joined which cuts the line *P'E'* at *X* which is the elevation of the point *O* where the centre line intersects the upper surface of the stratum. Hence to locate *O*, perpendicular is dropped from *X* on the centre line (Fig. 19.17).

Analytically, with *AB* and *AP* being known

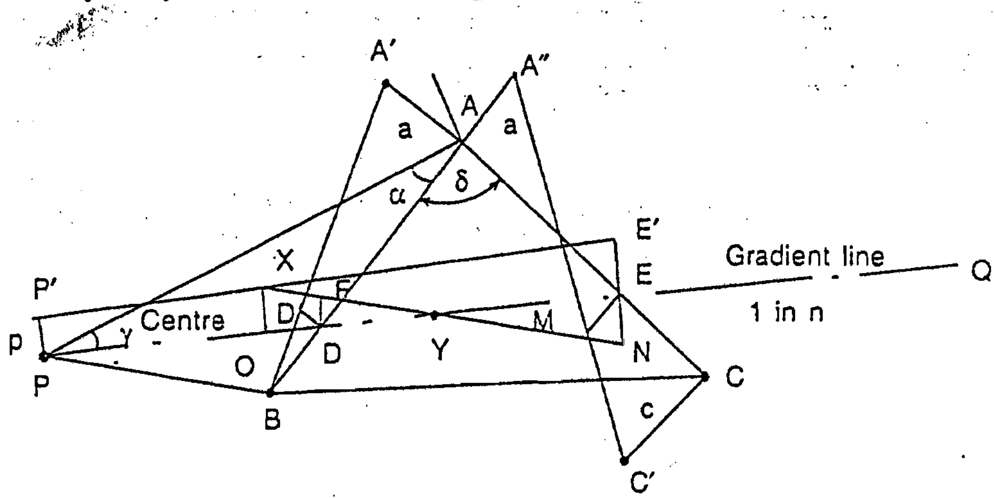


Fig. 19.17 Intersection of tunnel with rock stratum.

$$AD = AP\frac{\sin\gamma}{\sin(\alpha+\gamma)} \qquad PE = AP\frac{\sin(\alpha+\delta)}{\sin(\alpha+\gamma+\delta)}$$

$$AE = AP\frac{\sin\gamma}{\sin(\alpha+\gamma+\delta)} \qquad PD = AP\frac{\sin\alpha}{\sin(\alpha+\gamma)}$$

Then level of *D* on *AB* with respect to *B*

$$DD' = \frac{a}{AB}\cdot BD$$

Level of *D* in *PQ* relative to *P*

$$= p \div \frac{PD}{n}$$

From Fig. 19.17

$$OX = p + \frac{PO}{n} = \frac{OY}{DY} \times DF$$

$$= \frac{(PD + DY - PO)\, DF}{DY}$$

Cross multiplying

$$(pn + PO)\, DY = (n\, PD + n\, DY - n\, PO)\, DF$$

or $$PO.DY + n\, PO.DF = n(PD + DY)\, DF - pn\, DY$$

or $$PO(DY + nDF) = n(PD + DY)\, DF - np.DY$$

or $$PO = \frac{n\{(PD + DY) \cdot DF - p.DY\}}{DY + nDF}$$

Example 19.8 The coordinates and surface levels of three exploratory bore holes *A*, *B* and *C* are given below together with the depths to a metalliferous ore-body. A fourth bore hole is to be drilled at point *D*, the coordinates and surface level of which are also listed. All quantities are in meters. Calculate (i) the direction and rate of full dip of the ore body which may be assumed to be uniform; (ii) the bore hole depth at which the ore body would be intersected at point *D*.

Bore hole	Easting	Northing	Level *AOD*	Depth Ore body
A	2960	1920	90	400
B	4020	2850	260	300
C	4970	1830	100	500
D	3680	430	390	?

[Eng. Council]

Solution The points *A*, *B* and *C* are plotted in the Fig. 19.18. From coordinates length of

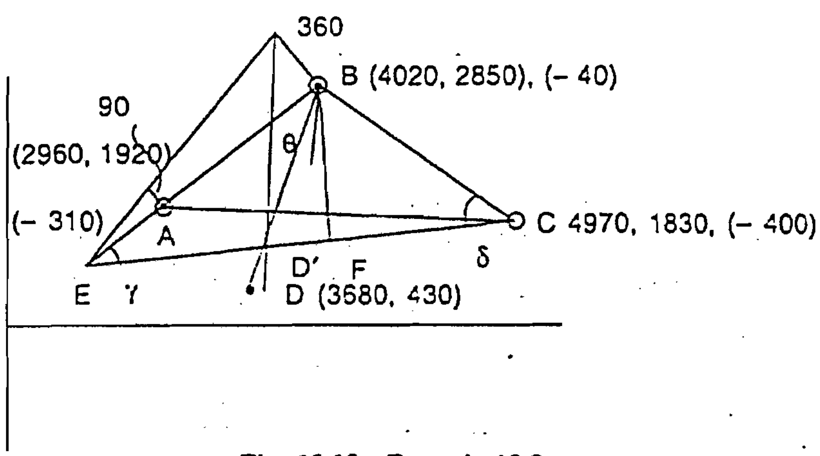


Fig. 19.18 Example 19.8.

$$AB = \sqrt{(4020 - 2960)^2 + (2850 - 1920)^2}$$
$$= 1410.14 \text{ m}$$

$$BC = \sqrt{(4970 - 4020)^2 + (2850 - 1830)^2}$$
$$= 1393.87 \text{ m}$$

$$CA = \sqrt{(4970 - 2960)^2 + (1920 - 1830)^2}$$
$$= 2012.00 \text{ m}$$

$$\frac{BE}{360} = \frac{AE}{90} = \frac{BE - AE}{270} = \frac{AB}{270}$$

$$BE = \frac{360}{270} \times AB = \frac{360}{270} \times 1410.14$$
$$= 1880.19$$

$$\text{W.C. bearing of } BA = 180° + \tan^{-1} \frac{4020 - 2960}{2850 - 1920}$$
$$= 180° + 48.74 = 228.74°$$

$$\text{W.C. bearing of } BC = 180 - \tan^{-1} \frac{4970 - 4020}{2850 - 1830}$$
$$= 180° - 42.96° = 137.04°$$
$$ABC = 228.74 - 137.04 = 91.70°$$

$$\tan \frac{1}{2} (\delta - \gamma) = \frac{BE - BC}{BE + BC} \tan \frac{1}{2} (88.30)$$
$$= \frac{486.32}{3274.06} \tan 44.15°$$

$$\frac{1}{2} (\delta - \gamma) = 8.20 \quad \text{or} \quad \delta - \gamma = 16.40$$
$$\delta + \gamma = 88.30$$
$$\delta = 52.35°$$
$$\gamma = 35.95°$$
$$BF = BC \sin 52.35°$$
$$= 1103.60 \text{ W.C.B of } BF = 137.04° + 37.25$$
$$= 174.29°$$

$$\text{dip} = \frac{360}{1103.60} = \frac{1}{3.06}$$

$$\text{Length } BD = \sqrt{(4020 - 3680)^2 \div (2850 - 430)^2}$$
$$= 2443.76 \text{ m}$$

$$\tan\theta = \frac{4020 - 3680}{2850 - 430}$$

$$\theta = 7.99°$$

$$\angle EBD' = 48.74 - 7.99$$

$$= 40.75°$$

$$\frac{BE}{\sin(35.95 + 40.75)} = \frac{BD'}{\sin 35.95}$$

or

$$BD' = \frac{BE \sin 35.95}{\sin 76.70}$$

$$= 1134.24 \text{ m}$$

Depth of ore at D

$$\frac{360}{1134.24} = \frac{x}{1309.52}$$

$$x = \frac{1309.52 \times 360}{1134.24}$$

$$= 415.632 \text{ m}$$

$$\text{Level } AOD \text{ of } D = 390.000$$

$$\text{Level of ore body} = -400 + (-415.63)$$

$$= -815.63$$

$$\text{bore hole depth} = 815.63 + 390.00$$

$$= 1205.63 \text{ m}$$

Alternative Solution of Example 19.8

Equation of a plane passing through three points:

A (2960, 1920, – 310)
B (4020, 2850, – 40)
C (4970, 1830, – 400)

Equation of a plane passing through these three points.

$$\begin{vmatrix} x & y & z & 1 \\ 2960 & 1920 & -310 & 1 \\ 4020 & 2850 & -40 & 1 \\ 4970 & 1830 & -400 & 1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} \frac{x}{1000} & \frac{y}{1000} & \frac{z}{1000} & 1 \\ 2.96 & 1.92 & -0.310 & 1 \\ 4.02 & 2.85 & -0.040 & 1 \\ 4.97 & 1.83 & -0.40 & 1 \end{vmatrix} = 0$$

or $\frac{x}{1000} \times \begin{vmatrix} 1.92 & -0.310 & 1 \\ 2.85 & -0.040 & 1 \\ 1.83 & -0.400 & 1 \end{vmatrix} - \frac{y}{1000} \times \begin{vmatrix} 2.96 & -0.31 & 1 \\ 4.02 & -0.04 & 1 \\ 4.97 & -0.40 & 1 \end{vmatrix}$

$+ \frac{z}{1000} \times \begin{vmatrix} 2.96 & 1.92 & 1 \\ 4.02 & 2.85 & 1 \\ 4.97 & 1.83 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 2.96 & 1.92 & -0.31 \\ 4.02 & 2.85 & -0.04 \\ 4.97 & 1.83 & -0.40 \end{vmatrix}$

Expanding the determinants

$$-.0594\,x + 0.6381\,y - 1.9647\,z - 1658.385 = 0$$

At D, $x = 3680$, $y = 430$, $z = -815.693$

$$390 - \text{depth of ore body} = -815.693$$

$$\text{depth of ore body} = 390 + 815.693 = 1205.693 \text{ m.}$$

with z coordinate zero, equation of the line $-0.0594\,x + 0.6381\,y = 1658.385$
perpendicular distance on the from point (x, y)

$$-\frac{0.0594}{\sqrt{0.0594^2 + 0.6381^2}} \cdot x + \frac{0.6381}{\sqrt{0.0594^2 + 0.6381^2}} \cdot y - \frac{1658.385}{\sqrt{0.0594^2 + 0.6381^2}}$$

From the point (4020, 2850)

$$\text{perpendicular distance} = -\frac{0.0594}{\sqrt{0.0594^2 + 0.6381^2}} \times 4020$$

$$+\frac{0.6381}{\sqrt{0.0594^2 + 0.6381^2}} \times 2850$$

$$-\frac{1658.385}{\sqrt{0.0594^2 + 0.6381^2}}$$

$$= -122.629 \text{ (negative sign is not significant)}$$

$$\text{slope} = \frac{40}{122.62} = \frac{1}{3.065} \text{ as before.}$$

Example 19.9 The table gives data concerning the position of a plane rock stratum at three stations A, B and C. Determine the coordinates of the point at which the formation centre line of a cutting and tunnel constructed at a downgrade of 3 in 106 from A would find the stratum when driven in a north easterly direction. At what depth will this point be below the surface, the cutting starting at ground level at A? If tunnelling commences when the formation is 18.0 m below ground level locate where the cutting finishes (Fig. 19.20).

Station	*A*	*B*	*C*
Ground level above OD (m)	150.0	177.0	192.0
Depth to rock (m)	13.5	34.5	40.5
Coordinates (N, E)	0.0	240.30	90.330

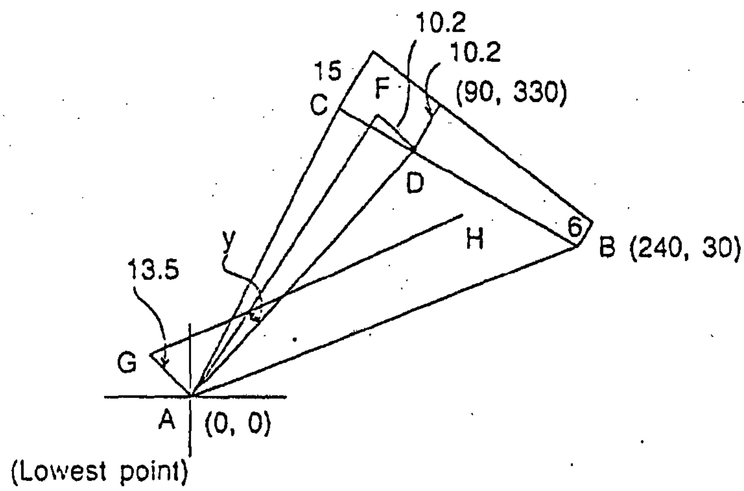


Fig. 19.19 Example 19.9.

Solution. Levels of rocks at A = 150.0 − 13.5 = 136.5 m

B = 177.0 − 34.5 = 142.5 m

C = 192.0 − 40.5 = 151.5 m

$$\text{Length of } AB = \sqrt{240^2 + 30^2}$$

$$= 241.867 \text{ m}$$

$$\theta_{AB} = \tan^{-1}\frac{30}{240} = \tan^{-1}\frac{1}{8} = 7.125°$$

$$\text{Length } BC = \sqrt{(240 - 90)^2 + (30 - 330)^2}$$

$$= 335.41$$

$$\theta_{BC} = \tan^{-1}\frac{300}{150}$$

$$= 63.43°$$

$$\angle ABD = 7.125 + 63.430$$

$$= 70.555°$$

$$\angle DAB = 45° - 7.125°$$

$$= 37.875°$$

$$\angle ADB = 180 - (70.555 + 37.875)$$

$$= 71.57°$$

Applying sine rule,

$$\frac{\sin 71.57°}{241.867} = \frac{\sin 70.555}{AD} = \frac{\sin 37.875}{DB}$$

$$AD = \frac{\sin 70.555}{\sin 71.57} \times 241.867$$

$$= 240.4$$

$$DB = \frac{\sin 37.875}{\sin 71.57} \times 241.867$$

$$= 156.51$$

$$\text{Depth at } D = 6 + \frac{9}{335.41} \times 156.51$$

$$= 10.2$$

Let the two lines AF and GH cut at a distance of x. Then

$$0 + \frac{x}{240.4} \cdot 10.2 = 13.5 - \frac{3}{106} \cdot x$$

$$x = 190.86 \text{ m}$$

$$y = \frac{10.2 \times 190.86}{240.4} = 8.098 \text{ m}$$

Level of the intersection point

$$= 136.5 + 8.098$$

$$= 144.598$$

Difference of surface between B and C

$$= 192.0 - 177.0$$

$$= 15.0$$

Ground level of point of intersection = 150 + 27 = 177.00 m

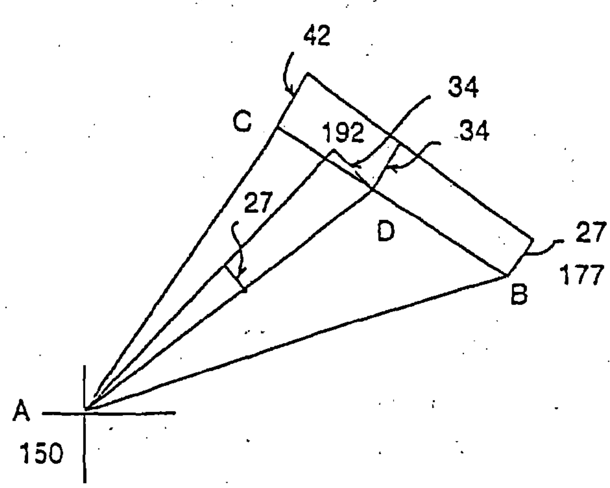


Fig. 19.20 Example 19.9.

Level of intersection point = 144.60 m

Hence depth of surface = 32.40 m

Let the formation be 18.0 m from G.L at a distance x from A.

$$\text{Ground level at } x = 150 + \frac{34 \times x}{240.4}$$

$$= 150 + 0.14\,x$$

$$\text{Formation level at } x = 150.00 - \frac{x \times 3}{106}$$

The difference is 18 m

$$\text{Therefore } 150 + 0.14\,x - \left(150 - \frac{x \times 3}{106}\right) = 18.00$$

or $$x = 106.95 \text{ m}$$

$$\text{Formation level at } x = 106.95$$

$$= 150 - \frac{3}{106} \times 106.95$$

$$= 146.973$$

$$\text{Ground level at } x = 106.95$$

$$= 150 + 0.14 \times 106.95$$

$$= 164.973,$$

Diff. = 18 m (checked)

Hence cutting finishes at a distance of 106.95 m from A.

Alternative Solution of Example 19.9

Coordinates of three points on rock.

$A' = 0, 0, 136.5$ m

$B' = 240, 30, 142.5$ m

$C' = 90, 330, 151.5$ m

Equation of a plane passing through these three points.

$$\begin{vmatrix} x & y & z & 1 \\ 0 & 0 & 136.5 & 1 \\ 240 & 30 & 142.5 & 1 \\ 90 & 330 & 151.5 & 1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} \frac{x}{100} & \frac{y}{100} & \frac{z}{100} & 1 \\ 0 & 0 & 1.365 & 1 \\ 2.4 & 0.30 & 1.425 & 1 \\ 0.90 & 3.30 & 1.515 & 1 \end{vmatrix} = 0$$

Expanding the determinant Equation of the plane

$$-.00153\,x - .00306\,y + .0765\,z - 10.44225 = 0$$

Equation of a line passing through the point (x_1, y_1, z_1) and having direction cosines l, m, n

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = \lambda \text{ (say)}$$

If the line passes through $A(0, 0, 150)$ and it moves in north easterly direction $l = \frac{1}{\sqrt{2}}, m = \frac{1}{\sqrt{2}}$ and the slope is 3/106, $n = -.02829$.

Then the equation of the line

$$\frac{x - 0}{\frac{1}{\sqrt{2}}} = \frac{y - 0}{\frac{1}{\sqrt{2}}} = \frac{z - 150}{-.02829} = \lambda$$

or $\quad x = 0.707\,\lambda,\ y = 0.707\,\lambda$ and $z = 150 - .02829\,\lambda$

Since it intersects the plane, substituting these values in the equation of the plane.

$$-.00153 \times (.707)\,\lambda - .00306\,(0.707\,\lambda)$$
$$+ .0765\,(-.02829\,\lambda + 150)$$
$$- 10.44225 = 0$$
$$\lambda = 190.920$$
$$x = .707 \times \lambda = 134.98$$
$$y = 134.98$$
$$z = 150 - .02829 \times 190.920$$
$$= 144.598 \text{ m}$$

Coordinates of corresponding points A, B, C on the ground

$A = 0, 0, 150$
$B = 240, 30, 177$
$C = 90, 330, 192$

Equation of a plane passing through these three points:

$$\begin{vmatrix} x & y & z & 1 \\ 0 & 0 & 150 & 1 \\ 240 & 30 & 177 & 1 \\ 90 & 330 & 192 & 1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} \frac{x}{100} & \frac{y}{100} & \frac{z}{100} & 1 \\ 0 & 0 & 1.5 & 1 \\ 2.4 & 0.3 & 1.77 & 1 \\ 0.90 & 3.30 & 1.92 & 1 \end{vmatrix} = 0$$

or $$-.00765\,x - .00765\,y + .0765\,z - 11.475 = 0$$

or $$.0765\,z = 11.475 + 2\,(.00765)\,(134.98)$$

or $$z = 176.996$$

difference = 176.996 − 144.598
= 32.398 m
= distance below the surface.

Equation of the line of cutting.

$$\frac{x}{0.707} = \frac{y}{0.707} = \frac{z - 150}{-.02829} = \lambda'$$

The equation of the plane of ground

$$-.00765\,x - .00765\,y + .0765\,z - 11.475 = 0$$

Let the formation is 18 m below ground level at coordinates of the formation y_1, z_1 and coordinates of the plane x_2, y_2, z_2. At that point

$$x_1 = x_2, \qquad y_1 = y_2, \qquad z_2 - z_1 = 18$$

Substituting

$$-.00765\,\lambda' \times 0.707 - .00765\,\lambda' \times .707$$
$$+ .0765\,(-.02829\,\lambda' + 150 + 18) - 11.475 = 0$$

or $$\lambda' = 106.075$$

giving $x_1 = 75$ $\quad y_1 = 75$ $\quad z_1 = 147$ $\quad z_2 = 165.00$

distance along $x\,y$ plane = $\sqrt{2} \times 75 = 106.05$ m.

PROBLEMS

19.1 Briefly discuss how in a typical tunnel survey the surface alignment and levels are transferred to the underground tunnel and how underground setting out is done. [AMIE Advanced Surveying Summer 1983]

19.2 (a) Describe the surveying operations necessary in transferring a given surface alignment down a shaft in order to align the construction work of a new tunnel.

(b) *A* and *B* were two vertical wires suspended in tunnel shaft and the bearing of *AB* was 55° 10′30″. A theodolite at *C* measured the angle *ACB* and its value was 0°20′25″. The distances *AC* and *CB* were 6.4782 m and 3.2998 m respectively. The point *C* was on the right hand side while proceeding from *A* to *B*. Calculate the perpendicular distance from *C* to *AB* produced, the bearing of *CA* and the angle to be set out from *BC* to establish *CE* parallel to *AB*.
[AMIE Advanced Surveying Winter 1983]

19.3 What are the broad steps in tunnel surveying? Explain the Weisbach triangle method of tunnel alignment underground.

[AMIE Advanced Surveying Winter 1993]

19.4 Two surface reference stations *X* and *Y* having coordinates of 1000.00 m E and 1000.00 m N and 1300.00 m E, 1500.00 m N respectively were observed during a shaft plumbing exercise. A theodolite was set up at surface station *A* near to the line *XY* and the readings in the following Table recorded.

Pointing direction	Horizontal circle reading
X	273°42′24″
Y	93°42′08″
Plumb wire *P*	98°00′50″
Plumb wire *Q*	98°00′40″

The distances from the theodolite to *X* and *P* were 269.12 m and 8.374 m whilst *P* and *Q* were 5.945 m apart, *P* being nearer to *A* than *Q*. Estimate the bearing of *PQ*. [Salford]

19.5 *A* and *B* are points on the centre line of a level mine roadway and *C* and *D* are points on the centre line of a lower roadway having a uniform gradient between *C* and *D*. It is proposed to connect the roadways by a drivage from point *B* on a bearing of 165°35′. Given the following data, calculate (i) the actual length and gradient of the drivage, (ii) the coordinates of the point at which it meets the lower roadway.

Point	Northing	Easting	Reduced Level
A	2653 m	1321 m	462.5 m
B	2763 m	1418 m	462.5 m
C	2653 m	1321 m	418.2 m
D	2671 m	1498 m	441.8 m

[C.E.I]

20

Computer Programs in Surveying*

20.1 INTRODUCTION

As in all other subjects computers are being widely used in surveying also. In this chapter a few computer programs on solution of examples in surveying are discussed. The listings of the programs are given at the end of the chapter, with sample input and output data. All the programs are in Fortran 77. Though the programs are interspersed with copious comments and explanations, it is assumed that the reader has some knowledge of computer programming.

20.2 EXPLANATION OF THE PROGRAMS

(a) *Program 1* Solves problem on normal tension using method of Bi-section. The normal tension in surveying is that tension which will stretch an unsupported measuring tape by an amount which is exactly equal to the decrease in length due to the sag. The normal tension as given in Chapter 3 (Eq. 3.16).

$$P_n = \frac{0.204\, W\sqrt{AE}}{\sqrt{P_n - P_s}}$$

This can be rewritten as:

$$P_n = (P_n^2 \cdot P_s + 0.204^2 \cdot W^2 \cdot AE)^{1/3}$$

This non-linear equation is solved for P_n using method of Bi-section. In Example 3.7, $L = 30$ m, $P_s = 89$ N, $A = 3$ mm^2, $w = 0.024$ kg/m.

and $$E = 155{,}000 \text{ N/mm}^2$$

Trial and error solution gives $P_n = 139$ N. Solution by Program 1 yields $P_n = 138.957323$ N.

*This chapter has been written in association with Dr. K.K. Bhar, Assistant Professor of C.E. B.E. College (D.U).

(b) *Program 2* Here the same problem is solved using Newton-Raphson Method. Solution by this program gives P_n = 138.9573 N.

Usually Newton-Raphson method works faster than Bi-section method. It required 11 iterations whereas the latter took 38 iterations. Although both worked well, Newton-Raphson method is preferred.

(c) *Program 3* It calculates the ordinates of a transition curve. For a transition curve, referring Eq. 12.23 and Eq. 12.25

$$x = l\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{9360} + \ldots\right)$$

and $$y = l\left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \frac{\phi^7}{75{,}600} + \ldots\right)$$

where $$\phi = \frac{l^2}{2LR}$$

But as $$\phi_s = \frac{L}{2R} \qquad \phi = \left(\frac{l}{L}\right)^2 \phi_s$$

For a given x value, l is to be determined from the non-linear Eq. 12.23 using any iterative method, e.g. Newton-Raphson Method and using that value of l, y is to be determined from Eq. 12.25.

Considering Eq. 12.23 let

$$\theta = \frac{\phi^2}{10} - \frac{\phi^4}{216} + \frac{\phi^6}{9360} + \ldots$$

Then $x = l(1 - \theta)$

or $$f(l) = x - l(1 - \theta) = 0$$

This equation is to be solved for l, using Newton-Raphson's method, which gives

$$l_{i+1} = l_i - \frac{f(l_i)}{f'(l_i)}$$

where i is the iteration index.

Now $$f(l) = x - l(1 - \theta)$$

$$= x - l\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{9360} + \ldots\right)$$

$$= x - l\left(1 - \left[\frac{l^2}{L^2}\cdot\phi_s\right]^2 \Big/ 10 + \left[\frac{l^2}{L^2}\cdot\phi_s\right]^4 \Big/ 216 - \ldots\right)$$

$$= x - l\left(1 - \frac{l^4}{L^4}\cdot\frac{\phi_s^2}{10} + \frac{l^8}{L^8}\cdot\frac{\phi_s^4}{216} - \ldots\right)$$

$$= x - \left(l - \frac{l^5}{10L^4} \cdot \phi_s^2 + \frac{l^9}{216\,L^8} \cdot \phi_s^4 - \ldots\right)$$

Therefore, $$f'(l) = 0 - \left(1 - \frac{5l^4\,\phi_s^2}{10L^4} + \frac{9l^8}{216L^8} \cdot \phi_s^4 - \ldots\right)$$

Neglecting the 2nd and higher term, we get. $f'(l) = -1$.

Hence $$l_{i+1} = l_i + f(l_i) = l_i + x_i - l_i(1 - \theta_i) = x_i + l_i\theta_i \qquad (20.1)$$

This is the recursive equation to be used to determine l for a particular x. In the computer program upto the fourth terms within the parenthesis (in the expressions for x and y, Eqs. 12.23 and 12.25) are considered.

First, discrete values of x are generated from the beginning and end of x value and the no. of divisions for x. For each of these discrete values of x, corresponding value of l is determined from Eq. 20.1 using an iterative scheme. This value of l and corresponding ϕ is then used to determine y from Eq. 12.5.

(d) *Program 4* In this program, the area under a curve is computed by trapezoidal rule.

(e) *Program 5* In this program, the area under a curve is computed by Simpson's 1/3rd rule.

Example 20.1 The following offsets were taken from a chain line to a hedge.

Distance (m)	0	30	60	90	120	150	180
Offset (m)	9.40	10.8	12.5	10.5	14.5	13.0	7.5

Compute the area included between the chain line, the hedge and the end offsets by (i) Trapezoidal rule, (ii) Simpson's rule, (iii) by Program 4 and (iv) by Program 5.

Solution

(i) Area by Trapezoidal rule:

$$= \frac{30}{2}\,[9.4 + 7.5 + 2\,(10.8 + 12.5 + 10.5 + 14.5 + 13.0)]$$

$$= 15\,[139.5] = 2092.5\text{ m}^2$$

(ii) Area by Simpson's rule

$$= \frac{30}{3}\,[9.4 + 7.5 + 4(10.8 + 10.5 + 13.0) + 2(12.5 + 14.5)]$$

$$= 2081\text{ m}^2$$

(iii) by Program 4, area = 2092.5 m^2

(iv) by Program 5, area = 2081 m^2

(f) *Program 6* This program calculates the independent coordinates of the stations of a closed traverse after applying corrections by Bowditch's rule. Finally the area of the traverse is computed in terms of the coordinates.

Example 20.2

(i) A survey was carried out in a closed traverse with six sides. With the traverse labelled anticlockwise as shown in Fig. 20.1, the data in Table 20.1 were obtained.

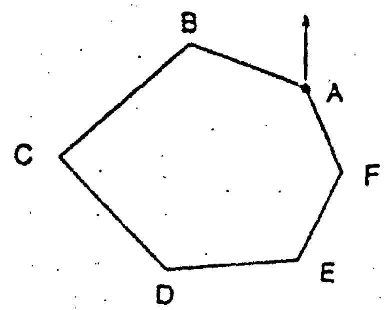


Fig. 20.1 Example 20.2.

Table 20.1 Example 20.2.

Station	Internal Angle	Length
A	130°18′45″	*AB* 14.248
B	110°18′23″	*BC* 85.771
C	99°32′35″	*CD* 77.318
D	116°18′02″	*DE* 28.222
E	119°46′07″	*EF* 53.099
F	143°46′20″	*FA* 65.914

The coordinates of point *A* are 1000 mE, 1000 mN and the whole circle bearing of line *AF* is 166°45′52″. After adjustment by Bowditch's method what are the coordinates of the other five traverse stations? [Salford/CIOB]

(ii) Compute also the area of the traverse in m².

Solution

(i) The solution is presented in the form of Gales Traverse in Table 20.2.

(ii) Area in terms of Coordinates. (Ref. Fig. 20.1)

$$\begin{aligned}\text{Area} &= \frac{1}{2}\,[Y_A(X_B - X_F) + Y_B(X_C - X_A)\\ &\quad + Y_C(X_D - X_B) + Y_D(X_E - X_C)\\ &\quad + Y_E(X_F - X_D) + Y_F(X_A - X_E)]\\ &= \frac{1}{2}\,[1000.00\,(987.311 - 1015.104)\\ &\quad + 1006.485\,(924.175 - 1000.00)\\ &\quad + 948.411\,(966.355 - 987.311)\\ &\quad + 883.624\,(994.374 - 924.175)\\ &\quad + 886.954\,(1015.104 - 966.355)\\ &\quad + 935.835\,(1000.000 - 994.374)]\\ &= -6725.988\ \text{m}^2\end{aligned}$$

By Program 6, Area = – 6726.048 m².

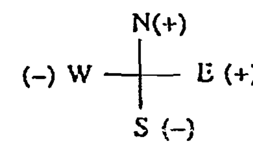


Table 20.2 Example 20.2 Gale's Traverse Table

Stn.	Line	Internal Angle	Corrected Angle	W.C. Bearing	Quadrantal bearing	Length	Latitude Observed	Latitude Corrected	Departure Observed	Departure Corrected	Ind. coordinates N	Ind. coordinates E
A		130°18′45″	130°18′43″								+ 1000.00	+ 1000.00
	AB			297°04′35″	N 62°55′25″ W	14.248	+ 6.485	+ 6.485	– 12.686	– 12.689		
B		110°18′23″	110°18′21″								+ 1006.485	+ 987.311
	BC			227°22′56″	S 47°22′56″ W	85.771	– 58.676	– 58.074	– 63.118	– 63.136		
C		99°32′35″	99°32′33″								+ 948.411	+ 924.175
	CD			146°55′29″	S 33°04′31″ E	77.318	– 64.789	– 64.787	+ 42.196	+ 42.180		
D		116°18′02″	116°18′00″								+ 883.624	+ 966.355
	DE			83°13′29″	N 83°13′29″ E	28.222	+ 3.329	+ 3.330	28.025	+ 28.019		
E		119°46′07″	119°46′05″								+ 886.954	+ 994.374
	EF			22°59′34″	N 22°59′34″ E	53.099	+ 48.880	+ 48.881	+ 20.741	+ 20.730		
F		143°46′20″	143°46′18″								+ 935.835	+1015.104
	FA			346°45′52″	N 13°14′08″ W	65.914	+ 64.163	+ 64.165	– 15.091	– 15.104		
A											+ 1000.00	+ 1000.00
	Σ	720°00′12″	720°00′00″			324.572	– 0.008	0.000	+ 0.067	0.000		

PROGRAM 1

```
c     ........................................................................
c     This program determines the normal tension related with chain
c     surveying. The basic inputs are the weight of the tape in Newton (N),
c     Ps is the standard tension in Newton, A is the sectional area in
c     mm2 and E is the modulus of elasticity in Newton/mm2. This program
c     uses the Bi-section method.
c     ........................................................................
      implicit double precision (a-h, o-z)
c     Opening input and output files
      open (10, file = 'data1 . in')
      open (11, file = 'data1. out')
c     Reading input values: (From file data1 .in)
c     PS: Standard tension, W: Weight, SA: Sectional area,
c     E: Modulus of elasticity, eps: precision factor
      read (10, *)
      read (10, *) PS, W, SA, E, eps
c     Form of the equation: PN^3-PN^2*PS-0.204^2*W^2*SA*E = 0
c     or: f(x) = x^3-ax^2-b = 0, where x = PN and
      a = PS
      b = W*W*SA*E*0.204*0.204
c     Method of bisection: Initial values of x are :x1 = 1000 N and x2 = 0 N
      x1 = 1000.0
      x2 = 0.0
      y1 = x1**3 - a*x1*x1 - b
      y2 = x2**3 - a*x2*x2 - b
c     Maximum number of iteration for bisection is taken as 100
      do 20 i = 1, 100
      x3 = 0.5* (x1 + x2)
      y3 = x3**3 - a*x3*x3 - b
      if (abs (y3) .le. eps) go to 30
      if (y1*y3 .gt. 0.0) then
         x1 = x3
         y1 = y3
      else
         x2 = x3
         y2 = y3
      endif
20    continue
      write (11, *) 'Does not converge after 100 iterations'
      write (*, *) 'Does not converge after 100 iterations'
      stop
30    write (11, 40) i, x3
      write (*, 40) i, x3
40    format (1x, 'Converged after', i3,' iterations'/
     11x, 'The normal tension is calculated as', f12.6,' Newton')
      end
```

Input file: data1. in
Values of standard tension (N), weight (N), sectional area (mm^2), E (N/mm^2) and eps
89.0 7.06032 3.0 155000.0 0.0001

Output file: datal. out
Converged after 38 iterations
The normal tension is calculated as 138.957323 Newton

PROGRAM 2

```
c     ..............................................................................
c     This program determines the normal tension related with chain
c     surveying. The basic inputs are the weight of the tape in Newton
c     Ps is the standard tension in Newton. A is the sectional area in
c     mm2 and E is the modulus of elasticity in Newton/mm2. This program
c     uses the Newton-Raphson method.
c     ..............................................................................
      implicit double precision (a-h, o-z)
c     Opening input and output files
      open (10, file = 'data1 .in')
      open (11, file = 'data1 .out')
c     Reading input values: (From file data1 .in)
c     PS: Standard tension, W: Weight, SA: Sectional area,
c     E: Modulus of elasticity, eps: precision factor
      read (10, *)
      read (10, *) PS, W, SA, E, eps
c     Form of the equation: PN^3-PN^2*PS-0.204^2*-W^2*SA*E = 0
c     or: f(x) = x^3-ax^2-b = 0, where x = PN and
      a = PS
      b = W*W*SA*E*0.204*0.204
c     Newton-Raphson Method: Initial value of x is: x1 = 1000 N
      x1 = 1000.0
      x = x1
c     Maximum number of iteration for Newton-Raphson Method
c     is taken as 100
      do 20 i = 1, 100
      fx = x**3 - a*x*x - b
      fdx = 3.0*x*x - 2.0*a*x
      if (abs (fdx) .le. 1.0e-05) go to 50
      x = x - fx/fdx
      if (abs (fx) .le. eps) go to 30
20    continue
      write (11, *) 'Does not converge after 100 iterations'
      write (*, *) 'Does not converge after 100 iterations'
      stop'
30    write (11, 40) i, x
      write (*, 40) i, x
40    format (1x, 'Converged after ',i3,' iterations'/
     11x, 'The normal tension is calculated as ',f12.6,' Newton')
      stop'
50    write (*, *) 'fdx is too small'
      stop'
      end
```

Input file: data1. in
Values of standard tension (N). weight (N). sectional area (mm^2). E (N/mm^2) and eps
89.0 7.06032 3.0 155000.0 0.0001

Output file: data1. out
Converged after 11 iterations
The normal tension is calculated as 138.957323 Newton

PROGRAM 3

```
c      ..............................................................................
c      Program to calculate the (x, y) coordinates of a transition curve
c      which are given as follows:
c      x = 1 (1-f^2/10 + f^4/216-f^6/9360 + ...) and
c      y = 1(f/3 - f^3/42 + f^5/1320 - f^7/75600 + ...). where
c      f = 1^2/(2LR) = (1/L)^2 fs; fs = L/2R.
c      For a given x, first to find 1 and then to calculate y
c      ..............................................................................
       real L. Li, R. Xo, Xm, delx, theta. fs. xr. eps
       real cord (50, 2)
c      Opening input and output files
       open (10, file = 'data2 .in')
       open (11, file = 'data2 .out')
c      Reading input values: (From file data2. in)
       read (10, *)
       read (10, *)
       read (10, *) L, R, Xo, Xm, Nx, iter, eps
       delx = (Xm - Xo)/Nx
       xi = Xo-delx
       fs = L/(2.0*R)
       do 100 i = 1, Nx + 1
       xi = xi + delx
       Li = xi
       do 70 k = 1, iter
       f = fs* (Li/L) **2
       theta = f**2/10.0 - f**4/216.0 + f**6/9360.0
       xr = xi-Li* (1.0-theta)
       if (abs (xr) .le. eps) go to 80
       Li=xi + Li *theta
70     continue
       write (*, *) 'Does not converge after ',iter,' iterations'
       write (11, *) 'Does not converge after ', iter,' iterations'
       stop
80     write (*, *) 'Converged after ',k,' iterations'
       yi = 1i* (f/3.0 - f**3/42.0 + f**5/1320.0 - f**7/75600.0)
       cord (i. 1) = xi
       cord (i. 2) = yi
100    continue
       write (11, 200)
200    format ('The coordinates of the transition curve are as follows'/
     1' ........................................'/
     2'           xi               yi'/
```

```
     1' .......................................')
        write (11, 210) (cord (i, 1), cord (i, 2), i = 1, Nx + 1)
210     format (f8.4, 2x, f10.4)
        write (11, 220)
220     format ('...................................')
        end
```

Input file: data2 .in
Length, Radius R, initial value of x, final value of x, no. delx values, no. of iterations, eps
75 300 0.0 74.882895 20 100 0.00001
Output file: data2 .out
The coordinates of the transition curve are as follows:

xi	yi
.0000	.0000
3.7441	.0004
7.4883	.0031
11.2324	.0105
14.9766	.0249
18.7207	.0486
22.4649	.0840
26.2090	.1334
29.9532	.1991
33.6973	.2835
37.4414	.3889
41.1856	.5177
44.9297	.6722
48.6739	.8547
52.4180	1.0678
56.1622	1.3137
59.9063	1.5948
63.6505	1.9137
67.3946	2.2728
71.1388	2.6745
74.8829	3.1215

PROGRAM 4

```
c       ..............................................................................
c       This Program computes the area under a curve by Trapezoidal
c       rule of numerical integration.
c       The trapezoidal rule can be stated as follows:
c       I = A = h[y1/2 + (y2 + y3 + ... + yn - 1) + yn/2];
c       where yi is the ith ordinate of the curve y = f (x), n is the total
c       number of ordinates and h is the interval between two successive
c       ordinates (constant). The basic inputs are: n, yi, i = 1, . . n and h.
c       ..............................................................................
        dimension y(100)
c       Opening input and output files
        open (10, file = 'data3 .in')
```

```
      open (11, file = 'data3 .out')
c     Reading input values: (From file data3. in)
c     n: Total number of ordinates; h is the uniform interval
      read (10, *)
      read (10, *) n, h
c     y(i): ordinates of the curve y = f(x)
      read (10, *)
      read (10, *) (y(i), i = 1, n)
c     Writing the given values
      write (11, 200)
200   format (20x, 'Trapezoidal Method of numerical integration'/
     12x, 'Given ordinates:— '/
     2' ............................. '/
     33x, 'No. ',' Ordinate'/
     2' .............................')
      write (11, 210) (i, y(i), i = 1, n)
210   format (2x, i3, 2x, f10.4)
      write (11, 220)
220   format (' ..........................')
c     Trapezoidal method
      sum = 0.5*(y(1) + y(n))
      do 10 i = 2, n - 1
      sum = sum + y (i)
10    continue
      A = h* sum
      write (11, 230) A
      write (*, 230) A
230   format (/2x, 'Using the given ordinates, the area under the curve'/
     12x, 'is computed as', f10.4, 'units')
      end
```

Input file: data3. in
No. of ordinates and interval
7 30.0
Ordinates
9.40 10.80 12.50 10.50 14.50 13.00 7.50

Output file: data3. out

Trapezoidal Method of numerical integration
Given ordinates:—

No.	Ordinate
1	9.4000
2	10.8000
3	12.5000
4	10.5000
5	14.5000
6	13.0000
7	7.5000

Using the given ordinates, the area under the curve
is computed as 2092.5000 units

PROGRAM 5

```
c       ......................................................................................
c       This Program computes the area under a curve by Simpson's
c       1/3rd rule of numerical integration.
c       The 1/3rd rule can be stated as follows:
c       I = A = (h/3) [y1 + 4y2 + 2y3 + 4y4 + 2y5 ... + 2yn - 2 + 4yn - 1) + yn];
c       where yi is the ith ordinate of the curve y = f (x), n is the total
c       number of ordinates (must be odd) and h is the interval between two
c       successive ordinates (constant).
c       The basic inputs are: n, yi, i = 1, . . n and h.
c       ......................................................................................
        dimension y(100)
c       Opening input and output files
        open (10, file = 'data4 .in')
        open (11, file = 'data4 .out')
c       Reading input values: (From file data4. in)
c       n: Total number of ordinates; h is the uniform interval
        read (10, *)
        read (10, *) n, h
c       y(i): Ordinates of the curve y = f(x)
        read (10, *)
        read (10, *) (y(i), i = 1, n)
c       Writing the given values
        write (11, 200)
200     format (20x, 'Simpson''s one-third rule of numerical integration'/
      12x, 'Given ordinates:— '/
      2' ............................. '/
      33x, 'No. ',' Ordinate'/
      2' .............................')
        write (11, 210) (i, y(i), i = 1, n)
210     format (2x, i3, 2x, f10.4)
        write (11, 220)
220     format (' ...........................')
c       Simpson's 1/3rd rule
        sum = (y(1) - y(n))
        do 10 i = 2, n - 1, 2
        sum = sum + 4.0*y(i) + 2.0*y (i + 1)
10      continue
        A = h* sum/3.0
        write (11, 230) A
        write (*, 230) A
230     format (/2x, 'Using the given ordinates, the area under the curve'/
      12x, 'is computed as', f10.4, 'units')
        end
```

Input file: data4. in
No. of ordinates and interval
7 30.0
Ordinates
9.40 10.80 12.50 10.50 14.50 13.00 7.50

Output file: data4. out

Simpson's one-third rule of numerical integration
Given ordinates:—

No.	Ordinate
1	9.4000
2	10.8000
3	12.5000
4	10.5000
5	14.5000
6	13.0000
7	7.5000

Using the given ordinates, the area under the curve
is computed as 2081.0000 units

PROGRAM 6

```
c    ...............................................................................
c    Computation of independent coordinates of the stations of a closed
c    traverse after applying Bowditch's rule. The basic inputs are:
c    (1) No. of stations, n, (2) included angle at each station, (3) length of
c    each side, (4) whole circle bearing (WCB) of the line joining stations
c    1 and n, and the coordinates of the first station.
c    ...............................................................................
     integer deg (15), min (15), sec (15)
     real al (15), cd (15, 2), angle (15), WCB (15), delE (15), delN (15)
c    ...............................................................................
     open (10, file = 'data6. in')
     open (11, file = 'data6. out')
c    n: Number of stations
     read (10, *)
     read (10, *) n
c    Included angle at each station in degree, minute and second
     read (10, *)
     read (10, *) (deg (i), min (i), sec (i), i = 1, n)
c    al: length of each side
     read (10, *)
     read (10, *) (al (i), i = 1, n)
c    WCB of the line joining stations 1 and n
     read (10, *)
     read (10, *) n1, n2, n3
     WCB1 = n1 *3600 + n2*60 + n3
c    Coordinates of the 1st station
     read (10, *)
     read (10, *) cd (1, 1), cd (1, 2)
c    Data reading complete
     write (11, 200)
200  format (10x, 'Computation of errors in a closed traverse'/
    115x, 'using Bowditch''s rule'/2x 'Given data:—'/48 ('-'/2x,
```

```
      2 'Station   Internal angle   Side   Length')
      write (11, 207)
      do 15 i = 1, n
      i1 = i + 1
      if (i1 .gt. n) i1 = 1
      write (11, 205) i, deg (i), min (i), sec (i), i, i1, al (i)
205   format (2x, i3, 10x i3, 'd', i2, 'm', i2, 's', 4x, i2, '-', i2, 2x, f8.3)
15    continue
      write (11, 207)
207   format (48 ('-'))
c     1. Angular error:
c        angle—observed angle in seconds
c        AT—Total included angle (theoretical)
c        AT1—Total included angle (observed)
      AT = (2*n - 4) *90*3600
      AT1 = 0.0
      do 20 i = 1, n
      angle (i) = deg (i) *3600 + min (i) *60 + sec (i)
      AT1 = AT1 + angle (i)
20    continue
c     err: Error in observed angles
      err1 = AT1 - AT
c     Distributing the error to each angle
      err1 = -err1/float (n)
c     Corrected angle
      write (11, 209)
209   format (/2x, 'Correction for internal angle:—')
      write (11, 211)
      write (11, 210)
210   format (2x, 'Angle    Observed value    Correction    Adjusted value'/
      1 '    (second)                                  (second)')
      write (11, 211)
211   format (54 ('-'))
      AT2 = 0.0
      do 22 i = 1, n
      an = angle (i) + err1
      AT2 = AT2 + an
      write (11, 212) i, angle (i), err1, an
      angle (i) = an
212   format (2x, i3, 5x, f10.0, 8x, f5.1, 6x, f10.0)
22    continue
      write (11, 211)
      write (11, 214) AT1, AT2
214   format (5x, 'Total', f10.0, 19x, f10.0)
c     Computation of whole circle bearings
      al = 180.0*60.0*60.0
      WCB (1) = WCB1 + angle (1)
      do 30 i = 2, n
      WCB(i) = WCB(i-1) + angle (i)
      if (WCB (i) .ge. al) then
         WCB (i) = WCB (i) - al
```

```
        else
           WCB (i) = WCB (i) + al
        endif
30      continue
c       Determination of easting and northing differences, delE, delN
c       fact-PI/(180*3600)
        fact = 4.848136811e - 06
        err2 = 0.0
        err3 = 0.0
        tl = 0.0
        do 32 i = 1, n
        tl = tl + al (i)
        dr = WCB (i) *fact
        delE (i) = al (i) *sin (dr)
        delN (i) = al (i) *cos (dr)
        err2 = err2 + delE (i)
        err3 = err3 + delN (i)
32      continue
c       Corrected easting and northings
        write (11, 215)
215     format (/2x, 'Corrections to eastings and northings:—'/
       1 66('-')/
       2' Line    WCB      Length    delE            delN'/
       3'        (second)        actual    corrtd   actual    corrtd
       4'/66('-'))
        s1 = 0.0
        s2 = 0.0
        s3 = 0.0
        s4 = 0.0
        do 34 i = 1, n
        a1 = delE (i) - err2*al (i)/tl
        a2 = delN (i) - err3*al (i)/tl
        s1 = s1 + a1
        s2 = s2 + a2
        s3 = s3 + delE (i)
        s4 = s4 + delN (i)
        i1 = i + 1
        if (i1 .gt. n) i1 = 1
        write (11, 216) i, i1, WCB (i), al (i), delE (i), a1, delN (i), a2
216     format (2x, i2, '-', i2, f10.0, f8.3, 4 (f10.3))
        delE (i) = a1
        delN (i) = a2
34      continue
        write (11, 217) s3, s1, s4, s2
217     format (66 ('-')/2x, 'Total', 18x, 4 (f10.3))
        do 36 i = 2, n
        cd (i, 1) = cd (i - 1, 1) + delE (i - 1)
        cd (i, 2) = cd (i - 1, 2) + delN (i - 1)
36      continue
        write (11, 220)
220     format (/2x, 'Corrected eastings and northings:—')
        write (11, 230)
```

```
      write (11, 222)
222   format (2x, 'Station   Easting   delE   Northing   delN')
      write (11, 230)
      write (11, 224) (i, cd (i, 1), delE (i), cd (i, 2), delN (i), i = 1, n)
224   format (2x, i3, 5x, f10.3, 3x, f8.3, 2x, f10.3, 3x, f8.3)
      write (11, 230)
230   format (54 ('-'))
c     Computation of the area of the traverse
      area = 0.0
      do 40 i = 1, n
      i1 = i - 1
      i2 = i + 1
      if (i1 .eq. 0) i1 = n
      if (i2 .gt. n) i2 = 1
      area = area + (cd (i, 2)* (cd (i2, 1) - cd (i1, 1)) *0.5)
40    continue
      write (11, 240) abs (area)
240   format (/2x, 'Area of the traverse =', f10.3 units')
      end
```

Input file: data6. in

```
Number of stations, n
6
Included angle at each station in degree, minute, second
130 18 45
110 18 23
99 32 35
116 18 2
119 46 7
143 46 20
Length of each side
14.248 85.771 77.318 28.222 53.099 65.914
WCB of the line joining stations 1 and n
166 45 52
Coordinates of  the 1st station
1000.0 1000.0
```

Output file: data6. out

Computation of errors in a closed traverse
using Bowditch's rule

Given data:—

Station	Internal angle			Side	Length
1	130d	18m	45s	1 – 2	14.248
2	110 d	18m	23s	2 – 3	85.771
3	99 d	32m	35s	3 – 4	77.318
4	116 d	18m	2s	4 – 5	28.222
5	119 d	46m	7s	5 – 6	53.099
6	143 d	46m	20s	6 – 1	65.914

Correction for internal angle:—

Angle	Observed value (second)	Correction	Adjusted value (second)
1	469125.	− 2.0	469123.
2	397103.	− 2.0	397101.
3	358355.	− 2.0	358353.
4	418682.	− 2.0	418680.
5	431167.	− 2.0	431165.
6	517580.	− 2.0	517578.
Total	2592012.		2592000.

Corrections to eastings and northings:—

Line	WCB (second)	Length	delE actual	delE corrtd	delN actual	delN corrtd
1–2	1069475.	14.248	− 12.686	− 12.689	6.485	6.486
2–3	818576.	85.771	− 63.118	− 63.135	− 58.076	− 58.074
3–4	528929.	77.318	42.196	42.180	− 64.789	− 64.787
4–5	299609.	28.222	28.025	28.019	3.330	3.330
5–6	82774.	53.099	20.741	20.730	48.881	48.882
6–1	1248352.	65.914	− 15.091	− 15.105	64.163	64.164
Total			.066	.000	− .006	.000

Corrected eastings and northings:—

Station	Easting	delE	Northing	delN
1	1000.000	− 12.689	1000.000	6.486
2	987.311	− 63.135	1006.486	− 58.074
3	924.175	42.180	948.411	− 64.787
4	966.355	28.019	883.624	3.330
5	994.374	20.730	886.954	48.882
6	1015.105	− 15.105	935.836	64.164

Area of the traverse = 6726.048 units.

Answers to Problems

CHAPTER 2

2.6 701.7 m, 0.0966 m.
2.7 (b) 77°14′23″, 44°40′39″, 53°04′58″
2.8 (b) 83°42′28.17″, 102°15′42.35″,
94°38′26.83″, 79°23′22.65″

CHAPTER 3

3.7 3068.32 m
3.8 30.94 m
3.9 3400.1126 m
3.11 661.55077 m
3.12 (c) 70.612 m
3.13 (b) 5.5 m
3.14 (b) 1999.44 m

CHAPTER 4

4.3 (a) ± 8 mm (b) ± 20 mm
4.4 (a) $\pm \frac{1}{4160.88}$, (b) $\pm \frac{1}{18630}$
(c) $\pm \frac{1}{95200}$
4.5 0.14 m, 2436.38 m, 1205.59 m, 1230.79 m
4.8 424.718 m
4.9 58.826 m
4.10 299710.38 km/s
4.11 9.989902 m
4.12 299726.11 km/s

CHAPTER 5

5.5 1°1′5.32″
5.6 − 320.0326 mm (concave)
5.10 99.00605 m
5.11 (b) 42.49 km
5.12 25.2 mm

CHAPTER 6

6.4 (b) 125.00 m, 0.0751335 m, – .1751 m
6.5 (b) 1.325 m
6.6 (b) $\frac{1}{25.43}$
6.7 (b) – 0.5825 m
6.8 (b) 99.310 (R.L. at stn. 8)
6.10 1.925 (F.S. at stn. 9)
6.11 (b) (i) downward
(ii) 3.750, 2.750
(iii) Diaphragm has to be brought downward
6.13 (d) collimation is inclined downward by 0.500 m

CHAPTER 9

9.2 (b) No local attraction, or equal local attractions at A and B.
9.3 (b) 745.044 steps, N 45°30′ W.
9.4 (b) At A = – 30′, B = + 1°05′, C = – 40′

	Corrected for Local attraction	Corrected for Declination
AB	48°55′	47°25′
BC	176°40′	175°10′
CD	104°55′	103°25′
DE	165°15′	163°45′
EA	259°30′	258°00′

9.5 (a) 200°00′
(b) 10° E
(c)

Line	True back bearing	Forward bearing
AB	280°00′	100°
AC	330°00′	150°
AD	20°00′	200°

9.7 (a) 206°00′, declination is 2° E
(b) (i) R = 2° W, S = 2° W
(ii) RS = 209°, SP = 314°
PQ = 54° QR = 134°

9.8 (b)

Line	True bearing	Magnetic bearing
BC	11°00′	15°30′
CA	251°	255°30′
AB	131°	135°30′

9.9 True bearing at
AB = 29°45′
BC = 122°45′
CD = 180°30′
DA = 286°00′

9.10 (b) N 32°30′ E
S 9°38′ E
S 37°54′ W
N 32°36′ W
9.11 (c) S 20° W
(d) True bearing at
AB = 60°10′
BC = 98°55′
CD = 39°40′
DA = 319°25′

CHAPTER 11

11.1 (a) 0.00, 139.6 mW
(c) 2.577 m. N 57°21′ E
11.2 126.044 m. 252.497 m
11.3 (a) + 846.41, + 200.00
+ 1492.82, − 119.62
+ 746.41, −59.81
(b) 278.39 m, 248°57′
11.4 (b) B 25.220 N, 100.00 E
C 24.193 N 230.00 E
D 109.403 N 280.00 E
E 357.428 N 280.00 E
(c) 6.0227 hectare
11.5 250, 350
11.6 134.46 m, N 70°10′12″ W
Length of BE = 45.848 m Bearing N 70°10′12″ W
11.7 (i) 2.828, S 45° E
(ii) 300.1485 N, 200.099 W
(iii) 400.1485 N, − 100.099 W
11.8 32.15, 38.31
11.10 523.68 m, 32°08′24″
11.11 163°16′12″
11.12 CD = 760.23 m, DE = 837.176 m
11.13 CD = N 82°31′48″ W, BC = N 62°58′48″ E

CHAPTER 12

12.1 Length of 1st subchord − 0.22 m. Δ_1 = 1′20″
12.2 Length of Circular curve − 600 m
12.3 R = 85 m. R_2 = 80.2 m, l_1 = 42.30 m. l_2 = 37.69 m
12.4 576.82 m
12.6 1654.908 m, 1420.653 m. 1852.613 m
12.7 46.324 m. 56.6 cm
12.8 457.29 m. 2490.29 m, 3121.647 m
12.9 453.052 m. 272.50 m, 905.72 m
12.10 (b) O_1 = 0.2 m O_2 = 1m

CHAPTER 13

13.8 (a) 602.736 m, (b) 1808 m, (c) 683.100 m

CHAPTER 14

14.1 24881.614 m^2
14.2 5.10077 hectare
14.3 21.656 m, 36.697 m, 549.119 m
14.4 (a) 106416.67 m^3, (b) 104375 m^3
14.5 10,000 m^3
14.6 (a) (i) 1445.50 m^2, (ii) 1458.67 m^2
(b) 23.228 m^3
14.7 1228.425 m^3
14.8 2556 m^3
14.9 16765.667 m^3
14.10 32816 m^3
14.11 95.71 m
14.15 4684.15 m^3
14.16 13,200 m, 75.24 × 10^5 m^3
14.17 (b) 5718.25 m^3

CHAPTER 15

15.1 414.735 m, 77.933 m
15.2 – S
15.3 (a) 70.094 m, (b) 106.989 m, (c) 1.413, 1.780, 2.148
15.4 2.48 mm
15.5 140 m, 500 m
15.6 (b) 1 in 10.5
15.7 (b) 1 in 5.376
15.8 (c) 1521.54 m, 6520.59 m
15.9 (c) 234.638 m, S 52°10′30″ E

CHAPTER 16

16.15 (c) (i) .02 mm (ii) 0.5 mm (iii) 1.5 mm

CHAPTER 17

17.4 (b) (i) 378, 500 m^3, (ii) 367, 333.33 m^3

CHAPTER 19

19.2 (b) .03994 m, 235°31′41.52″ 0°41′36.52″
19.4 35°16′00″
19.5 104.581 m, 1 in 3.5959, 2665.4155, 1343.087, 434.48.

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